# Optimal Rating Contingent Regulation

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**Abstract**

Rating contingent regulation is an important building block of world-wide regulatory frameworks, such as the recently proposed Basel III. We analyze an economy in which a government uses credit ratings provided by an independent, profit-maximizing rating agency to regulate banks' risk taking behavior. Regulation has to be set before uncertainty about the complexity of securities in the economy is realized. The economy features good and bad types of securities. Bad types are not only negative NPV projects but also exposed to aggregate risk, that is, an aggregate shock induces fluctuations in default rates of bad types. The government has an incentive to allow banks to hold risky securities since banks are better at recovering contractual debt payments from firms (relative to retail investors). On the other hand, banks have a private incentive to over-expose themselves to bad securities since the government cannot commit not to bail out banks in case of default. Exposure to bad securities and thereby to aggregate risk is necessary for the bank to profit from the government's put. The government cannot directly observe the types of securities held by banks and therefore uses ratings as measures of creditworthiness when regulating banks' asset mix. Uncertainty about the complexity of securities implies that the optimal ex ante sensitivity of capital requirements to ratings may lead to rating inflation if the aggregate amount of complex securities turns out to be high.

# 1. Introduction

Triggered by the recent financial crisis, the regulation of banks has gained new traction among academics, regulators, and politicians. One of the key challenges in effective regulation is time (in)consistency of regulation. While a regulator would like to commit ex-ante not to bail-out banks to set the right incentives, this threat is not credible since the regulator does not follow through in the event of a crisis. The implications are well understood: Banks have an incentive to engage in asset substitution. However, in contrast to almost all other industries, asset substitution does not (primarily) create a conflict of interest between equityholders and debtholders, but between equityholders and taxpayers due to the implicit government guarantee of debt. Even worse, this lack of downside participation might counter the disciplining effect of debt!

As a result of this identified time inconsistency problem a regulator needs to restrict risk-taking behavior of banks ex ante. This can be achieved via adjusting the risk-profile of the asset side and /or the adjustment of leverage, i.e., the liability side. Our paper is primarily concerned with the former tool, i.e., asset management of a bank. The starting point of our paper is the natural assumption that a regulator cannot directly observe the riskiness of assets, but needs to rely on an external (private) assessment of risk.[1] Since the introduction of Basel I guidelines rating agencies have become this main "objective" measure of risk in regulation of banks. This important role has been confirmed by the recently Basel III (2011) guidelines. In such a world, the business of rating agencies serves 2 purposes: providing information and selling regulatory treatment.

Our paper builds up on the model of Opp, Opp, and Harris (2011) which focuses on the *positive* implications of exogenous regulation on the rating agency's behavior and reveals that rating contingent regulation will affect the optimal choice of information production and disclosure rule of the rating agency. In particular, if the regulatory advantage of highly rated securities is sufficiently large, the rating agency will only focus on the business of regulatory certification (rating inflation without information production).

In the present paper, our perspective is *normative*, as we aim to solve for the optimum regulation of banks using ratings as measures of risk. Banks are socially valuable, as they possess a natural advantage at enforcing debt claims, i.e., a monitoring technology, relative to retail investors. While this technology allows banks to create value on the one hand, this special role causes regulators to bail-out banks ex-post; which might then cause banks to take excessive risks ex ante.

Regulators take into account the behavior of banks and in particular the rating agencies' incentive to provide informative ratings. Regulators face a key tradeoff. On the one hand, restricting banks to hold very safe assets may cause underinvestment in risky, but NPV positive securities. On the other hand, lax restrictions allow banks to overexpose them to risky securities, facilitated by rating agencies' practice of rating inflation. Since regulators have to set up the regulatory regime before the resolution of uncertainty about

[1] If a regulator could observe the riskiness of assets directly in a timely and unambiguous way, then he could simply prescribe banks which assets to hold and which assets to not hold.

the state of world has materialized, the regulator has to trade off these costs depending on their relative likelihood. In equilibrium, he might even be willing to incur rating inflation in some states of the world to increase valuable investments in other states of the world.

Our paper aims at understanding the repercussions of recent proposals to overhaul regulation of the financial sector. The tools we are analyzing are motivated by regulation currently in place. In contrast to Basel III, which still relies on rating contingent regulation, Dodd-Frank makes a radical proposal to remove all references to rating agencies in regulation and to replace them by all publicly available information. Another radical proposal is presented by Admati, DeMarzo, Hellwig, and Pfleiderer (2011), who argue that a dramatic increase in equity requirements would come a long way to improve banks' incentives. They criticize existing theories of bank financing with debt (such as Diamond (1984)) on the grounds that these theories cannot provide a quantitative theory of the extreme leverage of the banking industry.

Our paper is organized as follows. Section 2 presents an overview of the modeling assumptions. The main analysis is presented in Section 3. Section 4 concludes.

# 2. Model

## 2.1. Setup

We extend the setup of Opp, Opp, and Harris (2011) by introducing aggregate uncertainty, an optimizing regulator and a retail (non-bank) sector. All players in the economy (government, banks, firms, retail investors and a rating agency) are risk-neutral. All players will be described in the subsequent sections in great detail. There are 3 dates, $t = 0, 1, 2$. Our economy features aggregate uncertainty about defaults denoted by an aggregate state $a \in \{L, H\}$ and (independent) uncertainty about the complexity of securities. In particular, we focus on uncertainty over a cost function parameter $c \in \left\{c^l, c^h\right\}$ that shifts the rating agency's marginal cost of information acquisition. The two uncertainties play different roles. We introduce aggregate risk to allow for risk-taking of banks. We introduce uncertainty about complexity (at the time the regulator decides on the policy) so that the regulator cannot optimize regulation state by state, but instead needs to target the "average state". This translates into the following timeline.

$t = 0$ The government sets the regulatory environment (see Section 2.1.6)

$t = 1$ The complexity of securities is realized $c \in \left\{c^l, c^h\right\}$ and becomes public knowledge. Banks, firms, retail investors and the rating agency make decisions dependent on the current regulation and the complexity of securities. Investment takes place. The logical structure of time 1 actions will be discussed in Section 2.1.5.

$t = 2$ The aggregate state $a \in \{L, H\}$ and projects' payoffs are realized.

It is crucial, that all players (except for the government) make decisions conditional on the complexity of securities, i.e., they all act at time 1, but before the aggregate state is revealed. We will first consider the actions of these players.

### 2.1.1. Firms

There is a continuum of firms of measure 1. Let $v$ index firms, with $v \in \Omega = [0, 1]$. Each firm is owned by a risk-neutral entrepreneur who has no cash. The entrepreneur has access to a risky project that requires an initial investment of 1 and may either succeed or fail. If the project succeeds, the firm's net cash flow at the end of the period is $R > 1$. In case of failure, the cash flow is 0. Firms differ with regards to their probability of success. Firms are assumed to default on their contracts with banks if and only if their projects fail. The complexity of projects can be either high or low. Complexity refers to the rating agency's cost function parameter ($c \in \left\{c^l, c^h\right\}$). Projects' complexities are realized at date 1 and become public knowledge at that time.

Default probabilities are denoted by $d_n^a$ , where $n \in \{g, b\}$ denotes the type, and where $g$ represents "good" and $b$ stands for "bad." The default probabilities for bad types depend on an aggregate state $a$ that can be either low $(L)$ or high $(H)$. Bad types default with probability 1 in the low aggregate state ($d_b^L = 1$), but have a default probability below 1 in the high aggregate state, $a = H$. Define the unconditional default probability for bad types as

$$d_b \equiv 1 \cdot \Pr[L] + d_b^H \cdot \Pr[H], \tag{1}$$

such that $d_b^H = \frac{d_b-(1-\Pr[H])}{\Pr[H]}$. Good types' default probabilities $d_g^a$ are not exposed to aggregate uncertainty, implying $d_g \equiv d_g^L = d_g^H$. It follows that only bad types provide exposure to aggregate uncertainty.

**ASSUMPTION 1.** *The expected payoff of the good firm type is higher in each state of the world, i.e.,*

$$d_g < d_b^H.$$

Whereas entrepreneurs know their projects' types (and complexity) at date 1, they only learn the aggregate state $a$ at date 2, when it becomes publicly observable and verifiable. The fraction of good types in the population $\pi_g$ is common knowledge to all parties at date 0. The $NPV$ of a type-$n$ project (absent government intervention), as of date 1, is given by

$$V_n = R(1 - d_n) - 1. \tag{2}$$

The good type has positive $NPV$ projects ($V_g > 0$), whereas the bad type has negative $NPV$ projects ($V_b < 0$). The average project with default probability $\bar{d} = \pi_g d_g + \pi_b d_b$ is assumed to have negative $NPV$. Firms seek financing from competitive banks and competitive retail investors.

### 2.1.2. Storage Technology

All agents may keep capital as cash or in an equivalent safe technology that pays zero interest.

### 2.1.3. Banks

There is a continuum of banks of measure 1. Banks are ex ante identical and, at date 1, have legacy liabilities (debt, deposits, etc.) that will cause withdrawals of $W_2$ per bank at date 2. At date 1, the bank has total assets $M_1$ in the form of cash. In order to ensure that $M_1$ is a meaningful parameter, we exclude the possibility that banks can pay out cash as a dividend to equity holders at date 1.

**ASSUMPTION 2.** *Each bank has sufficient deposits at date 1, $M_1$, to meet all withdrawals $W_2$, conditional on only investing in cash $\left(\frac{W_2}{M_1} < 1\right)$.*

We may interpret $\frac{W_2}{M_1}$ as a measure of book leverage. Leverage is fixed exogenously. It can be motivated by various models in the spirit of Diamond (1984) that should be thought of as operating in the background of our model.[2] Alternatively, from a practical perspective one could ask the question: Conditional on some leverage $\frac{W_2}{M_1}$, what is the optimal regulation of the bank asset side? In the model, regulation will effectively be a constraint on cash. The optimal cash-holding requirement will depend on parameters of the model. In particular, the regulator's optimal minimum cash constraint $\omega_C^{\min}$ is a function of $\frac{W_2}{M_1}$.

If a bank is not able to service its withdrawals at date 2, and is not bailed out by the government, a social continuation value of $\xi > 0$ is lost (for further interpretation see below). Thus, it is always ex post optimal to bail out banks that are under water. As a result, debt holders always get paid back the entire amount $M_1$ regardless of the banks' investments. Since debt holders do not participate on the downside due to the bailout guarantee, the equity holders' maximization problem also maximizes total firm value (equity value + debt value).

Banks can invest in any project or in cash. Yet, the government may restrict banks' investment policy via rating contingent regulation. Note that the government could always tell banks to invest only in cash and thereby avoid rating contingent regulation altogether. In order for this to be not the optimal solution to the regulator problem, we need an economic reason why banks should hold these risky projects in the first place. Consider the following channel: retail investors' investment in risky securities is not a perfect substitute for banks' investment, that is, retail investors have a relative disadvantage that may be caused by banks' ability to increase ex post loan performance (Diamond and Rajan (2011) have a very similar assumption). In terms of the model,

[2] As will become clear later, the government's optimal capital requirement would be 100% in terms of our model (which is not realistic).

banks can recover the full face value $N$ in the good state, whereas retail investors can only recover a fraction $\phi < 1$ of the face value $N$, i.e. $\phi N < N$. Economic reasons for this difference might be: (1) banks' transaction cost are lower due to economies of scale. (2) Banks are able to avoid cash flow diversion by firm owners. Retail investors have higher cost of verifying the firm outcome, or higher cost of enforcing the contractual debt payments. These cost destroy a fraction $(1 - \phi)$ of the face value. In this context, $\xi$, the social loss in case of bank default, is equal to the loss caused by the fact that banks in default do not enforce contracts at date 2. Retail investors seizing the assets incur a deadweight loss of $(1 - \phi) \cdot N$ per funded security that does not default. If banks have invested in a mass $\mu^{bank}$ of risky securities with default rate $d^{bank}$, the social loss (ex post) is given by

$$\xi = (1 - \phi) \cdot N \cdot \mu^{bank} \cdot (1 - d^{bank}).$$

Thus, the competence of banks in monitoring is the reason why they are bailed out; which in turn makes it necessary to regulate them.

#### 2.1.4. Retail Investors

Retail investors are at a disadvantage relative to banks, since they can only recover a fraction $\phi$ of the face value. Retail investors therefore do not enjoy an implicit government guarantee and require to break even on their investments (relative to an investment in cash).

#### 2.1.5. Rating Agency

Firms can approach a rating agency that has access to an information production technology that generates noisy, private signals $s \in \{A, B\}$ of firm type, where $A$ ($B$) refers to the good (bad) signal. We consider the following signal structure (see Figure I):

$$\Pr\left(s = A | \, n = g\right) = \Pr\left(s = B | \, n = b\right) = 1 - \alpha\left(\iota\right), \tag{3}$$

where $\iota \in [0, \frac{1}{2}]$ denotes the rating agency's choice of information production. Importantly, the quality of the rating agency's signal, $1 - \alpha(\iota)$, is endogenous. Signals are informative if the error probability $\alpha(\iota)$ is smaller than 50%. It is convenient and without loss of generality to assume $\alpha$ is affine; that is,

$$\alpha\left(\iota\right) = \frac{1}{2} - \iota. \tag{4}$$

Since signal quality is strictly increasing in the level of information production, $\iota$, we will sometimes refer to $\iota$ itself as signal quality. The cost function will depend on the complexity of securities $c \in \{c_L, c_H\}$. We posit that the cost functions can be written as

$$C\left(\iota\right) = c\tilde{C}\left(\iota\right) \tag{5}$$

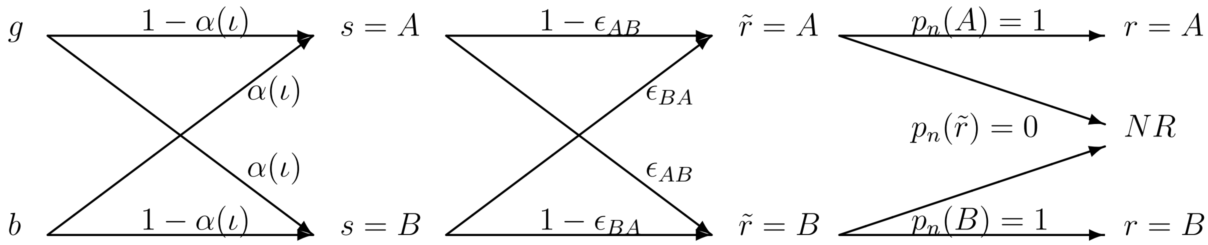


**FIGURE I**

Conditional on each type $n \in \{g,b\}$, the rating agency observes a quality signal $s \in \{A,B\}$. For each signal $s$, the rating agency reports an indicative rating $\tilde{r} \neq s$ with probability $\varepsilon_{s\tilde{r}}$. If the rating is purchased by the issuer, $p_n(\tilde{r}) = 1$, the rating $\tilde{r}$ becomes the public rating $r$. Otherwise, i.e., $p_n(\tilde{r}) = 0$, the firm remains unrated ($NR$).

where $\tilde{C}(\iota)$ is increasing and convex,

$$\tilde{C}(0) = 0, \text{ and} \tag{6}$$

$$\lim_{\iota \to \frac{1}{2}} \tilde{C}'(\iota) = \infty. \tag{7}$$

Consistent with practice, the publication of a rating involves two steps (see also Figure I). First, firms are provided with a free *indicative* rating $\tilde{r}$ by the rating agency. Second, the indicative rating becomes the public rating, $r = \tilde{r}$, if the issuer decides to purchase the rating, denoted as $p_n(\tilde{r}) = 1$, for a fee $f > 0$.[3] Since signals $s$ are not publicly observable, the rating agency can potentially offer ratings, $\tilde{r} \neq s$. As the indicative rating and the public rating coincide, subject to the firm purchasing decision, the probabilities of misreporting, $\varepsilon = (\varepsilon_{AB}, \varepsilon_{BA})$, conditional on the privately observed signal $s \in \{A,B\}$, completely characterize the disclosure rule of the rating agency. The term $\varepsilon_{AB}$ refers to the probability that an issuer with signal $A$ is offered a $B$-rating ($\varepsilon_{BA}$ is defined analogously). Full disclosure implies $\varepsilon = (0,0)$. Without loss of generality, we restrict ourselves to disclosure rules that ensure the $A$ category represents the superior rating class. This assumption can be formalized as

$$\tilde{d}_A(\iota, \varepsilon) \leq \tilde{d}_B(\iota, \varepsilon), \tag{8}$$

where $\tilde{d}_{\tilde{r}}$ is the default probability of a firm conditional on its *indicative* rating being $\tilde{r} \in \{A,B\}$. The idea behind this restriction is to avoid relabeling the notion of good and bad ratings.[4] This restriction is automatically satisfied if firms with an $A$ signal obtain an indicative $A$ rating.

In the following analysis, we assume the value of future business is high enough that the rating agency can effectively commit to any desired level of information acquisition

[3]The equilibrium implications would be identical if the rating agency charged rating-contingent fees.

[4]For example, the rule "always misreport," $\varepsilon = (1,1)$, is informationally equivalent to $\varepsilon = (0,0)$ and would make $B$ securities good securities.

$\iota \geq 0$ and any disclosure rule $\varepsilon \geq 0$ subject to the restriction imposed by equation 8. We provide a formal justification for this assumption in our earlier paper. As a result, we can summarize the logical sequence of events at time 1 as follows

1. The rating agency sets a fee $f$, information acquisition $\iota$, and the disclosure rule $\varepsilon$.
2. Firms solicit a rating.
3. The rating agency incurs information acquisition cost $C(\iota)$ and receives a private, noisy signal $s$.
4. The rating agency reports an indicative rating $\tilde{r}$ to firms.
5. Firms decide whether to agree to pay the fee $f$ to publish their ratings, and ratings of firms who do are published.
6. Investors decide whether to provide funding to firms.
7. Firms that agreed to pay the fee $f$ do so, and invest the remainder of the funds raised.

At time $t = 2$ the aggregate state and cash flows are realized and debt is repaid if possible.

To capture the notion that firms with good projects have access to alternative costly ways of signaling their type, we introduce type-dependent outside options $\bar{U}_n$, satisfying $\bar{U}_b = 0 < \bar{U}_g < V_g$ ( see Opp, Opp, and Harris (2011)). Economically, the presence of the outside option prevents the monopolistic rating agency from extracting the entire surplus from the projects that are financed. In the present paper, all results of interest go through if we consider the limit as $\bar{U}_g$ approaches zero.

### 2.1.6. Government

Ex post, it is always beneficial to bail out banks to avoid the social net-loss of $\xi$ per bank. The government cannot commit not to bail out banks that do not have sufficient funds at date 2. At date 0, the government sets the regulatory regime that is in place at dates 1 and 2. Government regulation specifies how much cash banks have to hold for each dollar invested in $A-$rated securities, and how much cash or $A$-rated securities must be held for any dollar invested in $B$-rated securities.[5] The government aims to maximize ex-ante welfare $W$ before the realization of the complexity of securities $c \in \left\{c^l, c^h\right\}$.

[5] Instead of specifying the complexity of securities as random we could alternatively specify the amount of bank capital $M_1$ as random. In order to obtain the result that the occurrence of rating inflation can be consistent with a maximizing regulator, one deep parameter (e.g. complexity, bank capital $M_1$, or the fraction of good types $\pi_g$) must be uncertain when the regulator sets the regime, and the regulator cannot write regulatory rules contingent on this uncertain variable.

# 3. Analysis

## 3.1. Firm Maximization

Consider the decision of a firm of type $n$ to purchase an indicative rating $\tilde{r}$, taking the strategies of all investors, the rating agency, and all other firms as given. Let $N_r$ denote the minimum face value the marginal investor is willing to accept to purchase a bond with (public) rating $r$. A bad type purchases a rating $\tilde{r}$ ($p_b^*(\tilde{r}) = 1$) as long as $N_r < R$, which yields a positive expected payoff. In contrast, a good type only purchases a rating $\tilde{r}$ if the expected payoff of approaching the capital market using this rating is greater than its outside option $\bar{U}_g$. Thus, for a good type to purchase a rating, the face value of public debt must be sufficiently low, that is, $N_r \leq \bar{N} < R$, where $\bar{N}$ ensures that a good firm is just indifferent between purchasing a rating and using the outside option. In other words, $\bar{N}$ satisfies

$$(1 - d_g)\left(R - \bar{N}\right) = \bar{U}_g. \tag{9}$$

Note $\lim_{\bar{U}_g \to 0} \bar{N} = R$. Since, whenever a good type purchases a rating $\tilde{r}$, $N_r \leq \bar{N} < R$, the bad type will also purchase that rating. This result is stated formally in the following lemma.

**LEMMA 1.** $p_g(\tilde{r}) = 1$ *implies* $p_b(\tilde{r}) = 1$.

Below, we analyze the case in which $p_n(A) = 1$ and $p_n(B) = 0$ for all types $n$. If the regulator prevents banks from holding $B$-rated securities then only non-banks (retail investors) can hold $B$-rated securities. Yet, retail investors will not purchase $B$-rated securities, implying that the rating agency cannot charge a positive fee for a $B$ rating. In fact, retail investors can only recover a fraction $\phi$ of the face value, which implies that they may not even want to hold $A$-rated securities, if $\phi$ is low enough.

If there is no regulation, banks could have an incentive to fund $B$-rated securities that are effectively subsidized by the government due to the bailouts. This could imply that firms that obtain an indicative rating $\tilde{r} = B$ may purchase the rating (i.e. $p_n(B) = 1$). The following analysis thus takes as given that the regulator optimally forbids banks to hold $B$-rated securities.

Given a level of information acquisition $\iota(v)$ for firm $v$, the probabilities with which the rating agency obtains the signals $s = A$ and $s = B$ for firm $v$, denoted as $\mu_A$ and $\mu_B$, respectively, satisfy

$$\mu_A(\iota) = \pi_g(1 - \alpha(\iota)) + \pi_b \alpha(\iota), \tag{10}$$

$$\mu_B(\iota) = \pi_g \alpha(\iota) + \pi_b(1 - \alpha(\iota)). \tag{11}$$

Given a disclosure rule $\varepsilon(v) = (\varepsilon_{AB}(v), \varepsilon_{BA}(v))$ for firm $v$, the probability that a firm obtains an *indicative* rating of $\tilde{r} = A$, denoted by $\tilde{\mu}_A$, satisfies

$$\tilde{\mu}_A(\iota, \varepsilon) = \mu_A(\iota) \cdot (1 - \varepsilon_{AB}) + \mu_B(\iota) \cdot \varepsilon_{BA}. \tag{12}$$

The total mass of securities with an indicative $A$ rating is given by:

$$\tilde{\mu}_A = \int_0^1 \tilde{\mu}_A \left(\iota\left(v\right), \varepsilon\left(v\right)\right) dv. \tag{13}$$

Prior to the realization of the aggregate state $a$, the default probability of a security with a *public* $A$ rating, $d_A = \tilde{d}_A$ (both firm types purchase an $A$ rating), follows directly from Bayes' Law, i.e.,

$$\begin{aligned} d_A =& \pi_g \frac{\int_0^1 \left(\left(1-\alpha\left(\iota\left(v\right)\right)\right)\left(1-\varepsilon_{AB}\left(v\right)\right)+\alpha\left(\iota\left(v\right)\right)\varepsilon_{BA}\left(v\right)\right) dv}{\tilde{\mu}_A} \cdot d_g \\ &+ \pi_b \frac{\int_0^1 \left(\left(1-\alpha\left(\iota\left(v\right)\right)\right)\varepsilon_{BA}\left(v\right)+\alpha\left(\iota\left(v\right)\right)\left(1-\varepsilon_{AB}\left(v\right)\right)\right) dv}{\tilde{\mu}_A} d_b. \end{aligned} \tag{14}$$

The ex post default rates of $A$-rated securities depend on the realization of the aggregate state $a \in \{H, L\}$, and are given by

$$\begin{aligned} d_A^a =& \pi_g \frac{\int_0^1 \left(\left(1-\alpha\left(\iota\left(v\right)\right)\right)\left(1-\varepsilon_{AB}\left(v\right)\right)+\alpha\left(\iota\left(v\right)\right)\varepsilon_{BA}\left(v\right)\right) dv}{\tilde{\mu}_A} \cdot d_g \\ &+ \pi_b \frac{\int_0^1 \left(\left(1-\alpha\left(\iota\left(v\right)\right)\right)\varepsilon_{BA}\left(v\right)+\alpha\left(\iota\left(v\right)\right)\left(1-\varepsilon_{AB}\left(v\right)\right)\right) dv}{\tilde{\mu}_A} d_b^a. \end{aligned} \tag{15}$$

where $d_b^L = 1$, $d_b^H = \frac{d_b - (1-\Pr[H])}{\Pr[H]}$.

## 3.2. Retail Investors' Maximization

In case a project is financed by retail investors, the required face value is given by

$$N_r^{retail}\left(f, d_r\right) = \frac{1+f}{\left(1-d_r\right)\phi}.$$

Funding by retail investors is feasible as long as $N_r^{retail}\left(f, d_r\right) \leq R$. If $\phi$ is sufficiently low, even good securities may not have a positive $NPV$ if financed by retail investors. If banks are constrained in their ability to fund firms (e.g. due to regulatory constraints), then retail investors may potentially choose to step in.

## 3.3. Banks' Maximization

### 3.3.1. Banks' Portfolio Returns and Pricing

Banks take as given the decisions of the rating agency, the regulator, and firms. Let $\omega_C$ denote the fraction of $M_1$ that the bank allocates to cash and $\omega_A$ the fraction of deposits allocated to $A$-rated securities. Note that the share of $B$-rated securities should be optimally set to zero, since $B$-rated securities are worse than average and the average NPV is negative. For ease of exposition, we assume this result right away to economize on notation. The regulator sets a minimum fraction of investment in cash ($\omega_C^{\min}$), that is, banks face the regulatory constraint $\omega_C \geq \omega_C^{\min}$. Finally, since portfolio shares add up, we have $\omega_C = 1 - \omega_A$.

Further, let $r_{A2}^a$ denote the net return on the bank's investments in $A$-rated securities in aggregate state $a$, which is realized at date 2. The bank defaults if its portfolio's gross return from date 1 to date 2, denoted by $(1 + r_{P2})$, is below the return required to meet all withdrawals at date 2, that is,

$$1 + r_{P2}^a = \omega_C \cdot 1 + (1 - \omega_C) \cdot (1 + r_{A2}^a) < \frac{W_2}{M_1}. \tag{16}$$

Rearranging this relation implies that absent government intervention, the bank defaults if the return on $A$-rated securities is ex post below a threshold $r_{A2}^* (\omega_C)$:

$$r_{A2}^a < r_{A2}^* (\omega_C) = \frac{\frac{W_2}{M_1} - \omega_C}{1 - \omega_C} - 1. \tag{17}$$

Since the government intervenes in case $r_{A2}^L < r_{A2}^* (\omega_C)$ and provides banks with just enough capital to meet all obligations $W_2$, it effectively subsidizes $A-$rated securities held by banks by subsidizing their portfolio return by $y_{A2}^L$, that is,

$$\omega_C + (1 - \omega_C) \cdot \left(1 + r_{A2}^L + y_{A2}^L\right) = \frac{W_2}{M_1}, \tag{18}$$

where

$$y_{A2}^L (\omega_C, r_{A2}) = \max\left[r_{A2}^* (\omega_C) - r_{A2}^L, 0\right]. \tag{19}$$

Notice that $y_{A2}^L (1, r_{A2}) = 0$, that is, if banks are forced to only hold cash they will never obtain a subsidy, since, by Assumption 2 we start with a sufficiently capitalized bank $\left(\frac{W_2}{M_1} < 1\right)$.

**ASSUMPTION 3.** *The banking sector always has enough total deposits $M_1$ to fund all risky securities, i.e. $M_1$ is large.*

Even though banks in principle have enough capital to fund all risky projects in the economy, they may be unable to hold them all due to regulatory constraints. Thus, banks either hold all $A$-rated securities in the economy (if they can at least (privately) break even on these securities), or banks also hit their regulatory constraint for $A$-rated securities, implying $\omega_C = \omega_C^{\min}$. In the former case banks are marginal investors for $A$-rated securities (Case A), in the latter case, retail investors may be marginal (Case B). We discuss the two cases below in more detail.

### 3.3.2. CASE A: Regulatory Constraint Is Not Binding $[(1-\omega_C^{\min})\cdot M_1 > \tilde{\mu}_A \cdot (1+f)]$

A bank is sufficiently capitalized in all states $a$ (i.e., will not default), if the following inequalities hold:

$$\omega_C + (1-\omega_C)\cdot(1+r_{A2}^a) \geq \frac{W_2}{M_1},\ \text{for } a \in \{H, L\} \tag{20}$$

where $r_{A2}^a$ denotes the net-return on $A$-rated securities absent government intervention, which may be written as

$$r_{A2}^a = \frac{N_A \cdot (1-d_A^a)}{1+f} - 1. \tag{21}$$

The gross return on $A$-rated securities is simply the expected repayment on $A$-rated securities, $N_A \cdot (1-d_A^a)$, divided by the amount of funds provided to the issuer, $1+f$, which the issuer in turn uses to pay the fee $f$.

If banks are insufficiently capitalized in the low state $a = L$, the government steps in and provides just enough capital to each bank to ensure its survival, that is, the bank's subsidized portfolio return just matches the return required to service all maturing obligations $W_2$,

$$\omega_C + (1-\omega_C)\cdot\left(1 + r_{A2}^L + y_{A2}^L\right) = \frac{W_2}{M_1}, \tag{22}$$

where

$$r_{A2}^L + y_{A2}^L(\omega_C) = r_{A2}^*(\omega_C). \tag{23}$$

**Pricing** Due to the bailout guarantee, equity holders maximize total firm value. For pricing purposes, we need to distinguish between two possibilities: banks are sufficiently capitalized in all states, or they are insufficiently capitalized in the low state.

If banks are sufficiently capitalized in all states $a$, competition among banks implies that the pricing restriction is given by

$$E_1[r_{A2}] = \Pr[a = H]\cdot r_{A2}^H + \Pr[a = L]\cdot r_{A2}^L = 0. \tag{24}$$

Solving equation 24 for $N_A$ yields:

$$N_A = \frac{1+f}{1-d_A}.$$

If banks are insufficiently capitalized in the low state $a = L$, the pricing restriction conditional on a bank portfolio decision $\omega_C$ is given by

$$E_1[r_{A2}] = \Pr[a = H]\cdot r_{A2}^H + \Pr[a = L]\cdot r_{A2}^*(\omega_C) = 0. \tag{25}$$

Substituting in the definition of $r_{A2}^H$ and solving equation 25 for the face value $N_A$ yields:

$$N_A = \frac{1+f}{1-d_A^H}\left(1 - \frac{\Pr[a = L]}{\Pr[a = H]} r_{A2}^*(\omega_C)\right). \tag{26}$$

In summary, we obtain:

$$
\begin{aligned}
&N_A^{bank}\left(f, d_A^L, d_A^H, \omega_C\right) \\
=&\begin{cases} \frac{1+f}{1-d_A} & \text{for } \omega_C + (1-\omega_C)\frac{1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1}, \\ \frac{1+f}{1-d_A^H}\left(1 - \frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*(\omega_C)\right) & \text{otherwise.} \end{cases}
\end{aligned}
\tag{27}
$$

Note that $\frac{\partial N_A^{bank}\left(f, d_A^L, d_A^H, \omega_C\right)}{\partial \omega_C} \geq 0$, that is, the higher the bank's cash holdings the higher is the face value it requires. This implies that, everything else equal, a bank can require the *lowest* face value when it minimizes its cash holdings, i.e., if it sets $\omega_C = \omega_C^{\min}$.

### 3.3.3. CASE B: Regulatory Constraint Is Binding $(1-\omega_C^{\min}) \cdot M_1 < \tilde{\mu}_A \cdot (1+f)$

Regulation can imply that retail investors are marginal investors of $A$-rated securities. Given that a mass $\tilde{\mu}_A$ of securities is rated $A$, a total investment of $\tilde{\mu}_A \cdot (1+f)$ is necessary to fund all $A$-rated securities in the economy. There is a continuum of banks of measure 1 with total funds $M_1$.

Due to competition among investors, the market value of an $A$-rated security is equal to the investment $(1+f)$. Yet, if retail investors are marginal (due to regulatory constraints), the banks' private valuation of the asset may be larger than the investment value, in other words, banks can make profits from investing in $A$-rated securities.

If banks are restricted to set $\omega_C \geq \omega_C^{\min}$, then the banking sector cannot fund all $A$-rated securities if

$$(1-\omega_C) \cdot M_1 < \tilde{\mu}_A \cdot (1+f). \tag{28}$$

If banks cannot hold all $A$-rated securities, two outcomes may obtain. Either retail investors step in and fund the remaining mass of $A$-rated securities, or credit rationing obtains in the sense that bank capital strictly limits the amount of funding of $A$-rated securities (which is the case when retail investors are unwilling to hold $A$-rated securities, for example due to a low $\phi$). Retail investors can step in and purchase remaining $A$-rated securities if

$$1+f \leq (1-d_A) \cdot \phi \cdot R. \tag{29}$$

In this case, by the law of one price, all $A$-rated securities offer a face value $N_A^{retail}(f, d_A)$. Notice that at this face value banks could strictly prefer to purchase more $A$-rated securities, but they are constrained by regulation. If retail investors cannot step in, then even though a mass $\tilde{\mu}_A$ of securities is rated $A$ only a mass $\mu_A^* < \tilde{\mu}_A$ can be actually funded, where

$$\mu_A^* = \frac{\left(1-\omega_C^{\min}\right) \cdot M_1}{1+f}. \tag{30}$$

In this case, banks are again marginal and the face value is given by $N_A^{bank}\left(f, d_A^L, d_A^H, \omega_C\right)$. Further,

$$\omega_C^{\text{int}} = \frac{M_1 - \tilde{\mu}_A \cdot (1+f)}{M_1} \tag{31}$$

is the (interior) value of $\omega_C$ in case the bank is not constrained by regulation.

In summary, banks' optimal cash holding share $\omega_C$ is given by:

$$\omega_C\left(f,\tilde{\mu}_A,d_A^L,d_A^H,\omega_C^{\min}\right) \tag{32}$$

$$=\begin{cases}\omega_C^{\text{int}} & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} \leq \bar{N},\\ \omega_C^{\min} & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\min}} \leq \bar{N},\\ 0 & \text{otherwise.}\end{cases} \tag{33}$$

Even though banks would be willing to provide funds as long as $N_A^{bank} > R$, good firms will not choose to participate if $N_A > \bar{N}$. Thus $\omega_C$ will be equal to zero if $N_A^{bank} > \bar{N}$. The mass of $A$-rated securities funded in equilibrium is given by

$$\mu_A^{funded} = \mu_A^{bank} + \mu_A^{retail}, \tag{34}$$

where $\mu_A^{bank}$ denotes the mass of securities funded by banks and $\mu_A^{retail}$ the mass funded by retail investors and where

$$\mu_A^{banks}\left(f,\tilde{\mu}_A,d_A^L,d_A^H,\omega_C^{\min}\right)$$

$$=\begin{cases}\tilde{\mu}_A & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} \leq \bar{N},\\ \mu_A^* & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\min}} \leq \bar{N},\\ 0 & \text{otherwise.}\end{cases} \tag{35}$$

The total mass of funded $A$-rated securities is given by

$$\mu_A^{funded}\left(f,\tilde{\mu}_A,d_A^L,d_A^H,\omega_C^{\min}\right)$$

$$=\begin{cases}\tilde{\mu}_A & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} \leq \bar{N}\\ & \text{or if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{retail}\left(f,d_A\right) \leq \bar{N},\\ \mu_A^* & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\min}} \leq \bar{N}\\ & \wedge\ N_A^{retail}\left(f,d_A\right) > \bar{N},\\ 0 & \text{otherwise.}\end{cases} \tag{36}$$

and the face value required in equilibrium is given by

$$N_A^{funded}\left(f,\tilde{\mu}_A,d_A^L,d_A^H,\omega_C^{\min}\right)$$

$$=\begin{cases}N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\min}} & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}\\ & \wedge\ N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\min}} \leq \bar{N}\\ & \wedge\ N_A^{retail}\left(f,d_A\right) > \bar{N},\\ N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}\\ & \wedge\ N_A^{bank}\left(f,d_A^L,d_A^H,\omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} \leq \bar{N},\\ N_A^{retail}\left(f,d_A\right) & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{retail}\left(f,d_A\right) \leq \bar{N},\\ \text{n.d.} & \text{otherwise.}\end{cases} \tag{37}$$

## 3.4. Rating Agency Maximization

The previous two subproblems imply the rating agency must set the fee $f$, information acquisition $\iota$, and disclosure rule $\varepsilon$ such that it induces good types to purchase an $A$ rating $\left(N_A \leq \bar{N}\right)$.

In equilibrium, fees $f$ are collected from all firms that are offered an indicative rating of $A$ and obtain funding, $\mu_A^{funded}$. Thus the solution to the following profit maximization problem determines the rating agency's equilibrium behavior:

$$\Pi = \max \left\{ \max_{f,\iota(v),\varepsilon(v)} \left\{ \mu_A^{funded} \left( f, \tilde{\mu}_A, d_A^L, d_A^H, \omega_C^{\min} \right) \cdot f - \int_0^1 C\left(\iota\left(v\right)\right) dv \right\}, 0 \right\}, \tag{38}$$

**LEMMA 2.** *For all firms $v$ for which the rating agency chooses $\iota(v) > 0$, the rating agency chooses an identical level of information acquisition $\iota(v) = \iota$.*

**Proof.** The lemma follows from the convexity of the information acquisition cost function.

Let $\Omega^\iota \subseteq \Omega$ denote the subset of firms for which the rating agency chooses to acquire information $(\iota(v) = \iota > 0)$ and let $\mu^\iota = \int_{\Omega^\iota} dv$ denote the corresponding mass of firms. Then it follows from Lemma 2 that for a mass $(1 - \mu^\iota)$ of firms the rating agency does not acquire information (i.e., it sets $\iota(v) = 0$). In what follows, we focus on disclosure rules $\varepsilon(v)$ that are symmetric for all firms in the sets $\Omega^\iota$ and $\Omega \setminus \Omega^\iota$, and denote them by $\varepsilon^\iota$ and $\varepsilon^{\iota-}$ respectively. This reduces the rating agency's choice variables to $(\mu^\iota, \iota, \varepsilon^\iota, \varepsilon^{\iota-}, f)$, which fully determine the variables $\left(f, \tilde{\mu}_A, d_A^L, d_A^H\right)$ that enter the rating agency's maximization problem. Thus, we may write:

$$\Pi = \max \left\{ \max_{f,\mu^\iota,\iota,\varepsilon^\iota,\varepsilon^{\iota-}} \left\{ \mu_A^{funded} \left( f, \tilde{\mu}_A, d_A^L, d_A^H, \omega_C^{\min} \right) \cdot f - C(\iota) \cdot \mu^\iota \right\}, 0 \right\}, \tag{39}$$

where

$$\tilde{\mu}_A = \mu^\iota \cdot \tilde{\mu}_A(\iota, \varepsilon^\iota) + (1 - \mu^\iota) \cdot \tilde{\mu}_A\left(0, \varepsilon^{\iota-}\right), \tag{40}$$

$$\begin{aligned} d_A^a &= \pi_g \frac{\mu^\iota \cdot \left[ (1 - \alpha(\iota))(1 - \varepsilon_{AB}^\iota) + \alpha(\iota)\varepsilon_{BA}^\iota \right] + \frac{(1-\mu^\iota)\left(1-\varepsilon_{AB}^{\iota-}+\varepsilon_{BA}^{\iota-}\right)}{2}}{\tilde{\mu}_A} \cdot d_g \\ &+ \pi_b \frac{\mu^\iota \cdot \left[ (1 - \alpha(\iota))\varepsilon_{BA}^\iota + \alpha(\iota)(1 - \varepsilon_{AB}^\iota) \right] + \frac{(1-\mu^\iota)\left(1-\varepsilon_{AB}^{\iota-}+\varepsilon_{BA}^{\iota-}\right)}{2}}{\tilde{\mu}_A} d_b^a \end{aligned} \tag{41}$$

In case the rating agency operates, good types' participation constraint is binding, that is,

$$N_A^{funded} \left( f, \tilde{\mu}_A, d_A^L, d_A^H, \omega_C^{\min} \right) = \bar{N}.$$

This equation allows to express the fee as a function of the other choice variables $\left(f\left(\tilde{\mu}_A, d_A^L, d_A^H, \omega_C^{\min}\right)\right)$ and is provided in the Appendix. It can be shown that for $d_b^H > d_g$ (see Assumption 1) only the following strategies may be optimal for the rating agency (in case the rating agency optimally operates at all):

1. Information production ($\iota > 0$) and truthful reporting of informative signals

   (a) All firms are investigated: $\mu^\iota = 1$, $\varepsilon^\iota = (0,0)$, $\tilde{\mu}_A = \mu_A(\iota)$.

   (b) Only a fraction of firms is investigated: $\mu^\iota = \frac{\mu_A^*}{\mu_A(\iota)} < 1, \varepsilon^\iota = (0,0), \varepsilon^{\iota -} = (1,0)$, $\tilde{\mu}_A = \mu^\iota \cdot \mu_A(\iota)$. Firms with $B$ signals and without investigation are rated $B$.

2. No information production ($\iota = 0$)

   (a) All firms are rated $A$: $\mu^\iota = 1$, $\iota = 0$, $\varepsilon^\iota = (0,1)$, $\tilde{\mu}_A = 1$.

   (b) Only a fraction of firms is rated $A$: $\mu^\iota = \mu_A^*$, $\iota = 0$, $\varepsilon^\iota = (0,1)$, $\varepsilon^{\iota -} = (1,0)$, $\tilde{\mu}_A = \mu_A^*$.

Thus, for all firms for which costly information acquisition is performed, i.e., $\mu^\iota$, the rating agency will truthfully report the signal. Conditional on incurring the cost, this is the optimal thing to do (see Opp, Opp, and Harris (2011)). For firms that are not investigated, $1 - \mu^\iota$, the rating agency effectively only chooses the mass of firms that it wants to rate as $A$. To be consistent with our notation, this simple labeling is performed in two steps. First, the rating agency obtains an uninformative signal $s \in \{A, B\}$, then it chooses $\varepsilon_{AB}$ and $\varepsilon_{BA}$ to make sure it obtains the desired supply of $A$ securities. In order to provide some intuition for the rating agency's maximization problem, we plot outcomes for these various rating agency policies in Figure II (see discussion below).

### 3.4.1. Discussion Figure II

**Left-hand Side Panel (Information Production $\iota > 0$ and Truthful Reporting $\varepsilon = 0$)** The graphs on the left-hand side consider outcomes given the rating agency commits to acquiring information $\iota \geq 0$ (varied on the horizontal axis) for all firms, i.e., $\mu^\iota = 1$, and reporting truthfully, i.e., $\varepsilon = 0$. The graph at the top plots the rating agency's profits and revenues for various levels of information acquisition $0 \leq \iota \leq 0.5$. The profit function has two humps in information acquisition. The global maximum for profits conditional on $\varepsilon = 0$ is at $\iota = 0$. Table I provides the benchmark model parameters chosen for all illustrations.

If the RA chooses sufficiently low information acquisition, $\iota < 0.25$, $A$-rated securities become sufficiently risky to bring banks "under water" in the low aggregate state $a = L$, implying a government subsidy of the banks' portfolios. The corresponding return subsidy is shown in the graph at the bottom (see dashed line for $y_{A2}^L$): the subsidy is zero for $\iota > .25$ and is positive and decreasing in information acquisition for $\iota < 0.25$. For $\iota > 0.25$, profits are increasing up to the local maximum of 0.43. Here, a marginal increase in information acquisition increases the rating agency's profits (up to the local maximum), because information production increases investment efficiency and leaves fewer rents for bad types, and, at the same time, has no marginal effect on government bailout subsidies (see bottom graph).

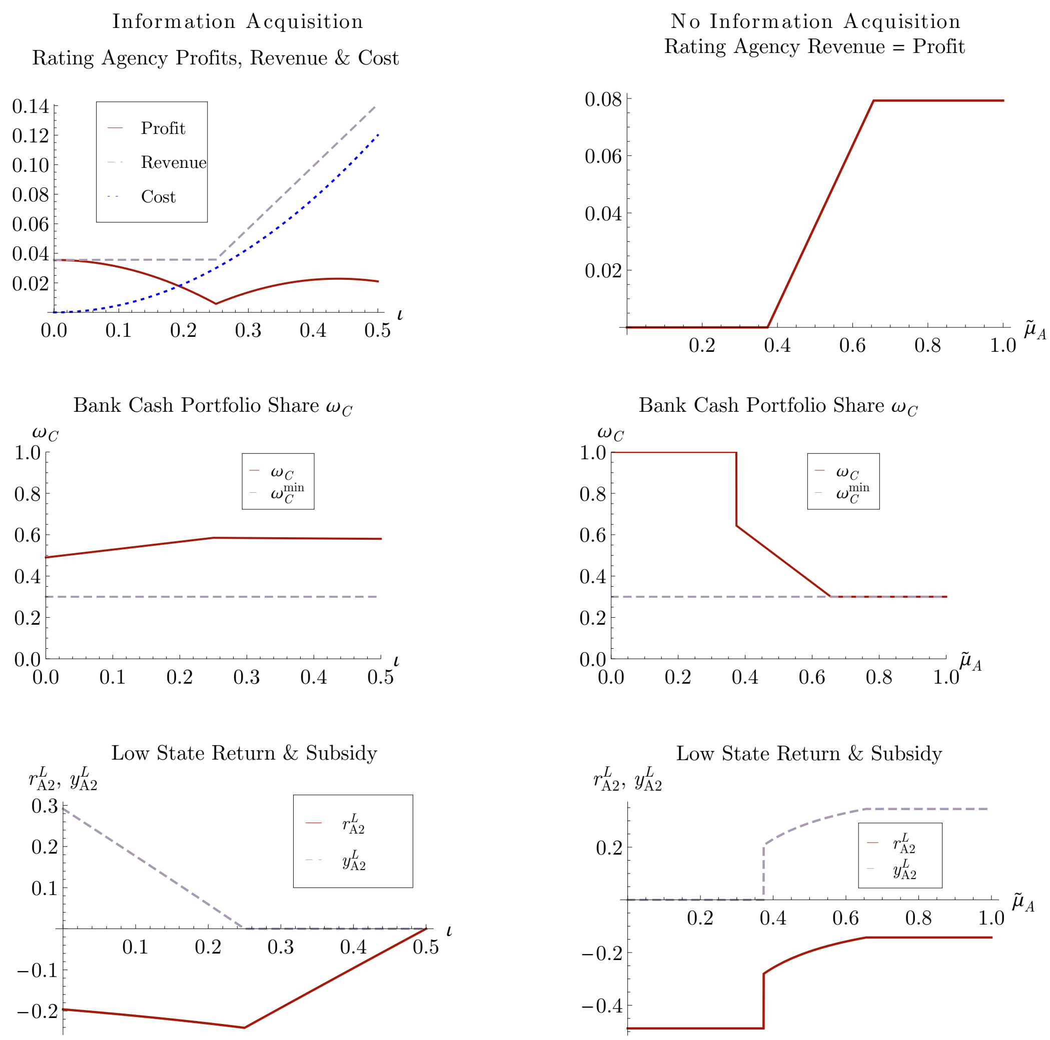


**FIGURE II**

The figure plots outcomes for two rating agency strategies: (1) information acquisition and truthful reporting $\iota \geq 0$, $\varepsilon = 0$ (graphs on the left-hand side) and (2) rating inflation, $\varepsilon = 1$, $\iota = 0$, with varying masses of $A$-rated securities, $\mu_A$ (graphs on the right-hand side). The graphs at the top plot rating agency profits and revenues. The graphs in the middle depict banks' portfolio share of cash holding $\omega_C$ (solid line), and the regulated minimum portfolio share of cash holdings $\omega_C^{\min}$ (dashed line). The graphs at the bottom plot the effective return on $A$-rated securities in the low aggregate state $a = L$, given that the government bails out banks in case they cannot meet their obligations (denoted by $r^L_{A2}$) and the effective return subsidy provided (denoted by $y^L_{A2}$). Here, the cost function satisfies $C(\iota) = 0.48\iota^2$.

The graph in the middle depicts the portfolio share of banks' cash holdings $\omega_C$. Regulation does not constrain banks' investment in $A$-rated securities, since $\omega_C^{\min} = 0.3 < \omega_C$ for all $\iota \geq 0$. Note that $\omega_C^{\min}$ is not the optimal solution to the regulator problem, but instead just an example of a possible regulation (we will need to solve for the optimal ex ante value of $\omega_C^{\min}$). Notice that in principle, if $\omega_C^{\min}$ was higher, banks' exposure to

$A$-rated securities could be constrained by regulation. Yet, since regulation is sufficiently lenient, the rating agency is better off acquiring no information at all. As we will see from the graphs on the right-hand side, the rating agency can do even better by not acquiring information and inflating ratings. Notice that in the graphs on the left-hand side the first important *"kink"* was reached: banks default in the low state (*default kink*).

**Right-hand Side Panel (No Information Production $\int \iota(v)\,dv = 0$, $\tilde{\mu}_A \in [0,1]$)** The graphs on the right-hand side consider the case in which the rating agency does not acquire any information ($\int \iota(v)\,dv = 0$) and provides a mass $\tilde{\mu}_A \in [0,1]$ of firms with an indicative $A$ rating. If the mass of $A$-rated securities is sufficiently small, banks would not default in the bad state of the world, i.e., the subsidy would be zero. This in turn would imply that banks would have to hold a negative NPV security (since the rating agency does not provide any information and the average project is NPV negative). As a result, the rating agency could not make positive profits and would not operate under this level of supply of $A$ securities. It can only make profits by supplying a sufficient mass of $A$ securities, i.e, $\mu_A > 0.38$, so that the subsidy outweighs the negative average NPV. Profits are increasing in the supply of $A$ securities for $\mu_A > 0.38$ until bank regulation becomes a binding constraint (second kink, *"regulatory kink"*). Regulation becomes binding around $\mu_A^* = 0.66$. At $\tilde{\mu}_A = \mu_A^*$ banks max out on their possible $A$-rated securities holdings ($\omega_C = \omega_C^{\min}$), implying that banks cannot purchase more $A$-rated securities (see middle graph). At that point, all the A-rated securities in the economy are held by banks and banks are still the marginal investor. By expanding the amount of $A$-rated securities beyond this threshold ($\tilde{\mu}_A > \mu_A^*$) retail investors would have to purchase the remaining $A$-rated securities. Yet, retail investors will not purchase any $A$-rated securities since they do not enjoy government subsidies and cannot fund the average project (the average NPV is negative and since $\varepsilon = 1$, $\iota = 0$ pooling obtains). Thus, we obtain rationing in the sense that only a mass $\mu_A^*$ of projects can be funded (they will be funded by banks), even when the rating agency chooses $\tilde{\mu}_A > \mu_A^*$. That is why the profit function flattens out at $\mu_A^*$.

Regulation effectively limits the amount of subsidies that can be extracted in the sense if banks could hold less cash than the considered $\omega_C^{\min} = 0.3$, they would expose themselves to even more $A$-rated securities, and the government would provide an even larger return subsidy in the low state $y_{A2}^L$. Yet, due to regulation, the subsidy is limited (see bottom graph for $y_{A2}^L$). The optimal solution for the rating agency conditional on choosing $\varepsilon = 1$ is thus to choose $\tilde{\mu}_A \geq \mu_A^*$, yielding a profit of about 0.08.

## 3.5. Regulator Maximization

The regulator attempts to maximize ex ante welfare $W$, where welfare is defined as the $NPV$ of all funded projects (including those funded via good types' outside option) minus the rating agency's information acquisition cost.

When the regulator decides on $\omega_C^{\min}$ she is uncertain about the complexity of securities in the future. The regulator solves

$$W = \max_{\omega_C^{\min}} \left\{ \Pr\left[c = c^h\right] \cdot W\left(\omega_C^{\min} | c^h\right) + \Pr\left[c = c^l\right] \cdot W\left(\omega_C^{\min} | c^l\right) \right\}.$$

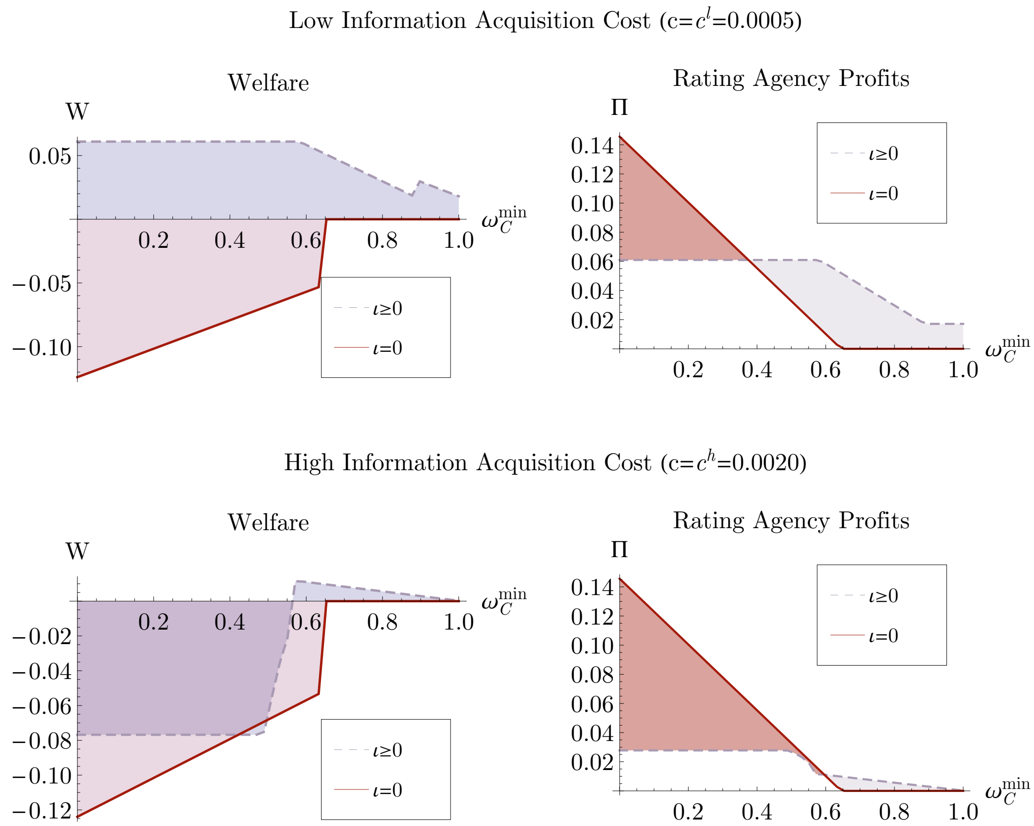


**FIGURE III**

The figure illustrates welfare $W$ (left-hand side graphs) and rating agency profits $\Pi$ (right-hand side graphs) as functions of the regulator's choice variable $\omega_C^{\min}$. Each graph depicts two curves illustrating outcomes under two different sets of rating agency strategies: (1) Information production and truthful reporting of informative signals ($\mu^\iota \geq 0$, $\iota \geq 0$, $\varepsilon^\iota = (0,0)$, $\varepsilon^{\iota-} = (1,0)$) or no operation, and (2) no information production and distorted reporting ($\mu^\iota > 0$, $\iota = 0$, $\varepsilon^\iota = (0,1)$, $\varepsilon^{\iota-} = (1,0)$, $\tilde{\mu}_A = \mu^\iota$) or no operation. The top-row graphs illustrates the case of low information acquisition cost ($c^l = 0.0005$). The bottom-row graphs illustrate the case of high information acquisition cost ($c^h = 0.0020$).

**Optimal Regulation with Uncertain Complexity (Figure III)** Figure III illustrates welfare $W$ (on the left-hand side) and rating agency profits $\Pi$ (on the right-hand side) as functions of the regulator's choice variable $\omega_C^{\min}$. Table I provides the benchmark model parameters chosen for all illustrations. Each figure depicts two curves illustrating outcomes under two different sets of rating agency strategies:

1. Information production and truthful reporting of informative signals ($\mu^\iota \geq 0$, $\iota \geq 0$, $\varepsilon^\iota = (0,0)$, $\varepsilon^{\iota-} = (1,0)$) or no operation.
2. No information production and distorted reporting ($\mu^\iota > 0$, $\iota = 0$, $\varepsilon^\iota = (0,1)$, $\varepsilon^{\iota-} = (1,0)$, $\tilde{\mu}_A = \mu^\iota$) or no operation.

The regulator anticipates that conditional on each regulatory regime (characterized by $\omega_C^{\min}$), the rating agency will choose the strategy that maximizes its profits given the realization of complexity ($c \in \{c^l, c^h\}$). The graphs illustrate outcomes given the rating agency chooses the (constrained) profit-maximizing strategy from each set of strategies. The global maximum of the rating agency's profits for each $\omega_C^{\min}$ is given by the maximum of the two plotted curves for $\Pi$.

First, consider the top-row which illustrates the case of low information acquisition cost $c = c^l = 0.0005$. The rating agency's profits (plotted on the right-hand side) are declining in regulatory constraints (that is, higher $\omega_C^{\min}$). For relatively lenient regulation (low $\omega_C^{\min}$), rating agency profits from rating inflation are higher. For stringent regulation (high $\omega_C^{\min}$) rating agency profits from information production ($\iota \geq 0$) are higher. By choosing $\omega_C^{\min} > 0.38$, the regulator can induce the rating agency to provide information.

In case the rating agency chooses to provide information ($\iota \geq 0$), welfare may decrease with stricter regulation (for $\omega_C^{\min} \geq 0.59$). On the other hand, in case the rating agency chooses not to acquire information at all ($\iota = 0$), welfare is weakly *increasing* in $\omega_C^{\min}$. Increased requirements for bank cash holdings limit welfare losses from rating inflation, implying a positive relationship between $\omega_C^{\min}$ and welfare in case of rating inflation. On the contrary, in case of information production, an increase in $\omega_C^{\min}$ may reduce welfare since skilled investors (banks) are constrained in their ability to fund projects. For very high regulatory requirements, welfare may actually increase in stricter regulation, which causes a small spike in welfare for $\omega_C^{\min}$ close to 1 resulting from retail investor financing of projects unfunded by banks. If regulation restricts banks holdings of highly rated securities sufficiently much, the rating agency prefers to investigate all securities and sets information acquisition and the fee such that retail investors can invest in $A$-rated securities. On the other hand, for lower values of $\omega_C^{\min}$, the optimal rating agency strategy implies that banks are the marginal investors for $A$-rated securities. Firms are willing to pay a higher fee to the rating agency if banks are marginal investors, since banks have better loan collection abilities and potentially benefit from government subsidies. Yet the benefit of being able to charge a higher fee is counteracted by the low volume of funded $A$-rated securities if only banks can hold $A$-rated securities and if banks are severely constrained by regulation.

If information acquisition cost are high (see lower panel of Figure III $c = c^h = 0.0020$), the rating agency prefers not to acquire information for a wider range of regulatory regimes $\omega_C^{\min}$. Profits from rating inflation are above those from information production for all $\omega_C^{\min} \leq 0.6$. Interestingly, since information acquisition cost are high, the rating agency even chooses a strategy with zero information acquisition as the constrained optimum from the set of strategies with weakly positive information production (implying that the term "information production" is somewhat misleading in this case). This behavior also explains why welfare from both strategies is negative for $\omega_C^{\min} < 0.56$.

If the regulator chooses $\omega_C^{\min}$ before complexity $c$ is realized, she faces a trade off. Conditional on $c = c^l$, she would like to set $\omega_C^{\min} > 0.38$ in order to ensure that the rating agency chooses to acquire information, and $\omega_C^{\min} < 0.59$ to ensure that banks are unconstrained in their holdings of $A$-rated securities. Yet, on the other hand, conditional on high complexity ($c = c^h$), all $\omega_C^{\min} \in [0.38, 0.59]$ would result in negative welfare. By setting $\omega_C^{\min} = 0.6$ the regulator would incur some welfare losses in the low information cost state ($c = c^l$), but avoid higher welfare losses in the high information cost state ($c = c^h$). If the high information cost state has a low ex ante probability, the regulator will set $\omega_C^{\min}$ below 0.6 and accept rating inflation with negative welfare in case the state with high information acquisition cost is realized in order to improve welfare in the state with low information acquisition cost.

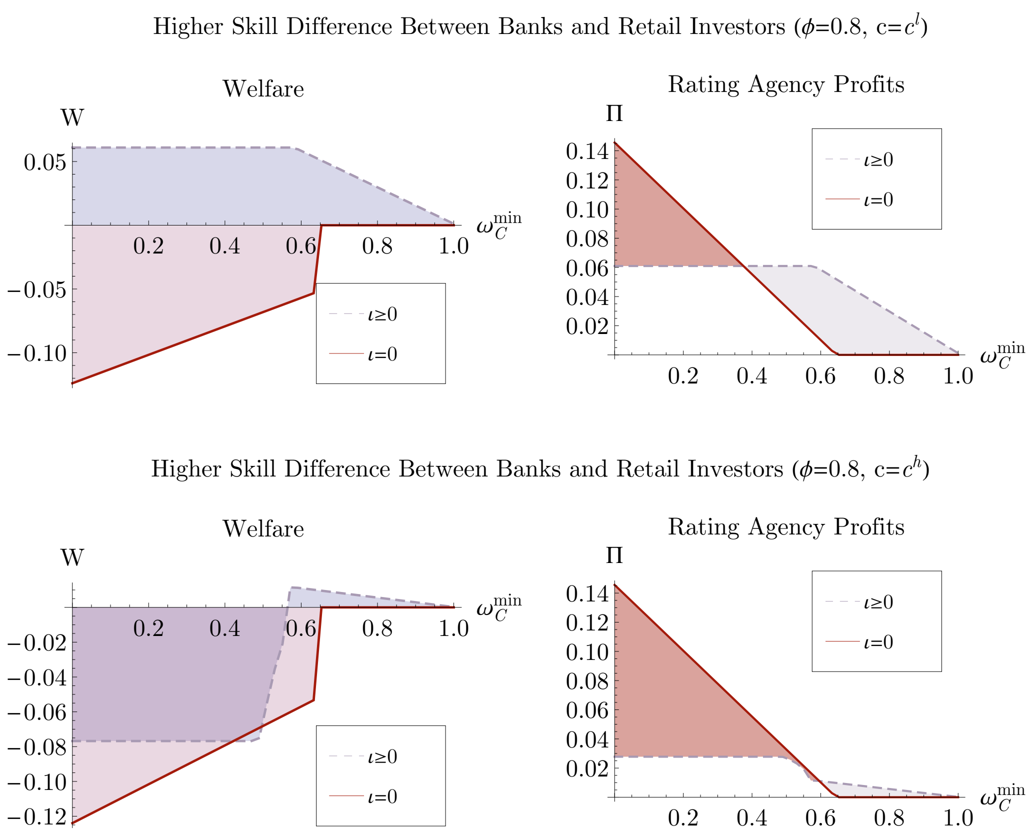


**FIGURE IV**

The figure illustrates welfare $W$ (LHS graph) and rating agency profits $\Pi$ (RHS graph) as functions of the regulator's choice variable $\omega_C^{\min}$. Each graph depicts two curves illustrating outcomes under two different regimes: (1) information production and truthful reporting ($\varepsilon = 0$, $\iota \geq 0$), and (2) rating inflation ($\varepsilon = 1$, $\iota = 0$). The graphs consider the case of increased skill differences between banks and retail investors in terms of ex post loan collection ability ($\phi = 0.8$ vs. 0.9 in the benchmark case plotted in figure III).

**Figure IV - Optimal Regulation with High Skill Differences** Figure IV illustrates the effect of a larger disadvantage of retail investors in their ability to collect loans, characterized by of a lower value for $\phi$ ($\phi = 0.8$ vs. 0.9 in figure III). For $\phi = 0.8$ retail investors do not hold $A$-rated securities in the given parameterization. Thus, when banks' holdings of $A$-rated securities become constrained by regulation, retail investors do not pick up the remaining $A$-rated securities in the market. As a result, banks stay the marginal investors of $A$-rated securities, and welfare does not spike for high values of $\omega_C^{\min}$ the way it did in the case of $\phi = 0.9$ and $c = c^l$ (compare the blue dashed lines in the top-left graphs in Figures III and IV).

**Regulator Trade-off in detail – See Figures V and VI at the end** The regulator's basic trade-off is that an increase in the minimum cash holdings $\omega_C^{\min}$ reduces banks ability to expose themselves to risky securities, making rating inflation less valuable. Yet, on the other hand, a high level of cash holdings constrains the amount of lending by the banks, which causes *regulation-induced credit rationing.*

**Figure V - Details on "Tough" Regulation** Figure V considers a case with a high level for $\omega_C^{\min} = 0.7$, implying that banks are heavily constrained in their holdings of $A$-rated securities. Regulation ensures that the rating agency's revenue from rating inflation (right-hand side graphs) is negative throughout, so that it would not operate under the rating inflation regime (thus plotted as 0). Instead, the rating agency optimally choose to acquire a positive level of information acquisition, i.e., $\iota = 0.36$ and full disclosure. Welfare at that point is strictly positive. Since our parametrization assumes that $\bar{U}_g = 0$, the rating agency profits are equal to welfare between $\iota = 0.18$ and $\iota = 0.43$.

**Figure VI - Details on "Lenient" Regulation** Figure VI considers the same economy as in Figure V but with a lower level of $\omega_C^{\min} = 0.3$, implying that banks are less constrained in their ability to invest in $A$-rated securities. Conditional on information production and truthful reporting (left-hand side graphs), we can see that welfare is increased relative to Figure III. The improvement is due to the fact that banks fund all $A$-rated securities in the economy, and they are more efficient than retail investors. Yet rating agency profits from rating inflation are positive now and so large that the rating agency will now choose to inflate. In this case, welfare is strictly negative.

# 4. Conclusion

This paper provides a unified framework to analyze the practice of governments to regulate banks' balance sheets by resorting to credit ratings as measures of creditworthiness. Since banks are the most efficient owners of risky securities, a government will ensure survival of banks under water. In turn, this creates incentives for the bank to take on imprudent risks. The government faces a key trade-off between restricting banks exposure to socially valuable risks and preventing risk-taking that is only privately valuable. Since the regulator needs to set regulations before uncertainty about the economic environment is revealed, policies may have differential welfare implications depending on the state of the world. A regulator might even be willing to incur rating inflation in some states of the world (with associated financing of NPV negative projects), if lax regulation enables more financing of good projects in other states of the world.

In contrast to Basel III guidelines, the recent Dodd-Frank proposals aims to eliminate regulations based on ratings and advocates to base regulation instead on all publicly available information. However, to make such regulation implementable, the regulator needs to come up with other, concrete measures of risk. As Bond, Goldstein, and Prescott (2010) point out, even market based measures will be distorted, i.e., reflect the value of regulation itself, once they are used for regulation. This classical Lucas-critique insight will apply to any relevant regulation that is outsourced to private parties. Other regulatory measures, such as a government-run credit rating agency, exogenous formulae also have obvious shortcomings. Thus, any regulation is almost by definition second best. Our paper has provided insights on the merits and potential drawbacks of using private ratings as measures of risk to control the asset side of banks. An important question for future research is to study the repercussions of dramatic increases of equity Admati, DeMarzo, Hellwig, and Pfleiderer (2011), i.e., liability management (leverage), which would make excessive risk taking of banks on the asset side potentially less relevant.

# 5. Appendix

## 5.1. Tables and Figures

**Table 1**

Common parameters and functional specifications used in Figures II-VI.

| | Variable Descriptions | Notation | Values |
|---|---|---|---|
| 1. | Probability of the high aggregate state | $p^H$ | 0.50 |
| 2. | Fraction of good types in the population | $\pi_g$ | 0.30 |
| 3. | Gross return if project succeeds | $R$ | 1.50 |
| 4. | Default rate of good types | $d_g$ | 0.02 |
| 5. | Average default rate of bad types | $d_b$ | 0.60 |
| 6. | Maximum face value for an $A$-rated bond | $\bar{N}$ | $R$ |
| 7. | Banks initial funds | $M_1$ | 1.05 |
| 8. | Bank leverage | $\frac{W_2}{M_1}$ | 0.90 |
| 9. | Rating agency cost function | $C(\iota)$ | $\frac{c}{(\iota-0.51)^2}$ |
| 10. | Rating agency cost function parameter | $c$ | 0.0005 |

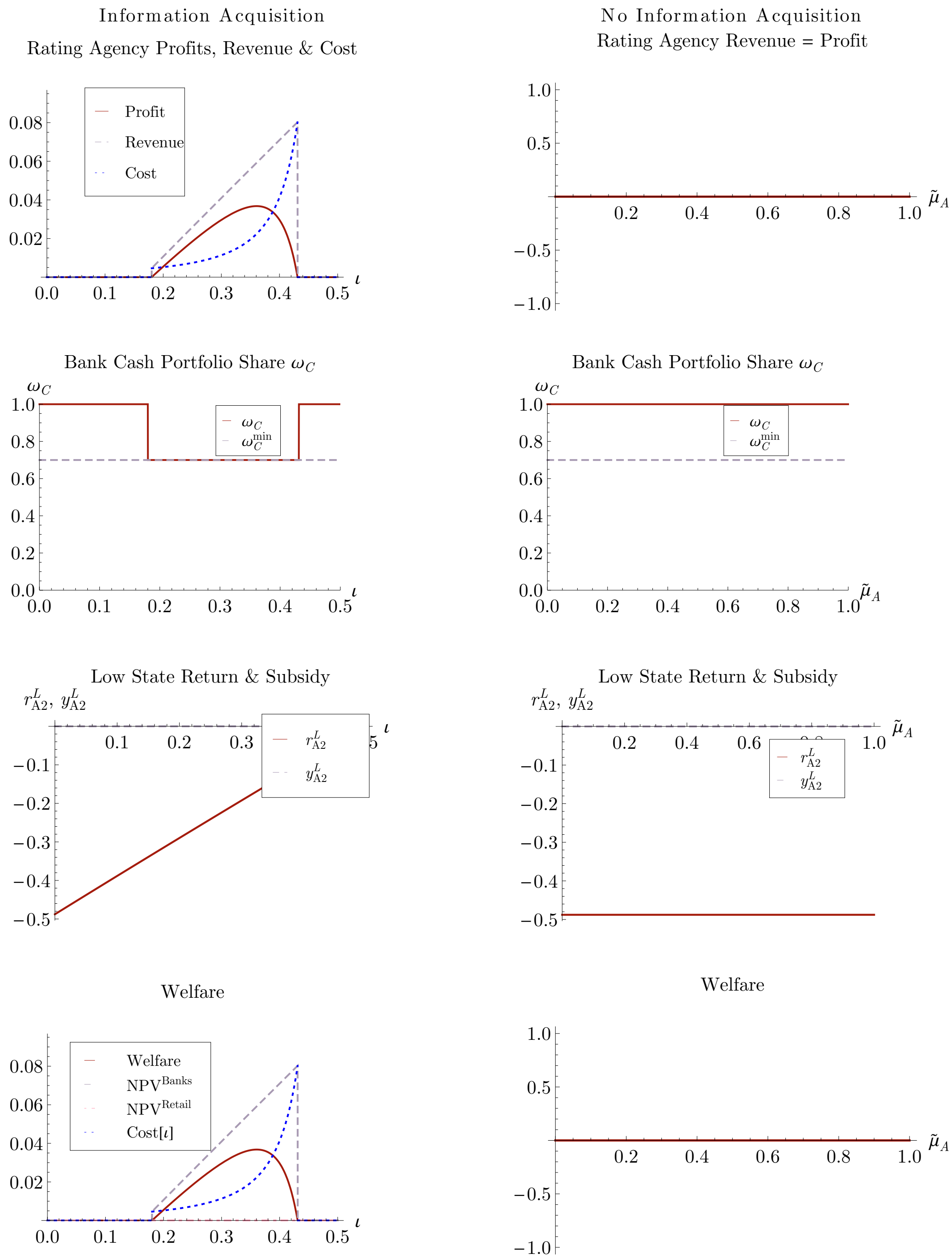


**FIGURE V**

The structure of the graphs is similar to Figure II with $\omega_C^{\min} = 0.7$, i.e., tough regulation. In addition to Figure II, we plot welfare in the bottom panel and use the cost function as in Table 1.

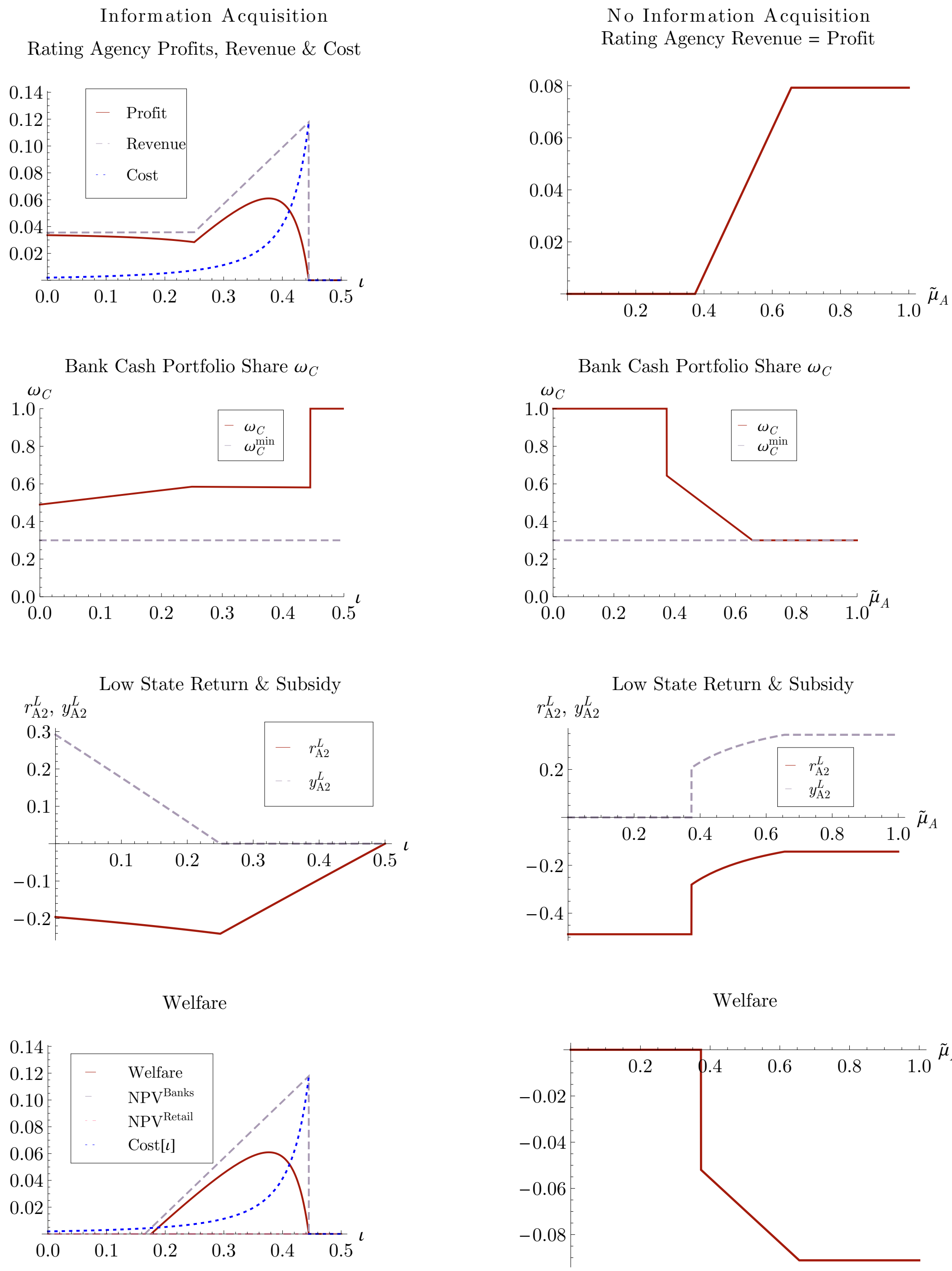


**FIGURE VI**

The structure of the graphs is similar to Figure II with $\omega_C^{\min} = 0.3$, i.e., lenient regulation. In addition to Figure II, we plot welfare in the bottom panel and use the cost function as in Table 1.

## 5.2. Rating Agency Fee

The binding participation constraint for good firms yields the following relations for the fee:

$$
\begin{array}{ll}
N_A^{bank}\left(f, d_A^L, d_A^H, \omega_C\right)|_{\omega_C=\omega_C^{\min}} = \bar{N} & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge N_A^{retail}\left(f, d_A\right) > \bar{N}, \\
N_A^{bank}\left(f, d_A^L, d_A^H, \omega_C\right)|_{\omega_C=\omega_C^{\text{int}}} = \bar{N} & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}, \\
N_A^{retail}\left(f, d_A\right) = \bar{N} & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}
\end{array}
$$

Solving for the fee yields

$$
f = \begin{cases}
\bar{N}\left(1-d_A\right)-1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge \frac{1+f}{(1-d_A)\phi} > \bar{N}, \quad \omega_C + \left(1-\omega_C\right)\frac{1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1} \\
\frac{\bar{N}\left(1-d_A^H\right)}{\left(1-\frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*\left(\omega_C^{\min}\right)\right)} - 1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}} \wedge \frac{1+f}{(1-d_A)\phi} > \bar{N}, \quad \omega_C + \left(1-\omega_C\right)\frac{1-d_A^L}{1-d_A} < \frac{W_2}{M_1} \\
\bar{N}\left(1-d_A\right)-1 & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}, \quad \omega_C + \left(1-\omega_C\right)\frac{1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1} \\
\frac{\bar{N}\left(1-d_A^H\right)}{\left(1-\frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*\left(\omega_C^{\text{int}}\right)\right)} - 1 & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}, \quad \omega_C + \left(1-\omega_C\right)\frac{1-d_A^L}{1-d_A} < \frac{W_2}{M_1} \\
\bar{N}\left(1-d_A\right)\phi-1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot(1+f)}{1-\omega_C^{\min}}
\end{cases}
$$

Substituting out the fees yields:

$$
\begin{aligned}
& f\left(\tilde{\mu}_A, d_A^L, d_A^H, \omega_C^{\min}\right) \\
= & \begin{cases}
\bar{N}\left(1-d_A\right)-1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot\bar{N}(1-d_A)}{1-\omega_C^{\min}} \\
& \wedge\ \omega_C^{\min} + \ \left(1-\omega_C^{\min}\right)\frac{1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1} \\
\bar{N}\frac{1-d_A^H}{1-\frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*\left(\omega_C^{\min}\right)} - 1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot\bar{N}\left(1-d_A^H\right)}{\left(1-\omega_C^{\min}\right)\left(1-\frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*\left(\omega_C^{\min}\right)\right)} \\
& \wedge\ \frac{\left(1-d_A^H\right)}{\phi\left(1-\frac{\Pr[a=L]}{\Pr[a=H]} r_{A2}^*\left(\omega_C^{\min}\right)\right)(1-d_A)} > 1 \\
& \wedge\ \omega_C^{\min}+\left(1-\omega_C^{\min}\right)\frac{1-d_A^L}{1-d_A} < \frac{W_2}{M_1} \\
\bar{N}\left(1-d_A\right)-1 & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot\bar{N}(1-d_A)}{1-\omega_C^{\min}} ok \\
& \wedge\ \frac{\left(1-\frac{\tilde{\mu}_A}{M_1}\bar{N}(1-d_A)\right)\left(d_A^L-d_A\right)+1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1} \\
\bar{N}\left(1-d_A^H\right)-1+\frac{\Pr[a=L]}{\Pr[a=H]}\frac{W_2-M_1}{\tilde{\mu}_A} & \text{if } M_1 > \frac{\tilde{\mu}_A\cdot\bar{N}\left(1-d_A^H\right)+\frac{\Pr[a=L]}{\Pr[a=H]}(W_2-M_1)}{1-\omega_C^{\min}} \\
& \wedge\ \frac{\left(\left(1-\frac{\tilde{\mu}_A}{M_1}\bar{N}\left(1-d_A^H\right)-\frac{\Pr[a=L]}{\Pr[a=H]}\left(\frac{W_2}{M_1}-1\right)\right)\cdot\left(d_A^L-d_A\right)+1-d_A^L\right)}{1-d_A} < \frac{W_2}{M_1} \\
\bar{N}\left(1-d_A\right)\phi-1 & \text{if } M_1 \leq \frac{\tilde{\mu}_A\cdot\bar{N}(1-d_A)\phi}{1-\omega_C^{\min}}
\end{cases}
\end{aligned}
$$

## 5.3. Rating Agency Maximization – Details

### 5.3.1. Banks Constrained. Retail Investors Marginal.

Banks are constrained if $M_1\left(1-\omega_C^{\min}\right)\leq\tilde{\mu}_A\cdot(1+f)$. Retail investors are marginal if $N_A^{retail}(f,d_A)\leq\bar{N}$. The rating agency would set the fee such that

$$f=\bar{N}(1-d_A)\phi-1.$$

which implies

$$M_1\leq\frac{\tilde{\mu}_A\cdot\bar{N}(1-d_A)\phi}{1-\omega_C^{\min}}.$$

The rating agency's (restricted) maximization problem would then amount to maximize

$$\begin{aligned}\Pi &= f\cdot\mu^{funded}-\mu^{\iota}\cdot C(\iota)\\ &= \tilde{\mu}_A\cdot\left(\bar{N}(1-d_A(\iota))\phi-1\right)-C(\iota)\end{aligned}$$

Since retail investors are marginal and do not benefit from a bailout, the surplus is increasing in rating precision (as in Opp, Opp, and Harris (2011)), implying that it is optimal to set $\varepsilon^{\iota}=\mathbf{0}$. Since all firms are investigated ($\mu^{\iota}=1$), the disclosure strategy $\varepsilon^{\iota-}$ is irrelevant.

### 5.3.2. Banks Constrained. Banks Marginal.

If $M_1\leq\frac{\tilde{\mu}_A\cdot\bar{N}(1-d_A)}{1-\omega_C^{\min}}$ and $\omega_C^{\min}+(1-\omega_C^{\min})\frac{1-d_A^L}{1-d_A}\geq\frac{W_2}{M_1}$ then $f=\bar{N}(1-d_A)-1$. Profits are given by

$$\begin{aligned}\Pi &= f\cdot\mu^{funded}-\mu^{\iota}C(\iota)\\ &= f\cdot\frac{\left(1-\omega_C^{\min}\right)\cdot M_1}{1+f}-\mu^{\iota}C(\iota)\\ &= \left(\bar{N}(1-d_A)-1\right)\cdot\frac{\left(1-\omega_C^{\min}\right)\cdot M_1}{\bar{N}(1-d_A)}-\mu^{\iota}C(\iota)\\ &= \left(1-\frac{1}{\bar{N}(1-d_A)}\right)\cdot\left(1-\omega_C^{\min}\right)\cdot M_1\;-\mu^{\iota}C(\iota)\end{aligned}$$

The revenues are only a function of $d_A$. The lower $d_A$ the higher are revenues. For any given $\iota$, $d_A$ is minimized by setting $\varepsilon_{BA}=0$ and $\varepsilon_{AB}=0$. Further, given only a mass $\mu^*=\frac{\left(1-\omega_C^{\min}\right)\cdot M_1}{\bar{N}(1-d_A)}$ of $A$-rated securities can be financed, it is optimal to set $\mu^{\iota}=\frac{\mu^*}{\mu_A(\iota)}$.

If $M_1\leq\frac{\tilde{\mu}_A\cdot\bar{N}\left(1-d_A^H\right)}{\left(1-\frac{\Pr[a=L]}{\Pr[a=H]}r_{A2}^*\left(\omega_C^{\min}\right)\right)\left(1-\omega_C^{\min}\right)}$ and $\frac{\left(1-d_A^H\right)}{\phi\left(1-\frac{\Pr[a=L]}{\Pr[a=H]}r_{A2}^*\left(\omega_C^{\min}\right)\right)(1-d_A)}>1$ and $\omega_C^{\min}+(1-$

$\omega_C^{\min})\frac{1-d_A^L}{1-d_A} < \frac{W_2}{M_1}$ then the fee is given by

$$f = \frac{\bar{N}\left(1-d_A^H\right)}{\left(1-\frac{\Pr[a=L]}{\Pr[a=H]}r_{A2}^*\left(\omega_C^{\min}\right)\right)} - 1.$$

Thus, profits are given by

$$\begin{aligned}
\Pi &= f\cdot\mu^{funded} - \mu^{\iota}C(\iota) \\
&= f\cdot\frac{\left(1-\omega_C^{\min}\right)\cdot M_1}{1+f} - \mu^{\iota}C(\iota) \\
&= \left(1-\frac{\left(1-\frac{\Pr[a=L]}{\Pr[a=H]}r_{A2}^*\left(\omega_C^{\min}\right)\right)}{\bar{N}\left(1-d_A^H\right)}\right)\cdot\left(1-\omega_C^{\min}\right)\cdot M_1 - \mu^{\iota}C(\iota)
\end{aligned}$$

which is a decreasing function of $d_A$. For any given $\iota$, $d_A$ is minimized by setting $\varepsilon_{BA}=0$ and $\varepsilon_{AB}=0$. Further, given only a mass $\mu^*$ of $A$-rated securities can be financed, it is optimal to investigate a measure $\mu^{\iota}=\frac{\mu^*}{\mu_A(\iota)}$ of firms. Alternatively, if information acquisition cost are too high then it may be optimal not to acquire information at all (since the bailout is provided funding may still take place absent any information production) and to rate a mass $\mu^{\iota}=\mu_A^*$ of firms $A$ (that is, $\mu^{\iota}=\mu_A^*$, $\iota=0$, $\varepsilon^{\iota}=(0,1)$, $\varepsilon^{\iota-}=(1,0)$, $\tilde{\mu}_A=\mu_A^*$).

### 5.3.3. Banks Unconstrained. Banks Marginal.

If

$$M_1 > \frac{\tilde{\mu}_A\cdot\bar{N}\left(1-d_A\right)}{1-\omega_C^{\min}}$$

and

$$\frac{\left(1-\frac{\tilde{\mu}_A}{M_1}\bar{N}\left(1-d_A\right)\right)\left(d_A^L-d_A\right)+1-d_A^L}{1-d_A} \geq \frac{W_2}{M_1}$$

then

$$f = \bar{N}\left(1-d_A\right)-1$$

and rating agency profits are given by

$$\begin{aligned}
\Pi &= f\cdot\mu^{funded} - \mu^{\iota}C(\iota) \\
&= \left(\bar{N}\left(1-d_A\right)-1\right)\cdot\tilde{\mu}_A - \mu^{\iota}C(\iota)
\end{aligned}$$

As shown in Opp, Opp, and Harris (2011) this is minimized by acquiring information on all firms and reports signals truthfully (that is, $\mu^{\iota}=1$, $\varepsilon^{\iota}=(0,0)$, $\tilde{\mu}_A=\mu_A(\iota)$) or rating all firms $A$ without acquiring information (that is, $\mu^{\iota}=1$, $\iota=0$, $\varepsilon^{\iota}=(0,1)$, $\tilde{\mu}_A=1$).

If

$$M_1 > \frac{\tilde{\mu}_A\cdot\bar{N}\left(1-d_A^H\right)}{1-\omega_C^{\min}-\frac{\Pr[a=L]}{\Pr[a=H]}\left(\frac{W_2}{M_1}-1\right)},$$

and

$$\left(1 - \frac{\tilde{\mu}_A}{M_1}\bar{N}\left(1 - d_A^H\right) - \frac{\Pr\left[a = L\right]}{\Pr\left[a = H\right]}\left(\frac{W_2}{M_1} - 1\right)\right) \cdot \left(\frac{d_A^L - d_A}{1 - d_A}\right) + \frac{1 - d_A^L}{1 - d_A} < \frac{W_2}{M_1},$$

then

$$f = \bar{N}\left(1 - d_A^H\right) - 1 + \frac{\Pr\left[a = L\right]}{\Pr\left[a = H\right]}\left(\frac{W_2}{M_1} - 1\right)\frac{M_1}{\tilde{\mu}_A},$$

and rating agency profits are given by

$$\begin{aligned} \Pi &= f \cdot \mu^{funded} - \mu^{\iota} C\left(\iota\right) \\ &= \left(\bar{N}\left(1 - d_A^H\right) - 1\right) \cdot \tilde{\mu}_A + \frac{\Pr\left[a = L\right]}{\Pr\left[a = H\right]}\left(\frac{W_2}{M_1} - 1\right) M_1 - \mu^{\iota} C\left(\iota\right) \end{aligned}$$

where $\frac{\Pr[a=L]}{\Pr[a=H]}\left(\frac{W_2}{M_1} - 1\right) M_1$ is independent of the choice variables. Thus, given $d_g > d_b^H$, again the same argument as in Opp, Opp, and Harris (2011) applies, implying that either the rating agency acquires information on all firms and reports signals truthfully (that is, $\mu^{\iota} = 1$, $\varepsilon^{\iota} = (0, 0)$, $\tilde{\mu}_A = \mu_A(\iota)$) or it rates all firms $A$ without acquiring information (that is, $\mu^{\iota} = 1$, $\iota = 0$, $\varepsilon^{\iota} = (0, 1)$, $\tilde{\mu}_A = 1$).

# References


Admati, A. R., P. M. DeMarzo, M. F. Hellwig, and P. C. Pfleiderer (2011): "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is not Expensive," Working paper series, Stanford University.

Bond, P., I. Goldstein, and E. S. Prescott (2010): "Market-Based Corrective Actions," *Review of Financial Studies*, 23(2), 781–820.

Diamond, D. W. (1984): "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, 51(3), 393–414.

Diamond, D. W., and R. Rajan (2011): "Illiquid Banks, Financial Stability, and Interest Rate Policy," NBER Working Papers 16994, National Bureau of Economic Research, Inc.

Opp, C., M. M. Opp, and M. Harris (2011): "Rating agencies in the face of regulation," Working paper series, University of California, Berkeley.