Sovereign Wealth Investing from a National Balance-Sheet Perspective

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(Preliminary Draft – Not for Quotation)

Abstract

This paper presents a parsimonious model that is roughly consistent with the current state of the global economy. It describes the consequences of a sovereign wealth asset allocation process based on each country's national balance sheet characteristics. This process was first described in Lee (2006) in which each sovereign wealth fund can formulate its own allocation strategy considering inflows and outflows characteristics in both its capital and current accounts, and any allocation decision should price in "once-in-a-decade" tail risk events. Now we have moved beyond stand-alone asset allocation models computed from the perspective of a single allocator, since sophisticated mathematical allocation solutions are often rendered infeasible by the unique global market constraints faced by mega-sized investors. By modeling the flows and constraints in the global financial system as a system of difference equations incorporating the likely investment choices of three key allocators of public surpluses – those that are abundant in natural resources, those that are abundant in labors and those that are abundant in intangible assets – this paper identifies scenarios that may result in system-wide instability. In particular, we aim to characterize the boundary conditions associated with unsustainable global imbalances. Building upon this aggregated model allows us to make an initial attempt to answer questions such as how sovereign wealth funds should respond to market crises and whether it is effective for them to sell investment holdings to fund rescue attempts. The financial market is based on cash, equities and commodities, without any *a priori* assumptions on asset price behaviors.

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1. Introduction

This paper presents a parsimonious model that is roughly consistent with the current state of the global economy. Such a model can be used to better understand possible policy options and the roles of sovereign wealth funds (SWFs), particularly those in Asia, in turbulent global financial markets.

1.1 Motivation

In Lee (2006), the author proposed a SWF asset allocation model using fourth-order mathematics by optimizing the following portfolio objective function:

$$ASR \equiv \frac{\sum_i e_i \pi_i}{z_\pi^- \sigma_\pi} + \frac{1}{2} \frac{\sum_i \pi_i \left(z_i^+ \sigma_i \right)^2}{z_\pi^- \sigma_\pi} - \frac{1}{2} z_\pi^- \sigma_\pi \quad (1)$$

where

e_i is expected return of the *i*-th asset

π_i is weight of the *i*-th asset

σ_i and σ_π are volatilities of the *i*-th asset and of the portfolio, respectively

$z^+ = \frac{\max\left(z_{cf}\left(z_C^+ \right), 0 \right)}{z_C^+}$ where z_C^+ is critical value from for probability α

$z^- = \frac{\max\left(z_{cf}\left(z_C^- \right), 0 \right)}{z_C^-}$ where z_C^- is critical value for probability $1-\alpha$

z_{cf} is the fourth-order Cornish-Fisher coefficient that corresponds to the "fat tail" adjustment for the asset return distribution (with S and K denoting observed skewness and kurtosis statistics):

$$z_{cf} = z_C + \frac{1}{6}\left(z_C^2 - 1 \right) S + \frac{1}{24}\left(z_C^3 - 3 z_C \right) K - \frac{1}{36}\left(2 z_C^3 - 5 z_C \right) S^2$$

This performance metric, called the Alternative Sharpe Ratio (Lee and Lee 2004), is similar to the "Omega function" proposed by Keating and Shadwick (2004) and loss functions used in actuarial mathematics. It can be shown that the metric provides a number of useful mathematical properties:

1. If a particular asset in the portfolio shows positive skewness (or upside characteristics), this portfolio performance metric will improve.
2. If a particular asset in the portfolio contributes to portfolio tail risk, this portfolio performance metric will be penalized.
3. If all assets in the portfolio have normally as well as independently and identically distributed returns, this portfolio performance metric will revert to the typical second-order portfolio performance metric of the Sharpe Ratio.

When applied properly, such a complex asset allocation technique can make the appropriate adjustments to account for tail-risk behavior in asset returns. In contrast, traditional asset allocation techniques are mostly based on second-order objective functions, which are known to be biased toward high-tail-risk strategies such as "short put options". Empirically, such techniques tend to generate abruptly different allocations after each realization of a tail-risk event. Given their massive size, it will be difficult for any mega-sized asset allocator such as a SWF to apply that type of second-order asset allocation techniques in practical portfolio management, because SWFs cannot easily liquidate, rebalance, or hedge out undesirable risk factor exposures at will. The defining limitation faced by a rational SWF asset allocator is that any major allocation decision, once executed, may not be easily reversible; therefore, a rational allocator needs to price in "once-in-a-decade" tail events ahead of such decisions. The author's proposed technique has been presented in publications linked to the World Bank (Lee 2006), the European Central Bank (Lee 2010) and most recently the Asian Development Bank (Lee 2011).

In this latest installment, the author attempts to move beyond stand-alone asset allocation models computed from the perspective of a single allocator. Typical asset allocation models *assume* the abundance of liquidity, in that it is feasible for the allocation decision to be executed without significant movements of the market. The unique challenges faced by SWFs are: i. Their fund size may approach that of one or more liquid markets; in fact, there is at least one case that a SWF comes quite close to dominating the entire market (Figure 1 illustrates the impressive number of component stocks in the Straits Times Index held by Temasek Holdings) and ii. Such a fund may become the primary remaining source of market liquidity in a crisis scenario. The reality is that some of these SWFs have grown sufficiently big that any major asset rebalancing will almost certainly cause significant market movements. A "good" allocation decision may turn "bad" if the very act of making the allocation can drive up the target asset price by a large amount, or the decision may provoke another mega-sized player to react in such a way that accentuate market instability.

The explicit goal of this paper is to eliminate the "smoke and mirrors" of complex mathematics used by most sophisticated asset allocation models in order to better understand the underlying issues using a relatively parsimonious approach. In particular, we have deliberately avoided superimposing any asset allocation model with complex market-impact dynamics, which is required in order to give a reasonsable description of the asset rebalancing process by a SWF. Doing so may result in a model with way too many degrees of freedom to be helpful. To the best of the author's knowledge, most SWFs stay away from complex quantitative asset allocation models, precisely because the incorporation and calibration of market impact models are far from satisfactory for typical transactions running in the billions of dollars. One goal in this exercise is to demonstrate that it is possible to construct an intellectually sound yet compact model, which can also produce practical recommendations that are appealing to the Chief Investment Officers of SWFs.

Rank	Constituent Name	Subsector Name	Weight in Index (%)
1	Singapore Telecom	Telecommunications	9.78
2	DBS Group Holdings	Banks	9.59
3	Oversea-Chinese Banking	Banks	8.47
4	United Overseas Bank	Banks	8.44
5	Wilmar International Limited	Food Products	6.76
6	Capitaland	Development	4.88
7	Jardine Matheson	Diversified Industrials	4.64
8	Hong Kong Land	Development	4.55
9	Keppel Corp	Diversified Industrials	4.1
10	Noble Group	Diversified Industrials	3.76
11	Singapore Airlines	Airlines	3.68
12	Singapore Exchange	Investment Services	3.3
13	City Developments	Hotels	2.8
14	Fraser and Neave	Diversified Industrials	2.45
15	Singapore Press Holdings	Publishing	2.37
16	Jardine Strategic	Diversified Industrials	2.36
17	Golden Agri-Resources	Farming & Fishing	2.02
18	Engineering	Aerospace	1.88
19	Genting Singapore	Recreational Services	1.77
20	CapitaMall Trust	Retail REITs	1.71
21	Jardine Cycle & Carriage	Specialty Retailers	1.52
22	Olam International	Food Products	1.51
23	CapitaMalls Asia	Development	1.44
24	SembCorp Industries	Diversified Industrials	1.38
25	Sembcorp Marine	Trucks	1.3
26	ComfortDelgro Corp	Travel & Tourism	1.27
27	Neptune Orient Lines	Marine Transportation	0.79
28	StarHub	Telecommunications	0.59
29	SMRT Corp	Travel & Tourism	0.59
30	SIA Engineering	Transportation Services	0.31
		Total	100

Figure 1: Components of the Straits Times Index *not* held by Temasek Holdings marked in Red (April 2011)

1.2 Recap on the Lee (2006) Approach

Research in optimal portfolio strategies by Merton (1998) describes how long-term wealth managers should consider not only contemporaneous asset holdings, but also the potential substitution effects due to anticipated inflow and outflow characteristics. Lee (2006) further expanded on the basic model of Merton to better understand asset allocation from a national balance sheet perspective, assuming that the global economy is populated by three types of countries, as shown in Figure 2:

1. Group A – Countries in Group A have abundant natural resources;
2. Group B – Countries in Group B have abundant productive labor and favor the production of manufactured goods; and

3. Group C – Countries in Group C have abundant intangible assets, such as intellectual property, scientific and technical leadership, high-value-added managerial skills, capital market expertise including stock markets, hedge funds and venture capital investments.

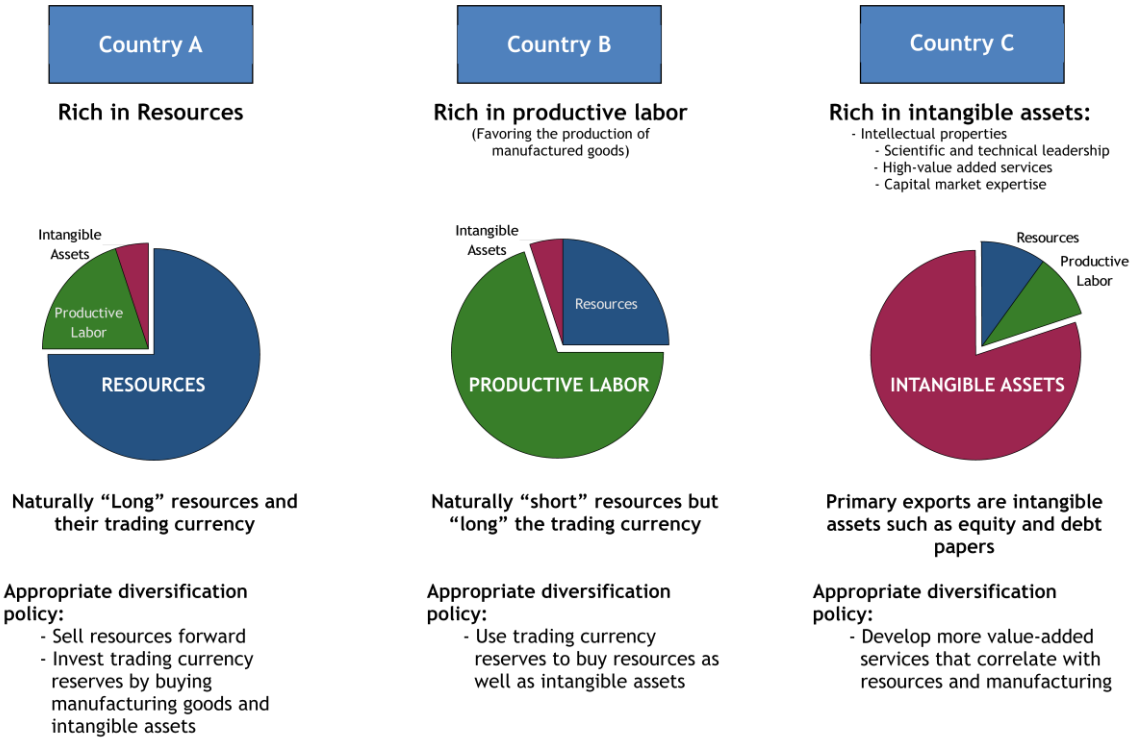

Figure 2: Model of Global Economy in Lee (2006)

Lee (2006) has assumed that one dominant country in Group C issues the major trade settlement currency in this global economy. The author was able to demonstrate through the use of empirical market data how each of these Groups will follow these asset allocation policy responses:

1. Group A – These resource-rich countries are naturally "long" resources and "long" the settlement currency (as a result of selling their natural resources). Their appropriate diversification policy is to sell resources forward and invest their settlement currency reserves by buying manufacturing goods and intangible assets.

2. Group B – These manufacturing powerhouses are naturally "short" resources and "long" the settlement currency (as a result of producing and selling their manufacturing goods). Their appropriate diversification policy is to buy resources as well as intangible assets.
3. Group C – The primary "exports" of these countries are intangible assets, such as equity and debt papers. Their appropriate diversification policy is to develop more value-added services (and hence intangible assets) related to resources and manufacturing. An example could be to develop an advanced oil/gas services sectors and expertise in managing the complex logistics of global manufacturing.

Table 1 shows one optimal allocation calculated for Groups A and B based on market data, under the initial assumptions that Group A is significantly "long" Crude Oil while Group B is significantly "short" Crude Oil and other resources. In fact, depending on the Commodity Index being used, it can be shown that Group A should sell resources forward while Group B should stockpile resources.

	Group A	**Group B**
Crude Oil*	**50%**	**-25%**
Commodity Index	5%	5%
Cash Equivalent	25%	100%
Equity	20%	20%
Balance Sheet Total	**100%**	**100%**

Table 1: Optimized Allocations for Groups A and B, based on the initial assumptions (from Lee 2006, as noted in *) that Group A is "long" Crude Oil while Group B is "short" Crude Oil

What is still missing from the analysis above is the potential impact of immigration and knowledge transfer – in particular, the flow of skills and expertise from Group C countries to Group A and B countries, making the distinctions among Groups A, B and C increasingly blurred. That there will

be Group A and Group B countries accumulating large public surpluses to invest in debt and equity papers issued by Group C countries is a natural outcome of this model. The picture portrayed above works well until one of the following boundary conditions are reached: i. Certain Group A countries start running low in natural resources; or ii. Certain Group C countries start running into a crisis of market confidence. Group B countries are in a slightly more enviable position because manufacturing can be retooled (despite the potential costs involved), and there will always be some demands for certain manufacturing goods. By comparison, corrective actions may be relatively limited when natural resources are depleted, or when the market no longer has confidence to support the valuation of intangible assets produced by a certain country.

There is one significant shortcoming in the techniques discussed so far. In the absence of practical market constraints, any optimal asset allocation computed purely from market data will recommend essentially the same optimum for all countries, with the primary difference being pre-existing positions from national balance sheets prior to any netting. However, constraints matter for mega-sized investors at the global level. The most obvious example is that there is only a finite amount of Crude Oil available globally even after counting unextracted reserves, so it may not be feasible for all Groups to allocate to Crude Oil simultaneously. In that case, the market prices used in deriving any such asset allocation will be misleading, because they do not reflect executable prices that a SWF may likely transact on, when a "response function" should be constructed for the scenario in which all SWFs are buying Crude Oil simultaneously. Similarly, Group C cannot issue significantly more new securities without the corresponding interest by Groups A and B to make allocations to such securities. A meaningful SWF asset allocation model must incorporate such relationships into its mathematical formulation, instead of solving the SWF allocation problem as an abstract stand-alone problem from the perspective of a single allocator.

Any mathematical allocation solution, however sophisticated, may be rendered infeasible by the unique global market constraints faced by mega-sized investors. Given that any asset allocation decisions by such investors, once made, are relatively static and can at best be used to provide general directions on asset allocations made by SWFs, this paper focuses on what matters the most by building a model based on the aggregated flows and the constraints on the global economy.

1.3 Overview

In the remainder of this piece, Section 2 provides a mathematical description of our global economy based on the consequences of the SWF asset allocation process as described in Lee (2006). Section 3 will seek to better understand the potential outcomes of policy interventions in such a global economy. Section 4 will make an attempt to understand the implications of our model to SWF investing. Section 5 concludes.

2. Model of Global Economy

To simplify the modeling approach used in Lee (2006), there are now only 3 countries (instead of 3 groups of countries) in our formulation of the global economy:

- Country A – Rich in resources.
- Country B – Rich in productive labor.
- Country C – Imports resources and manufactured goods and exports innovation and intellectual capital.

It is now recognized that any major asset allocation decision, once made by a SWF, is relatively static because of its typical size relative to liquid markets. As a natural next step, the goal of this paper is to fully understand the aggregated flows among countries and any applicable constraints on the global economy.

2.1 Basic Descriptions

General Assumptions:

All countries start with the same initial level of endowment.

Country C is the issuer of the single global trading currency, and the only issuer of public debts and public equities.

We will specify the following for each country:

- Net Asset Value on National Balance Sheet – Net Asset Value (NAV) will be computed from an asset management accounting perspective, in that we are only counting *tangible* accounting items.

- Cash – Our current assumption is that there is a single global trading currency. We also assume that only short-term bonds are issued and they are considered cash equivalent as is consistent with market conventions.
- Investments – Portions of reserves managed by SWFs (i.e. cash and investments make up a surplus country's total reserves)
- Commodities – We assume that this represents primarily metals and other mineral wealth, since soft commodities typically represent minor portions of the wealth of countries of interest to this analysis. From a national balance sheet perspective, both extracted as well as unextracted mineral reserves should be counted.
- Basic Trade Flows – In a closed global economy, the sum total of all net imports and net exports should be zero.
- Equity and Debts *Issued* – Issuance is listed on the liability column of the national balance sheet. The values of Equity and Debts are functions of factors such as price, holdings and discount rates. However, we do not see the need to create any complex mathematical specification at this stage, except to point out that equity has no theoretical limits on its value while the principal on debts can at best be valued at par. Notice that Equity here only denotes equity *issued* and publicly sold. As one may expect, there will be equity values on state assets such as infrastructure or state land that are not issued as equity or sold to the public. Such equity values are not counted as part of equity *issued*.
- PPI, or Proxy Price Index = (Cash + Investment) / Commodity, i.e. the aggregate amount of cash and investments available to pay for each unit of physical Commodity within a domestic economy. Notice that domestic prices of Commodities can be different from country to county, even though the same Commodity can be priced in different countries under a single global trading currency; in real-life commodity trading, that is known as "delivery spread". The proposed representation is consistent with the concept of asset inflation: If a country is

"printing money" or it is experiencing an investment bull market, it will experience inflation since the commodity base is somewhat static in general. Such a ratio will then increase, as is consistent with real-world observations. Our initial assumption is that Countries B and C will have the same purchasing power, while Country A enjoys a lower proxy price index because it is endowed with more abundant resources.

2.2 Country A – Rich in Resources

The asset and liability profiles of Country A can be described as follows:

$$Asset_A(t) = Cash_A(t) + Commodity_A(t) + Investment_A(t) \quad (2)$$

$$Liability_A(t) = Equity_A(t) + Debt_A(t) + NetAsset_A(t) \quad (3)$$

We assume that Country A does not need to issue debts and equities. Equations (2) and (3) above imply that Country A will start out as a net creditor, as in:

$$NetAsset_A(t) = Cash_A(t) + Commodity_A(t) + Investment_A(t) \quad (4)$$

Notice that $NetAsset_A$ appears on the liability side of the equation as it represents a claim on Country A's wealth by its citizens. This is expressed in standard net asset value representation used in asset management, in that we are only counting *tangible* accounting items.

The following initial conditions and temporal change relationships are roughly consistent with the findings of Lee (2006), but the numbers have been rounded off for the ease of analysis:

Initial Conditions:

$$Asset_A(0) = 100$$

$$Cash_A(0) = 25$$

$$Commodity_A(0) = 50$$

$$Investment_A(0) = 25$$

$$PPI_A(0) = \frac{25 + 25}{50} = 1$$

Temporal Changes:

$$NetExport_A(t) = -2.5$$

Negative sign means net imports because it represents a payment.

$$Commodity_A(t) - Commodity_A(t + 1) = 5$$

The positive sign represents the annual sales of commodity as a result of commodities production. Since the asset equation should balance, any net sales in commodity will translate into receipts in cash and investment.

$$Asset_A(t + 1) = Asset_A(t) + NetExport_A(t)$$

The relationship above assumes a single-period delay for the payment of account payables.

Net Asset Value:

$$Cash_A(t + 1) + Commodity_A(t + 1) + Investment_A(t + 1) = Cash_A(t) + Commodity_A(t) + Investment_A(t) + NetExport_A(t) \quad (5)$$

Equation (5) assumes that cash and investments do not change valuation and their proportional allocation remains static. This assumption will be revisited in Section 4. From Equation (5), we have:

$$Cash_A(t+1)+Investment_A(t+1) = Cash_A(t) + Investment_A(t) + NetExport_A(t) + Commodity_A(t) - Commodity_A(t+1) \quad (6)$$

Simplifying based on the temporal change relationships as stated above,

$$Cash_A(t+1)+Investment_A(t+1) = Cash_A(t) + Investment_A(t) + 2.5 \quad (7)$$

2.3 Country B – Rich in Labor

The asset and liability profiles of Country B can be described as follows:

$$Asset_B(t) = Cash_B(t) + Commodity_B(t) + Investment_B(t) \quad (8)$$

$$Liability_B(t) = Equity_B(t) + Debt_B(t) + NetAsset_B(t) \quad (9)$$

We assume that Country B does not need to issue debts and equities. Equations (8) and (9) imply that Country B will start out as a net creditor, as in:

$$NetAsset_B(t) = Cash_B(t) + Commodity_B(t) + Investment_B(t) \quad (10)$$

Notice that $NetAsset_B$ appears on the liability side of the equation as it represents a claim on Country B's wealth by its citizens. This is expressed in standard net asset value representation used in asset management, in that we are only counting *tangible* accounting items.

The following initial conditions and temporal change relationships are roughly consistent with the findings of Lee (2006), but the numbers have been rounded off for the ease of analysis:

Initial Conditions:

$$Asset_B(0) = 100$$

$$Cash_B(0) = 50$$

$$Commodity_B(0) = 25$$

$$Investment_B(0) = 25$$

$$PPI_B(0) = \frac{50 + 25}{25} = 3$$

Temporal Changes:

$$NetExport_B(t) = 5$$

Positive sign means net receipts from exports.

$$Commodity_B(t) - Commodity_B(t+1) = -2.5$$

Negative sign represents the annual purchase of commodity as a result of the consumption of commodities. Since the asset equation should balance, any net purchase in commodity will translate into payments in cash and investment.

$$Asset_B(t+1) = Asset_B(t) + NetExport_B(t)$$

The relationship above assumes a single-period delay for the collection of account receivables.

Net Asset Value:

$$Cash_B(t+1) + Commodity_B(t+1) + Investment_B(t+1) = Cash_B(t) + Commodity_B(t) + Investment_B(t) + NetExport_B(t) \quad (11)$$

Equation (11) assumes that cash and investments do not change valuation and their proportional allocation remains static. This assumption will be revisited in Section 4. From Equation (11), we have:

$$Cash_B(t+1)+Investment_B(t+1) = Cash_B(t) + Investment_B(t) + NetExport_B(t) + Commodity_B(t) - Commodity_B(t+1) \quad (12)$$

Simplifying based on the temporal change relationships as stated above,

$$Cash_B(t+1)+Investment_B(t+1) = Cash_B(t) + Investment_B(t) + 2.5 \quad (13)$$

2.4 Country C – Rich in Intellectual Properties/Innovation

The asset and liability profiles of Country C can be described as follows:

$$Asset_C(t) = Cash_C(t) + Commodity_C(t) + Investment_C(t) \quad (14)$$

$$Liability_C(t) = Equity_C(t) + Debt_C(t) + NetAsset_C(t) \quad (15)$$

Notice that $NetAsset_C$ appears on the liability column as it represents a claim on Country C's wealth by its citizens. This is expressed in standard net asset value representation used in asset management, in that we are only counting *tangible* accounting items.

The following initial conditions and temporal change relationships are constructed so that Country C will issue 100 units each of Equity and Debt to be held by all countries in the global economy:

$$Equity_C(0) = 100$$

$$Debt_C(0) = 100$$

Initial Conditions:

$$Asset_C(0) = 100$$

$$Cash_C(0) = 25$$

$$Commodity_C(0) = 25$$

$$Investment_C(0) = 50$$

$$PPI_C(0) = \frac{25 + 50}{25} = 3$$

Based on these initial conditions, Country C will issue more debts and equities than it has *tangible* assets. Therefore, Country C will start out in a negative net asset position:

$$NetAsset_C(t) = Cash_C(t) + Commodity_C(t) + Investment_C(t) - Equity_C(t) - Debt_C(t)$$

In real-world accounting, it is not uncommon for entities such as pre-revenue high-tech companies to issue more debt and equities than it has *tangible* assets, but one can expect intangible accounting items such as goodwill or book value of intellectual property to be recorded on the asset column of its balance sheet. In this case, those items will be counted as part of $NetAsset_C(t)$.

Temporal Changes:

$$NetExport_C(t) = -\big(NetExport_A(t) + NetExport_B(t)\big) = -2.5$$

In other words, the sum of net exports from all countries must be zero since global net imports and exports should balance.

$$Commodity_C(t) - Commodity_C(t+1) = -2.5$$

Negative sign represents the annual purchase of commodity as a result of the consumption of commodities. Since the asset equation must balance, net purchases in commodity will translate into payments in cash and investment.

$$Asset_C(t+1) = Asset_C(t) + NetExport_C(t)$$

The relationship above assumes a single-period delay for the payment of account payables.

Moreover, a closed global economy must observe the following additional temporal change relationships:

$$Equity_C(t) = Investment_A(t) + Investment_B(t) + Investment_C(t) \quad (16)$$

Equation (16) assumes that country C creates the world's only equity issues available.

$$Debt_C(t) = Cash_A(t) + Cash_B(t) + Cash_C(t) \quad (17)$$

Equation (17) assumes that country C creates the world's currency and cash equivalent i.e. the world's trading currency is in fact short-term obligations of Country C.

Net Asset Value:

$$Cash_C(t+1) + Commodity_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) =$$
$$Cash_C(t) + Commodity_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) + NetExport_C(t) \quad (18)$$

Equation (18) assumes cash and investments do not change valuation and their proportional allocation remains static. This assumption will be revisited in Section 4. From Equation (18), we have:

$$Cash_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) = Cash_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) + Commodity_C(t) - Commodity_C(t+1) + NetExport_C(t) \quad (19)$$

Simplifying based on the temporal change relationships as stated above,

$$Cash_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) = Cash_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) - 5 \quad (20)$$

2.5 Analysis in the Absence of Policy Actions

We will analyze the implications from this representation of the global economy.

Country C needs to issue debts and equities in order to finance its imports and consumptions. Absent of any structural changes in this global economy, its wealth will be drained over time to pay for excessive imports and overconsumption. Tangible wealth will be transferred gradually from Country C to Countries A and B, producers of raw materials and manufactured goods, respectively.

We will also assume for now that, barring seasonal and expected cyclical fluctuations, global production is roughly equal to global consumption. If Country C consistently imports excessively and/or overconsumes, this whole system can be sustained only if the economic values created by Country C (in terms of new equity/debt issuance and/or increase in valuation – these aspects will be revisited in Section 4) will be sufficient to pay for its imports and borrowing. In other words, Country C can pay for its imports either by borrowing or by making highly successful investments. There is nothing inherently wrong with such an approach as long as Country C can demonstrate consistent and exceptional investment skills.

When one day i. Country C may run up so much debt that the rest of the world loses confidence that it can realistically create sufficient economic values to repay its debts, or ii. Country C is raising new debt primarily to roll over existing debt and to pay for consumption, or iii. there is a gradual shifting of high value-added expertise in innovation and investment to Countries A and B, Country C may have a difficult time issuing equities and debts *ad infinitum*. From that point in time, this global economic structure will become unsustainable.

Alternatively, debt financing can also be seen as a means to create leverage on equity. Debt can be used by Country C to finance its economic value creation process, so it will get to retain a larger share of the total equities that it has issued. Again, this strategy will work only if the economic

value creation process by Country C can outpace the servicing burden on the total amount of debts it raised.

Furthermore, we assume that country A can enjoy more purchasing power initially because of its relative abundance (i.e. it owns the largest share of natural resources available globally), while the model assumes that Countries B and C have equal purchasing power at the outset. (As mentioned earlier, domestic prices of Commodities can be different from country to county even though the same Commodity can be priced in different countries under a single global trading currency.) The natural consequence is that Country B will experience asset inflation over time (i.e. cash + investment will outgrow its resource base, given that purchasing power is measured by $PPI(t) = \frac{Cash+Investment}{Commodity}$) while Country C will experience deflationary pressure due to the gradual downsizing of its net assets – *unless* it is able to create sufficient economic values that the increase in the values of its share of equity held can grow faster than the total amount of debts issued. This representation is in fact consistent with real-world observations.

Another possible unsustainable scenario: What if any potential increase in equity value is not a direct reflection of Country C's ability to innovate, but primarily driven by the demands of SWFs at Countries A and B due to the lack of other sound investment alternatives? At some point, Country C will be unable to justify its equity valuation, and the refusal by the SWFs of Countries A and B to support unreasonable equity valuation may trigger a downward spiral for Country C equities. Since it is *not* in the interest of Countries A and B to force Country C into an abrupt downward spiral, such a mechanism may create a scenario in which Countries A and B are forced to support Country C's continued issuance despite its inability to justify its high equity valuation.

Looking at the situation primarily from the perspective of Country C policymakers, this global economy is not sustainable (by Country C eventually defaulting or being forced into a painful debt restructuring) if any of the following boundary scenarios are reached:

- Scenario 1 – Country C issues more debts/equities (claims by other countries) than it is creating economic values, or the amount of debt becomes so excessive that the market perceives that Country C is raising debt primarily to roll over existing debts and to sustain consumption instead of creating leverage in its capital structure.
- Scenario 2 – SWFs in Countries A and B are investing simply to "buy up" the market, not to pick sound investments and reward companies that create values. (This model assumes that only the Investment portion of a country's surplus wealth will be actively managed by a SWF.) In other words, indexing and indexing *only* by SWFs may create a long-term problem in that such an investment process falls short of incentivizing Country C companies to create economic values by innovation. Consider the case in which Country C only issues equities, while Countries A and B buy only those equities but no debts. The typical company in Country C will be incentivized to pursue projects that produce stable returns instead of any high-risk, high-return projects (that are more typical among innovation-driven economic value creation). Overtime, Country C equities will generate a stable but hardly spectacular dividend yields similar to those of preferred stocks or convertible bonds. There will be minimal practical difference in the outcome of this system as compared to another one supported purely by debt. In the first case, Country C will eventually run out of valuation and revenues to issue new equity (because the valuation/revenue base in any economy that does not emphasize innovation will be relatively low and stable) so Country C will be left with limited means to finance its imports; in the latter case, Country C may be forced into default once it has issued so much debts that servicing the debt load becomes unsustainable, so countries A and B will simply refuse to buy additional new debts from Country C in the absence of a debt restructuring plan.
- Scenario 3 – Country C issues mostly equity; accordingly, Countries A and B are holding primarily equity. Country C is successful in creating economic values by innovation. However, if the SWFs of Countries A and B only focus on making innovation-driven investments, there is

still a significant risk of destabilizing the global economy. The reason is that innovation-driven investments tend to consist of a few "home runs" mixed with plenty of failures. If Country C goes through a drought period with limited innovations, the SWFs of Countries A and B (as public organizations) may face political pressures to refrain from making additional new investments until some of their existing investments begin to yield credible returns. Their refusal to further invest may cut off the source of funding for Country C to pay for its steady imports/consumptions. That may result in a classic mismatch between income and liability. Therefore, it may be unwise for SWFs to hold a high portion of its equity investments in venture capital or private equity to avoid the risk of potentially destabilizing the global financial system, when deficit countries can only rely on the occasion but exceptional yields generated by such investments to pay for its imports. Country C should encourage surplus countries to hold their surpluses in a diversified balance of debt, public equity and private equity/venture capital, in order to better match its refinancing profile, even though SWF investments should aim to incentivize genuine economic value creation processes.

3. Potential Outcomes of Policy Interventions

From Section 2, our model of the global economy is driven by a system of three difference equations:

$$Cash_A(t+1)+Investment_A(t+1) = Cash_A(t) + Investment_A(t) + 2.5 \quad (7)$$

$$Cash_B(t+1)+Investment_B(t+1) = Cash_B(t) + Investment_B(t) + 2.5 \quad (13)$$

$$Cash_C(t+1)+Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) = Cash_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) - 5 \quad (20)$$

Bankruptcy of Country C is defined as $Cash_C(t+1)+Investment_C(t+1) = 0$, a condition in which Country C runs out of any assets to pay its bills. To avoid reaching bankruptcy, country C must do one or both of the following:

1. Issues additional debts so as to increase the amount of cash in hand; or
2. Makes successful investments so that any increase in the valuation and/or dividend yield in its equity holdings will be sufficient to fund its "burn rate"

The first solution (commonly known as "printing money", as well as other clinical descriptions such as quantitative easing) has its limits. Countries A and B are unlikely to buy Country C's bonds indefinitely, when they realized they will only be repaid in a significantly devalued currency (in terms of purchasing power), or when Country C's debt load becomes clearly unsustainable, defined as Country C being perceived by the financial markets as having insufficient incoming cashflows to service its existing debts.

The above suggests that the driver to Country C's economic process should be option 2, but option 1 can be used i. to "smooth" potentially volatile valuation and dividend yield payout patterns, or ii. to better match Country C's income-liability profile, or iii. to provide leverage.

It is unrealistic to expect that the financial market will wait until any country actually reaches bankruptcy. When $Cash_C(t+1)+Investment_C(t+1) < Threshold_C$ (i.e., $Threshold_C$ represents a liquidity level for Country C whereby its debt load is no longer perceived by the financial market as sustainable), the market will drive up the borrowing cost of Country C by so much that it will trigger a crisis of market confidence that will ultimately impact Country C's ability to roll over its existing debts. This section aims to analyze what policy interventions may be carried out to mitigate or even reverse any such crisis of confidence in Country C.

Assuming that this model gives a reasonable description of today's global economy, we want to use the model to better understand the potential outcomes of several widely-discussed options in policy intervention.

3.1 Intervention Policy 1 – Restoration of a Gold Standard

One interpretation of imposing a Gold Standard is to fix Commodity Price Index = Cash / Commodity = K at a constant for all countries. Doing so means Country C makes a policy decision one day to fix its currency to K units of commodities. We will assume for simplicity that K will reflect the then prevailing price of Commodity at the point of conversion at time t_1.

At time $t \geq t_1$, the system of difference equations can be reduced to:

$$K * Commodity_A(t+1)+Investment_A(t+1) = K * Commodity_A(t)+Investment_A(t) + 2.5 \quad (21)$$

$$K * Commodity_B(t+1)+Investment_B(t+1) = K * Commodity_B(t)+Investment_B(t) + 2.5 \quad (22)$$

$$K * Commodity_C(t+1)+Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) = K * Commodity_C(t)+Investment_C(t) - Debt_C(t) - Equity_C(t) - 5 \quad (23)$$

Possible Outcome:

From this point on, any new debts issued by Country C will be effectively repaid in Commodity. Assuming fraud risk is negligible (examples of frauds will include: i. Country C issuing debts with no intention to repay, or ii. Country C makes an accounting misrepresentation that its debt is collateralized by a non-existent/unproven inventory of Commodities) then there is no practical difference between issuing debts and selling Commodities forward.

Now Countries A and B have no reason to buy such debts without Country C pledging its commodity reserves or another form of hard collateral, leaving Country C without any mechanism to borrow simply by "printing money". With these structural constraints on issuing debts, it will be more logical for Country C to raise funds by issuing as much as equity as supported by valuation and revenues linked to any new issuance.

Since equity value will then be measured against a rigid monetary base, any increase in equity value will be tied to actual increase in earnings or productivity (in Commodity terms). Thus, the mandate of the SWFs in Countries A and B become clearer – in any rigidly structured financial markets, there are fewer opportunities for SWFs to profit by placing "macro bets" (beta). Their SWFs will need to focus on picking companies that can create values (alpha or stock selection).

If the crisis is created partly by Country C being too aggressive with printing money, such a crisis will deteriorate into a crisis of market confidence that Country C will not be able to service the debts that it has already issued. Then, a stable monetary base may restore market confidence or at least do so temporarily, as long as the crisis scenario is a *liquidity* event, defined as one in which Country C will be able to service its debts once market confidence and "normal" level of liquidity return to the market, usually after Country C has shown concrete commitments to pursue the necessary fiscal and budgetary reforms.

However, fixing the size of the monetary base will also make it challenging for Country C to raise any additional funds that may be critical to restoring its economic health. If the issue is that this is a *solvency* event (i.e. without new funds, Country C will be driven into bankruptcy), then this is a blunt solution in which one possible undesirable outcome may be the total collapse of Country C's economic activities, since it will have no other way to pay for key imports such as fuel on which the daily functioning of its economy may depend.

Stabilizing the monetary base also makes it unlikely for Countries A and B to accumulate ever-growing public surpluses when Country C simply cannot print IOUs at will. The SWFs of Countries A and B can then focus on managing the legacy of past public surpluses.

3.2 Intervention Policy 2 - "Printing Money" *Ad Infinitum*

This policy is often given more clinical descriptions such as quantitative easing. The strategy allows the Commodity Price Index to increase in time, so that Commodity Price Index = Cash / Commodity = $K(t) \to \infty$ for large t.

This global economic system is still consisted of a similar set of equations, except that K is now denoted as a function of time and is generally increasing for $t \geq t_1$:

$$K(t+1) * Commodity_A(t+1) + Investment_A(t+1) =$$
$$K(t) * Commodity_A(t) + Investment_A(t) + 2.5 \quad (24)$$

$$K(t+1) * Commodity_B(t+1) + Investment_B(t+1) =$$
$$K(t) * Commodity_B(t) + Investment_B(t) + 2.5 \quad (25)$$

$$K(t+1) * Commodity_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) =$$
$$K(t) * Commodity_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) - 5 \quad (26)$$

Country C equity values will drop when dividend discount rates increases (to compensate for the rapid loss of purchasing power by cash). Over time, Commodity will be the one thing that is most likely to hold its value. As in any out-of-control inflationary scenario, the logical response is for Countries A and B to hoard their Commodities and/or delay any exports, so that the goods can fetch as high a price as possible under a rapidly devaluing currency.

The only natural hedge available to protect the values of its overall investment holdings is for the SWFs of Countries A and B to allocate to Commodities. (Country A can be less proactive in allocating to commodities since its revenue source is primarily from commodities: the simple alternative is for Country A to export fewer Commodities.) The net effect will be a simultaneous allocation to Commodities by investors or mass hoarding. Mass hoarding is known to result in distortions in the efficient allocation of economic resources, since Commodity prices will eventually reach a level that hinders productive economic activities. Therefore, it is not obvious how this policy is necessarily helping Country C's economy and the global economy restore their health.

3.3 Intervention Policy 3 – Issuing Alternative Trading Currencies

Instead of simply letting Country C print money *ad infinitum*, Countries A and B can act to put an end their monetary union with Country C. At a specific time t_1, Countries A and B may declare that they will create their own common currency to replace Country C's by fixing their common currency to the then prevailing price $K(t_1)$ of commodities, while $K_C(t_1 + \tau)$, $\tau \geq 1$, will continue to fluctuate as a function of time (likely to increase in time), as in:

$$K(t_1) * Commodity_A(t+1) + Investment_A(t+1) =$$
$$K(t_1) * Commodity_A(t) + Investment_A(t) + 2.5 \quad (27)$$

$$K(t_1) * Commodity_B(t+1) + Investment_B(t+1) =$$
$$K(t_1) * Commodity_B(t) + Investment_B(t) + 2.5 \quad (28)$$

$$K_C(t+1) * Commodity_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) =$$
$$K_C(t) * Commodity_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) - 5 \quad (29)$$

for $t \geq t_1$. If Country C continues to issue more debts, the new debts will be primarily absorbed by Country C's expanding monetary base, so the net result is for Country C to suffer inflation and rapid devaluation of its domestic currency (relative to the new common currency). Country C will therefore be disincentivized to run up unsustainable levels of debts in such a scenario, although it still has some ability to "print money".

What is not yet modeled in this system is that Country C will eventually lose it ability to import under the pressure of a depreciating currency and its exports are expected to become stronger, thus reducing any imbalance of payments within this global economic system over time. For $t \geq t_1$,

$$K(t_1) * Commodity_A(t+1) + Investment_A(t+1) =$$
$$K(t_1) * Commodity_A(t) + Investment_A(t) + BalancePayment_A(t) \quad (30)$$

$$K(t_1) * Commodity_B(t+1) + Investment_B(t+1) =$$
$$K(t_1) * Commodity_B(t) + Investment_B(t) + BalancePayment_B(t) \quad (31)$$

$$K_C(t+1) * Commodity_C(t+1) + Investment_C(t+1) - Debt_C(t+1) - Equity_C(t+1) =$$
$$K(t) * Commodity_C(t) + Investment_C(t) - Debt_C(t) - Equity_C(t) - BalancePayment_A(t) -$$
$$BalancePayment_B(t) \quad (32)$$

Barring any extreme scenario in which Country C will reach bankruptcy before the necessary adjustments can be made, it is now feasible for certain structural adjustments to take effects so that the global economy stands a chance to return to some form of equilibrium. This solution is

potentially more helpful than simply leaving Country C under the structural constraints of a rigid global Gold Standard, which provides an extremely limited policy toolkit to restore balanced economic growth.

Notice that it is in the interest of Countries A and B to help restore Country C's economy to health; otherwise, their Investments (which are priced in Country C's currency) may rapidly depreciate in value. Moreover, this solution does allow Countries A and B to provide bridge financing for any short-term liquidity scenario. After Countries A and B have exited a monetary union with Country C, the risk for them to purchase any significant amount of Country C debts and equity are: i. inducing a runaway currency depreciation and domestic asset inflation in Country C without necessarily restoring its economic health; and ii. Countries A and B will be exchanging their new common currency for a potentially depreciating currency, resulting in a form of global economic contagion.

In short, a structural solution by adjusting the relative prices of goods and services appears to be a pre-condition to reversing on-going and persistent imbalances. By comparison, it is unclear how central banks and public surplus managers necessarily have the (right) monetary and investment tools to solve fiscal and overconsumption challenges. Their intervention is appropriate only when the crisis is a liquidity but not solvency situation; however, it is notoriously difficult to make any such distinction *ex ante*. Only after Country C finds an effective solution to correct its spending habits, the SWFs of Countries A and B can more effectively deal with the legacy of past imbalances and to invest in Country C's economic recovery.

4. Implications to SWF Investing

In this section, we start by looking at how SWFs should allocate in markets with prices and unique assets (instead of exposures to generic asset classes). By now, the reader should understand that the primary mandate of SWFs is to deal with the legacies of past imbalances, not to stage market rescues or use monetary and investment tools to solve fiscal problems.

Given various market and global economic realities faced by the investment activities of SWFs, this Section aims to answer 2 questions:

1. SWFs' sheer size can create liquidity shocks to the market if they lead the market to allocate to a certain asset or asset class. When they are ready to sell such an asset or asset class *en masse*, the likely outcome is that everyone else will follow suit. Thus, their size makes any large, directional macro bets at best difficult to execute in practice; in contrast, smaller macro bets may not make much of a difference to the overall profit-and-loss of a SWF, but one wrong macro bet (regardless of size) may result in immediate outcries of misappropriating public funds. Are there rational circumstances for SWFs to make macro bets?
2. On one hand, SWF can act as responsible long-term holders to stem temporary runs on specific markets; on the other hand, SWFs do not have infinite resources to respond to *every* liquidity crisis. Can any general principle be identified to guide them so that they are intervening to stem market distortions, instead of providing artificial support to unworthy assets?

4.1 SWF Asset Allocation in Typical Markets

Accumulating massive reserves in cash or cash equivalent under a rigid currency regime can create potentially destabilizing effects. One possible outcome is to encourage deficit countries to issue bonds until they can no longer afford to roll over their debts. Also, a sustainable deficit country

should issue a combination of equities and debts with the goal of creating economic values to pay for its imports and to service its debts. As a result, responsible surplus countries should hold their surpluses in a diversified combination of assets instead of cash and cash equivalent only. This practical reality gives rise to the need for SWFs to have an appropriate allocation strategy, as well as one that may be different from those suggested by typical asset allocation models due to their size.

We will illustrate our analysis with the net asset representation of Country B.

Net Asset Representation of Country B:

From Section 2, we have:

$$NetAsset_B(t) = Cash_B(t) + Commodity_B(t) + Investment_B(t) \quad (10)$$

Similarly,

$$NetAsset_B(t+1) = Cash_B(t+1) + Commodity_B(t+1) + Investment_B(t+1) \quad (33)$$

As an enhancement to Equation (11) now that both the valuations and the allocations of Country B's assets may be changed:

$$NetAsset_B(t+1) - NetAsset_B(t) =$$
$$NetExport_B(t) + \Delta Cash_B(t, t+1) + \Delta Commodity_B(t, t+1) + \Delta Investment_B(t, t+1) \quad (34)$$

where

$$\Delta Commodity_B(t, t+1)$$

$$= Price_{Commodity}(t+1) * Wt_{Commodity}(t+1) - Price_{Commodity}(t) * Wt_{Commodity}(t)$$

$$= Price_{Commodity}(t+1) * Wt_{Commodity}(t+1) - Price_{Commodity}(t+1) * Wt_{Commodity}(t) +$$

$$Price_{Commodity}(t+1) * Wt_{Commodity}(t) - Price_{Commodity}(t) * Wt_{Commodity}(t)$$

$$=$$

$$Price_{Commodity}(t+1) * \Delta Wt_{Commodity}(t, t+1) + Return_{Commodity}(t+1, t) *$$

$$Wt_{Commodity}(t) \quad (35)$$

Analogous equations similar to Equation (35) can be derived for Cash and Investment, respectively. Notice that these portfolio weights can be thought of as unit holdings normalized to a specific point in time, which is a standard notation used in asset management. In other words, Equation (34) can be rewritten as:

$$NetAsset_B(t+1) - NetAsset_B(t) = NetExport_B(t) + DeltaAllocation_B(t, t+1) +$$

$$DeltaValuation_B(t, t+1) \quad (36)$$

where

$$DeltaAllocation_B(t, t+1) = Price_{Cash}(t+1) * \Delta Wt_{Cash}(t, t+1) + Price_{Commodity}(t+1) *$$

$$\Delta Wt_{Commodity}(t, t+1) + Price_{Investment}(t+1) * \Delta Wt_{Investment}(t, t+1) \quad (37)$$

and

$$DeltaValuation_B(t, t+1) = Return_{Cash}(t, t+1) * Wt_{Cash}(t) + Return_{Commodity}(t, t+1) *$$

$$Wt_{Commodity}(t) + Return_{Investment}(t, t+1) * Wt_{Investment}(t) \quad (38)$$

Moreover, since the net sum of any changes in weights due to any specific allocation decision at a specific point in time must sum to zero, i.e:

$$\Delta Wt_{Cash}(t, t+1) + \Delta Wt_{Commodity}(t, t+1) + * \Delta Wt_{Investment}(t, t+1) = 0 \quad (39)$$

Accordingly, Equation (37) can be rewritten as:

$$DeltaAllocation_B(t, t+1)$$

$$= Price_{Cash}(t+1) * \left(-\Delta Wt_{Commodity}(t, t+1) - \Delta Wt_{Investment}(t, t+1)\right)$$
$$+ Price_{Commodity}(t+1) * \Delta Wt_{Commodity}(t, t+1) + Price_{Investment}(t+1)$$
$$* \Delta Wt_{Investment}(t, t+1)$$

$$=$$

$$\left(Price_{Commodity}(t+1) - Price_{Cash}(t+1)\right) * \Delta Wt_{Commodity}(t, t+1) +$$
$$\left(Price_{Investment}(t+1) - Price_{Cash}(t+1)\right) * \Delta Wt_{Investment}(t, t+1) \quad (40)$$

Based on the relationships derived above, investment performance is determined by two terms: the change in allocation ($DeltaAllocation_B$) as well as the change in valuation ($DeltaValuation_B$) of each asset.

Allocating to an appreciating asset (relative to cash) is almost always positive. However, because of their size, SWFs can "buy up" specific market segments, which is likely to be accompanied by a move in the opposite direction when they must eventually "cash out". That also means that it is only effective for SWFs to make long-term macro bets based on price changes reflecting fundamental changes in relative market values as a result of economic value creation.

Nothing in the $DeltaValuation_B$ and $DeltaAllocation_B$ terms penalizes a SWF for making a long-term investment as long as such an investment can beat cash returns at the time of exit. As a result, SWFs are in positions to pick companies, industry sectors or market segments that create long-term values or reflect fundamental changes in market values (commonly known as "alpha investing"). Their ability to hold onto their investments for much longer horizons than typical pensions and endowments (given the lack of any explicit liability terms) can provide them with significant advantages in the competition for capital. In contrast, passive indexing (commonly

known as “beta investing”) does not necessarily incentivize economic value creation, and does not automatically help SWFs realize their investment goals over the long-term horizon.

Specifically, because there are other terms such as $NetExport_B$ in the national balance sheet (as well as other yet-to-be-specified terms such as Government Surpluses), professional managers in SWFs should be incentivized to pick not just valued-added investments, but also companies and sectors that complement existing profiles of national outputs.

In view of above, we can name one real-life example of an ineffective SWF investment: SWFs of oil exporters investing in companies active in oil services, which is a surprisingly common practice among oil-exporting countries. We can also name a real-life example of an effective SWF investment: SWF investing in rare-earth metals by analyzing fundamental supply and demand and technology trends.

4.2 SWF Allocation under Market in Distress

As mentioned, the financial markets go into distress when $Cash_C(t)+Investment_C(t) < Threshold_C$. Specifically, when the total returns (both yield and valuation) on cash and investments earned by Country C are smaller than its net outflows, investors are likely to respond by demanding punitive yields on Country C’s new debts issued.

In that case, when will it be appropriate for SWFs to step in?

Case 1:

The investments held by Country C have sound fundamentals, but they are yielding below average returns because of prevailing economic headwinds. Long-term returns are expected to be able to fund Country C’s outflows, so this is primarily a liquidity situation.

In most economic downturns, the second and the third terms in $DeltaValuation_B$ are expected to be negative:

$$DeltaValuation_B(t,t+1) = Return_{Cash}(t,t+1) * Wt_{Cash}(t) + Return_{Commodity}(t,t+1) * Wt_{Commodity}(t) + Return_{Investment}(t,t+1) * Wt_{Investment}(t) \quad (38)$$

In order to intervene, SWFs need to lock in losses in Commodity and Investment and reallocate those assets to Cash in order to fund a rescue attempts. Since $DeltaAllocation_B$ is given by:

$$DeltaAllocation_B(t,t+1) = \left(Price_{Commodity}(t+1) - Price_{Cash}(t+1)\right) * \Delta Wt_{Commodity}(t,t+1) + \left(Price_{Investment}(t+1) - Price_{Cash}(t+1)\right) * \Delta Wt_{Investment}(t,t+1) \quad (40)$$

both $\Delta Wt(t,t+1)$ terms are negative in order to raise cash to fund a rescue. Intervention will be appropriate when the SWF portfolio manager expects that $Price_{Cash}(t+1) \gg Price_{Commodity}(t+1)$ and/or $Price_{Cash}(t+1) \gg Price_{Investment}(t+1)$, and do so by a significant amount. The decision to raise cash under the circumstances suggests that i. it is better to "unload" risky assets and invest in cash instruments if the expectation is that market will further collapse and ii. *without* a rescue, both $Return_{Commodity}(t,t+1)$ and $Return_{Investment}(t,t+1)$ may suffer so badly that on balance it is still in the interest of a SWF to participate in a rescue effort in order to support valuations in the rest of its portfolio.

Similar arguments were used to launch the rescues of certain global banks by syndicates of SWFs.

Case 2:

If Country C's ability to generate cash is still a long way from meeting outflows purely from debt servicing cost, then the situation may deteriorate into a solvency issue. SWFs should demand debt restructuring as discussed in Section 3.3 before making further investments in Country C's debts.

This is similar to arguments used by some SWFs for their refusal to participate in the capital infusion of the European Financial Stability Fund.

4.3 SWF Participation in Rescue Efforts

The reality is that there is no hard and fast rule for any SWF to separate Case 1 from Case 2, just like no central bank has identified a clear signal to distinguish a liquidity situation from a solvency crisis. More often than not, a liquidity crisis may simply deteriorate into a solvency crisis as a self-fulfilling prophecy, when countries in distress face further attacks from speculators. Once borrowing costs rise dramatically, Country C will no longer able to roll over its existing debts and pay its bills. At the same time, the economic collapse of Country C may eventually hurt Countries A and B. So, when should a SWF step in and participate in a rescue effort?

Multi-period analysis from time t to $t+\tau$ gives rise to an inequality condition that appears similar to those found in stochastic control problems. Please note that $Price(t)$ of different assets are stochastic variables in the relationships below:

$$NetAsset_B(t+\tau) - NetAsset_B(t) = \sum_{i=0}^{\tau-1}\big(NetExport_B(t+i) + DeltaAllocation_B(t+i, t+i+1) + DeltaValuation_B(t+i, t+i+1)\big) > 0 \quad (41)$$

Expanding on the $DeltaAllocation_B$ and $DeltaValuation_B$ terms give rise to the following inequality:

$$NetAsset_B(t+\tau) - NetAsset_B(t) = \sum_{i=0}^{\tau-1}\Big\{NetExport_B(t) + \Big(Price_{Commodity}(t+i+1) - Price_{Cash}(t+i+1)\Big) * \Delta Wt_{Commodity}(t+i, t+i+1) + \big(Price_{Investment}(t+i+1) - Price_{Cash}(t+i+1)\big) * \Delta Wt_{Investment}(t+i, t+i+1) + Return_{Cash}(t+i, t+i+1) *$$

$$Wt_{Cash}(t+i) + Return_{Commodity}(t+i, t+i+1) * Wt_{Commodity}(t+i) +$$

$$Return_{Investment}(t+i, t+i+1) * Wt_{Investment}(t+i)\Big\} > 0 \quad (42)$$

Intervention may be desirable when this inequality is satisfied.

Analysis:

SWFs are likely to suffer *initially* as a result of participating in a rescue: They can only sell Commodities and Investments in a rapid declining market in order to fund any rescue attempt.

If there is a successful rescue in a market crisis, SWFs have the ability to benefit from "sell high, buy low" in the $DeltaAllocation_B$ summation term. Once the rescue effort has turned the situation around, SWFs can liquidate its cash and reallocate to Commodities and Investments at market lows. In integral form, $\Delta Wt_{Commodity}(t+i, t+i+1)$ and $\Delta Wt_{Investment}(t+i, t+i+1)$ are negative when $Price_{Commodity}(t+i+1)$ and $Price_{Investment}(t+i+1)$ collapse, but (depending on the specific timing of sound investment decisions) both terms can be positive when prices recover.

Realistically, one should expect that the $DeltaValuation_B$ term may suffer because not all investments will come out intact in most financial crises, so failed investments will still impact the balance sheets of SWFs. In integral form, the terms $Return_{Cash}(t+i, t+i+1) * Wt_{Cash}(t+i) + Return_{Commodity}(t+i, t+i+1) * Wt_{Commodity}(t+i) + Return_{Investment}(t+i, t+i+1) * Wt_{Investment}(t+i)$ may not show strong directions either way – most investments will recover but some may simply experience total or significant losses.

As long as $NetExport_B(t)$ can still come out to be flat or even slightly positive after all is said and done, a SWF has done its job in stabilizing the markets. From a self-interest viewpoint, Country B may not want to force Country C into any disorderly defaults because the $NetExport_B(t)$ term may not recover after a disorderly default, and there is no guarantee that the values of its Investments

will hold up after Country C goes into default. In contrast, Country A may be less concerned because it is endowed with Commodities, as long as Commodity exports are sufficiently inelastic.

Some practical implementation issues will include the following:

- In practice, picking any market "highs" and "lows" in order to maximize the $DeltaAllocation_B$ term can be at best challenging. There can also be a significant loss in $DeltaValuation_B$ during the initial phase of any rescue effort. Naturally, it does not help if the initial losses taken by a SWF to participate in a rescue attempt are so huge that it may destabilize or even topple a government. SWFs must be realistic about the size of the losses that they are prepared to absorb. This is consistent with the steep losses and subsequent rebound experienced by some SWFs after 2008, which is a natural result of SWFs' stabilization role, but nonetheless creates significant political difficulties for the governments concerned.
- Unlike in physics, markets do not obey any fixed laws of nature. In theory, one can model a potential rescue effort by using Bellman's method to solve the inequality condition above in real time to obtain a stochastically optimal solution, but market shocks are at best difficult to predict and calibrate. After all, Bellman's method is not known to be exceptionally good at solving stochastic control problems with large, "uncontrolled" jumps as often observed in real-life markets. In practice, it may be unwise for SWFs to overly rely on complex and opaque mathematical models, instead of using a parsimonious model constructed with straightforward mathematics to guide sound judgment.
- For SWFs to step in, the general pre-condition should be that the situation is a liquidity not solvency crisis. Since the situation is driven by liquidity (and therefore confidence), markets have to be convinced that any rescue attempt will be of a sufficient size that it can mitigate or even reverse any such crisis of market confidence. Feeble attempts to intervene can make a bad situation worse: if the SWFs cannot reverse the crisis, the net effect is for these SWFs to lock in

more losses by raising cash in a collapsing market, thereby using up “dry powders” to launch credible rescue attempts in the future and shaking up market confidence further.

5. Conclusions

The proposed approach of this paper emphasizes on:

- Parsimonious modeling, by focusing on modeling the flows and constraints in the global economy instead of confusing the underlying issues with the "smoke and mirrors" of complex mathematics commonly used by sophisticated asset allocation models;
- Providing a reasonable description that is roughly consistent with the current state of the global economy and a plausible explanation of known investment behavior among SWFs in the recent past; and
- Laying the foundation for a continuous-time model for future research.

This paper aims to illustrate only the general framework used by the proposed approach. Any real-world implementation is expected go beyond SWF investing to incorporate other macro drivers in the global economy, such as open market operations by central banks and government fiscal policies. It may be impractical to address that level of details all in a single paper.

Some of the accessible policy insights gained from this paper have also been shared in a series of Op Ed articles published by *The Straits Times* in Singapore. They include:

1. SWFs are artifacts of global imbalances. Neither returning to a gold standard nor allowing deficit economies to issue debts or print money *ad infinitum* is a constructive solution. Another "shock therapy" is for resource-rich and exporting countries to create their own common currency to force deficit economies to instill fiscal disciplines. These solutions may cause unintended adverse consequences in theory, and they come with potentially unmanageable execution risk.
2. It is not obvious how SWFs, as major concentrations of public wealth, are better equipped than central banks and multilateral monetary agencies to deal with any solvency situations. However,

they can step in to help resolve liquidity crises, before any such situation deteriorates into a self-fulfilling prophecy. SWFs also need to be realistic about the amount of losses that it can absorb, at least during the initial stages of any rescue attempts.

3. SWFs can play a role in stabilizing the markets. From a self-interest point of view, export-driven surplus economies may be ill-advised to force deficit economies into any disorderly defaults, because the net exports terms on their national balance sheets may not recover after any such disorderly defaults, while there is no reason to believe that the values of their investments will necessarily hold up after disorderly defaults. In contrast, resource-rich countries may be less concerned because it is endowed with commodities, as long as those exports are sufficiently inelastic.

Bibliography

Ang, A., Goetzmann, W., and S. Schaefer, 2009, "A Broad Evaluation of Active Management of the Norwegian Government Pension Fund – Global," M2 Presswire

Bernstein, S., Lerner, J., and A. Schoar, 2009, "The Investment Strategies of Sovereign Wealth Funds," Harvard Business School Working Paper

Dyck, A. and A. Morse, 2010, "How and Why Do Sovereign Wealth Funds Tilt Their Portfolios," University of Chicago, Working Paper

Keating, C. and W.F. Shadwick, 2005, Omega: Function and Metrics, Gilmore Drummond

Johan, S., Knill, A. & N. Mauck, 2011, Determinants of Sovereign Wealth Fund Investment in Private Equity, European Financial Management (September)

Lee, B., 2006, "Robust Portfolio Construction in a Sovereign Wealth Context," Sovereign Wealth Management, Central Banking Publications

Lee, B., 2009, "Brief History of Hedge Fund Failures," BNP Paribas Hedge Fund Centre Case Study, Singapore Management University

Lee, B., 2010, "Temasek's Seatown: a fund within a fund," Op Ed, The Business Times, 31 March 2010

Lee, B., Rogal, D., and F. Weinberger, 2010, "Strategic Asset Allocation and Portfolio Construction for Sovereign Wealth Managers," Central Bank Reserves and Sovereign Wealth Management, Macmillan

Lee, B., and H. Hwang, 2011, "Reevaluating the Roles of Large Public Surpluses and Sovereign Wealth Funds in Asia," Asian Development Bank Institute Working Paper

Lee, B., 2011, “How Asia Can Best Use Its Surpluses,” Op Ed, The Straits Times, 31 August 2011

Lee, B., 2011, “The Unfolding of a Greek Tragedy,” Op Ed, The Straits Times, 12 October 2011

Lee, B., 2011, “More Global Imbalances Ahead,” Op Ed, The Straits Times, 11 November 2011

Lee, B., and Y. Lee, 2004, “The Alternative Sharpe Ratio,” Intelligent Hedge Fund Investing, Risk Books

Merton, R.C. 1998, “Optimal Investment Strategies for University Endowment Funds,” Worldwide Asset and Liability Modeling, Cambridge University Press

Sun, T., and H. Hesse, 2009, “Sovereign Wealth Funds and Financial Stability – an Event Study Analysis,” IMF Working Paper