The Theory Of Optimal Delegation With An Application To Tariff Caps*

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Abstract

We consider a general representation of the delegation problem, with and without money burning, and provide sufficient and necessary conditions under which an interval allocation is optimal. We also provide a partial characterization for cases were money burning is optimal. We apply our results to the theory of trade agreements among privately informed governments and establish conditions under which tariff caps are optimal.

1 Introduction

In many important settings, a principal faces an informed but biased agent and contingent transfers between the principal and agent are infeasible. The principal then chooses a permissible set of actions and "delegates" the agent to select any action from this set. The optimal form of delegation reflects an interesting tradeoff. The principal may wish to grant flexibility to the agent in order to utilize the agent's superior information as to the state of nature; however, the principal may also seek to restrict the agent's selection so as to limit the expression of the agent's bias.

The "delegation problem" contrasts with most of the mechanism-design literature, which assumes that contingent transfers are feasible. Contingent transfers may be infeasible, or at least severely restricted, in a number of settings of economic and political interest. For

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example, in a setting in which a regulator selects permissible prices or outputs for a monopolist with private information, legal rules may preclude contingent transfers between a regulator and a regulated firm.[1] Legal rules also limit contingent transfers in a variety of political settings. In other settings, contingent transfers may be discouraged due to social or ethical considerations.

The delegation problem was first defined and analyzed by Holmstrom (1977). He provides conditions for the existence of an optimal solution to the delegation problem. He also characterizes optimal delegation sets in a series of examples, under the restriction that the delegation set takes the form of a single interval.[2] As Holmstrom (1977) argues, interval delegation is commonly observed. It is thus of special importance to understand when interval delegation is an optimal solution to the delegation problem.[3]

In this paper, we consider a general representation of the delegation problem and provide conditions under which interval delegation is an optimal solution to this problem. We posit that the agent's action is taken from an interval in the real line and that the state has a continuous distribution over a bounded interval on the real line. While most of the delegation literature has focused on quadratic preferences, we consider a more general set of preferences. The principal's welfare function is continuous in the action and state and is twice differentiable and strictly concave in the action. The state enters into the agent's welfare function in a multiplicative fashion, as it is standard, and the agent's welfare function is twice differentiable and concave in the action. We assume that the agent's preferred action is interior and strictly increasing in the state. We do not impose any conditions on the direction of the bias of the agent. We also analyze a modified delegation problem with a two-dimensional delegation set, where an action may be permitted only when an associated level of money is burned.[4]

To establish our findings, we utilize and extend the Lagrangian methods developed by Amador, Werning and Angeletos (2006) in their analysis of a consumption-savings model. The Lagrangian method that they propose, however, is not directly applicable to the general setting that we consider. First, in the setting without money burning, our constraint set features a continuum of equality constraints. Second, and more generally, our set up allows

[1] For further discussion, see Alonso and Matouschek (2008) and the references cited therein. The regulation example is further analyzed by Amador and Bagwell (2011).

[2] Holmstrom (1977, p. 44) also establishes for a specific example with quadratic preferences that a single interval is optimal over all compact delegation sets.

[3] There is a large literature that followed Holmstrom's original work. See, for example, Melumad and Shibano (1991), Mylovanov (2008), Martimort and Semenov (2006) and, more recently, Frankel (2010) and Armstrong and Vickers (2010).

[4] For recent work on money burning within a standard delegation setting, see Ambrus and Egorov (2009) and Kovac and Mylovanov (2009).

that the Lagrangian may fail to be concave with respect to the action.[5] The key to our approach is to construct valid Lagrange multipliers such that the Lagrangian is concave in the action when evaluated at those multipliers. We can then check first-order conditions for the maximization of the Lagrangian and thereby identify sufficient conditions for the optimality of interval delegation. Finally, we use simple perturbations to determine necessary conditions.

Our first proposition establishes sufficient conditions for an optimal solution to the delegation problem to take the form of interval delegation, where the sufficient conditions are expressed in terms of the welfare functions and the distribution of the state of nature. We then consider the modified delegation problem in which money burning is allowed. In this problem, the principal selects permissible pairs of actions and money burning, where money burning entails an equal loss to the welfares of the principal and the agent. Our second proposition establishes sufficient conditions for an optimal solution to the delegation problem with money burning to take the form of an interval delegation (with no money burning). When the principal's welfare function is at most as concave as the agent's, we obtain the same sufficient conditions as for the setting without money burning. However, when the principal's welfare function is more concave than the agent's, the sufficient conditions for the optimality of interval delegation become tighter when money burning is feasible.

We next consider necessary conditions for the optimality of interval delegation. If the principal's welfare function is at least as concave as the agent's, then we show for the delegation problem with money burning that our sufficient conditions are also necessary for the optimality of interval delegation. For other circumstances, we consider specific variations that enable us to identify necessary conditions. These variations are enough to identify a family of welfare functions for which the sufficient conditions of our first two propositions are also necessary for the optimality of interval delegation. This preference family includes, as special cases, the preferences commonly used in the literature.[6]

Our techniques can also be used to characterize optimal solutions to delegation problems even when interval delegation is not optimal. To illustrate this, we consider the delegation problem with money burning and provide sufficient conditions for the optimality of delegation sets that feature money burning. The solutions that we propose and verify are motivated

[5]In Amador et al. (2006), concavity of the Lagrangian obtained directly from the structure of the problem, and the constraint set did not feature a continuum of equality constraints. This allowed Amador et al. (2006) to obtain both necessary and sufficient conditions from Lagrangian methods.

[6]There are other papers that have obtained the optimality of interval (or pooling) allocations in different settings. For example, see Athey, Atkeson and Kehoe (2005) in the context of a monetary policy game and Athey, Bagwell and Sanchirico (2004) and McAfee and McMillan (1992) in the context of collusion. Because we do not consider additional expectational constraints or allow for multiple agents, our results do not directly apply to these papers.

by the findings of Ambrus and Egorov (2009), who provide analytical characterizations of solutions with money burning when welfare functions are quadratic and the distribution function is uniform. We utilize our Lagrangian approach to provide sufficient conditions for solutions with money burning in a set up that allows for general welfare and distribution functions.

As noted above, we assume that money burning entails equal losses in the welfares of the agent and principal. This assumption can be motivated in two ways. First, as Ambrus and Egorov (2009) argue, we may associate money burning with bureaucratic expenses that must be incurred by the agent if certain actions are taken.[7] If the agent has an ex ante participation constraint and ex ante (non-contingent) transfers are feasible, then the principal must compensate the agent for the expected expense associated with money burning. The principal's welfare thus falls as well when money burning is used. Second, for some settings, two players may attempt to maximize the expected value of their joint welfare, with the understanding that one of the players will subsequently observe the state and choose an action from the permissible set to maximize his welfare.[8] In this context, the "principal's" welfare corresponds to the players' joint welfare, and the agent's welfare is the welfare of the player who is subsequently informed and chooses an action. Any money burning expense incurred by the agent then also lowers the principal's welfare.

We also establish that important characterizations of optimal delegation in previous work can be captured as special cases of our findings. In particular, Alonso and Matouschek (2008) analyze the optimal delegation problem when money burning is not allowed. They consider a setting with quadratic welfare functions and provide necessary and sufficient conditions for interval delegation to be optimal.[9] Their welfare functions are included in the family of welfare functions for which we provide necessary and sufficient conditions for the optimality of interval delegation. Likewise, we show that the minimum savings result of Amador et al. (2006) can be captured as a special case of our second proposition. Finally, our sufficient conditions for solutions with money burning include the sufficient

[7] In a linear-quadratic set up, allowing for money burning is equivalent to allowing for stochastic allocations. For related work, see Kovac and Mylovanov (2009) and Goltsman, Hörner, Pavlov and Squintani (2009). Within the context of Symmetric Perfect Public Equilibria in a repeated game with privately observed and i.i.d. shocks, money burning can be interpreted as symmetric punishments: the provision of continuation payoffs that lie strictly below the symmetric maximum payoff (see Athey et al., 2004). Athey et al. (2005) provide a related repeated game interpretation in a monetary policy game. In the context of a consumption-savings problem, Amador et al. (2006) interpret money burning as the possibility of selecting a consumption-savings bundle that lies in the interior of the consumer's budget set.

[8] One such setting is the trade-agreement application which we discuss below.

[9] We discuss Alonso and Matouschek's (2008) findings regarding the optimality of interval delegation in greater detail in Section 5.1. Alonso and Matouschek (2008) also characterize the value of delegation, provide associated comparative statics results and obtain a characterization when interval delegation is not optimal.

conditions identified by Ambrus and Egorov (2009) for the special case of quadratic welfare functions and a uniform distribution for the state of nature. To further illustrate the power of our approach, we generalize the findings of Ambrus and Egorov (2009) and identify a broader family of distribution functions under which the essential features of their analytical characterizations are preserved.[10]

Using our findings, we also develop a new application of delegation theory to the theory of trade agreements among governments with privately observed political pressures. We consider a simple two-country model of trade. The importing government sets a tariff, and governments negotiate a trade agreement to maximize their expected joint welfare. A trade agreement defines a set of permissible import tariffs and associated money burning levels, where we may think of money burning in this context as any wasteful bureaucratic procedures that a government must follow in the course of selecting certain tariffs. After the trade agreement is formed and the delegation set is selected, the importing government privately observes the level of political pressure from its import-competing industry and then selects its preferred tariff from the set of permissible tariffs. We can capture this scenario as a delegation problem with money burning, in which the "principal's" objective is to maximize expected joint government welfare, the agent's objective is to maximize the welfare of the importing government for any given level of political pressure, and the state variable is the level of political pressure.

Employing our second proposition, we establish conditions under which an optimal trade agreement does not employ money burning and takes the form of a tariff cap. Our finding thus provides an interpretation of a key design feature of the GATT/WTO trade agreement, whereby governments negotiate "tariff bindings" or "bound tariff levels" rather than precise tariffs. A bound tariff is simply a tariff cap. Our analysis also provides an interpretation of a practice that is sometimes observed, whereby a WTO member government applies a tariff that falls below its negotiated bound level. This phenomenon is called "binding overhang". In our model, a government that is subjected to high political pressure applies a tariff that equals the cap, but a government applies a tariff below the cap when its political pressure is sufficiently low. Our analysis thus indicates conditions under which binding overhang is expected to occur with positive probability in an optimal trade agreement. Finally, we note that our assumption that contingent transfers are unavailable can be motivated in the trade agreement setting, since side-payments do not figure prominently in the rules of the WTO and explicit monetary transfers are rarely used between governments in WTO dispute

[10] In addition to their analytical characterization of optimal delegation in the quadratic-uniform specification of the model, Ambrus and Egorov (2009) identify properties of an optimal delegation contract under more general specifications and examine a model in which contingent transfers are allowed but the amount of transfer from the principal to the agent is bounded from below.

resolutions.

We illustrate our tariff-cap finding in two particular specifications of the trade model. The first specification is a linear-quadratic model of trade.[11] We show that a tariff cap is optimal under this specification if the density function that determines political pressure is non-decreasing, and also under a condition that allows for decreasing densities. The second specification is an endowment model with log utility. We establish related conditions under which a tariff cap is optimal for this specification as well. We also confirm with this specification that our approach can handle non-quadratic preferences.

The paper is organized as follows. The basic model is presented in Section 2. In Section 3, we present sufficient and also necessary conditions for interval delegation to solve the delegation problem without and with money burning, respectively. We analyze two cases in which money burning is optimal in Section 4. In Section 5, we discuss in more detail the relationship between our findings and those of Alonso and Matouschek (2008), Amador et al. (2006) and Ambrus and Egorov (2009). Our tariff-cap application is found in Section 6, and Section 7 concludes. The Appendix contains several proofs. Additional details and proofs are found in an Online Appendix.

2 Basic Set Up

We start from a standard delegation problem: there is a principal and an agent. The principal has a welfare function given by $w(\gamma, \pi)$, while the agent has a welfare given by $\gamma\pi + b(\pi)$. The value of π represents an action or allocation, and the value of γ represents a state or shock that is private information to the agent. We assume that γ has a continuous distribution $F(\gamma)$ with bounded support $\Gamma = [\underline{\gamma}, \overline{\gamma}]$ and with an associated continuous density $f(\gamma) > 0$, while $\pi \in \Pi$, where Π is an interval of the real line. For $\overline{\pi}$ in the extended reals, we define, without loss of generality, $\Pi = [0, \overline{\pi}]$ if $\overline{\pi} < \infty$ or $\Pi = [0, \overline{\pi})$ if $\overline{\pi} = \infty$. For the remainder of the paper, we impose the following conditions on the primitives:

Assumption 1. *The following holds:* (i) $w : \Gamma \times \Pi \to \mathbb{R}$ *is continuous on* $\Gamma \times \Pi$*;* (ii) $w(\gamma, \pi)$ *is twice differentiable and concave in* π*;* (iii) $b : \Pi \to \mathbb{R}$ *is twice differentiable and strictly concave in* π*;* (iv) $\gamma\pi + b(\pi)$ *has a unique interior maximum over* $\pi \in \Pi$ *for all* $\gamma \in \Gamma$*,*

[11] Bagwell and Staiger (2005) consider the linear-quadratic model as well and characterize the optimal tariff cap. They do not, however, establish conditions under which the optimal trade agreement takes the form of a tariff cap. Bagwell (2009) shows that optimal delegation for this model does not take the form of a tariff cap, when political pressures are of two types. Tariff bindings and binding overhang have also received some attention in other modeling frameworks that feature contracting costs, lobbying, and risk preferences. See Horn, Maggi and Staiger (2010), Maggi and Rodríguez-Clare (2007), and Bagwell and Sykes (2004), respectively.

denoted by $\pi_f(\gamma)$ with π_f twice differentiable and $\pi_f'(\gamma) > 0$; (v) *$w_\pi(\gamma, \pi)$ is continuous on $\Gamma \times \Pi$; and* (vi) *if $\bar{\pi} < \infty$ then $b'(\bar{\pi})$ and $w_\pi(\gamma, \bar{\pi})$ are finite for all γ.*

Note that conditions (i)-(v) in Assumption 1 are standard. Condition (vi) could be relaxed, but it simplifies some of the arguments below.

We will consider two problems of interest:

Problem without money burning: In the first case, the goal is simply to choose an allocation as a function of the private information, $\pi : \Gamma \to \Pi$, so as to maximize the principal's welfare function:

$$\max \int_\Gamma w(\gamma, \pi(\gamma)) dF(\gamma) \quad \text{subject to:} \tag{P1}$$
$$\gamma \in \arg\max_{\tilde{\gamma} \in \Gamma} \left\{ \gamma \pi(\tilde{\gamma}) + b(\pi(\tilde{\gamma})) \right\}, \text{ for all } \gamma \in \Gamma$$

where this last constraint arises from γ being private information of the agent.[12]

Problem with money burning: In the second case, we allow for the possibility of *burning money*: the undertaking of an additional action that reduces everyone's utility. The problem is then to choose an action allocation, $\pi : \Gamma \to \Pi$, and an amount of money burned, $t : \Gamma \to \mathbb{R}$, to solve the following problem:

$$\max \int_\Gamma \Big(w(\gamma, \pi(\gamma)) - t(\gamma) \Big) dF(\gamma) \quad \text{subject to:} \tag{P2}$$
$$\gamma \in \arg\max_{\tilde{\gamma} \in \Gamma} \left\{ \gamma \pi(\tilde{\gamma}) + b(\pi(\tilde{\gamma})) - t(\tilde{\gamma}) \right\}, \text{ for all } \gamma \in \Gamma$$
$$t(\gamma) \geq 0\,;\ \forall \gamma$$

Before proceeding to solve problems (P1) and (P2), let us first consider what the optimal allocation would be if the principal were constrained to choose among interval allocations. This is what we do in the next section.

3 Interval Delegation

3.1 Optimality within the class of Interval Allocations

We would like to find the optimal pair (γ_L, γ_H) so that the objective is maximized among all allocations of the form:

[12] All integrals used in the paper are Lebesgue integrals.

$$\pi^\star(\gamma) = \begin{cases} \pi_f(\gamma_L) & ;\gamma \in [\underline{\gamma}, \gamma_L] \\ \pi_f(\gamma) & ;\gamma \in (\gamma_L, \gamma_H) \\ \pi_f(\gamma_H) & ;\gamma \in [\gamma_H, \overline{\gamma}] \end{cases} \tag{1}$$

We refer to such an allocation as an *interval allocation*. We have the following result:

Lemma 1 (Optimal Interval)**.** *The interval $[\gamma_L, \gamma_H]$ with $\gamma_H > \gamma_L$ is optimal within the class of interval allocations only if the following conditions hold:*

(a) *If $\gamma_H = \overline{\gamma}$, then $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \geq 0$,*

(b) *If $\gamma_H < \overline{\gamma}$ then $\int_{\gamma_H}^{\overline{\gamma}} w_\pi(\gamma, \pi_f(\gamma_H))f(\gamma)d\gamma = 0$,*

(c) *If $\gamma_L = \underline{\gamma}$, then $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$,*

(d) *If $\gamma_L > \underline{\gamma}$ then $\int_{\underline{\gamma}}^{\gamma_L} w_\pi(\gamma, \pi_f(\gamma_L))f(\gamma)d\gamma = 0$.*

Proof. In Appendix B. □

To understand the above lemma, note that $w_\pi(\gamma, \pi_f(\gamma))$ indicates the direction of the bias of the agent. For example, a value of $w_\pi(\gamma, \pi_f(\gamma)) > 0$ means that the principal would prefer a higher action than the one most preferred by the agent of type γ. Conditions (a) and (c) imply that if agents are not being pooled at the extremes, it must be because the principal prefers an even more extreme action. Conditions (b) and (d) show that if agents are pooled at the extremes, then the pooling points must be such that the average bias among the pooled agents is zero (i.e. the action is efficient on average).

3.2 Sufficient Conditions without Money Burning

Our goal in this section is to obtain sufficient conditions for a solution to problem (P1) to be an interval allocation.

By writing the incentive constraints in their usual integral form plus a monotonicity restriction, we can rewrite problem (P1) as:[13]

$$\max_{\pi:\Gamma\to\Pi} \int w(\gamma,\pi(\gamma))dF(\gamma) \quad \text{subject to:} \tag{P1'}$$

$$\gamma\pi(\gamma) + b(\pi(\gamma)) = \int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} + \underline{U}, \ \text{ for all } \gamma \in \Gamma \tag{2}$$

$$\pi \text{ non-decreasing} \tag{3}$$

where $\underline{U} \equiv \underline{\gamma}\pi(\underline{\gamma}) + b(\pi(\underline{\gamma}))$.

We follow and extend the Lagrangian approach used by Amador et al. (2006). Differently from that paper, here we have to deal with a (possible) failure of concavity of the Lagrangian (which we discuss below), together with a continuum of equality constraints, constraints (2). Following Amador et al. (2006), we first embed the monotonicity constraint (3) into the choice set of $\pi(\gamma)$. Then, we write constraints (2) as two inequalities:

$$\int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} + \underline{U} - \gamma\pi(\gamma) - b(\pi(\gamma)) \le 0, \ \text{ for all } \gamma \in \Gamma, \tag{4}$$

$$-\int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} - \underline{U} + \gamma\pi(\gamma) + b(\pi(\gamma)) \le 0, \ \text{ for all } \gamma \in \Gamma. \tag{5}$$

The problem is then to choose a function $\pi \in \Phi$ so as to maximize (P1') subject to (4) and (5) and where the choice set is given by $\Phi \equiv \{\pi|\pi : \Gamma \to \Pi \text{ and } \pi \text{ non-decreasing}\}$.

By assigning cumulative Lagrange multiplier functions Λ_1 and Λ_2 to constraints (4) and (5) respectively, we can write the Lagrangian for the problem:

$$\mathcal{L}(\pi|\Lambda_1,\Lambda_2) \equiv \int_{\Gamma} w(\gamma,\pi(\gamma))dF(\gamma) - \int_{\Gamma}\left(\int_{\underline{\gamma}}^{\gamma} \pi(\gamma')d\gamma' + \underline{U} - \gamma\pi(\gamma) - b(\pi(\gamma))\right) d(\Lambda_1(\gamma) - \Lambda_2(\gamma)) \tag{6}$$

The Lagrange multipliers Λ_1 and Λ_2 are restricted to be non-decreasing functions.

For a sufficiency result, we rely on Theorem 1 stated in Appendix A. Basically, we need to construct non-decreasing Lagrange multipliers that satisfy complementary slackness and are such that an interval allocation maximizes the resulting Lagrangian. As usual, to check whether an allocation maximizes the Lagrangian, first order conditions are particularly useful. However, differently from the case in Amador et al. (2006), the Lagrangian above is not necessarily concave in π, and first order conditions are not in general sufficient for opti-

[13] See Milgrom and Segal (2002).

mality. The key is to note that the sufficiency theorem requires that the optimal allocation maximizes the Lagrangian at some (valid) Lagrangian multipliers. Hence, our objective is to construct Lagrange multipliers Λ_1 and Λ_2 so that the resulting Lagrangian is concave in π when evaluated at those specific multipliers. We can then check the first order conditions of the Lagrangian, which are now sufficient for an allocation to be a maximizer. The proof of the following proposition details the arguments and explicitly constructs such multipliers:

Proposition 1 (Sufficiency)**.** *Define* $\kappa \equiv \min_{\gamma,\pi}\left\{\frac{w_{\pi\pi}(\gamma,\pi)}{b''(\pi)}\right\}$. *Suppose there exists* γ_L ,$\gamma_H \in \Gamma$ *with* $\gamma_L < \gamma_H$ *such that the following holds:*

(c1) $\kappa F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma)$ *is non-decreasing for all* $\gamma \in [\gamma_L, \gamma_H]$,

(c2) *if* $\gamma_H < \overline{\gamma}$,

$$(\gamma - \gamma_H)\kappa \geq \int_{\gamma}^{\overline{\gamma}} w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))\frac{f(\tilde{\gamma})}{1 - F(\gamma)}d\tilde{\gamma} \ , \ \forall \gamma \in [\gamma_H, \overline{\gamma}]$$

with equality at γ_H,

(c2') *if* $\gamma_H = \overline{\gamma}$, $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \geq 0$,

(c3) *if* $\gamma_L > \underline{\gamma}$,

$$(\gamma - \gamma_L)\kappa \leq \int_{\underline{\gamma}}^{\gamma} w_\pi(\tilde{\gamma}, \pi_f(\gamma_L))\frac{f(\tilde{\gamma})}{F(\gamma)}d\tilde{\gamma}, \forall \gamma \in [\underline{\gamma}, \gamma_L]$$

with equality at γ_L,

(c3') *if* $\gamma_L = \underline{\gamma}$, $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$.

Then, the allocation $\pi^\star(\gamma)$*, given by* (1)*, is optimal.*

Proof. Our objective here is to be able to apply the Theorem 1 in Appendix A which is a modified version of the sufficiency Theorem 1 of Section 8.4 in Luenberger (1969, p. 220). Towards that goal, let $\Lambda(\gamma) \equiv \Lambda_1(\gamma) - \Lambda_2(\gamma)$. Integrating by parts the Lagrangian, we get:[14]

[14]Note that $h(\gamma) \equiv \int_{\underline{\gamma}}^{\gamma} \pi(\gamma')d\gamma'$ exists (as π is bounded and measurable by monotonicity) and is absolutely continuous; and $\Lambda(\gamma) \equiv \Lambda_1(\gamma) - \Lambda_2(\gamma)$ is a function of bounded variation, as it is the difference between two non-decreasing and bounded functions. It follows then that $\int_{\underline{\gamma}}^{\overline{\gamma}} h(\gamma)d\Lambda(\gamma)$ exists (it is the Riemman-Stieltjes integral), and integration by parts can be done as follows: $\int_{\underline{\gamma}}^{\overline{\gamma}} h(\gamma)d\Lambda(\gamma) = h(\overline{\gamma})\Lambda(\overline{\gamma}) - h(\underline{\gamma})\Lambda(\underline{\gamma}) - \int_{\underline{\gamma}}^{\overline{\gamma}} \Lambda(\gamma)dh(\gamma)$. Since $h(\gamma)$ is absolutely continuous, we can replace $dh(\gamma)$ with $\pi(\gamma)d\gamma$.

$$\mathcal{L}(\pi|\Lambda) = \int_\Gamma \Big[w(\gamma,\pi(\gamma))f(\gamma) - (\Lambda(\overline{\gamma}) - \Lambda(\gamma))\pi(\gamma)\Big]d\gamma + \int_\Gamma \Big(\gamma\pi(\gamma) + b(\pi(\gamma))\Big)d\Lambda(\gamma) - \underline{U}(\Lambda(\overline{\gamma}) - \Lambda(\underline{\gamma})).$$

A proposed multiplier. Let us propose some non-decreasing multipliers Λ_1 and Λ_2 so that their difference, Λ, satisfies:

$$\Lambda(\gamma) = \begin{cases} 1 + \kappa(1 - F(\gamma)) & \gamma \in [\gamma_H, \overline{\gamma}] \\ 1 - w_\pi(\gamma, \pi_f(\gamma))f(\gamma) & \gamma \in (\gamma_L, \gamma_H) \\ 1 - \kappa F(\gamma) & \gamma \in [\underline{\gamma}, \gamma_L] \end{cases}$$

Note that Λ is well defined even when γ_L and γ_H are not interior. Below we will show that the hypothesis of Proposition 1 guarantees that $\kappa F(\gamma) + \Lambda(\gamma) \equiv R(\gamma)$ is non-decreasing; hence, it follows that $\Lambda(\gamma)$ can be written as the difference of two non-decreasing functions, $R(\gamma) - \kappa F(\gamma)$.[15]

Concavity of the Lagrangian. We now check that the Lagrangian when evaluated at the multipliers is indeed concave. First, we will check that the jumps in Λ at γ_L and γ_H are non-negative. The jumps are:

$$1 - \kappa F(\gamma_L) \leq 1 - w_\pi(\gamma_L, \pi_f(\gamma_L))f(\gamma_L)$$
$$1 - w_\pi(\gamma_H, \pi_f(\gamma_H))f(\gamma_H) \leq 1 + \kappa(1 - F(\gamma_H))$$

To show this, we use conditions (c2), (c2'), (c3) and (c3') as follows.

If $\gamma_L > \underline{\gamma}$, we know that the inequality in (c3) must be satisfied with equality at γ_L. Hence we can sign the derivative at γ_L, and we get that:

$$w_\pi(\gamma_L, \pi_f(\gamma_L))\frac{f(\gamma_L)}{F(\gamma_L)} \leq \kappa,$$

which delivers that the jump at γ_L is non-negative. If $\gamma_L = \underline{\gamma}$, then (c3') directly implies that the jump at $\underline{\gamma}$ is non-negative. A similar argument, using (c2) and (c2'), works to show that the jump at γ_H is non-negative.

[15] For our purposes only the difference between the multipliers matters: we just need to know that there exists some non-decreasing functions whose difference delivers Λ.

Using that $\Lambda(\overline{\gamma}) = \Lambda(\underline{\gamma}) = 1$, we can write the Lagrangian as:

$$\mathcal{L}(\pi|\Lambda) = \int_{\Gamma} \Big[w(\gamma, \pi(\gamma)) - \kappa(\gamma\pi(\gamma) + b(\pi(\gamma))) \Big] f(\gamma) d\gamma - \int_{\Gamma} (1 - \Lambda(\gamma))\pi(\gamma) d\gamma \\ + \int_{\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma))) d(\kappa F(\gamma) + \Lambda(\gamma)). \quad (7)$$

By the definition of κ, we see that $w(\gamma, \pi(\gamma)) - \kappa b(\pi(\gamma))$ is concave in $\pi(\gamma)$; further, condition (c1) and the fact that jumps at γ_H and γ_L are non-negative implies that $\kappa F(\gamma) + \Lambda(\gamma)$ is non-decreasing. Hence, the above Lagrangian is concave at the proposed multiplier.

Maximizing the Lagrangian. We now proceed to show that the proposed allocation $\pi^\star$ maximizes the Lagrangian. For this, we use the sufficiency part of Lemma A.2 in Amador et al. (2006) which concerns the maximization of concave functionals in a convex cone.

Given that $\mathcal{L}$ is a concave functional on π, if $\overline{\pi} = \infty$ then we can appeal to this Lemma directly as Φ is a convex cone. We can then say that if

$$\partial\mathcal{L}(\pi^\star; \pi^\star|\Lambda) = 0, \quad (8)$$

$$\partial\mathcal{L}(\pi^\star; x|\Lambda) \leq 0; \text{ for all } x \in \Phi \quad (9)$$

then $\pi^\star$ maximizes the Lagrangian, where the first order condition is in terms of Gateaux differentials.[16] For the case where $\overline{\pi} < \infty$ a little bit of care if needed, but after invoking Assumption 1, the same result applies.[17]

[16]Given a function $T : \Omega \to Y$, where $\Omega \subset X$ and X and Y are normed spaces, if for $x \in \Omega$ and $h \in X$ the limit

$$\lim_{\alpha \downarrow 0} \frac{1}{\alpha} \left[T(x + \alpha h) - T(x) \right]$$

exists, then it is called the Gateaux differential at x with direction h and is denoted by $\partial T(x; h)$.

[17] In this case, we extend b and w to the entire positive ray of the real line in the following way:

$$\hat{w}(\gamma, \pi) = \begin{cases} w(\gamma, \pi) & ; \text{ for } \pi \in \Pi \\ w(\gamma, \overline{\pi}) + w_\pi(\gamma, \overline{\pi})(\pi - \overline{\pi}) & ; \text{ for } \pi > \overline{\pi} \end{cases}$$

for all $\gamma \in \Gamma$ and $\pi \in [0, \infty)$. And we similarly define $\hat{b}$. These extensions are possible from the boundedness of the derivatives as stated in part (vi) of Assumption 1. Then we let $\hat{\Phi} = \{\pi | \pi : \Gamma \to \mathbb{R}_+ \text{ and } \pi \text{ non-decreasing}\}$. Note that $\hat{\Phi}$ is a convex cone, and both $\hat{b}$ and $\hat{w}$ are continuous, differentiable, and concave. We then define the extended Lagrangian, $\hat{\mathcal{L}}(\pi|\Lambda)$, as in (7) but using $\hat{w}$ and $\hat{b}$ instead of w and b. We can now use Lemma A.2 in Amador et al. (2006), which states that the Lagrangian $\hat{\mathcal{L}}$ is maximized at $\pi^\star$ if $\hat{\mathcal{L}}$ is a concave functional defined in a convex cone $\hat{\Phi}$; $\partial\hat{\mathcal{L}}(\pi^\star; \pi^\star|\Lambda) = 0$; and $\partial\hat{\mathcal{L}}(\pi^\star; x|\Lambda) \leq 0$; for all $x \in \hat{\Phi}$. Now note that if $x \in \hat{\Phi}$, then for all sufficiently small $\alpha > 0$, we have that $\alpha x \in \Phi$ and $\partial\hat{\mathcal{L}}(\pi^\star; x|\Lambda) = \frac{1}{\alpha}\partial\hat{\mathcal{L}}(\pi^\star; \alpha x|\Lambda)$. Hence, it is sufficient to check the above first order conditions for all x in Φ. From Assumption 1 part (iv), it follows that $\pi^\star + \alpha x \in \Phi$ for all small enough $\alpha > 0$, as π_f is interior. Then, by the definition of the Gateaux differential, we have that $\partial\hat{\mathcal{L}}(\pi^\star; x|\Lambda) = \partial\mathcal{L}(\pi^\star; x|\Lambda)$ for all $x \in \Phi$, and conditions (8)-(9) are sufficient for optimality.

For our problem, taking the Gateaux differential in direction $x \in \Phi$ and using that $b'(\pi_f(\gamma)) = -\gamma$, we get that:[18]

$$\partial\mathcal{L}(\pi^\star; x|\Lambda) = \int_\Gamma \left[w_\pi(\gamma, \pi^\star(\gamma))f(\gamma) - (1 - \Lambda(\gamma))\right] x(\gamma)d\gamma \\ + \int_{\underline{\gamma}}^{\gamma_L} (\gamma - \gamma_L)\, x(\gamma)d\Lambda(\gamma) + \int_{\gamma_H}^{\overline{\gamma}} (\gamma - \gamma_H)\, x(\gamma)d\Lambda(\gamma), \quad (10)$$

which can be rewritten as:

$$\partial\mathcal{L}(\pi^\star; x|\Lambda) = \int_{\underline{\gamma}}^{\gamma_L} \left[w_\pi(\gamma, \pi_f(\gamma_L))f(\gamma) - \kappa F(\gamma) - \kappa(\gamma - \gamma_L)f(\gamma)\right] x(\gamma)d\gamma \\ + \int_{\gamma_H}^{\overline{\gamma}} \left[w_\pi(\gamma, \pi_f(\gamma_H))f(\gamma) + \kappa(1 - F(\gamma)) - \kappa(\gamma - \gamma_H)f(\gamma)\right] x(\gamma)d\gamma.$$

Integrating by parts, we have:[19]

$$\begin{aligned}\partial\mathcal{L}(\pi^\star; x|\Lambda) = &\left[\int_{\underline{\gamma}}^{\gamma_L} \left[w_\pi(\gamma, \pi_f(\gamma_L))f(\gamma) - \kappa F(\gamma) - \kappa(\gamma - \gamma_L)f(\gamma)\right] d\gamma\right] x(\gamma_L) \\ &- \int_{\underline{\gamma}}^{\gamma_L} \left[\int_{\underline{\gamma}}^{\gamma} \left[w_\pi(\tilde{\gamma}, \pi_f(\gamma_L))f(\tilde{\gamma}) - \kappa F(\tilde{\gamma}) - \kappa(\tilde{\gamma} - \gamma_L)f(\tilde{\gamma})\right] d\tilde{\gamma}\right] dx(\gamma) \\ &+ \left[\int_{\gamma_H}^{\overline{\gamma}} \left[w_\pi(\gamma, \pi_f(\gamma_H))f(\gamma) + \kappa(1 - F(\gamma)) - \kappa(\gamma - \gamma_H)f(\gamma)\right] d\gamma\right] x(\gamma_H) \\ &+ \int_{\gamma_H}^{\overline{\gamma}} \left[\int_{\gamma}^{\overline{\gamma}} \left[w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))f(\tilde{\gamma}) + \kappa(1 - F(\tilde{\gamma})) - \kappa(\tilde{\gamma} - \gamma_H)f(\tilde{\gamma})\right] d\tilde{\gamma}\right] dx(\gamma)\end{aligned}$$

We require that this differential be non-positive for all non-decreasing x and zero when evaluated at $x = \pi^\star$. Note that for $\gamma \in [\underline{\gamma}, \gamma_L] \cup [\gamma_H, \overline{\gamma}]$, if $x = \pi^\star$, then $dx(\gamma) = 0$. So we

[18] Existence of the Gateaux differential follows from Lemma A.1 of Amador et al. (2006). To be able to use that lemma, first note that the Lagrangian, equation (7), is written as the sum of three terms. The middle one is linear in π, so we can obtain directly the Gateaux differential. The remaining two terms are then integrals with integrands that are concave and satisfy the hypothesis of Lemma A.1. Existence of the integrals in the right hand side of equation (10) follows from Λ being of bounded variation and x being monotone in γ, and thus integrable, together with $w_\pi(\gamma, \pi^\star(\gamma))$ bounded and continuous in γ. The continuity of w_π follows from Assumption 1 and that $\pi^\star$ is continuous. It follows also that w_π is bounded as it is a continuous real function in a compact set.

[19] Integration by parts works, as one of the functions involved in each case is continuous. Existence of the integrals follow from $w_\pi(\gamma, \pi^\star(\gamma))$ being bounded and continuous in γ, as stated in footnote 18.

need that:

$$\int_{\underline{\gamma}}^{\gamma} [w_\pi(\tilde{\gamma}, \pi_f(\gamma_L))f(\tilde{\gamma}) - \kappa F(\tilde{\gamma}) - \kappa(\tilde{\gamma} - \gamma_L)f(\tilde{\gamma})]\, d\tilde{\gamma} \geq 0 \ \forall \gamma \in [\underline{\gamma}, \gamma_L]$$

$$\text{with equality at } \gamma_L$$

$$\int_{\gamma}^{\overline{\gamma}} [w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))f(\tilde{\gamma}) + \kappa(1 - F(\tilde{\gamma})) - \kappa(\tilde{\gamma} - \gamma_H)f(\tilde{\gamma})]\, d\tilde{\gamma} \leq 0 \ \forall \gamma \in [\gamma_H, \overline{\gamma}]$$

$$\text{with equality at } \gamma_H$$

Note that the above equations are implied by:

$$\int_{\underline{\gamma}}^{\gamma} \left[w_\pi(\tilde{\gamma}, \pi_f(\gamma_L)) \frac{f(\tilde{\gamma})}{F(\gamma)} \right] d\tilde{\gamma} \geq \kappa(\gamma - \gamma_L) \ \forall \gamma \in [\underline{\gamma}, \gamma_L] \text{ with equality at } \gamma_L$$

$$\int_{\gamma}^{\overline{\gamma}} \left[w_\pi(\tilde{\gamma}, \pi_f(\gamma_H)) \frac{f(\tilde{\gamma})}{1 - F(\gamma)} \right] d\tilde{\gamma} \leq \kappa(\gamma - \gamma_H), \forall \gamma \in [\gamma_H, \overline{\gamma}] \text{ with equality at } \gamma_H$$

if γ_H or γ_L is interior, respectively. Thus if γ_H or γ_L is interior, then (c2) or (c3) is sufficient for the satisfaction of the respective above equation. If not, that is if $\gamma_L = \underline{\gamma}$ or $\gamma_H = \overline{\gamma}$, then the respective above equation is automatically satisfied.

Hence, using concavity of Lagrangian plus Lemma A.2 in Amador et al. (2006), we have shown that the proposed allocation $\pi^\star$ maximizes the Lagrangian (6) given the multipliers.

Applying Luenberger's Sufficiency Theorem. We now apply Theorem 1 in Appendix A. To apply this theorem, we set (i) $x_0 \equiv \pi^\star$; (ii) $X \equiv \{\pi | \pi : \Gamma \to \Pi\}$; (iii) f to be given by the negative of the objective function, $\int_\Gamma w(\gamma, \pi(\gamma))dF(\gamma)$, as a function of π; (iv) $Z \equiv \{(z_1, z_2) | z_1 : \Gamma \to \mathbb{R} \text{ and } z_2 : \Gamma \to \mathbb{R} \text{ with } z_1, z_2 \text{ integrable }\}$; (v) $\Omega \equiv \Phi$; (vi) $P \equiv \{(z_1, z_2) | (z_1, z_2) \in Z \text{ such that } z_1(\gamma) \geq 0 \text{ and } z_2(\gamma) \geq 0 \text{ for all } \gamma \in \Gamma\}$; (vii) G to be the mapping from Φ to Z given by the left hand sides of inequalities (4) and (5); (viii) the linear operator T is given by:

$$T((z_1, z_2)) \equiv \int_\Gamma z_1(\gamma)d\Lambda_1(\gamma) + \int_\Gamma z_2(\gamma)d\Lambda_2(\gamma)$$

and Λ_1 and Λ_2 being non-decreasing functions implies that $T(z) \geq 0$ for $z \in P$. We have that

$$T(G(x_0)) \ \equiv \int_\Gamma \left(\int_{\underline{\gamma}}^{\gamma} \pi^\star(\gamma')d\gamma' + \underline{U} - \gamma\pi^\star(\gamma) - b(\pi^\star(\gamma)) \right) d(\Lambda_1(\gamma) - \Lambda_2(\gamma)) = 0.$$

where the last equality follows from the fact that there is no money burned. We have found conditions under which the proposed allocation, $\pi^\star$, minimizes $f(x) + T(G(x))$ for $x \in \Omega$. Given that $T(G(x_0)) = 0$, then the conditions of Theorem 1 hold and it follows that $\pi^\star$ solves $\min_{x\in\Omega} f(x)$ subject to $-G(x) \in P$, which is problem ($P1$). □

3.3 Sufficient Conditions with Money Burning

In our previous section, we uncovered sufficient conditions for interval delegation to be optimal when money burning was ruled out by assumption. In this section, we extend these results to find sufficient conditions when money burning is possible. This will naturally imply a tightening of the conditions found before.

Using the integral form for the incentive constraints, Problem (P2) becomes:

$$\max_{\left\{\substack{\pi:\Gamma\to\Pi,\\ t:\Gamma\to\mathbb{R}}\right\}} \int (w(\gamma,\pi(\gamma)) - t(\gamma))dF(\gamma) \quad \text{subject to:}$$

$$\gamma\pi(\gamma) + b(\pi(\gamma)) - t(\gamma) = \int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} + \underline{U}, \text{ for all } \gamma \in \Gamma$$

$$\pi \text{ non-decreasing, and } t(\gamma) \geq 0, \text{ for all } \gamma \in \Gamma$$

where $\underline{U} \equiv \underline{\gamma}\pi(\underline{\gamma}) + b(\pi(\underline{\gamma})) - t(\underline{\gamma})$.

Solving the integral equation for $t(\gamma)$ and substituting into both the objective and the non-negativity constraint, we get the following equivalent problem:

$$\max_{\left\{\substack{\pi:\Gamma\to\Pi,\\ t(\underline{\gamma})\geq 0}\right\}} \int \Big(v(\gamma,\pi(\gamma))f(\gamma) + (1-F(\gamma))\pi(\gamma)\Big)d\gamma + \underline{U} \quad \text{subject to:} \tag{P2'}$$

$$\gamma\pi(\gamma) + b(\pi(\gamma)) - \int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} - \underline{U} \geq 0; \quad \text{for all } \gamma \in \Gamma \tag{11}$$

$$\pi \text{ non-decreasing} \tag{12}$$

where v is defined such that $v(\gamma,\pi(\gamma)) \equiv w(\gamma,\pi(\gamma)) - b(\pi(\gamma)) - \gamma\pi(\gamma)$.

Note that once we have solved this program for $\pi(\gamma)$ and $t(\underline{\gamma})$, we can recover $t(\gamma)$ via

$$t(\gamma) = \gamma\pi(\gamma) + b(\pi(\gamma)) - \int_{\underline{\gamma}}^{\gamma} \pi(\tilde{\gamma})d\tilde{\gamma} - \underline{U} \tag{13}$$

There are two main differences between problem (P2') and the problem in the previous section, (P1'). First, the objective function in (P2') is not necessarily concave, as we have not imposed any assumptions on the function v. The second difference is that problem (P2') has just one inequality constraint, (11), while problem (P1') had two. However, the sufficiency result will proceed in a similar fashion. We will embed the monotonicity restriction, constraint (12), into the choice set of the problem. Then we will construct a (cumulative) Lagrange multiplier and form a Lagrangian. We will impose conditions that guarantee that the Lagrange multiplier is non-decreasing and the Lagrangian is concave in π. Next, we will use appropriate first order conditions to guarantee that an interval allocation maximizes the Lagrangian within the set of non-decreasing functions. With this in hand, we will once again use Theorem 1 in Appendix A to show that an interval allocation is an optimal solution to problem (P2'). Our main result is summarized in the following proposition:

Proposition 2 (Sufficiency). *Define* $\tilde{\kappa} = \min\left\{\min_{\gamma,\pi}\left\{\frac{w_{\pi\pi}(\gamma,\pi)}{b''(\pi)}\right\}, 1\right\}$. *Suppose there exists* $\gamma_L, \gamma_H \in \Gamma$ *such that the following holds:*

(c1) $\tilde{\kappa}F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma)$ *is non-decreasing for all* $\gamma \in [\gamma_L, \gamma_H]$,

(c2) *if* $\gamma_H < \overline{\gamma}$,

$$(\gamma - \gamma_H)\tilde{\kappa} \geq \int_{\gamma}^{\overline{\gamma}} w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))\frac{f(\tilde{\gamma})}{1 - F(\gamma)}d\tilde{\gamma}\ ,\ \forall \gamma \in [\gamma_H, \overline{\gamma}]$$

with equality at γ_H,

(c2') *if* $\gamma_H = \overline{\gamma}$, $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) \geq 0$,

(c3) *if* $\gamma_L > \underline{\gamma}$,

$$(\gamma - \gamma_L)\tilde{\kappa} \leq \int_{\underline{\gamma}}^{\gamma} w_\pi(\tilde{\gamma}, \pi_f(\gamma_L))\frac{f(\tilde{\gamma})}{F(\gamma)}d\tilde{\gamma}, \forall \gamma \in [\underline{\gamma}, \gamma_L]$$

with equality at γ_L,

(c3') *if* $\gamma_L = \underline{\gamma}$, $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$,

Then, the allocation $(\pi^\star(\gamma), t^\star(\gamma))$, *where* $\pi^\star$ *is given by* (1) *and* $t^\star(\gamma) = 0$ *for all* $\gamma \in \Gamma$, *is optimal when money burning is possible.*

Proof. Let the associated Lagrangian be defined as follows:

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) \equiv \int_{\gamma\in\Gamma}\Big(v(\gamma, \pi(\gamma))f(\gamma) + (1 - F(\gamma))\pi(\gamma)\Big)d\gamma + \underline{U}$$
$$- \int_{\gamma\in\Gamma}\left(\int_{\underline{\gamma}}^{\gamma}\pi(\gamma')d\gamma' + \underline{U} - \gamma\pi(\gamma) - b(\pi(\gamma))\right)d\tilde{\Lambda}(\gamma)$$

where $\tilde{\Lambda}$ is the (cumulative) Lagrange multiplier associated with equation (11). It is required that $\tilde{\Lambda}$ be non-decreasing.

Integrating by parts, and setting $\tilde{\Lambda}(\overline{\gamma}) = 1$ without loss of generality, we get:

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) \equiv \int_{\gamma\in\Gamma} \Big(v(\gamma, \pi(\gamma))f(\gamma) + (\tilde{\Lambda}(\gamma) - F(\gamma))\pi(\gamma)\Big) d\gamma \\ + \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\tilde{\Lambda}(\gamma) + \tilde{\Lambda}(\underline{\gamma})\underline{U} \quad (14)$$

A proposed multiplier. Our proposed multiplier in this case is:

$$\tilde{\Lambda}(\gamma) = \begin{cases} (1-\tilde{\kappa})F(\gamma) + \tilde{\kappa} & ;\ \text{for } \gamma \in [\gamma_H, \overline{\gamma}] \\ F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma) & ;\ \text{for } \gamma \in (\gamma_L, \gamma_H) \\ (1-\tilde{\kappa})F(\gamma) & ;\ \text{for } \gamma \in [\underline{\gamma}, \gamma_L] \end{cases}$$

Monotonicity of the Lagrange multiplier. Now let us show that the Lagrange multiplier is non-decreasing. In the flexibility region, $\gamma \in (\gamma_L, \gamma_H)$, the Lagrange multiplier can be written as $\tilde{\Lambda}(\gamma) = \tilde{\kappa}F(\gamma) - w_\pi(\gamma, \pi_f(\gamma))f(\gamma) + (1-\tilde{\kappa})F(\gamma)$. Under (c1) and the definition of $\tilde{\kappa}$, which ensures $\tilde{\kappa} \leq 1$, this is the sum of two non-decreasing functions and hence is non-decreasing. In the interior of the pooling regions, $\gamma < \gamma_L$ or $\gamma > \gamma_H$, the Lagrange multiplier is also non-decreasing since $\tilde{\kappa} \leq 1$. We now need to check that at the jumps, $\{\gamma_L, \gamma_H\}$, the Lagrange multiplier is non-decreasing. At γ_L we have that the Lagrange multiplier has a jump of size:

$$\tilde{\kappa}F(\gamma_L) - w_\pi(\gamma_L, \pi_f(\gamma_L))f(\gamma_L).$$

We need to consider two cases. If $\gamma_L = \underline{\gamma}$, then the jump is equal to $-w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}))f(\underline{\gamma})$ which is non-negative by (c3'). If $\gamma_L > \underline{\gamma}$, then condition (c3) holds at γ_L with equality. Taking the derivative of that condition, it must be then that:

$$\tilde{\kappa} - w_\pi(\gamma_L, \pi_f(\gamma_L))f(\gamma_L)/F(\gamma_L) \geq 0$$

which implies that the jump in the multiplier is non-negative. A similar argument, using (c2) and (c2'), shows that the Lagrange multiplier has a non-negative jump at γ_H, and so we have shown that the proposed Lagrange multiplier is non-decreasing.

Concavity of the Lagrangian. We first check that the Lagrangian is concave in the allocation at the proposed multiplier. Note that we can write the Lagrangian as:

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) \equiv \int_{\gamma\in\Gamma} \Big\{ \big[w(\gamma, \pi(\gamma)) - \tilde{\kappa}\gamma\pi(\gamma) - \tilde{\kappa}b(\pi(\gamma))\big] f(\gamma) + (\tilde{\Lambda}(\gamma) - F(\gamma))\pi(\gamma) \Big\} d\gamma$$
$$+ \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\Big((\tilde{\kappa} - 1)F(\gamma) + \tilde{\Lambda}(\gamma)\Big)$$

where we use that $\tilde{\Lambda}(\underline{\gamma}) = 0$. By the definition of $\tilde{\kappa}$ we have that $w(\gamma, \pi) - \tilde{\kappa}b(\pi)$ is concave in π. To see this note that the second derivative is: $w_{\pi\pi}(\gamma, \pi) - \tilde{\kappa}b''(\pi) = b''(\pi)(w_{\pi\pi}(\gamma, \pi)/b''(\pi)$ $-\tilde{\kappa})$. The last term in brackets is non-negative given our definition of $\tilde{\kappa}$, and hence the function $w(\gamma, \pi) - \tilde{\kappa}b(\pi)$ is concave in π. Finally, we note that from (c1) and the fact that jumps in the multiplier are non-negative, it follows that $(\tilde{\kappa} - 1)F(\gamma) + \tilde{\Lambda}(\gamma)$ is non-decreasing, which is needed in the second integral to guarantee that the concavity of b is not reversed.

Maximizing the Lagrangian. That the Lagrangian is maximized at the proposed allocation is similar to the argument used in our proof for Proposition 1 for the no-money burning case. To see this, first note that $t(\underline{\gamma})$ does not appear in the Lagrangian, given the proposed Lagrange multiplier. This implies that we can restrict attention to maximizing the Lagrangian over just $\pi(\gamma)$ for $\gamma \in \Gamma$. Now, let $\Lambda(\gamma) = 1 - F(\gamma) + \tilde{\Lambda}(\gamma)$. Then the Lagrangian can be rewritten as:

$$\mathcal{L}(\pi, t(\underline{\gamma})|\Lambda) \equiv \int_{\gamma\in\Gamma} \Big\{ \Big[w(\gamma, \pi(\gamma)) - \tilde{\kappa}\big(\gamma\pi(\gamma) + b(\pi(\gamma))\big)\Big] f(\gamma) - (1 - \Lambda(\gamma))\pi(\gamma) \Big\} d\gamma$$
$$+ \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\Big((\tilde{\kappa}F(\gamma) + \Lambda(\gamma)\Big)$$

which equivalent to the Lagrangian in the proof Proposition 1 with $\tilde{\kappa}$ instead of κ, and where Λ is the Lagrange multiplier as defined in that section. The same argument used there shows that given the conditions of the Proposition, which are written in terms of $\tilde{\kappa}$, the Lagrangian is maximized at an interval allocation.

Applying Luenberger's Sufficiency Theorem. We now apply Theorem 1. To apply this theorem, we set (i) $x_0 \equiv (\pi^\star, 0)$; (ii) f to be given by the negative of the objective function, $f \equiv -\int_\Gamma \big(v(\gamma, \pi(\gamma))f(\gamma) + (1 - F(\gamma))\pi(\gamma)\big)d\gamma - \underline{U}$; (iii) $X \equiv \{\pi, \underline{t}|\underline{t} \in \mathbb{R}_+$ and $\pi : \Gamma \to \Pi\}$; (iv) $Z \equiv \{z|z : \Gamma \to \mathbb{R}$ with z integrable$\}$; (v) $\Omega \equiv \{\pi, \underline{t}|\underline{t} \in \mathbb{R}_+, \pi : \Gamma \to \Pi$; and π non-decreasing$\}$; (vi) $P \equiv \{z|z \in Z$ such that $z(\gamma) \geq 0$ for all $\gamma \in \Gamma\}$; (vii) G to be the mapping from Ω to Z given by the negative of the left hand side of inequality (11);

(viii) $T(z)$ be the linear mapping:

$$T(z) = \int_\Gamma z(\gamma) d\tilde{\Lambda}(\gamma),$$

where $T(z) \geq 0$ for $z \in P$ follows from $\tilde{\Lambda}$ non-decreasing. We have that

$$T(G(x_0)) \equiv \int_\Gamma \left(\int_{\underline{\gamma}}^{\gamma} \pi^\star(\tilde{\gamma}) d\tilde{\gamma} + \underline{U} - \gamma \pi^\star(\gamma) - b(\pi^\star(\gamma)) \right) d\tilde{\Lambda}(\gamma) = 0.$$

which follows from $t(\gamma) = 0$ for all γ. We have found conditions under which the proposed allocation, $x_0 = (\pi^\star, 0)$, minimizes $f(x) + T(G(x))$ for $x \in \Omega$. Given that $T(G(x_0)) = 0$, the conditions of Theorem 1 hold and it follows that $(\pi^\star, 0)$ solves $\min_{x \in \Omega} f(x)$ subject to $-G(x) \in P$ which is Problem (P2). □

3.4 Necessary Conditions and Equivalence Results

When v is concave, which implies that $\tilde{\kappa} = 1$, problem (P2') is a maximization problem with a concave maximand and a convex constraint set. Because of this, we can strengthen the results to show that the sufficient conditions in this case are also necessary:

Proposition 3 (Necessity when $\tilde{\kappa} = 1$). *Suppose that $\tilde{\kappa} = 1$ where $\tilde{\kappa}$ is as defined in Proposition 2. Then, in the case with money burning, the conditions in Proposition 2 are also necessary for an interval allocation to be optimal.*

Proof. In Appendix C. □

For the case without money burning, or when $\tilde{\kappa} < 1$ in the case with money burning, we cannot appeal to the Lagrangian theorem used in the proof of Proposition 3 to show that sufficient conditions are also necessary. In the latter case, the conditions for the theorem fail because of the lack of convexity of the problem. In the first case, this issue gets compounded by the presence of a continuum of equality constraints that violate an interiority requirement. In what follows, we proceed to obtain two sets of necessary conditions from simple perturbations. The first set will be used for the flexibility region, while the second one will apply to the pooling regions. We will show that these necessary conditions will help us find a family of preferences for which the conditions in Propositions 1 and 2 can be shown to be both necessary and sufficient. This class of preferences includes the standard quadratic preferences used in the delegation literature and the preferences studied by Amador et al. (2006).

A Necessary Condition for the Flexibility Region

The next proposition characterizes a necessary condition for the flexibility region. This necessary condition is obtained by removing a vanishing interval of choices within the flexibility region, and checking that the resulting change in welfare is non-positive:

Proposition 4 (Necessity in the Flexible Region)**.** *An interval allocation is optimal only if*

$$\left(\frac{w_{\pi\pi}(\gamma, \pi_f(\gamma))}{b''(\pi_f(\gamma))}\right) f(\gamma) - \frac{d}{d\gamma}\Big[w_\pi(\gamma, \pi_f(\gamma)) f(\gamma)\Big] \geq 0, \tag{15}$$

for all $\gamma \in [\gamma_L, \gamma_H]$.

Proof. In Appendix D. □

We can illustrate Proposition 4 by considering the case in which the agent's bias is strictly positive in that $w_\pi(\gamma, \pi_f(\gamma)) < 0$. Over the region of flexibility, if we undertake a small variation in which the prescribed actions are removed for types γ to $\gamma + \epsilon > \gamma$, then there would be an indifferent type $\gamma(\epsilon)$ such that types between γ and $\gamma(\epsilon)$ select $\pi_f(\gamma)$ and types between $\gamma(\epsilon)$ and $\gamma + \epsilon$ select $\pi_f(\gamma + \epsilon)$. First, notice that this variation induces types between γ and $\gamma(\epsilon)$ to make a less-biased choice while types between $\gamma(\epsilon)$ and $\gamma + \epsilon$ make a more-biased choice. As suggested by the second term in (15), the variation is less likely to offer an improvement if the density and/or the bias is greater for higher types. Second, notice that the variation increases the variance of the allocation relative to the flexible allocation. Since $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma))$, this variance effect is captured by the first term in (15). As suggested by this term, the variation is less likely to offer an improvement when the concavity of the principal's welfare relative to the concavity of the agent's welfare is greater.

Now note that we can write (15) as:

$$\left(\frac{w_{\pi\pi}(\gamma, \pi_f(\gamma))}{b''(\pi_f(\gamma))} - \tilde{\kappa}\right) f(\gamma) + \frac{d}{d\gamma}\Big[\tilde{\kappa} F(\gamma) - w_\pi(\gamma, \pi_f(\gamma)) f(\gamma)\Big] \geq 0, \tag{16}$$

where we know that

$$\tilde{\kappa} \leq \kappa \equiv \min_{\gamma,\pi}\left\{\frac{w_{\pi\pi}(\gamma, \pi)}{b''(\pi)}\right\}.$$

It follows that the first term of (16) is non-negative, and thus the necessary condition is weaker than condition (c1) in Propositions 1 and 2, as expected.

A Necessary Condition for the Pooling Regions

Now we proceed to obtain a necessary condition that will apply at the pooling regions:

Proposition 5 (Necessity in the Pooling Region)**.** *Let* $g(\pi_0|\pi) \equiv \frac{b(\pi)-b(\pi_0)}{\pi_0-\pi}$. *An interval allocation with* $\gamma_H > \gamma_L$ *is optimal only if:*

(a) if $\gamma_H < \overline{\gamma}$*, then:*

$$\int_{g(\pi_0|\pi_f(\gamma_H))}^{\overline{\gamma}} \left(\frac{w(\tilde{\gamma}, \pi_0) - w(\tilde{\gamma}, \pi_f(\gamma_H))}{\pi_0 - \pi_f(\gamma_H)} \right) f(\tilde{\gamma}) d\tilde{\gamma} \leq 0 \tag{17}$$

for all $\pi_0 \in [\pi_f(\gamma_H), \pi_f(\overline{\gamma})]$*, and with equality at* $\pi_0 = \pi_f(\gamma_H)$*,*

(b) if $\gamma_H = \overline{\gamma}$*,* $w_\pi(\overline{\gamma}, \pi^f(\overline{\gamma})) \geq 0$*,*

(c) if $\gamma_L > \underline{\gamma}$*,*

$$\int_{\underline{\gamma}}^{g(\pi_0|\pi_f(\gamma_L))} \left(\frac{w(\tilde{\gamma}, \pi_0) - w(\tilde{\gamma}, \pi_f(\gamma_L))}{\pi_0 - \pi_f(\gamma_L)} \right) f(\tilde{\gamma}) d\tilde{\gamma} \geq 0 \tag{18}$$

for all $\pi_0 \in [\pi_f(\underline{\gamma}), \pi_f(\gamma_L)]$*, and with equality at* $\pi_0 = \pi_f(\gamma_L)$*,*

(d) if $\gamma_L = \underline{\gamma}$*,* $w_\pi(\underline{\gamma}, \pi^f(\underline{\gamma}) \leq 0$*.*

Proof. In Appendix E. □

Proposition 5 is the direct result of considering a perturbation that offers a new choice, π_0, in the pooling region, where type $g(\pi_0|\pi)$ is indifferent between continuing to pool at π (where π in this case can be either $\pi_f(\gamma_H)$ or $\pi_f(\gamma_L)$) and taking the new option, π_0. The conditions in the proposition guarantee that the resulting change in the allocation generated by the perturbation does not increase welfare. As it stands, it is not a particularly helpful proposition, but as we will show in the next section, it will provide what is needed for an important equivalence result.

3.5 An Equivalence Result for a Family of Welfare Functions

Using the necessity results in Propositions 3, 4 and 5, we can show that there is a family of utility functions for which the sufficiency conditions obtained in Propositions 1 and 2 are also necessary for the optimality of an interval allocation:

Proposition 6 (Equivalence)**.** *Let* $w(\gamma, \pi) = A\big[b(\pi) + B(\gamma) + C(\gamma)\pi\big]$*. Then* (a) *in the case without money burning, the conditions of Proposition 1 are also necessary for optimality,* (b) *in the case with money burning, the conditions of Proposition 2 are also necessary for optimality.*

Proof. In Appendix F. □

To see how part (a) of the above result arises, note that if preferences are such that $w_{\pi\pi}(\gamma, \pi)/b''(\pi) = \kappa$ for all γ and π, then the necessary condition (16) coincides with the sufficient condition (c1) of Proposition 1. The above proposition uncovers such a family of preferences. The proof also shows that, for this family of preferences, the necessary conditions for the pooling region also coincide with the respective sufficient conditions in Proposition 1. A similar argument, together with Proposition 3, shows part (b).

3.5.1 Why is $A = 1$ special?

In the case with money burning, the value of $\kappa = 1$ is special as the sufficient conditions are different for values of κ above or below 1. This is in contrast with the case where money burning is not allowed, where this difference does not arise.

To understand the economics behind this, let us focus on the case where preferences take the form stated in Proposition 6, so that the sufficient conditions of Proposition 2 are also necessary. Note that in this case, when $A \leq 1$ the conditions in Propositions 4 and 5 are sufficient for optimality; while when $A > 1$, they no longer are.

As the value of A increases, the cost of burning money to the principal is reduced, and hence money burning becomes a more attractive tool to provide incentives.[20] It is then not surprising that Propositions 4 and 5 are no longer sufficient when A is large, as these propositions were derived from perturbations that do not use money burning. Below we provide an example of a perturbation that uses money burning and that cannot improve upon the interval allocation when $A < 1$ but cannot be ruled out as an improvement when $A > 1$ by just appealing to the necessary conditions in Propositions 5.

For the pooling region, the necessary condition in Proposition 5 is equivalent to:

$$\mathbb{E}[C(\gamma)|\gamma > \gamma_0] - \gamma_0 \leq 0 \tag{19}$$

Let us consider the following deviation. Suppose that we were to move along the indifference curve for some agent that is pooled at the top: $\gamma_0 > \gamma_H$. The indifference curve of agent γ_0 is given by combinations of π and t_a such that $t_a(\pi|\gamma_0) = b(\pi) + \gamma_0\pi - U_0$ where $U_0 = b(\pi_f(\gamma_H)) + \gamma_0\pi_f(\gamma_H)$.

[20] For example, in the limit, as A goes to ∞, it seems reasonable to conjecture that the principal will choose the allocation of $\pi(\gamma)$ that maximizes $w(\gamma, \pi)$, while using money burning to provide the incentives for the agent. The underlying intuition (i.e. when A is bigger, money burning is more attractive) is discussed in the quadratic-uniform model of Ambrus and Egorov (2009).

We can only consider deviations where the money burning constraint is satisfied, that is, where $t_a(\pi|\gamma_0) \geq 0$. This requires that $\pi > \pi_f(\gamma_H)$ must satisfy that:

$$\frac{b(\pi) - b(\pi_f(\gamma_H))}{\pi - \pi_f(\gamma_H)} + \gamma_0 \geq 0 \tag{20}$$

Note that if we were to offer a new point in the allocation, $\{\pi, t_a(\pi|\gamma_0)\}$ where $\pi > \pi_f(\gamma_H)$, all agents with type $\gamma > \gamma_0$ will choose this new allocation, while all other agents will remain at their previous positions. We can now ask whether the principal prefers this perturbation to the original pooling allocation.

To do this, let us consider the indifference curve for the principal that pools all agents above γ_0:

$$t_w(\pi|\gamma_0) \equiv A\big(b(\pi) + \mathbb{E}\big[C(\gamma)\big|\gamma > \gamma_0\big]\pi + \mathbb{E}\big[B(\gamma)\big|\gamma > \gamma_0\big]\big) - W_0$$

where $W_0 = A\big(b(\pi_f(\gamma_H)) + \mathbb{E}\big[C(\gamma)\big|\gamma > \gamma_0\big]\pi_f(\gamma_H) + \mathbb{E}\big[B(\gamma)\big|\gamma > \gamma_0\big]\big)$.

The difference between the indifference curves of the principal and agent γ_0 can be computed as:

$$\begin{aligned} t_w(\pi|\gamma_0) - t_a(\pi|\gamma_0) = \\ = (\pi - \pi_f(\gamma_H))\left[(A-1)\left(\frac{b(\pi) - b(\pi_f(\gamma_H))}{\pi - \pi_f(\gamma_H)} + \gamma_0\right) + A\big(\mathbb{E}\big[C(\gamma)|\gamma > \gamma_0\big] - \gamma_0\big)\right] \end{aligned} \tag{21}$$

In the case that $0 < A < 1$, using that $\pi > \pi_f(\gamma_H)$, inequality (19) and inequality (20), we have that $t_w(\pi|\gamma_0) - t_a(\pi|\gamma_0) \leq 0$. Hence the principal will always find the above deviation sub-optimal: for it to remain indifferent, the perturbation will have to burn less money than it is required by incentive compatibility.

When $A > 1$, we cannot rule out the above perturbation anymore by just appealing to the condition of Proposition 5. A stronger condition is now required for sufficiency, and this is provided by Proposition 2, part (c2). This condition, in this case with $\tilde{\kappa} = 1$, becomes:

$$(\gamma_0 - \gamma_H) \geq A\big(b'(\pi_f(\gamma_H)) + \mathbb{E}\left[C(\gamma)|\gamma > \gamma_0\right]\big),$$

or equivalently,

$$0 \geq (A-1)\big(\gamma_0 + b'(\pi_f(\gamma_H))\big) + A\big(\mathbb{E}\left[C(\gamma)|\gamma > \gamma_0\right] - \gamma_0\big).$$

Using that $b'(\pi_f(\gamma_H)) \geq \frac{b(\pi) - b(\pi_f(\gamma_H))}{\pi - \pi_f(\gamma_H)}$ by concavity of b, it follows that the above inequality, together with (21), implies that $t_w(\pi|\gamma_0) \leq t_a(\pi|\gamma_0)$, and hence, that the deviation is not an improvement over the original.

Note that the type of deviation we have consider here is never an improvement when money burning is not allowed, as that is exactly the way we proved Proposition 5.[21]

4 When Money Burning is Optimal: Two cases

So far, we have studied sufficient and necessary conditions for an interval allocation to be optimal. In this section we show how the techniques we have developed can also be used to characterize situations where an interval allocation is no longer optimal. Motivated by the recent work of Ambrus and Egorov (2009), we conjecture the existence of particular allocations that feature money-burning, and provide sufficient conditions for these allocations to be optimal. Ambrus and Egorov (2009) managed to provide an analytical characterization of optimal allocations that feature strict money burning within a set up with quadratic utility and a uniform distribution for shocks. In this section, we use the Lagrangian approach to generalize their findings for general welfare and distribution functions.

4.1 Money Burning: Case 1

We now look for an allocation with the following properties: from $\underline{\gamma}$ to some γ_x, the allocation provides full flexibility where no money is burned; and from γ_x to $\overline{\gamma}$, the allocation burns money but still provides flexibility (no agents are pooled). The next assumption imposes conditions that turns out to be sufficient for such an allocation to be optimal.

Assumption 2. *The following holds:* (i) *$v(\gamma, \pi)$ is concave in π for all $\gamma \in \Gamma$;* (ii) $v_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}))$ ≤ 0*;* (iii) *there exists a continuous function $x(\gamma)$ and a value for $\gamma_x \in \Gamma$ such that:* (a) $v_\pi(\gamma, x(\gamma))$ $f(\gamma) = F(\gamma) - 1$ *for all $\gamma \geq \gamma_x$;* (b) *$x(\gamma) \leq \pi_f(\gamma)$ for all $\gamma > \gamma_x$ and with equality at γ_x;* (c) *$x(\gamma)$ is non-decreasing for all $\gamma \geq \gamma_x$;* (d) *$F(\gamma) - v_\pi(\gamma, \pi_f(\gamma))f(\gamma)$ is non-decreasing for all $\gamma < \gamma_x$.*

Our proposed allocation in this case is:

$$\pi_x(\gamma) = \begin{cases} \pi_f(\gamma) & ; \text{ for } \gamma \in [\underline{\gamma}, \gamma_x] \\ x(\gamma) & ; \text{ for } \gamma \in (\gamma_x, \overline{\gamma}] \end{cases}$$

[21] To see this here, note that if money burning is ruled out, then condition (20) must hold with equality (no money can be burned in the deviation). Using this in equation (21), it follows that $t_w(\pi|\gamma_0) - t_a(\pi|\gamma_0) \leq 0$, and hence the principal will never prefer the deviation.

with $t_x(\underline{\gamma}) = 0$.

Let us now briefly discuss the conditions in Assumption 2. Assumption 2i guarantees that problem (P2') is a convex problem, and hence the Lagrangian will be concave. Assumption 2ii guarantees that the bias at the bottom of the distribution is positive, and thus, the principal does not want to pool agents at the bottom. Assumption 2iii describes the conditions that point γ_x and the allocation in the money burning region, $x(\gamma)$, must satisfy. Part (d) imposes a condition equivalent to condition (c1) of Proposition 2 for the flexibility region. Part (c) guarantees monotonicity of the proposed allocation. Part (b) ensures that the allocation is continuous and burns non-negative amounts of money for all types above γ_x. Part (a) guarantees that the proposed allocation satisfies the first order conditions of the Lagrangian.[22]

The following is a sufficiency result:

Proposition 7. *The proposed allocation $(\pi_x, t_x(\underline{\gamma}))$ solves Problem (P2') if Assumption 2 holds.*

Proof. Just as in the proof of Proposition 2, let us write the associated Lagrangian to Problem (P2'):

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) \equiv \int_{\gamma\in\Gamma} \Big\{ v(\gamma, \pi(\gamma))f(\gamma) + (\tilde{\Lambda}(\gamma) - F(\gamma))\pi(\gamma) \Big\} d\gamma \\ + \int_{\gamma\in\Gamma} (\gamma\pi(\gamma) + b(\pi(\gamma)))\, d\tilde{\Lambda}(\gamma) + \tilde{\Lambda}(\underline{\gamma})\underline{U} \quad (22)$$

where we used without loss of generality that $\tilde{\Lambda}(\overline{\gamma}) = 1$. Recall that the Lagrange multiplier, $\tilde{\Lambda}$, must be non-decreasing.

A proposed multiplier. Our Lagrange multiplier now takes the following form:[23]

$$\tilde{\Lambda}(\gamma) = \begin{cases} 0 & ;\ \text{for } \gamma = \underline{\gamma} \\ F(\gamma) - v_\pi(\gamma, \pi_f(\gamma))f(\gamma) & ;\ \text{for } \gamma \in (\underline{\gamma}, \gamma_x] \\ 1 & ;\ \text{for } \gamma \in (\gamma_x, \overline{\gamma}]. \end{cases}$$

Monotonicity of the Lagrange multiplier. Using Assumption 2iii parts (a), (d) and the equality condition of part (b), we have that $\tilde{\Lambda}$ is non-decreasing for all $\gamma \in (\underline{\gamma}, \overline{\gamma})$. The

[22]Note that, for $\gamma \geq \gamma_x$, the function $x(\gamma)$ maximizes point-wise the integrand in problem (P2').

[23]Alternatively, we could have set $\tilde{\Lambda}(\underline{\gamma}) = -v_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}))f(\underline{\gamma})$, and the proof would have proceeded as in Proposition 8.

Lagrange multiplier has a jump at $\underline{\gamma}$ which equals $-v_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}))f(\underline{\gamma})$ which is non-negative by Assumption 2ii.

Concavity of the Lagrangian. Given our proposed multiplier, it follows that $\underline{U}$ drops from equation (22). Concavity of Lagrangian in π follows from $\tilde{\Lambda}$ non-decreasing and Assumption 2i which guarantees concavity of v in π.

Maximizing the Lagrangian. Again, as in the proof of Proposition 2, we first note that $t(\underline{\gamma})$ does not appear in the Lagrangian, given the proposed multiplier. We can then restrict attention to maximizing the Lagrangian over $\pi(\gamma)$ for $\gamma \in \Gamma$. Given concavity of the Lagrangian, we can use first order conditions to guarantee that the Lagrangian is maximized at the proposed allocation $\pi_x(\gamma)$ within the set of non-decreasing functions. The conditions as before require that $\partial\mathcal{L}(\pi_x; \pi_x|\tilde{\Lambda}) = 0$ and $\partial\mathcal{L}(\pi_x; y|\tilde{\Lambda}) \leq 0$ for all y non-decreasing.

The Gateaux differential of the Lagrangian in direction y is:[24]

$$\partial\mathcal{L}(\pi_x; y|\tilde{\Lambda}) = \int_{\gamma_x}^{\overline{\gamma}} \Big(v_\pi(\gamma, x(\gamma))f(\gamma) + (1 - F(\gamma))\Big) y(\gamma) d\gamma$$

By Assumption 2iii part (a) it follows then that $\partial\mathcal{L}(\pi_x; y|\tilde{\Lambda}) = 0$ for all y, and thus the first order conditions for π_x to maximize the Lagrangian are satisfied.

Money burning is non-negative. We now check that $t(\gamma) \geq 0$ for all γ. Recall that:

$$t(\gamma) = \gamma\pi_x(\gamma) + b(\pi_x(\gamma)) - \int_{\underline{\gamma}}^{\gamma} \pi_x(\tilde{\gamma})d\tilde{\gamma} - \underline{U}$$

for all γ. Given that $t(\gamma) = 0$ for $\gamma \in [\underline{\gamma}, \gamma_x]$ as the flexible allocation is being offered in that region we have that:

$$t(\gamma) = \begin{cases} 0 & ; \text{ for } \gamma \in [\underline{\gamma}, \gamma_x] \\ \gamma x(\gamma) - \gamma_x x(\gamma_x) + b(x(\gamma)) - b(x(\gamma_x)) - \int_{\gamma_x}^{\gamma} x(\tilde{\gamma})d\tilde{\gamma} & ; \text{ for } \gamma \in (\gamma_x, \overline{\gamma}] \end{cases}$$

Integrating by parts the last integrand we get that:

$$t(\gamma) = b(x(\gamma)) - b(x(\gamma_x)) + \int_{\gamma_x}^{\gamma} \tilde{\gamma} dx(\tilde{\gamma})$$

for $\gamma \geq \gamma_x$. By Assumption 2iii part (c), we know that x is monotone for $\gamma \geq \gamma_x$. By Assumption 2iii part (b), and concavity of b, we know that $b'(x(\gamma)) + \gamma \geq 0$. Then, it follows

[24] Arguments similar to those in the proof of Proposition 1 guarantee existence of the integral.

that

$$t(\gamma) \geq b(x(\gamma)) - b(x(\gamma_x)) - \int_{\gamma_x}^{\gamma} b'(x(\tilde{\gamma}))dx(\tilde{\gamma}) = 0$$

for all $\gamma \geq \gamma_x$.[25]

Complementarity Slackness. Now we check complementarity slackness which requires that:

$$\int_{\Gamma} \left(\int_{\underline{\gamma}}^{\gamma} \pi_x(\gamma')d\gamma' + \underline{U} - \gamma\pi_x(\gamma) - b(\pi_x(\gamma)) \right) d\tilde{\Lambda}(\gamma) = 0$$

which follows from $t(\gamma) = 0$ for $\gamma \in [\underline{\gamma}, \gamma_x]$ and $\tilde{\Lambda}(\gamma) = 1$ for $\gamma \in [\gamma_x, \overline{\gamma}]$.

Applying Luenberger's Sufficiency Theorem. Here we follow the exact mapping used in the proof of Proposition 2 to use Theorem 1 in Appendix A and show that proposed allocation $(\pi_x, 0)$ solves Problem (P2'). □

In the online appendix we show that the conditions of Assumption 2 are generically inconsistent with the conditions for Proposition 2, as expected.

4.2 Money Burning: Case 2

We next look for an allocation with the following properties: from $\underline{\gamma}$ to some γ_y, the allocation provides no flexibility; and from γ_y to $\overline{\gamma}$, the allocation burns money while providing some flexibility. The next assumption imposes conditions that turn out to be sufficient for such an allocation to be optimal.

Assumption 3. *The following holds:* (i) $v(\gamma, \pi)$ *is concave in* π *for all* $\gamma \in \Gamma$*;* (ii) *there exists a continuous function* $x(\gamma)$ *and a value* $\gamma_y \in \Gamma$ *such that* (a) $v_\pi(\gamma, x(\gamma))f(\gamma) = F(\gamma) - 1$*, for all* $\gamma \geq \gamma_y$*;* (b) $x(\gamma) \leq \pi_f(\gamma)$*, for all* $\gamma \geq \gamma_y$*;* (c) $x(\gamma)$ *is non-decreasing for all* $\gamma \geq \gamma_y$*;* (d) *the following holds:*

$$\underline{\gamma} + b'(x(\gamma_y)) + \int_{\underline{\gamma}}^{\gamma} [v_\pi(\tilde{\gamma}, x(\gamma_y))f(\tilde{\gamma}) + 1 - F(\tilde{\gamma})]d\tilde{\gamma} \geq 0,$$

for all $\gamma \in [\underline{\gamma}, \gamma_y]$ *with equality at* γ_y.

[25] The last equality follows from the continuity and the monotonicity of x (see Carter and Brunt, 2000, Theorem 6.2.1 and the subsequent discussion.)

Our proposed allocation in this case is:

$$\pi_y(\gamma) = \begin{cases} x(\gamma_y) & ; \text{ for } \gamma \in [\underline{\gamma}, \gamma_y] \\ x(\gamma) & ; \text{ for } \gamma \in (\gamma_y, \overline{\gamma}] \end{cases}$$

with $t_y(\underline{\gamma}) = 0$.

The next proposition is an alternative sufficiency result for money burning:[26]

Proposition 8. *The proposed allocation $(\pi_y(\gamma), t_y(\underline{\gamma}))$ solves Problem (P2') if Assumption 3 holds.*

Proof. Just as in the proof of Proposition 7, we begin by deriving the Lagrangian:

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) = \int_{\gamma\in\Gamma} \Big[v(\gamma, \pi(\gamma))f(\gamma) + (\tilde{\Lambda}(\gamma) - F(\gamma))\pi(\gamma)\Big] d\gamma \\ + \int_{\gamma\in\Gamma} \big(\gamma\pi(\gamma) + b(\pi(\gamma))\big) d\tilde{\Lambda}(\gamma) + \tilde{\Lambda}(\underline{\gamma})\underline{U}.$$

where we used without loss of generality that $\tilde{\Lambda}(\overline{\gamma}) = 1$. Recall that the Lagrange multiplier, $\tilde{\Lambda}$, must be non-decreasing.

A proposed multiplier and monotonicity. Our proposed Lagrange multiplier function is $\tilde{\Lambda}(\gamma) = 1$ for all $\gamma \in \Gamma$. This function is trivially non-decreasing.[27]

Concavity of the Lagrangian. When this multiplier function is imposed, the second integral disappears, permitting the Lagrangian to be expressed as

$$\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda}) = \int_{\gamma\in\Gamma} \Big[v(\gamma, \pi(\gamma))f(\gamma) + (1 - F(\gamma))\pi(\gamma)\Big] d\gamma + \underline{U}. \tag{23}$$

Given that v is concave under Assumption 3i and that $\underline{U} \equiv \underline{\gamma}\pi(\underline{\gamma}) + b(\pi(\underline{\gamma})) - t(\underline{\gamma})$ is concave (via b) in $\pi(\underline{\gamma})$ and linear in $t(\underline{\gamma})$, we see that $\mathcal{L}(\pi, t(\underline{\gamma})|\tilde{\Lambda})$ is concave in the policy functions.

Maximizing the Lagrangian. Our next step then is to consider the Gateaux differentials of the Lagrangian:

$$\partial\mathcal{L}(\pi, \underline{t}; y, \underline{t}_0|\tilde{\Lambda}) = \int_{\gamma\in\Gamma} \Big[v_\pi(\gamma, \pi(\gamma))f(\gamma) + (1 - F(\gamma))\Big] y(\gamma) d\gamma + \underline{\gamma}y(\underline{\gamma}) + b'(\pi(\underline{\gamma}))y(\underline{\gamma}) - \underline{t}_0$$

[26] In the online appendix we provide a set of sufficient conditions that, although less general, are easier to check than those in Assumption 3. Note also that Assumption 3ii part (d) implies that $\underline{\gamma} + b'(x(\gamma_y)) \geq 0$ and thus $x(\gamma_y) \leq \pi_f(\underline{\gamma})$.

[27] It seems from this multiplier that the non-negativity constraint is not binding for all γ. If this were true, then clearly the solution should make t as large and negative as possible! However, recall that there is still a non-negativity constraint that is been imposed through the choice set, that is, that $t(\underline{\gamma}) \geq 0$.

for any $y : \Gamma \to \Pi$, y non decreasing and $\underline{t}_0 \geq 0$. Evaluating the Gateaux differential at the proposed allocation:

$$\begin{aligned}
\partial \mathcal{L}(\pi_y, 0; \pi_y, 0|\tilde{\Lambda}) &= \int_{\gamma\in\Gamma} \Big[v_\pi(\gamma, \pi_y(\gamma)) f(\gamma) + (1 - F(\gamma)) \Big] \pi_y(\gamma) d\gamma + \underline{\gamma}\pi_y(\underline{\gamma}) + b'(\pi_y(\underline{\gamma}))\pi_y(\underline{\gamma}) \\
&= x(\gamma_y) \left\{ \int_{\underline{\gamma}}^{\gamma_y} \Big[v_\pi(\gamma, x(\gamma_y)) f(\gamma) + (1 - F(\gamma)) \Big] d\gamma + \underline{\gamma} + b'(x(\gamma_y)) \right\} = 0
\end{aligned}$$

where we have used Assumption 3ii part (a) to obtain the second equality. The last equality follows from part (d).

The differential at any other direction is:

$$\begin{aligned}
\partial \mathcal{L}(\pi_y, 0; y, \underline{t}_0|\tilde{\Lambda}) &= \int_{\underline{\gamma}}^{\gamma_y} \Big[v_\pi(\gamma, x(\gamma_y)) f(\gamma) + (1 - F(\gamma)) \Big] y(\gamma) d\gamma + \underline{\gamma} y(\underline{\gamma}) + b'(x(\gamma_y)) y(\underline{\gamma}) - \underline{t}_0 \\
&= y(\gamma_y) \int_{\underline{\gamma}}^{\gamma_y} \Big[v_\pi(\gamma, x(\gamma_y)) f(\gamma) + (1 - F(\gamma)) \Big] d\gamma \\
&\qquad - \int_{\underline{\gamma}}^{\gamma_y} \left\{ \int_{\underline{\gamma}}^{\gamma} \Big[v_\pi(\tilde{\gamma}, x(\gamma_y)) f(\tilde{\gamma}) + (1 - F(\tilde{\gamma})) \Big] d\tilde{\gamma} \right\} dy(\gamma) \\
&\qquad\qquad + \underline{\gamma} y(\underline{\gamma}) + b'(x(\gamma_y)) y(\underline{\gamma}) - \underline{t}_0 \\
&= - \int_{\underline{\gamma}}^{\gamma_y} \left\{ \int_{\underline{\gamma}}^{\gamma} \Big[v_\pi(\tilde{\gamma}, x(\gamma_y)) f(\tilde{\gamma}) + (1 - F(\tilde{\gamma})) \Big] d\tilde{\gamma} \right\} dy(\gamma) \\
&\qquad - \big(y(\gamma_y) - y(\underline{\gamma}) \big) \big(\underline{\gamma} + b'(x(\gamma_y)) \big) - \underline{t}_0 \\
&= - \int_{\underline{\gamma}}^{\gamma_y} \left\{ \int_{\underline{\gamma}}^{\gamma} \Big[v_\pi(\tilde{\gamma}, x(\gamma_y)) f(\tilde{\gamma}) + (1 - F(\tilde{\gamma})) \Big] d\tilde{\gamma} + \underline{\gamma} + b'(x(\gamma_y)) \right\} dy(\gamma) - \underline{t}_0 \leq 0
\end{aligned}$$

where the second equality follows from integration by parts (which can be done, as one of the functions involved is continuous and the other one is monotonic); and the third equality follows from using Assumption 3ii part (d). The fourth equality follows by just noticing that $\underline{\gamma} + b'(x(\gamma_y))$ is a constant, and the last inequality follows from Assumption 3ii part (d) and $\underline{t}_0 \geq 0$. Taken together, the above first order conditions imply that $(\pi_y, 0)$ maximizes the Lagrangian.

Money burning is non-negative. Our next step is to show that the induced value for $t(\gamma)$ is nonnegative. Using (13), we have that

$$t(\gamma) = \begin{cases} 0 & ; \text{ for } \gamma \in [\underline{\gamma}, \gamma_y] \\ \gamma x(\gamma) - \gamma_y x(\gamma_y) + b(x(\gamma)) - b(x(\gamma_y)) - \int_{\gamma_y}^{\gamma} x(\tilde{\gamma}) d\tilde{\gamma} & ; \text{ for } \gamma \in (\gamma_y, \overline{\gamma}] \end{cases}$$

The proof that $t(\gamma)$ is non-negative follows exactly the same steps as in the proof of Proposition 7, where we now use Assumption 3ii part (b) and (c).

Complementarity Slackness. Complementarity slackness follows from $\tilde{\Lambda}(\gamma) = 1$ for all γ.

Applying Luenberger's Sufficiency Theorem. Here we follow the exact mapping used in the proof of Proposition 2 to use Theorem 1 in Appendix A and show that proposed allocation $(\pi_y, 0)$ solves Problem (P2'). □

5 Relation to Previous Literature

We now discuss how our theorems can be used to obtain previous results found in the literature. In particular, we show that our equivalence result delivers as special cases the three main results in the literature so far: the optimality of interval delegation in linear-quadratic delegation models, the optimality of a minimum savings rule for quasi-hyperbolic consumers as studied by Amador et al. (2006), and the optimality of money burning allocations in Ambrus and Egorov (2009) (which we also generalize).

5.1 Relation to Alonso and Matouschek (2008)

Alonso and Matouschek (2008) study the optimal delegation problem in the absence of money burning. We show that our Proposition 6 can be used to derive Alonso and Matouschek (2008)'s characterization of sufficient and necessary conditions for interval delegation to be optimal.

In their main analysis, Alonso and Matouschek (2008) assume that the principal's welfare function is quadratic, and that, for any given state of nature, the agent's welfare function is single-peaked and symmetric around the agent's preferred action.[28] Alonso and Matouschek (2008) solve the following problem:

$$\max_{\pi(\gamma)} \left\{ w(\gamma, \pi) \equiv -(\pi - \pi_P(\gamma))^2/2 \right\} \tag{24}$$

subject to the agent choosing according to any utility function of the form $u(\gamma, \pi) = v_A(\pi - \pi_f(\gamma), \gamma)$, where v_A is single-peaked and symmetric around zero with respect to its first argument and where $\pi_f(\gamma)$ is strictly increasing.

[28] Alonso and Matouschek (2008) also identify sufficient conditions for interval delegation when the principal's preferences take a more general form, while maintaining the symmetry of the agent's preferences. However, the preferences that they allow require the absence of any bias for an intermediate type. This requirement is not met in many applications, including the trade-agreement application that we discuss later on. Our approach permits weaker sufficient conditions, holds for a more general class of preferences, and identifies a family of preferences for which the sufficient conditions are also necessary.

In this set up without money burning, it is without loss of generality to choose a quadratic utility for the agent, $v_A = -(\pi - \pi_f(\gamma))^2/2$.[29] It is also without loss of generality to assume that $\pi_f(\gamma) = \gamma$.[30]

The agent's utility can then be written as:

$$u(\gamma, \pi) \equiv \gamma\pi + b(\pi); \text{ and } b(\pi) \equiv -\pi^2/2$$

where we have removed the part of the utility that is not affected by choices.

This utility specification satisfies the conditions for Proposition 6 part (a) with $A = 1$, $C(\gamma) = \pi_P(\gamma)$ and $B(\gamma) = -(\pi_P(\gamma))^2/2$. Hence, the conditions we provide in Proposition 1 are both sufficient and necessary. We thus obtain Alonso and Matouschek (2008)'s result regarding the optimality of what they called *threshold delegation* as a special case of our results.[31]

5.2 Relation to Amador et al. (2006)

Amador et al. (2006) study the following hyperbolic consumption-savings problem:

$$\max_{u(.),w(.)} \left\{ \int_\Theta (\theta u(\theta) + \beta w(\theta)) dF(\theta) \right\} \quad \text{subject to:}$$

$$\theta \in \arg\max_{\tilde{\theta}\in\Theta} \left\{ \theta u(\tilde{\theta}) + \beta\delta w(\tilde{\theta}) \right\} \tag{25}$$

$$C(u(\theta)) + K(w(\theta)) \leq y; \ \forall \theta \in \theta \tag{26}$$

where C and K are strictly increasing and convex cost functions of the current utility level, u, and of the future utility level, w, respectively. The value of $\beta \in (0, 1)$ represents the standard geometric discount factor, and $\delta \in (0, 1)$ the hyperbolic adjustment. The value of θ is a shock to the marginal utility of current consumption which is private information to the agent. The constraint (25) is the incentive compatibility constraint and the constraint (26) is the budget constraint.

We will map this to into our setting with money burning. To do this, we let $t(\theta) \equiv W(y - C(u(\theta))) - w(\theta)$, where W is defined to be the inverse of K. Then we can write the

[29] Under a single-peaked and symmetric utility specification, the agent with type γ prefers π_0 to π_1 if and only if $|\pi_0 - \pi_f(\gamma)| < |\pi_1 - \pi_f(\gamma)|$. This ranking also holds for the quadratic specification, and hence guarantees that any allocation satisfies incentive compatibility under the original utility specification if and only if it does so under the quadratic one.

[30] Alonso and Matouschek (2008) show that it is without loss of generality to assume $\pi_f(\gamma) = \alpha + \beta\gamma$ for any α and $\beta > 0$. We can then choose $\alpha = 0$ and $\beta = 1$.

[31] See the online appendix for an exact statement of the above.

problem as:

$$\max \left\{ \int_\Theta \left(\frac{\theta}{\beta} u(\theta) + W(y - C(u(\theta))) - t(\theta) \right) dF(\theta) \right\} \quad \text{subject to:}$$

$$\theta \in \arg\max_{\tilde{\theta} \in \Theta} \left\{ \frac{\theta}{\delta\beta} u(\tilde{\theta}) + W(y - C(u(\tilde{\theta}))) - t(\tilde{\theta}) \right\}$$

and $t(\theta) \geq 0$ for all θ.

Using our notation, the above problem is equivalent to problem (P2) with:

$$\gamma \equiv \theta/(\delta\beta)\,;\ \pi \equiv u\,;\ b(\pi) \equiv W(y - C(\pi))\,;\ w(\gamma, \pi) \equiv \delta\gamma\pi + b(\pi)$$

Note that under this mapping, $b(\pi)$ is strictly concave, and $w(\gamma, \pi)$ is also strictly concave in π and satisfies the conditions of Proposition 6 part (b) with $A = 1$. Hence, we can use Proposition 2 to derive necessary and sufficient conditions for the optimality of interval delegation, which delivers the minimum-savings results of Amador et al. (2006).

5.3 Relation to Ambrus and Egorov (2009)

Ambrus and Egorov (2009) analyze a delegation problem with a principal and a privately informed agent. An initial transfer between the principal and the agent is used to satisfy the agent's ex ante participation constraint, but transfers between the principal and agent are otherwise infeasible.[32] A contract specifies incentive compatible actions and money-burning levels for the agent as functions of the agent's private information or type. They explicitly solve for optimal contracts in the *quadratic-uniform model*, in which the principal and agent have quadratic utility functions and the type is distributed uniformly over $[0, 1]$.

The quadratic-uniform model studied by Ambrus and Egorov (2009) can be mapped into our problem (P2) by assuming that

$$w(\gamma, \pi) = -\frac{\alpha + 1}{2} \left(\pi - \gamma - \frac{\beta}{\alpha + 1} \right)^2; \quad u(\gamma, \pi) = \gamma\pi + b(\pi); \quad \text{and} \quad b(\pi) = \beta\pi - \frac{\pi^2}{2}$$

where $\alpha > 0$ and $0 < \beta < 1$.[33] It follows that $\pi_f(\gamma) = \gamma + \beta$. Note that the preferences above satisfy our conditions for the equivalent result in Proposition 6 with $A = 1 + \alpha$, $B(\gamma) = -[\gamma + \beta/(1 + \alpha)]^2/2$, and $C(\gamma) = \gamma - \alpha\beta/(1 + \alpha)$, so the conditions of Proposition 2 are both necessary and sufficient for the optimality of the interval delegation allocation.

[32] They also consider an extended model in which contingent transfers are allowed.

[33] While Ambrus and Egorov highlight cases in which $\beta < 1$, they also discuss the possibility that $\beta \geq 1$. Below, we maintain the assumptions that $\alpha > 0$ and $0 < \beta < 1$.

Note as well that $v_\pi(\underline{\gamma}, \pi_f(\underline{\gamma}) = -\alpha\beta < 0$ and that $v(\gamma, \pi)$ is concave in π. Ambrus and Egorov (2009) also assume that $F(\gamma) = \gamma$ with $\underline{\gamma} = 0$ and $\underline{\gamma} = 1$.

We can now obtain Ambrus and Egorov (2009)'s characterization using our propositions. When $\alpha \leq 1$, we can use Proposition 2 to show that the cap allocation where $\gamma_L = \underline{\gamma}$ and $\gamma_H = 1 - 2\alpha\beta/(1+\alpha)$ is optimal. When $1 < \alpha \leq \frac{1}{\beta}$ we can then use our Proposition 7 to show that the money burning allocation

$$\pi_x(\gamma) = \begin{cases} \pi_f(\gamma) & ; \gamma \leq 1 - \alpha\beta \\ \gamma + (1-\gamma)/\alpha & ; \gamma > 1 - \alpha\beta \end{cases}$$

is optimal as Assumption 2 holds for $x(\gamma) = \gamma + (1-\gamma)/\alpha$ and $\gamma_x = 1 - \alpha\beta$. Finally, when $\alpha > \frac{1}{\beta}$, we can use Proposition 8 to show that the money burning allocation:

$$\pi_y(\gamma) = \begin{cases} \gamma_y + (1-\gamma_y)/\alpha & ; \gamma \leq \gamma_y \\ \gamma + (1-\gamma)/\alpha & ; \gamma \geq \gamma_y \end{cases}$$

where $\gamma_y = \frac{1}{\alpha}\left(\sqrt{1 + \frac{2\alpha}{\alpha-1}(\alpha\beta - 1)} - 1\right) \in (0,1)$ is optimal as Assumption 3 holds for $x(\gamma) = \gamma + (1-\gamma)/\alpha$. And this completes Ambrus and Egorov (2009)'s characterization of the optimal delegation allocation for the quadratic-uniform model.

5.3.1 Generalizing Ambrus and Egorov (2009)

In this subsection we show how we can generalize the Ambrus and Egorov (2009)'s results. The goal here is to illustrate how our propositions can be easily applied to obtain new results. In what follows, we maintain the quadratic preferences of Ambrus and Egorov (2009) but relax their uniform distribution assumption.

When $\alpha \leq 1$, we can use Proposition 2 to show that an interval allocation is optimal. For this we need to find $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that conditions (c1), (c2) and (c3') are satisfied when $\gamma_L = \underline{\gamma}$. Note that condition (c3') is satisfied, since $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) = -\alpha\beta < 0$. The following proposition provides sufficient conditions for (c1) and (c2) to hold:

Corollary 1. *Consider the generalized Ambrus and Egorov (2009) quadratic model with $\alpha \leq 1$. If* (i) $\mathbb{E}[\gamma] - \underline{\gamma} > \frac{\alpha\beta}{\alpha+1}$, *then there exists $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that $\mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = \alpha\beta/(1+\alpha)$. If, in addition,* (ii) *$F(\gamma) + \alpha\beta f(\gamma)$ is non-decreasing for $\gamma \in [\underline{\gamma}, \gamma_H]$, and* (iii) *$(1+\alpha)\mathbb{E}[\tilde{\gamma}|\tilde{\gamma} \geq \gamma] - \gamma \leq \alpha(\gamma_H + \beta)$ for $\gamma \in [\gamma_H, \overline{\gamma}]$, then a cap allocation is optimal.*

Proof. In appendix G. □

In the online appendix we show that part (iii) of the Corollary 1 holds if the function $d(\gamma) \equiv \mathbb{E}[\tilde{\gamma}|\tilde{\gamma} \geq \gamma] - \gamma$ is weakly convex. This convexity obtains for example, if the density function $f(\gamma)$ is non-decreasing and not too convex.[34] Using this, we can show, for example, that in the case of the power distribution, where $F(\gamma) = \gamma^n$ and $0 = \underline{\gamma} < \overline{\gamma} = 1$, a cap allocation is optimal if $n/(n+1) > \alpha\beta/(1+\alpha)$, $n \geq 1$ and $\alpha \leq 1$. The previously discussed example of a uniform distribution is captured as a special case when $n = 1$.

When $1 < \alpha \leq \frac{1}{\beta f(\underline{\gamma})}$, we can use Proposition 7 to obtain conditions under which a money burning allocation is optimal:

Corollary 2. *Consider the generalized Ambrus and Egorov (2009) quadratic model with* $1 < \alpha \leq \frac{1}{\beta f(\underline{\gamma})}$. *Let* $x(\gamma) \equiv \frac{1}{\alpha}\left(\frac{1-F(\gamma)}{f(\gamma)} + \alpha\gamma\right)$. *Then, there exists* $\gamma_x \in [\underline{\gamma}, \overline{\gamma})$ *such that* $x(\gamma_x) = \pi_f(\gamma_x)$. *If* (i) $\frac{1-F(\gamma)}{f(\gamma)} \leq \alpha\beta$ *for* $\gamma \in [\gamma_x, \overline{\gamma}]$, (ii) $\frac{1-F(\gamma)}{f(\gamma)} + \alpha\gamma$ *is non-decreasing for* $\gamma \in [\gamma_x, \overline{\gamma}]$, *and* (iii) $F(\gamma) + \alpha\beta f(\gamma)$ *is non-decreasing for* $\gamma \in [\underline{\gamma}, \gamma_x)$, *then the allocation* π_x*:*

$$\pi_x(\gamma) = \begin{cases} \pi_f(\gamma) & ; \forall \gamma \leq \gamma_x \\ x(\gamma) & ; \forall \gamma > \gamma_x \end{cases}$$

is optimal.

Proof. In Appendix H. $\square$

The conditions in Corollary 2 are satisfied for the uniform distribution, but also include a large family of non-uniform distributions as well.[35] To illustrate, we consider a family of linear densities: $f(\gamma) = a + 2(1-a)\gamma$ where γ is distributed over $[0,1]$ and $a \in (0,2)$. This family includes the uniform density ($a = 1$), increasing densities ($a < 1$) and decreasing densities ($a > 1$). As we show in the online appendix, if $\alpha > 1$ and $1 \geq a\alpha\beta$, then all of the conditions of Corollary 2 are satisfied.

We conclude with the characterization of the last parameter region, $\alpha > \frac{1}{\beta f(\underline{\gamma})}$, where we can also show that an allocation with money burning is optimal:

Corollary 3. *Consider the generalized Ambrus and Egorov (2009) quadratic model with* $\alpha > \frac{1}{\beta f(\underline{\gamma})}$. *If* (i) $\frac{1-F(\gamma)}{f(\gamma)} + \alpha\gamma$ *is strictly increasing for* $\gamma \in \Gamma$, (ii) $\frac{1-F(\gamma)}{f(\gamma)} \leq \frac{1}{f(\underline{\gamma})}$ *for* $\gamma \in \Gamma$,

[34] As we show in the online appendix, the exact result is that d is weakly convex if (i) $f'(\gamma) \geq 0$ for all $\gamma \in \Gamma$, (ii) if there exists $\gamma \in (\underline{\gamma}, \overline{\gamma})$ such that $f'(\gamma) > 0$, then $f'(\overline{\gamma}) > 0$, and (iii) $f''(\gamma) \leq 2f'(\gamma)^2/f(\gamma) + f'(\gamma)f(\gamma)/(1-F(\gamma))$ for all $\gamma \in \Gamma$.

[35] We note, however, that the power distribution considered above fails to satisfy condition (iii) when γ is close to zero.

and (iii) $(1+\alpha)(\overline{\gamma}-\mathbb{E}[\gamma])>\beta$, *then an allocation of the form*

$$\pi_y(\gamma)=\begin{cases} x(\gamma_y) & ;\forall\gamma\leq\gamma_y \\ x(\gamma)=\frac{1}{\alpha}\left(\frac{1-F(\gamma)}{f(\gamma)}+\alpha\gamma\right) & ;\forall\gamma>\gamma_y \end{cases}$$

for some $\gamma_y\in(\underline{\gamma},\overline{\gamma})$ *is optimal.*

Proof. In Appendix I. □

As an illustration of a non-uniform distribution that satisfies the assumptions of Corollary 3, consider the linear density $f(\gamma)=a+2(1-a)\gamma$ where γ is distributed over $[0,1]$ and $a\in[1,2)$. The assumption that $\alpha>1/(\beta f(\underline{\gamma}))$ and assumption (iii) of Corollary 3 hold if $1<a\alpha\beta$ and $a>6\beta/(1+\alpha)-2$, respectively. These inequalities can simultaneously hold if $\alpha\geq 1/2$. As we show in the online appendix, assumptions (i) and (ii) of Corollary 3 hold for this distribution when $a\in[1,2)$.

6 Application to Tariff Caps

We now apply our findings and characterize an optimal trade agreement between governments with private political pressures. We assume that a trade agreement identifies a menu of permissible tariffs and is negotiated before private political pressures are realized. After a government learns its private information, it then applies its preferred tariff from the permissible set. An optimal trade agreement maximizes ex ante joint government welfare subject to incentive compatibility constraints.

Following Bagwell and Staiger (2005), we analyze trade agreements in the context of a simple two-country setting. Our analysis differs from that of Bagwell and Staiger in three main respects. First, Bagwell and Staiger characterize the optimal tariff cap; however, they do not establish conditions under which an optimal trade agreement takes the form of a tariff cap. Second, Bagwell and Staiger analyze a linear-quadratic model. We consider more general settings and capture the linear-quadratic model as a special case. Third, we consider a multi-dimensional policy space, in which a trade agreement can specify tariffs as well as levels of money burning. For example, we might imagine that a trade agreement allows a higher import tariff only if certain wasteful bureaucratic procedures are followed by the importing country.[36]

[36] Our model is also related to work by Feenstra and Lewis (1991). A key difference is that we do not allow monetary transfers between governments.

6.1 Mapping into our modeling framework

In Appendix E of our online appendix, we describe the trade model in detail. In what follows, we present the basic set up and results that allow us to write the trade agreement problem as a delegation problem.

There are two countries, home and foreign, and two goods, x and n, where n is a numeraire. The home country imports good x, and we look for the optimal tariff agreement for this good. We assume that consumers in both countries have a symmetric utility function that is quasi-linear in the numeraire: $u(c^x) + c^n$, where c^x and c^n represent the amounts consumed of goods x and n, respectively. The function u is strictly increasing, strictly concave and thrice continuously differentiable. Letting p denote the home relative price of x to n, with p_* representing the relative price in the foreign country, we assume that there are competitive supply functions of good x in both countries, which we represent as $Q(p)$ and $Q_*(p_*)$, respectively. For prices that elicit strictly positive supply, we assume that these functions are strictly increasing and twice continuously differentiable. We also assume that $Q(p) < Q_*(p)$ for any p such that there is strictly positive world supply.[37]

We abstract from export policies and assume that the home country has available a specific (i.e., per unit) import tariff, τ, for good x. As we describe in further detail in our online appendix, the market-clearing prices in the home and foreign countries are then determined as functions of τ by the requirements that the home country import volume for good x equals the foreign country export volume for this good and that $p = p_* + \tau$.

We may now represent the welfare functions of both governments. Let π denote the producer surplus (profit) at home for good x that is induced by a given tariff. We can write the home government's welfare as

$$\gamma\pi + b(\pi),$$

where $b(\pi)$ is the sum of consumer surplus and tariff revenue in the home country and where $\pi \in [0, \overline{\pi}]$. The shock $\gamma \in \Gamma$ represents a political economy shock that determines the weight that the home government puts on the welfare of its (import-competing) producers. The welfare of the foreign government, $v(\pi)$, is determined as the sum of consumer surplus and producer surplus in the foreign country.

When trade volume is positive, a higher import tariff raises p and lowers p_*, where

[37] As usual, the numeraire is produced in each country under constant returns to scale using labor (the only factor), where the supply of labor is inelastic. The wage and the price of the numeraire may then be set at unity. The numeraire is freely traded across countries so as to ensure that trade is balanced. In our online appendix, we consider a slightly more general and symmetric trade model with three goods. The foreign country then imports a good y from the home country, where the supply assumptions on good y are the mirror image of those stated here for good x. Separability in the utility function, plus quasi-linearity, allows us to study the two good problem independently, as we do here.

the latter effect is the traditional terms-of-trade externality. A higher import tariff is then associated with a higher level of profit in the home country and a lower level of foreign welfare. We assume henceforth that trade volume is positive at tariffs that deliver $\pi \in [0, \overline{\pi})$, from which it follows that $v'(\pi) < 0$ for all $\pi \in [0, \overline{\pi})$. We assume that $b''(\pi) < 0$; however, as we show in our online appendix, if $Q'' \leq 0$, $Q''_* \leq 0$ and $u''' \geq 0$, then $v'' > 0$. We are thus careful below not to exclude the possibility of a strictly convex foreign welfare function.

The home and foreign governments negotiate a trade agreement before the political economy parameter, γ, is realized. Thus, at the time of negotiation, the home government is uncertain about its future preferences. We assume that γ is distributed over the support $\Gamma \equiv [\underline{\gamma}, \bar{\gamma}]$ according to a strictly positive density $f(\gamma)$. We represent the c.d.f as $F(\gamma)$. Once the value of γ is realized, the home government is privately informed of this value.[38]

We may imagine that a trade agreement allows a higher import tariff only if certain wasteful bureaucratic procedures are followed by the importing country. Hence, we model a trade agreement as a pair, $(\pi(\gamma), t(\gamma))$, that determines for each γ the profit allocated to the domestic producers and the level of wasteful activities or money burning. We look for a trade agreement that is incentive compatible and efficient. The optimal trade agreement solves the following problem:

$$\max_{\pi(\gamma), t(\gamma)} \left\{ \int_{\gamma \in \Gamma} \Big(\gamma\pi(\gamma) + b(\pi(\gamma)) + v(\pi(\gamma)) - t(\gamma) \Big) dF(\gamma) \right\} \quad \text{subject to:} \tag{PT}$$

$$\gamma \in \arg\max_{\widetilde{\gamma} \in \Gamma} \left\{ \gamma\pi(\widetilde{\gamma}) + b(\pi(\widetilde{\gamma})) - t(\widetilde{\gamma}) \right\} \tag{27}$$

Once the optimal profit function is determined, we can easily back out the associated tariff function.[39]

We may map problem (PT) into our general framework. Letting $w(\gamma, \pi) = \gamma\pi + b(\pi) + v(\pi)$, we assume that (i) b and v are twice differentiable with $b'' + v'' \leq 0$; (ii) $b'' < 0$; (iii) $\gamma\pi + b(\pi)$ has a unique interior maximum for all $\gamma \in \Gamma$, denoted by $\pi_f(\gamma)$ with $\pi'_f(\gamma) > 0$;

[38] In the three good model that we describe in our online appendix, the negotiation also concerns the foreign import tariff and precedes the realization of the foreign political economy parameter, γ_*, that defines the weight that the foreign government attaches to the profit of its import-competing producers (of good y). When the value of γ_* is realized, the foreign government is privately informed of its value. We assume that γ^* and γ are independently and identically distributed.

[39] The statement of the problem reflects our assumptions that governments do not have available contingent sidepayments (monetary transfers) and that they seek a trade agreement that maximizes the sum of their expected welfares. The solution generates a particular outcome on the efficiency frontier when sidepayments are not allowed. In the three-good model described in our online appendix, an analogous solution applies for good y, where the foreign government has private information about the weight that it attaches to its import-competing industry. If the instrument space is expanded so that governments can make non-contingent sidepayments during their negotiation, and thus before they obtain private information, then all efficient payoffs can be achieved by solving program (PT) and specifying an appropritate ex ante transfer. Grossman and Helpman (1995) make a similar point, in their analysis of "trade talks."

and (iv) if $\bar{\pi} < \infty$, then $v'(\bar{\pi})$ and $b'(\bar{\pi})$ are finite. These assumptions imply Assumption 1 and so problem (PT) is a special case of our problem with money burning, (P2).[40] Likewise, if we eliminated the money burning variable, $t(\gamma)$, from problem (PT), then it would be a special case of our problem without money burning, (P1).

6.2 The Optimal Trade Agreement

In this subsection, we utilize our earlier propositions to establish assumptions under which the optimal tariff cap represents an optimal trade agreement.

From Lemma 1, it follows that $\gamma_L = \underline{\gamma}$, as condition (d) cannot hold for any interior γ_L given that $v' \leq 0$.[41] Intuitively, the home government does not take into account the effect of its actions on foreign welfare. This leads to an upward bias in the home government tariff decisions, and hence the agreement should not pool the types at the bottom of the distribution, as these types are already choosing tariffs that are too high. Instead, we expect now that $\gamma_H < \overline{\gamma}$, which follows from $v'(\pi_f(\overline{\gamma})) < 0$.[42] We assume that $v'(\pi_f(\underline{\gamma})) - \underline{\gamma} + \mathbb{E}[\gamma] > 0$. This equation guarantees that γ_H is interior. That is, there exists $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that:

$$v'(\pi_f(\gamma_H)) - \gamma_H + \mathbb{E}[\gamma|\gamma > \gamma_H] = 0 \tag{28}$$

Our next step is to determine sufficient conditions for the optimal tariff cap allocation to represent an optimal trade agreement. Let the proposed allocation be $(\pi^\star, t^\star(\gamma) \equiv 0)$ where

$$\pi^\star(\gamma) = \begin{cases} \pi_f(\gamma) & \text{, for } \gamma < \gamma_H \\ \pi_f(\gamma_H) & \text{, for } \gamma \geq \gamma_H \end{cases}$$

and where γ_H is as in (28).

In the present setting, the value $\widetilde{\kappa}$ takes the following form:

$$\widetilde{\kappa} \equiv \min\left\{ \min_{\pi \in [0,\bar{\pi}]} \left\{ \frac{v''(\pi) + b''(\pi)}{b''(\pi)} \right\}, 1 \right\}.$$

Recall now that, under Assumption 1, $b''(\pi) < 0$ and $v''(\pi) + b''(\pi) \leq 0$. If we were to assume that v is a (weakly) concave function of π, then $\widetilde{\kappa} = 1$ would clearly follow. Recall,

[40] To see this, note (i)–(iii) are the same as Assumption 1 (i)–(iv). Note that $w_\pi(\gamma, \pi) = \gamma + b'(\pi) + v'(\pi)$ is continuous in γ so Assumption 1(v) is satisfied. Finally (iv) implies Assumption 1(vi).

[41] To see this note that $w_\pi(\gamma, \pi_f(\gamma_L)) = v'(\pi_f(\gamma_L)) + \gamma - \gamma_L < 0$ for all $\gamma \leq \gamma_L$, and hence condition (d) of Lemma 1 cannot hold.

[42] Note that $w_\pi(\overline{\gamma}, \pi_f(\overline{\gamma})) = v'(\pi_f(\overline{\gamma})) < 0$, where the inequality follows since $\pi_f(\overline{\gamma})$ is interior; hence condition (a) of Lemma 1 implies that $\gamma_H < \overline{\gamma}$.

however, that under reasonable circumstances v may be strictly convex. Notice also that $\widetilde{\kappa}$ falls as the convexity of v increases relative to the concavity of b.

We may represent the trade problem (PT) in terms of problem (P2') for the case in which v does not directly depend upon γ. Since in the trade agreement application v may be strictly convex in π, the associated Lagrangian for problem (P2') is not automatically concave in π. Our Proposition 2 allows for such a possibility, however, and may be applied directly to this problem. Indeed, we now use Proposition 2 to state easy-to-check conditions for the optimal tariff cap allocation to represent an optimal trade agreement:

Proposition 9. *Let γ_H be as in (28). Suppose further that $f(\gamma)$ is non-decreasing and that $\widetilde{\kappa} \geq 1/2$. Then the proposed allocation $(\pi^\star, 0)$ solves the problem,* (PT).

Proof. In Appendix J. □

The proof of Proposition 9 utilizes Proposition 2 with $\gamma_L = 0$ and $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ defined as in equation (28). Using $b'(\pi_f(\gamma)) + \gamma = 0$, we may re-state condition (c1) of Proposition 2 as

$$H(\gamma) \equiv \widetilde{\kappa} F(\gamma) - v'(\pi_f(\gamma)) f(\gamma) \text{ is non-decreasing for all } \gamma \in [\underline{\gamma}, \gamma_H]. \tag{29}$$

Next, after some manipulation, we may re-write condition (c2) of Proposition 2 as:

$$G(\gamma) \equiv v'(\pi_f(\gamma_H)) - \gamma + \int_{\gamma}^{\bar{\gamma}} \widetilde{\gamma} \frac{f(\widetilde{\gamma})}{1 - F(\gamma)} d\widetilde{\gamma} + (\gamma - \gamma_H)(1 - \widetilde{\kappa}) \text{ is non-positive for all } \gamma \in [\gamma_H, \bar{\gamma}], \tag{30}$$

with equality at γ_H. The key task is thus to show that, under the assumptions of Proposition 9, (29) and (30) are satisfied. In the Appendix, we perform this task and thereby prove Proposition 9.

6.3 Two specific settings

Proposition 9 establishes the optimality of a tariff cap for a family of welfare and distribution functions. To further explore the optimality of tariff caps, we now consider two particular specifications for the trade model. The first specification is the linear-quadratic model of trade that Bagwell and Staiger (2005) utilize. We show that tariff caps are optimal for this model when the density function is non-decreasing, and we also utilize the structure of the model to establish the optimality of tariff caps under a condition that allows for decreasing densities. The second specification considers an endowment model with log utility. We show that our findings apply here as well and confirm thereby that our analysis may be usefully applied to specific settings without quadratic payoffs.

Linear-Quadratic Example. Following Bagwell and Staiger (2005), we now assume $u(c) = c - c^2/2$, $Q(p) = p/2$ and $Q_*(p_*) = p_*$. The flexible or Nash tariff, $\tau_f(\gamma)$, is the tariff that maximizes domestic government welfare, given the realized value of the political economy parameter, γ. For a given value of γ, the fully efficient (i.e., first best) tariff, $\tau_e(\gamma)$, is the tariff that maximizes the sum of home and foreign government welfare. For $\gamma \in [1, 7/4)$, the flexible and efficient tariff functions satisfy $\tau_f(\gamma) > \tau_e(\gamma)$.[43] Thus, for political economy parameters in this range, the flexible tariff is higher than efficient. Intuitively, when contemplating a higher tariff, the domestic government doesn't internalize the negative terms-of-trade externality that is experienced by the foreign government. When $\gamma = 7/4$, the domestic political economy parameter is so high that the efficient tariff eliminates all trade. The flexible and efficient tariffs then agree: $\tau_f(7/4) = 1/6 = \tau_e(7/4)$, where 1/6 is the prohibitive tariff that eliminates all trade.

We provide here conditions for the linear-quadratic model under which the optimal trade agreement is given by an optimal tariff cap. We assume that political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$. Letting π denote domestic profits as before, we can write the welfare functions as:

$$b(\pi) = \frac{1}{2}(-1 + 9\sqrt{\pi} - 17\pi), \qquad v(\pi) = \frac{1}{4}(2 - 6\sqrt{\pi} + 9\pi)$$

where $\overline{\pi} = 1/9$.[44]

Inspecting the above, we may confirm that Assumption 1 holds.[45] Further, the definition of $\widetilde{\kappa}$ implies that $\widetilde{\kappa} = 2/3 > 1/2$. If $\mathbb{E}\gamma > [7 + 8\underline{\gamma}]/12$, then we may also verify that γ_H is interior.[46] Thus, if $f(\gamma)$ is non-decreasing, then we may conclude from Proposition 9 that the optimal trade agreement is represented by an optimal tariff cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$.[47]

An interesting special case is that the density is uniform.[48] In this case, $\gamma_H = 3\overline{\gamma} - \frac{7}{2}$ and

[43]In particular, $\tau_f(\gamma) = (8\gamma - 5)/[4(17 - 2\gamma)]$, which strictly exceeds $\tau_e(\gamma) = 4(\gamma - 1)/(25 - 4\gamma)$ for $\gamma \in [1, 7/4)$.

[44]In this example, $\overline{\pi} = 1/9$ is obtained when the prohibitive tariff of 1/6 is imposed. The expressions for $b(\pi)$, $v(\pi)$ and $\overline{\pi}$ are derived in our online appendix.

[45]There is slight complication that arises because of the infinite derivative at zero of w. However, this can be easily handled as the boundedness of the derivative is only used in footnote 18 when proving the existence of an integral. In this case, as long as $\pi^\star(\gamma) > 0$ for all γ, the derivative remains bounded and the integral exists just as before.

[46]To show that $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ exists, we recall that a sufficient condition for an interior maximizer is that $v'(\pi_f(\underline{\gamma})) - \underline{\gamma} + \mathbb{E}\gamma > 0$. Calculations confirm that this inequality holds if and only if $\mathbb{E}\gamma > [7 + 8\underline{\gamma}]/12$. When $\underline{\gamma} = 1$, this inequality reduces to Bagwell and Staiger (2005)'s assumption that $\mathbb{E}\gamma > 5/4$.

[47]In fact, in this case we can confirm that γ_H is unique. Given that $\widetilde{\kappa} > 1/2$, if $f(\gamma)$ is non-decreasing, we may see from the proof of Proposition 9 that $v'(\pi_f(\gamma^c)) - \gamma^c + \mathbb{E}[\gamma|\gamma > \gamma^c]$ is strictly decreasing, which implies that γ_H is unique.

[48]For further discussion of the optimal tariff cap under a uniform distribution, see Bagwell and Staiger (2005).

the optimal tariff cap is:

$$\bar{\tau} = \frac{1}{6} - \frac{7(7-4\overline{\gamma})}{24(4-\overline{\gamma})}.$$

Recalling that a tariff of 1/6 is prohibitive, we see that the optimal tariff cap allows for positive trade volume since $\overline{\gamma} < 7/4$. The optimal tariff cap binds for higher types (i.e., for $\gamma \geq \gamma_H$), while lower types apply their flexible (Nash) tariffs and thus exhibit "binding overhang". Interestingly, as $\overline{\gamma}$ approaches 7/4, $\overline{\tau}$ approaches 1/6 and so γ_H approaches $\overline{\gamma}$. Thus, when the distribution function is uniform and the highest level of support approaches the limiting case in which zero trade volume is efficient, the optimal trade agreement entails full flexibility for all types! In this limiting case, governments with private information are unable to design a trade agreement that improves upon the non-cooperative (Nash) benchmark.

The following summarizes the above results:

Corollary 4. *If $Q(p) = p/2$, $Q_\star(p) = p$, $u(c) = c - c^2/2$, and the political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$ according to a non-decreasing density satisfying $\mathbb{E}\gamma > [7 + 8\underline{\gamma}]/12$, then an optimal trade agreement is represented as an optimal tariff cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$. In the special case of a uniform distribution, the optimal tariff cap is at $\bar{\tau} = \frac{1}{6} - \frac{7(7-4\bar{\gamma})}{24(4-\bar{\gamma})}$, and full flexibility is thus used for all types as $\bar{\gamma}$ approaches 7/4.*

Proof. In the text. □

The linear-quadratic example also provides a tractable setting in which to explore the possibility of non-increasing densities. For this example, we can establish weaker sufficient conditions for (29) and (30).

Corollary 5. *If $Q(p) = p/2$, $Q_\star(p) = p$, $u(c) = c - c^2/2$, and the political shocks are distributed over $[\underline{\gamma}, \bar{\gamma}]$ where $1 \leq \underline{\gamma} < \bar{\gamma} < 7/4$ according to a differentiable density satisfying $\mathbb{E}\gamma > [7 + 8\underline{\gamma}]/12$ and $f(\gamma) - 3v'(\pi_f(\gamma))f'(\gamma) \geq 0$, then an optimal trade agreement is represented as an optimal tariff cap with $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$.*

Proof. In Appendix K. □

Notice that the assumption $f(\gamma) - 3v'(\pi_f(\gamma))f'(\gamma) \geq 0$ holds if the density is non-decreasing, since $v' < 0$. Corollary 5, however, includes as well densities that are decreasing over ranges or even over the entire support, provided that the rate of decrease is not so great as to violate the stated inequality. In particular, in the linear-quadratic example, we can derive that $v'(\pi_f(\gamma)) = -1/3(7/4 - \gamma)$ and thus re-write the inequality in Corollary 5 as $f(\gamma) + \left(\frac{7}{4} - \gamma\right) f'(\gamma) \geq 0$. This condition clearly holds even for densities that decline over the entire support, provided that the rate of decline is sufficiently small. As well, the condition holds for any concave density for which $f(\bar{\gamma}) + (\frac{7}{4} - \bar{\gamma})f'(\bar{\gamma}) \geq 0$, or for any convex density for which $f(\underline{\gamma}) + (\frac{7}{4} - \underline{\gamma})f'(\underline{\gamma}) \geq 0$.[49]

Together, Corollaries 4 and 5 confirm that the optimal tariff cap identified by Bagwell and Staiger (2005) in fact represents an optimal trade agreement for a broad family of distributions and when the possibility of money burning is also allowed. Indeed, Corollaries 4 and 5 hold as stated whether or not money burning is allowed in the analysis, since $\widetilde{\kappa} = \kappa = 2/3$ in the linear-quadratic model of trade. Note that this distinguishes our approach from the delegation literature. Although we could have obtained versions of the corollaries above by appealing to the results from Alonso and Matouschek (2008), these results would only hold if money burning were ruled out. When money burning is allowed, the delegation literature results cannot shed light on the solution. The results from the delegation literature are not useful also when preferences are not linear-quadratic, a case we discuss next.

A Log Utility with Endowments Example. The linear-quadratic example is tractable and offers a convenient setting with which to illustrate our findings. An important benefit of our general analysis is that we can employ our findings to characterize an optimal trade agreement for other examples, too. In Appendix L, we consider an example with log utility and endowments (inelastic supply), where the endowment of good x in the foreign country exceeds that in the home country. Similar results apply for this example as well.

7 Conclusions

We consider a general representation of the delegation problem, and we provide conditions under which an interval allocation is an optimal solution to this problem. We analyze both the delegation problem without money burning and the delegation problem with money burning. We also characterize optimal solutions to the delegation problem when interval allocations are not optimal. As we show, important characterizations of optimal delegation in previous work can be captured as special cases of our findings. We also develop a new application of delegation theory to the theory of trade agreements among privately informed governments. We establish conditions under which tariff caps are optimal and thereby provide interpretations of negotiations over tariff bindings and also binding overhang.

To establish our findings, we utilize and extend the Lagrangian methods developed by Amador et al. (2006). Our analysis allows that the Lagrangian may fail to be concave with

[49] To see this, note that the derivative of $f(\gamma) + (\frac{7}{4} - \gamma)f'(\gamma)$ with respect to γ is $(\frac{7}{4} - \gamma)f''(\gamma)$.

respect to the action, which is a possibility that arises naturally in the trade application, for example. We expect that our techniques will be useful for other studies of applied mechanism design when contingent transfers are infeasible.

A A Modified Version of Luenberger's Sufficiency Theorem

Here we provide a slightly modified version of Theorem 1 in section 8.4 of Luenberger (1969) that makes explicit the complementary slackness condition:

Theorem 1. *Let f be a real valued functional defined on a subset Ω of a linear space X. Let G be a mapping from Ω into the linear space Z having non-empty positive cone P. Suppose that* (i) *there exists a linear function $T : Z \to \mathbb{R}$ such that $T(z) \geq 0$ for all $z \in P$,* (ii) *there is an element $x_0 \in \Omega$ such that*

$$f(x_0) + T(G(x_0)) \leq f(x) + T(G(x)), \text{ for all } x \in \Omega,$$

(iii) $-G(x_0) \in P$, *and* (iv) $T(G(x_0)) = 0$. *Then x_0 solves:*

$$\min f(x) \text{ subject to: } -G(x) \in P, \quad x \in \Omega$$

Proof. Note that from (iii), x_0 is in the constraint set of the minimization problem. Suppose that there exists an $x_1 \in \Omega$ with $f(x_1) < f(x_0)$ and $-G(x_1) \in P$, so that x_0 is not a minimizer. Then, by hypothesis (i), $T(-G(x_1)) \geq 0$. Linearity implies that $T(G(x_1)) \leq 0$. Using this together with (iv), it follows that $f(x_1) + T(G(x_1)) < f(x_0) = f(x_0) + T(G(x_0))$, which contradicts hypothesis (ii). □

B Proof of Lemma 1

The welfare function can, under an interval allocation, be written as:

$$Obj(\gamma_L, \gamma_H) = \int_{\underline{\gamma}}^{\gamma_L} w(\gamma, \pi_f(\gamma_L))f(\gamma)d\gamma + \int_{\gamma_L}^{\gamma_H} w(\gamma, \pi_f(\gamma))f(\gamma)d\gamma + \int_{\gamma_H}^{\overline{\gamma}} w(\gamma, \pi_f(\gamma_H))f(\gamma)d\gamma$$

Taking the necessary first order condition for an interior γ_L we have:

$$\frac{dObj}{d\gamma_L} = \left(\int_{\underline{\gamma}}^{\gamma_L} w_\pi(\gamma, \pi_f(\gamma_L)) f(\gamma) d\gamma \right) \pi_f'(\gamma_L) = 0$$

and using that $\pi_f'(\gamma_L) > 0$ by assumption, we have that condition (d) is satisfied.

The second derivative is:

$$\frac{d^2Obj}{d\gamma_L^2} = \left(\int_{\underline{\gamma}}^{\gamma_L} w_\pi(\gamma, \pi_f(\gamma_L)) f(\gamma) d\gamma \right) \pi_f''(\gamma_L) + \left(w_\pi(\gamma_L, \pi_f(\gamma_L)) f(\gamma_L) \right) \pi_f'(\gamma_L) + \left(\int_{\underline{\gamma}}^{\gamma_L} w_{\pi\pi}(\gamma, \pi_f(\gamma_L)) f(\gamma) d\gamma \right) (\pi_f'(\gamma_L))^2$$

Now note that if $\gamma_L = \underline{\gamma}$, then $dObj/d\gamma_L = 0$ and $d^2Obj/d\gamma_L^2 = w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) f(\underline{\gamma}) \pi_f'(\underline{\gamma})$. So if $\gamma_L = \underline{\gamma}$ is optimal, it must be that $w_\pi(\underline{\gamma}, \pi_f(\underline{\gamma})) \leq 0$. This proves condition (c). The proofs for conditions (a) and (b) follow a similar argument, so we omit them.

C Proof of Proposition 3

The proof here follows the proof of Proposition 4, the necessity result, in Amador et al. (2006).

We proceed to use Theorem 1 (a necessity theorem) in Luenberger (1969, p. 217). Let (i) f to be given by the negative of the objective function, $f \equiv -\int_\Gamma \big(v(\gamma, \pi(\gamma)) f(\gamma) + (1 - F(\gamma)) \pi(\gamma) \big) d\gamma - \underline{U}$; (ii) $X \equiv \{\pi, \underline{t} | \underline{t} \in \mathbb{R}_+ \text{ and } \pi : \Gamma \to \Pi\}$; (iii) $Z \equiv \{z | z : \Gamma \to \mathbb{R}$ and z continuous $\}$ with the norm $||z|| = \sup |z(\gamma)|$; (iv) $\Omega \equiv \{\pi, \underline{t} | \underline{t} \in \mathbb{R}_+, \pi : \Gamma \to \Pi$, non decreasing and continuous$\}$; (v) $P \equiv \{z | z \in Z$ such that $z(\gamma) \geq 0$ for all $\gamma \in \Gamma\}$; (vi) G to be the mapping from Ω to Z given by the negative of the left hand side of inequality (11).

Note that we are restricting the choice set to the be the set of non-decreasing and **continuous** functions π. This is because we are looking for necessary conditions for the optimal allocation to be an interval, which is continuous.

Note that given that $\tilde{\kappa} = 1$ (by the hypothesis of the Proposition), f is convex. Note as well that G is convex, Ω is convex, P contains an interior point (e.g. $z(\gamma) = 1$ for all $\gamma \in \Gamma$), and that the positive dual of Z is isomorphic to the space of non-decreasing functions on Γ by the Riesz representation theorem (see Luenberger, 1969, chapter 5, p. 113). Note as well that if $(\pi, \underline{t})$ is optimal and lies in Ω, then it must be optimal within Ω. To see that there exists an interior point to the constraint set, just consider the allocation x_1 that bunches every type at some π_1 and burns a strictly positive amount. That allocation is in Ω and generates a function $G(x_1)$ that is in the interior of the negative cone $-P$. Given that the proposed allocation is continuous, it follows that the hypothesis of Theorem 1 of Luenberger (1969, p. 217) hold and there exists a non-decreasing function Λ_0, such that the Lagrangian, $\mathcal{L}(\pi, \underline{t}|\Lambda_0)$ is maximized at $(\pi^\star, t^\star(\underline{\gamma}))$ within Ω.[50] Without loss of generality, we normalize $\Lambda_0(\overline{\gamma}) = 1$.

In a similar fashion as in the proof of Proposition 1, we can now use Lemmas A.1 and A.2 of Amador et al. (2006) (together with the extension used in footnote 17), and argue that if the Lagrangian is maximized at some $(\pi^\star, t^\star(\underline{\gamma})) \in \Omega$, then it must be the case that:

$$\partial\mathcal{L}(\pi^\star, t^\star(\underline{\gamma}); \pi^\star, t^\star(\underline{\gamma})|\Lambda_0) = 0;$$
$$\partial\mathcal{L}(\pi^\star, t^\star(\underline{\gamma}); x, y|\Lambda_0) \leq 0$$

for all $(x, y) \in \Omega$, and where as before the derivative is in terms of Gateaux differentials.

Taking the Gateaux differential of the Lagrangian in (14), we get:[51]

$$\begin{aligned}\partial\mathcal{L}(\pi^\star, \underline{t}^\star|x, y|\Lambda_0) = &\int_{\gamma\in\Gamma} \Big(v_\pi(\gamma, \pi^\star(\gamma))f(\gamma) + (\Lambda_0(\gamma) - F(\gamma))\Big)x(\gamma)d\gamma \\ &+ \int_{\underline{\gamma}}^{\gamma_L} (\gamma - \gamma_L)x(\gamma)d\Lambda_0(\gamma) + \int_{\gamma_H}^{\overline{\gamma}} (\gamma - \gamma_H)x(\gamma)d\Lambda_0(\gamma) + \Lambda_0(\underline{\gamma})\Big((\underline{\gamma} - \gamma_L)x(\underline{\gamma}) - y\Big)\end{aligned}$$

Let us define g to be such that:

$$\begin{aligned}g(\gamma) \equiv &\int_{\gamma}^{\overline{\gamma}} \Big(v_\pi(\tilde{\gamma}, \pi^\star(\tilde{\gamma}))f(\tilde{\gamma}) + (\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma}))\Big)d\tilde{\gamma} \\ &+ \int_{\gamma}^{\overline{\gamma}} \Big[\mathbb{I}(\tilde{\gamma} < \gamma_L)(\tilde{\gamma} - \gamma_L) + \mathbb{I}(\tilde{\gamma} > \gamma_H)(\tilde{\gamma} - \gamma_H)\Big]d\Lambda_0(\tilde{\gamma})\end{aligned}$$

Then, integrating by parts the derivative above (which can be done given that x is continuous), it follows that:

$$\partial\mathcal{L}(\pi^\star, \underline{t}^\star; x, y|\Lambda_0) = \big[g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L)\big]x(\underline{\gamma}) + \int_{\underline{\gamma}}^{\overline{\gamma}} g(\gamma)dx(\gamma) - \Lambda_0(\underline{\gamma})y$$

The first order conditions require the above to be non-positive for all non-decreasing and

[50] Note that the Lagrangian in our case corresponds to the negative of the Lagrangian as defined in Luenberger (1969).

[51] See footnote 18 for existence of the Gateaux differential.

non-negative functions x and non-negative y, and hence:

$$g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) \leq 0; \quad g(\gamma) \leq 0; \quad \text{and } \Lambda_0(\underline{\gamma}) \geq 0$$

Now, using that $\partial\mathcal{L}(\pi^\star, \underline{t}^\star; \pi^\star, \underline{t}^\star|\Lambda_0) = 0$, it follows that (i) $g(\gamma) = 0$ for all $\gamma \in [\gamma_L, \gamma_H]$; (ii) $g(\underline{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) = 0$. From (i) we get that

$$\Lambda_0(\gamma) = F(\gamma) - v_\pi(\gamma, \pi^\star(\gamma))f(\gamma) \tag{31}$$

for $\gamma \in [\gamma_L, \gamma_H]$.[52] And using (ii) as well:

$$\int_{\underline{\gamma}}^{\gamma_L} \Big(v_\pi(\tilde{\gamma}, \pi^\star(\tilde{\gamma}))f(\tilde{\gamma}) + (\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma}))\Big)d\tilde{\gamma} + \int_{\underline{\gamma}}^{\gamma_L} (\tilde{\gamma} - \gamma_L)d\Lambda_0(\tilde{\gamma}) + \Lambda_0(\underline{\gamma})(\underline{\gamma} - \gamma_L) = 0 \tag{32}$$

From $g(\gamma) \leq 0$ for $\gamma \in [\underline{\gamma}, \gamma_L) \cup (\gamma_H, \overline{\gamma}]$ and $g(\gamma) = 0$ for $\gamma \in [\gamma_L, \gamma_H]$, it follows that:

$$\int_{\gamma}^{\overline{\gamma}} \Big(v_\pi(\tilde{\gamma}, \pi^\star(\tilde{\gamma}))f(\tilde{\gamma}) + (\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma}))\Big)d\tilde{\gamma} + \int_{\gamma}^{\overline{\gamma}} (\tilde{\gamma} - \gamma_H)d\Lambda_0(\tilde{\gamma}) \leq 0 \quad \text{for } \gamma \in (\gamma_H, \overline{\gamma}] \tag{33}$$

$$\int_{\gamma}^{\gamma_L} \Big(v_\pi(\tilde{\gamma}, \pi^\star(\tilde{\gamma}))f(\tilde{\gamma}) + (\Lambda_0(\tilde{\gamma}) - F(\tilde{\gamma}))\Big)d\tilde{\gamma} + \int_{\gamma}^{\gamma_L} (\tilde{\gamma} - \gamma_L)d\Lambda_0(\tilde{\gamma}) \leq 0 \quad \text{for } \gamma \in [\underline{\gamma}, \gamma_L) \tag{34}$$

Now note that $\int_\gamma^{\overline{\gamma}} \Lambda_0(\tilde{\gamma})d\tilde{\gamma} = \Lambda_0(\tilde{\gamma})(\tilde{\gamma} - \gamma)\Big|_\gamma^{\overline{\gamma}} - \int_\gamma^{\overline{\gamma}}(\tilde{\gamma} - \gamma)d\Lambda_0(\tilde{\gamma}) = (\overline{\gamma} - \gamma) - \int_\gamma^{\overline{\gamma}}(\tilde{\gamma} - \gamma)d\Lambda_0(\tilde{\gamma})$. And from the first of the two inequalities above we get that:

$$\int_{\gamma}^{\overline{\gamma}} \Big(v_\pi(\tilde{\gamma}, \pi_f(\gamma_H))f(\tilde{\gamma}) + 1 - F(\tilde{\gamma})\Big)d\tilde{\gamma} + (\gamma - \gamma_H)(1 - \Lambda_0(\gamma)) \leq 0 \quad \text{for } \gamma \in (\gamma_H, \overline{\gamma}]$$

And thus, the best chance we have for the above inequality to hold, given that Λ is non-decreasing and $\Lambda(\overline{\gamma}) = 1$, is that $\Lambda_0(\gamma) = 1$ for all $\gamma \in (\gamma_H, \overline{\gamma})$. Hence, a necessary condition

[52] For all $\gamma \in [\gamma_L, \gamma_H]$, we note that $g(\gamma) = \int_\gamma^{\gamma_H} h(\gamma)d\gamma$ for some integrable function h. From properties of absolute continuity it follows that $g(\gamma) = 0$ for all $\gamma \in [\gamma_L, \gamma_H]$ only if $h(\gamma) = 0$ almost everywhere in $[\gamma_L, \gamma_H]$. For simplicity's sake, we do not write the "a.e." conditioning in what follows, although the reader should keep it in mind.

is that:

$$\int_{\gamma}^{\overline{\gamma}} \Big(v_\pi(\tilde{\gamma}, \pi_f(\gamma_H))f(\tilde{\gamma}) + 1 - F(\tilde{\gamma})\Big)d\tilde{\gamma} \leq 0 \text{ for } \gamma \in (\gamma_H, \overline{\gamma}] \tag{35}$$

Now subtracting from (32), equation (34), we get:

$$\int_{\underline{\gamma}}^{\gamma} \Big(v_\pi(\tilde{\gamma}, \pi^\star(\tilde{\gamma}))f(\tilde{\gamma}) - F(\tilde{\gamma})\Big)d\tilde{\gamma} + \Lambda_0(\gamma)(\gamma - \gamma_L) \geq 0 \text{ for } \gamma \in [\underline{\gamma}, \gamma_L)$$

The best chance of satisfying this equation is when $\Lambda_0(\gamma) = 0$ for all $\gamma < \gamma_L$ (given that $\Lambda_0(\gamma)$ is non-decreasing and we have shown above that $\Lambda_0(\underline{\gamma}) \geq 0$). Then a necessary condition for optimality is that:

$$\int_{\underline{\gamma}}^{\gamma} \Big(v_\pi(\tilde{\gamma}, \pi_f(\gamma_L))f(\tilde{\gamma}) - F(\tilde{\gamma})\Big)d\tilde{\gamma} \geq 0 \text{ for } \gamma \in [\underline{\gamma}, \gamma_L) \tag{36}$$

Now we are ready to show that the conditions of Proposition 2 are also necessary when $\tilde{\kappa} = 1$.

Note that (c1) of Proposition 2 follows from (31) and the restriction that Λ_0 most be non-decreasing. Part (c2) of the proposition follows from (35) and where the equality restriction follows from Lemma 1. Part (c2') follows also from Lemma 1. Part (c3) follows from (36) and where the equality restriction follows from Lemma 1. Part (c3') follows from Lemma 1. Hence, when $\tilde{\kappa} = 1$, the sufficient condition of Proposition 2 are also necessary. $\square$

D Proof of Proposition 4

Suppose that within the flexibility region we were to remove the prescribed allocations for some types x to $x + \epsilon$. This is a feasible change to the allocation and is incentive compatible as follows: there is a type $\gamma(\epsilon)$ that is now indifferent between the allocations for types x and $x + \epsilon$, and $\gamma(\epsilon)$ satisfies:

$$\gamma(\epsilon) = -\frac{b(\pi_f(x+\epsilon)) - b(\pi_f(x))}{\pi_f(x+\epsilon) - \pi_f(x)}. \tag{37}$$

All types between x and $\gamma(\epsilon)$ now choose the allocation for type x, while all types between $\gamma(\epsilon)$ and $x + \epsilon$ choose the allocation for type $x + \epsilon$.

The following lemma characterizes some useful properties of the γ function:

Lemma 2. *The function $\gamma(\epsilon)$ is such that:* (i) $\gamma(0) = x$*;* (ii) $\gamma'(0) = 1/2$*; and* (iii) $\gamma''(0) =$

$\frac{1}{6}\frac{\pi_f''(x)}{\pi_f'(x)}$.

Proof. Recall also that in the flexible allocation we have that $b'(\pi_f(x)) = -x$. The first point follows from taking the limit of (37). The second follows from:

$$\gamma'(\epsilon) = \frac{x + \epsilon - \gamma(\epsilon)}{\pi_f(x+\epsilon) - \pi_f(x)}\pi_f'(x+\epsilon),$$

and taking the limit. Using L'Hopital's Rule, we get $\gamma'(0) = -\gamma'(0) + 1$ and so $\gamma'(0) = 1/2$. Taking another derivative we get that:

$$\gamma''(\epsilon) = \frac{1 - 2\gamma'(\epsilon)}{\pi_f(x+\epsilon) - \pi_f(x)}\pi_f'(x+\epsilon) + \gamma'(\epsilon)\frac{\pi_f''(x+\epsilon)}{\pi_f'(x+\epsilon)}$$

And taking limits as $\epsilon \to 0$, we get $\gamma''(0) = -2\gamma''(0) + \frac{1}{2}\frac{\pi_f''(x)}{\pi_f'(x)}$, which delivers that $\gamma''(0) = \frac{1}{6}\frac{\pi_f''(x)}{\pi_f'(x)}$. $\square$

We can now compute the effect on welfare of the removing the allocations prescribed for types x to $x+\epsilon$: all types between x and $\gamma(\epsilon)$ choose x in the new allocation, while all types between $\gamma(\epsilon)$ and $x+\epsilon$ choose $x+\epsilon$. The change in welfare, ΔW, is given by the following equation:

$$\Delta W(\epsilon) = \int_x^{\gamma(\epsilon)} w(\gamma, \pi_f(x))dF(\gamma) + \int_{\gamma(\epsilon)}^{x+\epsilon} w(\gamma, \pi_f(x+\epsilon))dF(\gamma) - \int_x^{x+\epsilon} w(\gamma, \pi_f(\gamma))dF(\gamma)$$

Then we can prove the following result:

Lemma 3. *The following holds:* (i) $\Delta W(0) = 0$*;* (ii) $\Delta W'(0) = 0$*;* (iii) $\Delta W''(0) = 0$*; and* (iv) $\Delta W'''(0) = \frac{\pi_f'(x)}{4}\left[\frac{d}{dx}w_\pi(x, \pi_f(x))f(x)\right] + \frac{1}{4}\pi_f'(x)f(x)w_{\pi\pi}(x, \pi_f(x))\pi_f'(x)$.

Proof. Taking the first derivative with respect to ϵ we get that:

$$\begin{aligned}\Delta W'(\epsilon) = \Big(w(\gamma(\epsilon), \pi_f(x)) - w(\gamma(\epsilon), \pi_f(x+\epsilon))\Big)f(\gamma(\epsilon))\gamma'(\epsilon) \\ + \int_{\gamma(\epsilon)}^{x+\epsilon} w_\pi(\gamma, \pi_f(x+\epsilon))\pi_f'(x+\epsilon)dF(\gamma)\end{aligned}$$

Thus, $\Delta W'(0) = 0$ since $\gamma(0) = x$ by Lemma 2. Taking one more derivative we get:

$$\begin{aligned}
\Delta W''(\epsilon) = & \left[\frac{d}{d\epsilon}(f(\gamma(\epsilon))\gamma'(\epsilon))\right](w(\gamma(\epsilon), \pi_f(x)) - w(\gamma(\epsilon), \pi_f(x+\epsilon))) \\
& + f(\gamma(\epsilon))(\gamma'(\epsilon))^2[w_\gamma(\gamma(\epsilon), \pi_f(x)) - w_\gamma(\gamma(\epsilon), \pi_f(x+\epsilon))] \\
& - 2f(\gamma(\epsilon))\gamma'(\epsilon)w_\pi(\gamma(\epsilon), \pi_f(x+\epsilon))\pi_f'(x+\epsilon) \\
& + w_\pi(x+\epsilon, \pi_f(x+\epsilon))\pi_f'(x+\epsilon)f(x+\epsilon) \\
& + \int_{\gamma(\epsilon)}^{x+\epsilon}\left[w_{\pi\pi}(\gamma, \pi_f(x+\epsilon))(\pi_f'(x+\epsilon))^2 + w_\pi(\gamma, \pi_f(x+\epsilon))\pi_f''(x+\epsilon)\right]dF(\gamma)
\end{aligned}$$

from where, using $\gamma(0) = x$ and $\gamma'(0) = 1/2$ by Lemma 2, it follows that $\Delta W''(0) = 0$.

Taking another derivative, and using our knowledge of $\gamma(\epsilon)$ through Lemma 2, we can get, after some algebra, that:

$$\begin{aligned}
\Delta W'''(0) = & \frac{1}{4}w_\pi(x, \pi_f(x))\pi_f'(x)f'(x) + \frac{1}{4}w_{\pi\gamma}(x, \pi_f(x))\pi_f'(x)f(x) \\
& + \frac{1}{2}w_{\pi\pi}(x, \pi_f(x))(\pi_f'(x))^2 f(x),
\end{aligned}$$

which delivers the result. □

A necessary condition for optimality is that $\Delta W'''(0) \leq 0$. Now, using that $\pi_f'(x) = -1/b_{\pi\pi}(\pi_f(x))$ and that $\pi_f'(\gamma) > 0$, the result of the proposition follows.

E Proof of Proposition 5

Let us just prove (a) and (b), as (c) and (d) follow a similar argument.

That condition (b) is a necessary condition follows directly from Lemma 1.

For condition (a), the fact that the condition must hold with equality at γ_H follows immediately from Lemma 1 as the allocation in the general case must be optimal as well when restricted to the class of interval allocations. To see this note that when $\pi_0 = \pi_f(\gamma_H)$ we have that $g(\pi_f(\gamma_H)|\pi_f(\gamma_H)) = -b'(\pi_f(\gamma_H)) = \gamma_H$ and the term inside the brackets in the integral in condition (a) then becomes $w_\pi(\tilde{\gamma}, \pi_f(\gamma_H))$; and thus the condition is the same as in Lemma 1.

To prove that the inequality in condition (a) must hold, consider the perturbation that introduces the choice π_0 (with no money burned) into the allocation. All agents between γ_H and $\gamma_0 \equiv g(\pi_0|\pi_f(\gamma_H))$ will remain in their old choice, $\pi_f(\gamma_H)$, while all agents between γ_0 and $\overline{\gamma}$ will now choose the new choice π_0. (This follows from noticing that type γ_0 remains indifferent between the two.) The effect on welfare of this perturbation is equal to the left hand side of inequality (17), multiplied by $\pi_0 - \pi_f(\gamma_H)$, and hence the inequality most hold or we would have found an improvement. $\square$

F Proof of Proposition 6

For part (a): For this case we know that $A = \kappa$. We next note that the following inequalities are each equivalent to the sufficient condition (c2) of Proposition 1:

$$\int_{\gamma}^{\overline{\gamma}} w_\pi(\tilde{\gamma}, \pi_f(\gamma_H)) \frac{f(\tilde{\gamma})}{1 - F(\gamma)} d\tilde{\gamma} \leq (\gamma - \gamma_H)A$$

$$\int_{\gamma}^{\overline{\gamma}} \left(w_\pi(\tilde{\gamma}, \pi_f(\gamma_H)) - A(\gamma - \gamma_H) \right) \frac{f(\tilde{\gamma})}{1 - F(\gamma)} d\tilde{\gamma} \leq 0$$

$$\int_{\gamma}^{\overline{\gamma}} w_\pi(\tilde{\gamma}, \pi_f(\gamma)) f(\tilde{\gamma}) d\tilde{\gamma} \leq 0$$

which should hold for all $\gamma \in (\gamma_H, \overline{\gamma})$ and with equality at γ_H. Note that we have used that $w_\pi(\tilde{\gamma}, \pi_f(\gamma_H)) - A(\gamma - \gamma_H) = w_\pi(\tilde{\gamma}, \pi_f(\gamma))$ given our assumption about w and using that $b'(\pi_f(\gamma)) = -\gamma$.

The necessary condition (17) of Proposition 5 is now:

$$A \int_{\gamma_0}^{\bar{\gamma}} \left(\left(\frac{b(\pi_0) - b(\pi_f(\gamma_H))}{\pi_0 - \pi_f(\gamma_H)} \right) + C(\tilde{\gamma}) \right) f(\tilde{\gamma}) d\tilde{\gamma} \leq 0$$

which should hold for all $\pi_0 \in [\pi_f(\gamma_H), \pi_f(\overline{\gamma})]$ and with equality at $\pi_0 = \pi_f(\gamma_H)$, where $\gamma_0 = g(\pi_0 | \pi_f(\gamma_H))$. Using the definition of g, we have that the above is equivalent to:

$$A \int_{\gamma_0}^{\bar{\gamma}} \left(-\gamma_0 + C(\tilde{\gamma}) \right) f(\tilde{\gamma}) d\tilde{\gamma} \leq 0.$$

Using $b'(\pi_f(\gamma_0)) = -\gamma_0$, and thus that $w_\pi(\tilde{\gamma}, \pi_f(\gamma_0)) = A(-\gamma_0 + C(\tilde{\gamma}))$, it follows that this is the same as the sufficient condition (c2) as represented above. Note that (c2') is the same as the condition (b) of Proposition 5.

A similar argument shows that (c3) and (c3') are equivalent to (c) and (d) of Proposition 5. Finally, the necessary condition in Proposition 4 is equivalent to condition (c1) of Proposition 1 using the preferences specified above. Taken together, the above shows that the sufficient conditions of Proposition 1 are also necessary.

For part (b): There are two cases to consider. The first case, where $A \geq 1$, implies that

$\tilde{\kappa} = 1$ and this is already covered by our Proposition 3. For the second case, where $A < 1$, the result is the same as for part (a), as $A = \tilde{\kappa}$. $\square$

G Proof of Corollary 1

Finally, we may express condition (c2) of Proposition 2 as

$$(1+\alpha)\mathbb{E}(\tilde{\gamma}|\tilde{\gamma} \geq \gamma) - \alpha(\gamma_H + \beta) - \gamma \leq 0, \forall \gamma \in [\gamma_H, \overline{\gamma}], \tag{38}$$

with equality at $\gamma = \gamma_H$. Let $q(\gamma, \gamma_H) \equiv (1+\alpha)\mathbb{E}(\tilde{\gamma}|\tilde{\gamma} \geq \gamma) - \alpha(\gamma_H + \beta) - \gamma$. Note that $q(\gamma, \gamma)$ is continuous in γ; equals $(1+\alpha)(\mathbb{E}[\gamma] - \underline{\gamma}) - \alpha\beta$ at $\underline{\gamma}$, which is strictly positive by hypothesis (i); and equals $-\alpha\beta$ at $\overline{\gamma}$, which is negative. Then there exists a $\gamma_H \in (\underline{\gamma}, \overline{\gamma})$ such that $q(\gamma_H, \gamma_H) = 0$, or equivalently such that $\mathbb{E}[\gamma|\gamma \geq \gamma_H] - \gamma_H = \alpha\beta/(1+\alpha)$. Hypothesis (iii) then implies that (38) holds for all $\gamma \in [\gamma_H, \overline{\gamma}]$.

In the text, we have already argued that condition (c3') of Proposition 2 holds. Condition (c1) holds if $F(\gamma) + \alpha\beta f(\gamma)$ is non-decreasing for all $\gamma \in [\underline{\gamma}, \gamma_H]$ which is true by hypothesis (ii) of Corollary 1. We may now apply Proposition 2. $\square$

H Proof of Corollary 2

As we discuss in the text, conditions i and ii of Assumption 2 hold for the quadratic preferences we are using in this section.

To show part (iii) of Assumption 2, note that the inequality $x(\gamma) \leq \pi_f(\gamma)$ can be written as

$$\frac{1}{\alpha}\left(\frac{1 - F(\gamma)}{f(\gamma)}\right) \leq \beta. \tag{39}$$

There exists a value of $\gamma_x \in [\underline{\gamma}, \overline{\gamma})$ such that the above holds with equality. This follows because the left hand side is zero when $\gamma_x = \overline{\gamma}$ and is weakly bigger than β when $\gamma_x = \underline{\gamma}$ by the hypothesis of Corollary 2. Continuity of $f(\gamma)$ then implies existence of such a γ_x. From hypothesis (i) of Corollary 2, it follows that (39) holds for all $\gamma > \gamma_x$. It follows also that for $\gamma > \gamma_x$, $v_\pi(\gamma, x(\gamma))f(\gamma) = F(\gamma) - 1$ by the definition of x. The function $x(\gamma)$ is non-decreasing for all $\gamma \geq \gamma_x$ given the hypothesis (ii) of Corollary 2. And finally, hypothesis (iii) of the corollary implies that that $F(\gamma) - v_\pi(\gamma, \pi_f(\gamma))f(\gamma)$ is non-decreasing. Hence Assumption 2 holds, and we can apply Proposition 7 to prove the corollary. $\square$

I Proof of Corollary 3

As argued before, Assumption 3i holds. Note that $x(\gamma) = \frac{1}{\alpha}((1-F(\gamma))/f(\gamma) + \alpha\gamma)$ satisfies 3iia and 3iic by definition of x and hypothesis (i) of Corollary 3. Note also that $x(\gamma) - \pi_f(\gamma) = \frac{1}{\alpha}\frac{1-F(\gamma)}{f(\gamma)} - \beta \leq \frac{1}{\alpha}\frac{1}{f(\underline{\gamma})} - \beta < 0$, by hypothesis (ii) of Corollary 3, so that 3iib holds. Finally note that the condition in 3iid is equivalent to:

$$p(\gamma,\gamma_y) \equiv (1-F(\gamma))\gamma + \beta + (1+\alpha)\mathbb{E}[\tilde{\gamma}|\tilde{\gamma} < \gamma]F(\gamma) - \left(\frac{1-F(\gamma_y)}{\alpha f(\gamma_y)} + \gamma_y\right)(1+\alpha F(\gamma)) \geq 0$$

for all $\gamma \in [\underline{\gamma}, \gamma_y]$ with equality at γ_y. Note that $pp(\gamma_y) \equiv p(\gamma_y, \gamma_y) = (1-F(\gamma_y))\gamma_y + \beta + (1+\alpha)\mathbb{E}[\tilde{\gamma}|\tilde{\gamma} \leq \gamma_y]F(\gamma_y) - (1+\alpha F(\gamma_y))[\gamma_y + (1-F(\gamma_y))/(\alpha f(\gamma_y))]$. Note also that $pp(\gamma_y)$ is continuous in γ_y and that $pp(\underline{\gamma}) = \beta - \frac{1}{\alpha f(\underline{\gamma})} > 0$ and $pp(\overline{\gamma}) = \beta + (1+\alpha)(\mathbb{E}[\gamma] - \overline{\gamma}) < 0$ where this last holds by hypothesis (iii). It follows then that there exists $\gamma_y \in (\underline{\gamma}, \overline{\gamma})$ such that $pp(\gamma_y) = 0$.

Note as well that

$$\begin{aligned}
\partial p(\gamma,\gamma_y)/\partial\gamma &= v_\pi(\gamma, x(\gamma_y))f(\gamma) + 1 - F(\gamma) \\
&= f(\gamma)\left\{\left[\frac{1-F(\gamma)}{f(\gamma)} + \alpha\gamma\right] - \left[\frac{1-F(\gamma_y)}{f(\gamma_y)} + \alpha\gamma_y\right]\right\} \leq 0
\end{aligned}$$

for $\gamma \leq \gamma_y$, where the inequality follows from hypothesis (i) of Corollary 3. Given that $p(\gamma_y, \gamma_y) = 0$ and that $\partial p(\gamma,\gamma_y)/\partial\gamma \leq 0$ for all $\gamma < \gamma_y$, it follows that $p(\gamma,\gamma_y) \geq 0$ for all $\gamma < \gamma_y$. Thus Assumption 3iid holds. We now apply Proposition 8 to prove the corollary. □

J Proof of Proposition 9

Let $d(x) \equiv \mathbb{E}[\gamma|\gamma > x] - x = \int_x^{\bar{\gamma}} \frac{1-F(\gamma)}{1-F(x)} d\gamma$. The following lemma is useful:

Lemma 4. *If $f(x)$ is non-decreasing, then $g(x) \equiv \frac{d(x)}{1-F(x)}$ is such that $g(x) \leq \frac{1}{2f(x)}$*

Proof. Note that

$$g'(x) = \frac{d'(x)}{1-F(x)} + \frac{d(x)}{1-F(x)}\frac{f(x)}{1-F(x)} = \frac{g(x)f(x) - 1}{1-F(x)} + \frac{g(x)f(x)}{1-F(x)} = \frac{2g(x)f(x) - 1}{1-F(x)}$$

where we used that $d'(x) = -1 + d(x)\frac{f(x)}{1-F(x)}$. We also know that $\lim_{x\to\bar{\gamma}} g(x) = \frac{1}{2f(\bar{\gamma})}$ (which follows from applying L'Hopital's rule on $d(x)/(1-F(x))$). From the ODE it follows then that if $g(x_0) > \frac{1}{2f(x_0)}$ for some x_0 then $g'(x_0) > 0$ and given that $f(x)$ is non-decreasing, this implies that $g(x) > \frac{1}{2f(x_0)} \geq \frac{1}{2f(\bar{\gamma})}$ for all $x > x_0$, which is a contradiction of the limit condition. □

Now we are ready to prove Proposition 9. Note that equation (30) can be written as

$$G(\gamma^c) = \mathbb{E}[\gamma|\gamma > \gamma^c] - \mathbb{E}[\gamma|\gamma > \gamma_H] - \tilde{\kappa}(\gamma^c - \gamma_H) \leq 0$$

for all $\gamma \geq \gamma_H$ by using (28). Note that $G(\gamma_H) = 0$ and

$$G'(\gamma^c) = \frac{d}{d\gamma^c}\left(\mathbb{E}[\gamma|\gamma > \gamma^c]\right) - \tilde{\kappa} = d'(\gamma^c) + 1 - \tilde{\kappa} = \frac{d(\gamma^c)f(\gamma^c)}{1 - F(\gamma^c)} - \tilde{\kappa} \leq \frac{1}{2} - \tilde{\kappa}$$

where the last inequality follows from f non-decreasing and Lemma 4. Letting $\tilde{\kappa} \geq \frac{1}{2}$ implies that $G'(\gamma^c) \leq 0$ for all $\gamma^c > \gamma_H$ which proves that $G(\gamma^c) \leq 0$ for all $\gamma^c > \gamma_H$.

Now, we just need to check that H is non-decreasing, as defined in equation (29). Note that we can rewrite H as:

$$\begin{aligned} H(\gamma) &= \tilde{\kappa}F(\gamma) - \int_{\underline{\gamma}}^{\gamma} v'(\pi_f(\tilde{\gamma}))df(\tilde{\gamma}) - \int_{\underline{\gamma}}^{\gamma} f(\tilde{\gamma})v''(\pi_f(\tilde{\gamma}))\pi_f'(\tilde{\gamma})d\tilde{\gamma} - v'(\pi_f(\underline{\gamma}))f(\underline{\gamma}) \\ &= \int_{\underline{\gamma}}^{\gamma} f(\tilde{\gamma})\Big(\tilde{\kappa} + \frac{v''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))}\Big)d\tilde{\gamma} + \int_{\underline{\gamma}}^{\gamma} (-v'(\pi_f(\tilde{\gamma})))df(\tilde{\gamma}) - v'(\pi_f(\underline{\gamma}))f(\underline{\gamma}) \end{aligned}$$

where we used that $\pi_f'(\gamma) = -1/b''(\pi_f(\gamma))$.

By the hypothesis that $\tilde{\kappa} \geq 1/2$, it follows that

$$\frac{v''(\pi_f(\tilde{\gamma})) + b''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))} \geq \tilde{\kappa} \geq \frac{1}{2} \Rightarrow \frac{v''(\pi_f(\tilde{\gamma}))}{b''(\pi_f(\tilde{\gamma}))} + \tilde{\kappa} \geq 2\tilde{\kappa} - 1 \geq 0$$

And thus the first integral above is increasing in γ. The second integral is also increasing in γ as $-v' \geq 0$ and f is non-decreasing. It follows that H is the sum of a constant plus two non-decreasing functions in γ, so H is also non-decreasing. Thus, we have proved that the conditions of Proposition 2 hold.

K Proof of Corollary 5

The following Lemma will be used:

Lemma 5. *In the linear-quadratic case, if $\widetilde{\kappa}F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing for all $\gamma \in \Gamma$, then (29) and (30) are satisfied.*

Proof. Let $X(\gamma) = (1 - F(\gamma))G(\gamma)$. Then we can show that

$$X'(\gamma) = -\widetilde{\kappa} + \widetilde{\kappa}F(\gamma) - [v'(\pi_f(\gamma_H)) + (1 - \widetilde{\kappa})(\gamma - \gamma_H)]f(\gamma).$$

In the linear-quadratic case, we have that $v'(\pi_f(\gamma)) = v'(\pi_f(\gamma_H)) + (1 - \widetilde{\kappa})(\gamma - \gamma_H)$, and thus

$$X'(\gamma) = -\widetilde{\kappa} + \widetilde{\kappa}F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$$

which is non-decreasing by the hypothesis of the lemma. This implies then that $X(\gamma)$ is a convex function of γ. Note that $X(\overline{\gamma}) = 0$ and $X'(\overline{\gamma}) = -v'(\pi_f(\overline{\gamma}))f(\overline{\gamma}) > 0$. It then follows that $X(\gamma)$ has at most another 0 for $\gamma < \overline{\gamma}$, which corresponds to γ_H. This also implies that $X(\gamma) < 0$ for all $\gamma \in (\gamma_H, \overline{\gamma})$ and thus $G(\gamma) < 0$ as well, which proves (30). The hypothesis of the lemma directly implies (29). □

To prove Corollary 5, we recall that $\mathbb{E}\gamma > [7 + 8\underline{\gamma}]/12$ ensures that γ_H is interior. We thus just need to show that $\kappa F(\gamma) - v'(\pi_f(\gamma))\gamma)f(\gamma)$ is non-decreasing in $\gamma \in \Gamma$ and invoke Lemma 5. Assuming differentiability of f, and using that in our example $\widetilde{\kappa} = 2/3$ and that $v''/b'' = -1/3$, we get that $\widetilde{\kappa}F(\gamma) - v'(\pi_f(\gamma))f(\gamma)$ is non-decreasing if

$$\frac{2}{3}f(\gamma) + \frac{v''(\pi_f(\gamma))}{b''(\pi_f(\gamma))}f(\gamma) - v'(\pi_f(\gamma))f'(\gamma) \geq 0,$$

or equivalently $\frac{1}{3}f(\gamma) - v'(\pi_f(\gamma))f'(\gamma) \geq 0$, the condition stated in Corollary 5.

L An Example with Endowment and Logarithmic Utility

In what follows we develop an endowment example and show that Proposition 9 allows us to characterize the optimal trade agreement. Assume that $u(c) = \log(c)$ and that $Q(p) = 1$ and $Q_\star(p) = A$ where $A > 1$. Then we can write that:

$$B = -p - p_\star z - \log(p); \qquad V = p_\star z - \log(p_\star); \qquad \Pi = p$$

where $p = (1 + z)^{-1}$, $p_\star = (A - z)^{-1}$, and z is the volume of trade.

Note that free trade is $z = \frac{1}{2}(A - 1)$. Writing everything in terms of π delivers:

$$b(\pi) = -\pi + \frac{\pi - 1}{(A+1)\pi - 1} - \log(\pi), \quad \text{and } v(\pi) = \frac{1 - \pi}{(A+1)\pi - 1} - \log\left(\frac{\pi}{(A+1)\pi - 1}\right)$$

and where $z = \frac{1}{\pi} - 1$.

The free trade allocation corresponds to $\pi^{ft} = \frac{2}{1+A}$. Zero trade corresponds to $\bar{\pi} = 1$. We will restrict attention to a set of admissible $\pi \in [\pi^{ft}, 1]$, which is equivalent to restricting tariffs to be non-negative.

Note that $v'(\pi) = \frac{\pi - 1}{\pi((A+1)\pi - 1)^2} \leq 0$ and note as well that:

$$b''(\pi) = \frac{1}{\pi^2} - \frac{2A(1+A)}{((A+1)\pi - 1)^3}$$

which is negative for all $\pi \in [\pi^{ft}, 1]$ if $1 \leq A < 1 + \sqrt{3}$. Similarly one can show that:

$$v''(\pi) + b''(\pi) = \frac{2 + (A+1)\pi((A+1)\pi - 4)}{\pi^2((A+1)\pi - 1)^2},$$

from which it follows that $w(\gamma, \pi)$ is concave in π for all $\pi \in [\pi^{ft}, 1]$ if $1 \leq A \leq 1 + \sqrt{2}$. This last condition guarantees that Assumption 1 is satisfied.

Using the above, the value of κ can be found to be:

$$\tilde{\kappa} = \begin{cases} \frac{2(A+1)}{7A-1} & ; \text{for } 1 \leq A \leq \frac{4+\sqrt{41}}{5} \\ \frac{-1-2A+A^2}{-2-2A+A^2} & ; \text{for } \frac{4+\sqrt{41}}{5} \leq A \leq 1 + \sqrt{2} \end{cases}$$

Note that $\tilde{\kappa} \geq 0$ for all $A \in [1, 1+\sqrt{2}]$. Also for A close to 1, $\tilde{\kappa} \approx 2/3$, which implies that we can apply Proposition 9, which requires $\kappa \geq 1/2$, and show that a tariff cap is optimal for distributions with non-decreasing densities when A is close to 1.

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