Risk Aversion Heterogeneity, Risky Jobs and Wealth Inequality*

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Abstract

This paper considers the macroeconomic implications of a set of empirical studies finding a high degree of dispersion in preference heterogeneity. It develops a model with both uninsurable idiosyncratic income risk and risk aversion heterogeneity to quantify their effects on wealth inequality. The results show that with the available estimates of the risk aversion distribution from PSID data the model can match the observed degree of wealth inequality in the U.S., accounting for the wealth Gini index in several cases. The model replicates well several features of the wealth distribution. However, the share of wealth held by the top 1% is still substantially underestimated. It is also shown that models without risk aversion heterogeneity underestimate the size of precautionary savings, and that the results are robust to both different income process specifications and to self-selection into risky jobs.

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1 Introduction

How much do observable preference heterogeneity in risk aversion and labor income risk account of the U.S. wealth inequality? This paper addresses this question within a macroeconomic model with incomplete markets, heterogeneous agents and (with or without) self-selection into risky jobs.

It is a well known fact that wealth is highly concentrated in the U.S., its Gini index being estimated in the $0.78-0.82$ range for the $1992-2007$ period. At the same time, income and labor earnings are less unequally distributed, as discussed by Wolff (1998), Cagetti and De Nardi (2008), and Diaz-Gimenez, Glover, and Rios-Rull (2011).

The wealth distribution and its determinants have important implications for capital accumulation and growth, the design of optimal taxation schemes and their welfare consequences. These issues have been studied, for example, by Imrohoroglu (1998), Ventura (1999), Heathcote (2005), and Conesa, Kitao and Krueger (2009).

The role of entrepreneurship, Quadrini (2000) and Cagetti and De Nardi (2006), uninsurable income risk, Castaneda, Diaz-Gimenez, and Rios-Rull (2003), and intergenerational links, De Nardi (2004), have been found to be key in explaining the high concentration of wealth.[1] One aspect that has not been fully explored in accounting for wealth inequality is the role of preference heterogeneity. This is interesting, particularly in light of the findings of a few recent studies trying to elicit individuals' preferences and determine some measures of their dispersion.

This paper takes these empirical results seriously and quantifies how much wealth inequality is accounted for by agents' heterogeneity in their risk aversion. An otherwise standard heterogeneous agents macroeconomic model with incomplete markets and uninsurable idiosyncratic income risk is extended to allow for risk aversion heterogeneity. The framework is used to compute the effects of the latter on measures of wealth inequality and, more generally, on the overall wealth distribution, with or without endogenous sorting into jobs that differ in their implied labor income risk.

A recent body of empirical research has aimed at estimating individuals' risk aversion. These contributions have relied on either special modules included in large and well established surveys, or on experimental set-ups. Surveys such as the Health and Retirement Study (HRS) and the Panel Study of Income Dynamics (PSID) for the U.S. (Barsky, Juster, Kimball, and Shapiro (1997), Kimball, Sahm, and Shapiro (2008), and Kimball, Sahm, and Shapiro (2009)), and the Survey of Household Income and Wealth (SHIW) for Italy (Chiappori and Paiella (2011)), have been used to provide estimates of the risk aversion distributions for the underlying populations. In the HRS and, to a less

[1] There is a vast literature developing models that try to account for the dispersion of wealth, reviewed in Quadrini and Rios-Rull (1997), and more recently by Cagetti and De Nardi (2008), and Heathcote, Storesletten and Violante (2009).

extent, in the PSID the respondents have been asked to answer a sequence of questions describing hypothetical lotteries on their *lifetime* income. Under appropriate assumptions, these answers provide a direct measure of the respondent's risk aversion. Differently, in the SHIW it is possible to exploit the households' portfolio composition and, crucially, its change over time (which is a rare feature for large surveys) to identify the distribution of risk aversion. In this case the empirical methodology does not measure preferences directly, rather it backs them out from actual intrinsically risky economic choices. The alternative experimental set-ups have been conducted either in a lab environment with a limited number of Danish individuals, Harrison, Lau, and Rutstrom (2007), or with an on-line interface targeting a large set of Dutch households, von Gaudecker, van Soest, and Wengstrom (2011). These experiments were designed in order for the participants to win or lose some *small* monetary stakes, by selecting the lottery they preferred from a menu of options that differed in their degree of risk.

The possible studies' limitations notwithstanding, a common and seemingly robust finding is the high dispersion of people's attitude towards risk. Both the survey and the experimental approaches to measuring preference dispersion have found considerable heterogeneity in the estimated risk aversion parameter. Figures 1 in Harrison, Lau, and Rutstrom (2007) and Kimball, Sahm, and Shapiro (2008), Figures 4 and 6 in von Gaudecker, van Soest, and Wengstrom (2011), and Figure 2 in Chiappori and Paiella (2011) provide compelling evidence on this result.

Identifying and estimating the risk aversion distribution is a challenging procedure. Pervasive measurement error and limited stakes in the experimental lotteries are unavoidable obstacles that make it a very hard empirical problem. However, in order to avoid some potential drawbacks of the analysis that will be discussed below, it is preferable to rely on sources of risk aversion heterogeneity that are empirically grounded, rather than treating it as unobserved heterogeneity.

First, I am going to consider a model where all individuals face the same stochastic income process. In this framework, three different specifications for preference heterogeneity are going to be proposed. The first case (KSS) will rely on the preference distribution computed in Kimball, Sahm, and Shapiro (2009), which estimates the distribution of risk tolerance from PSID data. Since this specification could potentially lead to counterfactual predictions on the implied consumption growth rate, a second specification (LN) will implicitly assume that the PSID respondents overestimate the amount of risk implied by the lotteries they are facing in the questionnaires. This alternative distribution of preferences is going to exploit the same dispersion found in the PSID data, while matching the available estimates for the (median) elasticity of intertemporal substitution. Finally, the third case (CP) will make use of the estimates on some moments of the risk aversion distribution provided by Chiappori and Paiella (2011), who rely on the SHIW, a panel dataset that tracks a representative sample of

Italian households. This additional case is considered because their estimated distribution of risk preferences differs from the others in some essential features.

Not only the individuals' attitude towards risk is an important ingredient in shaping the consumption and saving decisions, but also the amount of risk faced in the economy will affect their behavior. For all cases, several assumptions on the parameters representing the stochastic process for labor earnings are going to be considered. The results seem to be robust to several specifications.

Interestingly, with the available estimates of the risk aversion distribution, the model can match some salient features of the U.S. wealth distribution, while missing others. Compared to a model without preference heterogeneity, the degree of wealth concentration increases substantially. For a standard homogeneous preferences model the wealth Gini index lies in the $0.42 - 0.55$ range (depending on the assumed stochastic process for income risk), while it increases by no less than 20 points in the heterogenous preferences model. With a highly persistent income process, the three risk preference specifications lead to a Gini index ranging from 0.75 to 0.80, which is extremely close to what is observed in the data. Although this result is quite robust to the income process parameters, substantially less persistent processes imply a lower wealth concentration. In quite an extreme case, an income risk process that postulates heterogenous income profiles (an element that is not included in the model), the Gini index ranges from 0.65 to 0.70.

When focusing on the same income process, the three risk aversion distributions show results in terms of wealth concentration and shares of wealth held by a set of quantiles that are remarkably similar. Several features of the wealth distribution are replicated well, such as the bottom and top quintiles, unlike the share of wealth held by the top 1%, which is substantially underestimated in all specifications. Given that the wealthiest households in the U.S. hold approximately one third of the total wealth, this is an important finding. Preference heterogeneity alone does not fully account for the determination of the top of the wealth distribution.

Some additional results show that precautionary savings are underestimated in models without preference heterogeneity, with the bias being between 5.7 and 9.2 percentage points. Furthermore, considering the homogeneous preferences counterpart of the heterogeneous preferences economy does lead to equilibria and allocations that are quite different. The interest rate in the heterogeneous preferences economies is between 0.35 and 0.46 percentage points lower, leading always to larger capital stocks and output.

Although convenient, the assumption that individuals do not sort themselves into jobs that differ in their income volatility according to their preference for risk seems to be strong, and at odds with some recent evidence discussed in Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011). However, in order to carefully study the issue, data on how many potential careers are available in the economy for the workers to choose would be needed. The PSID data cannot provide this information, but they can be used to analyze how people's actual earnings instability changes with their risk aversion. This paper also provides a first attempt to tackle this complex empirical issue. It will be assumed that only two different careers are available for every worker to embark on, whose entailed risks are perfectly known to the workers before they enter the labor market. The workers are going to sort themselves into the two different jobs, by choosing one of the two different stochastic processes.

A calibrated version of the model with endogenous sorting confirms all the findings obtained in the simpler version of the model, with the results being quantitatively very similar.

1.1 Related literature

In a seminal paper, Becker (1980) showed that in a deterministic growth model with infinitely lived agents that differ in their discount factors, the whole capital stock is held by the most patient dynasty. However, this result is mainly driven by the absence of uncertainty and by the assumption of complete markets.

There is limited quantitative work on the aggregate effects of preference heterogeneity in stochastic growth models. Notable exceptions are Krusell and Smith (1998), Cagetti (2003), Coen-Pirani (2004), Guvenen (2006), and Hendricks (2007). However, there are a few differences between my framework and theirs.

Krusell and Smith (1998) consider an RBC model with heterogenous agents. They propose two versions of their model, one with preference heterogeneity and one without. Limited ad-hoc heterogeneity in the discount factors across generations is shown to have a major impact on wealth inequality, with the model being able to account for the observed Gini index only in this case.

Cagetti (2003) and Hendricks (2007) propose life-cycle models with incomplete markets, stochastic incomes, but without aggregate uncertainty. They focus on preference heterogeneity arising from differences in discount factors, and Cagetti (2003) allows for (some) heterogeneity in the risk aversion parameter as well. A structural estimation/calibration procedure is implemented to match a set of moments computed from the PSID and SCF samples. Cagetti (2003) estimates the preference parameters by matching the median age/wealth profile for three educational groups. However, he considers a partial equilibrium model. Hendricks (2007) picks the discount factors and their distribution to match the wealth Gini index profiles over the life-cycle. Their findings show that this type of preference heterogeneity accounts for the dispersion in wealth holdings of observationally equivalent households, and matches the high concentration of wealth.

Coen-Pirani (2004) and Guvenen (2006) study economies with aggregate fluctuations, multiple assets (a risk free asset and a risky one) and with a recursive utility framework. Their agents can potentially differ in both their risk aversions and in their Elasticities of Intertemporal Substitution (EIS). Although interesting, I don't follow this approach, because there are no reliable estimates for the EIS distribution for the overall U.S. population.[2] Coen-Pirani (2004) considers an endowment economy with preference heterogeneity only in the risk aversion parameter: his findings show that, contrary to the results obtained using standard expected utility, for some parameter values the long run distribution of wealth is dominated by the more risk averse agents. Guvenen (2006) proposes an RBC model with heterogenous EIS reconciling the findings of empirical studies using aggregate consumption data that estimate low EIS, and those of calibrated models designed to match growth and fluctuations facts that require a higher EIS. His results arise because of limited participation in stock markets together with an EIS increasing in wealth. The EIS estimated on simulated aggregate consumption data is small, because it reflects the EIS of the majority of the households, that are asset poor.

Compared to the literature, I allow for a form of preference heterogeneity which is empirically grounded, for GE effects, and for endogenously chosen risky careers. However, for tractability, I do not consider the effect of aggregate shocks.

In the empirical literature, Lawrance (1991) and Vissing-Jorgensen (2002) provide estimates of the consumption Euler equations that find heterogeneity in time preferences and in the EIS. Furthermore, the results in Chiappori and Paiella (2011) support the notion that individuals' relative risk aversion is constant: they find no significant response of the portfolio structure to changes in financial wealth.

Finally, at a more micro-level, Mazzocco and Saini (2010) test for efficient risk sharing when households have heterogeneous risk preferences. Relying on data for rural India, they strongly reject the hypothesis of identical risk preferences, and argue that risk-sharing is carried out at the caste and not at the village level.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 is devoted to the definition of the equilibrium concept used in the model. Section 4 presents the calibration procedure. Section 5 provides the main results and predictions of the baseline model, while Section 6 is devoted to the extension with endogenous sorting into risky jobs. Section 7 concludes. A set of appendices discuss the details of the numerical methods and provide some additional results.

[2] The results discussed in Barsky, Juster, Kimball, and Shapiro (1997) are based on a very small sample, and apply to a selected sample, namely people in the later stages of their lives.

2 The Economy

2.1 Risk Aversion Heterogeneity and Inequality: the Mechanics

The intuition behind this paper is easily understood with a simple graph.

[Figure 1 about here]

Figure 1 compares the endogenous wealth distributions that would arise in two different economies. Both economies share the same features, namely incomplete markets, an occasionally binding exogenous borrowing constraint and uninsurable labor income risk, as in Aiyagari (1994). However, they differ in one aspect: in the first economy all agents share the same risk aversion parameter γ, while in the second economy there are both high risk aversion types (γ_h), and low risk aversion ones (γ_l).[3] Figure 2 displays utility functions of the CRRA class for different degrees of constant relative risk aversion γ: the higher γ, the more concave the utility function and the stronger the precautionary savings motive (Huggett and Ospina (2001)).

[Figure 2 about here]

For the sake of the argument, let's assume that both types have the same mass $\mu_{\gamma_h} = \mu_{\gamma_l} = \frac{1}{2}$, and that the types are permanent, meaning that there is no evolution in their risk aversion.

In the first economy the wealth distribution is non degenerate because agents want to self-insure against the future risk represented by income fluctuations. This leads to a wealth distribution which could be represented as the one in the middle of Figure 1. Asset rich agents are the ones who experienced a long sequence of good income shocks, unlike the asset poor ones, allowing them to pile up a large stock of wealth. In the second economy, agents face the same uncertainty, i.e. the same stochastic process for labor income. However, the endogenous wealth distribution is going to change from the first economy: low risk averse types have a lower desire to smooth consumption across states of the world, hence they are going to accumulate less assets for self-insurance purposes.

[3] The analysis will rely on utility functions of the CRRA class. This choice is dictated by the set of results discussed in Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009), obtained assuming this type of preferences, and in Chiappori and Paiella (2011), who directly test this assumption. Notice that the first two papers provide estimates for the risk tolerance parameter τ, not the risk aversion γ. However, there is a simple relationship between the two: $\gamma = \frac{1}{\tau}$. Consequently, with CRRA utility, risk tolerance and elasticity of intertemporal substitution coincide. Moreover, recall that when a random variable is lognormally distributed, its reciprocal is lognormally distributed as well, with the same parameter σ_γ^2 but opposite μ_γ: $\gamma \sim LN\left(\mu_\gamma, \sigma_\gamma^2\right) \rightarrow \tau = \frac{1}{\gamma} \sim LN\left(-\mu_\gamma, \sigma_\gamma^2\right)$.

Differently, the more risk averse agents are going to save more, in order to achieve a better consumption smoothing. If the two agent types were to live in isolation, their wealth distributions would look like the ones in Figure 1. The wealth distributions of the two types are still non-degenerate, because labor income risk is still present and with concave utility functions some self-insurance is going to take place. In the absence of General Equilibrium (GE) effects, the economy would display a wealth distribution represented by the mixture of the two underlying ones. An outcome is clear: moving from the first economy to the second one, wealth inequality is going to increase, because the supports of the heterogenous types wealth distributions will start diverging. The higher the difference in their risk aversion, the stronger the tendency for the two groups to accumulate different wealth levels.

However, the economy with heterogenous types doesn't necessarily aggregate into its homogeneous types counterpart, with the average (or median) risk aversion: risk aversion heterogeneity is likely to lead to different aggregate capital supplies, triggering GE effects. A strong enough precautionary saving motive of the high risk aversion types leads to an increase in the aggregate capital supply, driving down the equilibrium interest rate. As a consequence, the saving motive for intertemporal reasons is reduced, because of the decreased rate of return: this GE effect leads to a further change in the wealth inequality.

This simple example explains the mechanics of how wealth inequality responds when preference heterogeneity in risk aversion is included in the framework. However, this also raises a potential problem. With enough flexibility, virtually any degree of wealth inequality (far enough from perfect equality) can be achieved. By picking appropriately the degree of uncertainty, the risk aversion parameters, and the distribution of types any degree of wealth concentration can be obtained, a point made by Quadrini and Rios-Rull (1997).

By way of an example, let's consider the case of maximum degree of inequality, that is a wealth Gini coefficient that is arbitrarily close to 1. In order to achieve this outcome, it is sufficient to impute a high degree of risk aversion for the high types, say $\gamma_h = 10$, and assign them just an almost negligible positive mass $\mu_{\gamma_h} = \epsilon_\mu > 0$. At the same time, the low types are assigned a small degree of risk aversion, say $\gamma_l = \epsilon_\gamma > 0$, and a mass $\mu_{\gamma_l} = 1 - \epsilon_\mu \approx 1$. This economy would display a wealth Gini coefficient close to 1, as the high degree of risk aversion for the high types would lead them to accumulate a lot of assets and at the same time with little dispersion, while the low types with preferences close to linear would not engage in extensive precautionary savings.

Given these considerations, the computational experiment carried out in this paper could be flawed. Treating preferences as unobserved heterogeneity can potentially lead to identification issues. Different preference distributions, with different implied equilibria, could match the very same moments. Moreover, considering preference heterogeneity as parametric unobserved heterogeneity, whose parameters are pinned down by a set of moments in the wealth distribution, can lead to some unpleasant implications. According to such theories, trends and fluctuations in the wealth distribution could be explained by changes in preferences. A by-product of this approach is that it could lead to virtually impossible relative assessments of different policy interventions, and their welfare implications. Unless the researcher knows how preferences evolve over time, and what triggers their change, welfare analysis is not a viable option.

In order to avoid the potential drawbacks of this framework, a lot of discipline is imposed in the model's parameterization, in order to avoid issues of the type mentioned above.

As for the amount of uncertainty that the agents are going to face, several estimates for wage processes that are routinely used in the quantitative literature on macroeconomics with heterogenous agents are going to be considered. If the wage processes are truly exogenous with respect to the agents' degree of risk aversion, this step is a valid one. Since this assumption could be violated, an extension considers a model with endogenous sorting into risky jobs.

As explained above, the analysis will rely on sources of risk aversion heterogeneity that are based on a set of empirical findings trying to measure it. Although feasible, a calibration of the preference distribution by matching some features of the saving behavior (hence some moments of the wealth distribution) will not be attempted, to impose a lot of discipline in the quantitative implementation of the model.

2.2 The Baseline Model

First, a model with only one exogenous stochastic income process is going to be proposed. A simple extension with endogenous sorting into two jobs that differ in their degree of risk will follow.

2.3 Demographics

Time is discrete. The economy is populated by a measure one of infinitely lived agents (workers) denoted by i.[4]

[4] Data limitation suggests to consider infinitely lived agents. In the PSID the risk aversion question was asked in 1996 only, making it hard to know if the higher risk aversion observed for parents vs. their offspring is due to a cohort effect, or if it is an intrinsic demographic trait, possibly due to risk aversion increasing with age instead. Furthermore, Kimball, Sahm, and Shapiro (2009) find a positive correlation between the risk aversion of parents and that of their children.

2.4 Preferences

Agents' utility function is defined over stochastic consumption sequences c_t

$$\begin{aligned} E_0 U(c_0, c_1, ...; \gamma_i) &= E_0 \sum_{t=0}^{\infty} \beta^t u(c_t; \gamma_i) \\ u(c_t; \gamma_i) &= \frac{c_t^{1-\gamma_i} - 1}{1-\gamma_i}, i \in [0,1] \end{aligned} \tag{1}$$

the future is discounted at rate $\beta \in (0,1)$, which is the common discount factor. The per period utility $u(.)$ is strictly increasing, and strictly concave. Differently from models without preference heterogeneity, it depends explicitely on the risk aversion parameter γ_i. Every agent i is born with an innate attitude towards risk, as captured by their CRRA parameter, which is a permanent feature.[5]

2.5 Endowments

There is a stochastic process for the effective units of labor ε a worker is going to supply in the labor market. This process is assumed to be an exogenous continuous first order Markov process.[6]

2.6 Technology

The production side of the model is represented by a constant returns to scale technology of the Cobb-Douglas form, which relies on aggregate capital K and labor L to produce the final output Y:

$$Y = F(K, L) = K^{\alpha} L^{1-\alpha}.$$

Capital depreciates at the exogenous rate δ and firms hire capital and labor every period from competitive markets. The first order conditions of the firm provide the expressions for the net real return to capital r and the wage rate per efficiency unit w:

$$r = \alpha \left(\frac{L}{K}\right)^{1-\alpha} - \delta, \tag{2}$$

$$w = (1-\alpha) \left(\frac{K}{L}\right)^{\alpha}. \tag{3}$$

[5] In the HRS it is possible to keep track of how an individual answers the same risk aversion question over time. As shown by Kimball, Sahm, and Shapiro (2008), many people change their answers across waves, and they propose an econometric methodology addressing the survey response error. Treating the switches as measurement error is a way of rationalizing this outcome.

[6] The analysis will focus on steady states, hence from now on time subscripts will be supressed.

Notice that the marginal productivity of labor is always positive, hence firms will rely on the total sum of the efficiency units of labor. It follows that in the steady-state:

$$L = \int \varepsilon d\mu_L(\varepsilon)$$

where $\mu_L(\varepsilon)$ is the stationary distribution over the labor endowments implied by the markov process.

2.7 Other market arrangements

The final good market is competitive. Firms hire capital every period from a competitive market. Capital is supplied by rental firms that borrow from workers at the risk-free rate r and invest in physical capital.

There are no state-contingent markets to insure against income risk, but workers can self-insure by saving into the risk-free asset. The agents also face a borrowing limit, denoted as $b \geq 0$.

3 Stationary Equilibrium

First the problem of a worker and the problem of the firm are defined. The individual state variables are the labor endowment $\varepsilon \in \mathcal{E} = [0, \overline{\varepsilon}]$, and asset holdings $a \in \mathcal{A} = [-b, \overline{a}]$. Moreover, risk aversion $\gamma \in \Gamma = [\underline{\gamma}, \overline{\gamma}]$ represents a permanent state for the agents. The stationary distribution is denoted by $\mu(\varepsilon, a; \gamma)$.

3.1 Problem of the agents

This Section first defines the problem of the agents in their recursive representation, then it provides a formal definition of the recursive competitive equilibrium.

3.1.1 Problem of the workers

The value function of an agent whose current asset holdings are equal to a, whose current labor endowment is ε, and with innate risk aversion γ is denoted with $V(\varepsilon, a; \gamma)$. The problem of these agents can be represented as follows:

$$V(\varepsilon, a; \gamma) = \max_{c,a'} \left\{ u(c;\gamma) + \beta E_{\varepsilon'|\varepsilon} V(\varepsilon', a'; \gamma) \right\} \tag{4}$$

$$s.t.$$

$$c + a' = (1+r)\, a + w\varepsilon$$
$$\log \varepsilon' = \rho_y \log \varepsilon + \eta', \eta \sim iid\ N(0, \sigma_y^2)$$
$$a_0 \ given, \quad c \geq 0, \quad a' > -b$$

Agents have to set optimally their consumption/savings plans. They enjoy utility from consumption, and face some uncertain events in the future. In the next period they will still have the same risk aversion parameter, but their labor income can go up or down, depending on the future realizations of the earnings shock η.

3.2 Recursive Stationary Equilibrium

Definition 1 *A recursive stationary equilibrium is a set of decision rules* $\left\{c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma), k = \frac{K}{L}\right\}$, *value functions* $V(\varepsilon, a; \gamma)$, *prices* $\{r, w\}$ *and a set of stationary distributions* $\mu(\varepsilon, a; \gamma)$ *such that:*

- Given relative prices $\{r, w\}$, the individual policy functions $\{c(\varepsilon, a; \gamma), a'(\varepsilon, a; \gamma)\}$ solve the household problem (4) and $V(\varepsilon, a; \gamma)$ are the associated value functions.
- Given relative prices $\{r, w\}$, k solves the firm's problem (2)-(3).
- The asset market clears
$$K = \int_{\mathcal{E}\times\mathcal{A}\times\Gamma} a d\mu(\varepsilon, a; \gamma)$$
- The goods market clears
$$F(K, L) = \int_{\mathcal{E}\times\mathcal{A}\times\Gamma} c(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma) + \delta K$$
- The stationary distributions $\mu(\varepsilon, a; \gamma)$ satisfy

$$\mu(\varepsilon', a'; \gamma) = \sum_{\varepsilon} \int_{a: a'(\varepsilon, a; \gamma) = a'} \pi\left(\varepsilon', \varepsilon\right) d\mu(\varepsilon, a; \gamma), \forall \gamma \in \Gamma \tag{5}$$

In equilibrium the measure of agents in each state is time invariant and consistent with individual decisions, as given by the equation (5) above.[7]

[7] Notice that the equation already exploits the Markov Chain representation of the continuous process for ε.

3.3 Discussion

As with many other dynamic problems, the Euler equations help giving an intuition of the main intertemporal trade-offs that the agents in this economy are facing. They give a more formal argument underlying the intuition provided in Figure (1). The optimal consumption functions $c(\varepsilon, a; \gamma)$ satisfy the following optimality condition, where the equality holds whenever the agents are not borrowing constrained:

$$\begin{aligned} u_c(c(\varepsilon, a; \gamma)) &\geq \beta (1+r) E_{\varepsilon'|\varepsilon} \left[u_c(c(\varepsilon', a'; \gamma)) \right] \\ c(\varepsilon, a; \gamma)^{-\gamma} &\geq \beta (1+r) E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma)^{-\gamma} \right] \end{aligned}$$

$c(.)^{-\gamma}$ is a convex function, hence $E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma)^{-\gamma} \right] \geq \left\{ E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma) \right] \right\}^{-\gamma}$ because of Jensen's inequality. In order to take care of this property, it is always possible to define a positive number $\eta(\gamma)$ such that $E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma)^{-\gamma} \right] = \left\{ E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma) \eta(\gamma) \right] \right\}^{-\gamma}$. More precisely, the CRRA parameter affects the strenght of Jensen's inequality, and for any γ there is an associated $\eta(\gamma)$ satisfying $0 < \eta(\gamma) < 1$ and $\frac{d\eta(\gamma)}{d\gamma} < 0$. Substituting this expression in the optimality condition and rearranging gets:

$$\frac{E_{\varepsilon'|\varepsilon} c(\varepsilon', a'; \gamma)}{c(\varepsilon, a; \gamma)} \geq \frac{\left[\beta (1+r) \right]^{\frac{1}{\gamma}}}{\eta(\gamma)}.$$

The LHS represents the expected consumption growth, which depends on β, γ and r. It is easy to show that, for a given interest rate, the expected consumption growth is increasing in both the discount factor and the risk aversion.[8] The higher γ, the more convex the function $c(.)^{-\gamma}$, and the lower the number $\eta(\gamma)$, which increases the expected consumption growth because of the increased precautionary savings, which at the same time is highly non-linear in γ. This property can rationalize a result that will be discussed later. Krusell and Smith (1998) found that approximate aggregation holds in their economy even in the case with heterogeneous β's. In the economy considered here, this will not be the case. The aggregate allocations of the heterogeneous preferences economy are going to be quantitatively quite different from the ones of the homogeneous preferences economy. An explanation could be the following. With the calibration used in Krusell and Smith (1998), i.e. $\gamma = 1$,

[8] Notice that $\frac{dLHS}{d\beta} = \frac{[\beta(1+r)]^{\frac{1}{\gamma}}}{\beta\gamma\eta(\gamma)} > 0$ and $\frac{dLHS}{d\gamma} = -\frac{[\beta(1+r)]^{\frac{1}{\gamma}} \ln[\beta(1+r)]}{\gamma^2 \eta(\gamma)} - \frac{[\beta(1+r)]^{\frac{1}{\gamma}} \eta'(\gamma)}{[\eta(\gamma)]^2} > 0$. The sign of the last expression is obtained by recognizing that in incomplete markets economies $\beta(1+r) < 1$, and $\ln[\beta(1+r)] < 0$. Moreover $\frac{d\eta(\gamma)}{d\gamma} < 0$, given that $\eta(\gamma) = \frac{\left\{ E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma)^{-\gamma} \right] \right\}^{-\frac{1}{\gamma}}}{E_{\varepsilon'|\varepsilon} \left[c(\varepsilon', a'; \gamma) \right]}$ and thanks to the properties of generalized means $M_{(.)}$ for non-negative random variables. The numerator of $\eta(\gamma)$ is the generalized mean of order $-\gamma$ while the denominator is the mean of order 1, and generalized means are such that $M_{(p)} < M_{(q)}$, for $p < q$. Finally, $\eta(\gamma)$ is bounded above by 1 because $\gamma > 0$, and it is bounded below by 0 because $M_{-\infty} = \min c(.) \geq 0$.

expected consumption growth is linear in β for unconstrained individuals, who are almost all the agents in the economy. As a consequence, the aggregate allocations of the heterogeneous preferences economy are going to be extremely close to the ones of the homogeneous preferences one. This result does not hold for an economy where the heterogeneity pertains the γ's in which, because of the non-linearity, the average of all type-γ consumption growth can be pretty far from the consumption growth of the average γ.

4 Parameterization

This section describes the calibration strategy. The length of the model period is set to one year.

The calibration starts by taking as given the two elements characterizing the uncertainty in this economy: how volatile and persistent agents' labor incomes are and how agents relate to the uncertainty arising from the postulated stochastic income process. Several cases are going to be considered and discussed in the following.

The remaining parameters are set such that in the steady state equilibrium the incomplete markets economy matches some characteristics of aggregate level data. It is worth stressing that no parameters are chosen to match selected features of the wealth distribution. The complete parameterization of the model is reported in Table 1.

[Table 1 about here]

4.1 Parameterizing Uncertainty

Uncertainty plays a double role in the model. On the one hand the agents are going to face stochastic income sequences with a given persistence and variance, on the other hand they are going to react to these possible income histories differently, according to their innate CRRA parameter.

4.1.1 Income Uncertainty

For each risk aversion distribution, four different specifications of the exogenous process for the labor efficiency endowments are going to be considered. These are reported in Table 2.

[Table 2 about here]

The stochastic process for labor earnings is a key element in the analysis. Different degrees of uncertainty, both in terms of shocks size and how persistent these shocks are, matter for the incentives to save and for the degree of wealth inequality. In order to perform some robustness checks and to gauge how much earnings uncertainty matters for the results, four different processes are considered. These are all similar in their econometric specification: they all share the same AR(1) process, which has two parameters. The persistence parameter is denoted by ρ_y, while the variance of the innovations by σ_y^2. Table 2 outlines the complete parameterizations of the earnings processes, together with some statistics and outcomes implied by them, the Gini index and the Coefficient of Variation of labor earnings in particular.[9]

The label *FREN* in Table 2 refers to the specification estimated on PSID data by French (2005), while the label *FLIN* refers to the specification estimated by Floden and Linde (2001). Finally, *GRIP* and *GHIP* are the two estimates provided by Guvenen (2009). The former refers to the Restricted Income Profile case, while the latter to the Heterogenous Income Profiles, in his terminology.

These four processes were selected because they imply quite different persistences and variances of the innovations: ρ_y ranges from 0.82 in the *GHIP* case, to 0.988 in the *GRIP* case, while σ_y ranges from 0.12 in the *FREN* case, to 0.21 in the *FLIN* case.

4.1.2 Risk Aversion Distribution

As for the risk aversion distributions, three different specifications are going to be considered. These are reported in Table 3.

[Table 3 about here]

First, the distribution for the risk aversion parameter estimated by Kimball, Sahm, and Shapiro (2009) is going to be used in the model, and this case will be denoted as KSS. Under the assumption that the risk preference parameter is lognormally distributed in the population, $\gamma \sim LN\left(\mu_\gamma, \sigma_\gamma^2\right)$, the two parameters' point estimates are $\mu_\gamma = 1.05$ and $\sigma_\gamma^2 = 0.76$.[10]

[9] Tauchen's discretization of the AR(1) process implies a stationary distribution of the markov chain that is symmetric. This is why the values for $\mu_L(\varepsilon_7) - \mu_L(\varepsilon_{11})$ are not reported.

[10] Tables 3 and 4 in Kimball, Sahm, and Shapiro (2008) show the estimates for these two parameters with HRS data: σ_γ is drastically reduced (from 1.76 to 0.73) when correcting the estimator for response error. In the PSID it is possible to apply the same correction procedure only by using some information from the HRS data, because the risk aversion question was asked only once. As for the other parameter, the value of $\mu_\gamma = 1.98$ is substantially larger in the HRS. These alternative estimates, in turn, lead to first order effects on the wealth Gini index. However, the HRS respondents are not a representative sample of the overall economically active U.S. population. Exploiting these estimates for the whole U.S. economy could lead to large biases.

Although Kimball, Sahm, and Shapiro (2008) and Kimball, Sahm, and Shapiro (2009) implemented a sophisticated econometric technique, inferring a continuous preference distribution from a limited sequence of lotteries and implied outcomes is a challenging task. Many problems can potentially bias the results. For example, a parametric assumption could give too little flexibility, possibly forcing the tails to be too fat or too thin. Or, more fundamentally, the survey's respondents might not be able to assess accurately the amount of risk involved by the lifetime lotteries they are facing in the questionnaires. This could lead to a systematic overestimate of the risk parameter.

With these caveats in mind, a finding based on the PSID (and HRS) data is that people show very high risk aversions. According to the parameters' estimates, there are very few individuals whose γ is below 1, with the average CRRA being 4.2. More in detail, 11.4% of the population has $\gamma \leq 1$, 22.9% has $\gamma \leq 1.5$, 34.1% has $\gamma \leq 2$, and 74.0% has $\gamma \leq 5$. With the class of preferences assumed in this paper, this result can lead to a counterfactual implication. The implied elasticity of intertemporal substitution can be at odds with the values typically estimated in the literature, and surveyed by Attanasio and Weber (2010). Although the average EIS, equal to 0.51, represents a value consistent with the empirical findings, the median EIS, equal to 0.35, seems to be low. According to such a low EIS, if the economy were to be considered out of the steady-state, the aggregate consumption growth rate would be too small.

In an attempt to tackle this issue, and to take into consideration the available empirical evidence on the EIS, a different approach is taken. The overall idea is to use the available estimates of the EIS and at the same time exploit the preference dispersion found in the PSID responses. Implicitly, this approach takes a stand on the information contained in the PSID data. It provides valuable information on the variance, but it is as if the respondents systematically overestimated the amount of risk they were subject to. More in detail, it is still assumed that the risk aversion parameter is lognormally distributed in the population, but an alternative parameterisation is considered. This case is denoted as LN in Table 3.

With two parameters to be pinned down, two moments are needed. The Log-normality assumption is particularly useful because the following formulas for the median and variance of the random variable γ apply:

$$Median\left[\gamma\right] \equiv \gamma^{Med} = \exp\left\{\mu_{\gamma}\right\} \rightarrow \frac{1}{\gamma^{Med}} = \exp\left\{-\mu_{\gamma}\right\} = EIS^{Med} \tag{6}$$

$$Var\left[\gamma\right] \equiv \gamma^{Var} = \exp\left\{2\mu_{\gamma} + 2\sigma_{\gamma}^{2}\right\} - \exp\left\{2\mu_{\gamma} + \sigma_{\gamma}^{2}\right\} \tag{7}$$

From equation (6) it follows that data on the median EIS (EIS^{Med}) uniquely identify the parameter μ_{γ}. When $EIS^{Med} = 0.5$, it follows that $\widehat{\mu}_{\gamma} = -\ln(EIS^{Med}) = 0.69$. With an estimate of the parameter μ_γ, and with information on the CRRA variance, it is possible to uniquely identify σ^2_γ as well. Rearranging (7) gets:

$$\exp\left\{2\sigma^2_\gamma\right\} - \exp\left\{\sigma^2_\gamma\right\} = \frac{\gamma^{Var}}{\exp\left\{2\widehat{\mu}_\gamma\right\}} \tag{8}$$

Equation (8) is a non-linear equation in σ^2_γ, which admits a unique positive root $\widehat{\sigma}^2_\gamma$. Using the value of $\gamma^{Var} = 19.7$ found in the PSID, together with $\widehat{\mu}_\gamma = 0.69$, give $\widehat{\sigma}^2_\gamma = 1.02$.

Finally, a third specification denoted as CP will make use of the estimates on some moments of the risk aversion distribution provided by Chiappori and Paiella (2011), because it differs in some crucial aspects from the other two cases. This study does not rely on U.S. data, but on the SHIW, a panel dataset that tracks a representative sample of Italian households. The authors back out the distribution of the risk aversion parameter from actual choices on the portfolio composition. They report two statistics: the median risk aversion ($\gamma^{Med} = 1.7$), and the third quartile ($\gamma^{0.75} = 3.0$). These two pieces of information are used to identify a parametric distribution. A $Beta\left(\mu_\gamma, \sigma_\gamma\right)$ specification proved to be an excellent solution. A minimum distance estimator was implemented, with the support of the distribution being $[0.5, 14]$. The two estimated parameters are $\mu_\gamma = 0.48$ and $\sigma_\gamma = 4.04$.[11]

[Figure (3) about here]

Figure (3) provides the plots of the three preference heterogeneity densities used in the solution of the model. Comparing these distributions it is clear that the KSS case has the highest mode, and that it has a lot of mass for high values of γ. The LN case lies between the other two, with a lot of mass concentrated for relatively low values of γ. Finally, the CP case shows a density whose mode is at the lower bound of its support, representing a population that is more risk tolerant than the other two.

[11] A lognormal specification was attempted at first, but the results were poor. Comparing the fitted distribution with Figure 2 in Chiappori and Paiella (2011) showed that in order to match the center of the distribution, the lognormal specification was clearly missing the behavior at the lower and upper ends of the support. Eyeballing the fitted *Beta* distribution, it is clear that it now captures the patterns of the one estimated by Chiappori and Paiella (2011), as Figure 5 in Appendix C shows. Notice that I did not attempt to use the results provided by Harrison, Lau, and Rutstrom (2007) and von Gaudecker, van Soest, and Wengstrom (2011). As for the former paper, the number of people participating in the experiments was very limited, making the estimation for the whole preference distribution prone to large errors. Differently, the latter did not rely on expected utility theory, making their preference distributions inconsistent with the framework used here.

4.2 Calibration

The remaining parameters are calibrated as follows. The capital depreciation rate is set to replicate an investment/output ratio of approximately 25%. This is achieved with $\delta = 0.08$. I assume a Cobb-Douglas production function, hence the capital share is captured by the parameter $\alpha = 0.36$, a common value for the U.S. economy.

The calibrated rate of time preference β deserves some comments. Consider as our benchmark the economy with the estimated distribution of risk aversion from the PSID survey, and with (say) the *GRIP* stochastic process. As expected, in this economy high risk aversion agents accumulate a lot of assets, when facing good earnings shocks. This leads to a supply of savings in the economy which is substantially higher than in its homogeneous preference parameters counterpart, with (say) $\gamma = 2.0$. If a typical value for β with a yearly time period (say $\beta = 0.96$) were used, the equilibrium interest rate would be close to 0% (even negative, for some income processes), to dissuade saving. In order to avoid this counterfactual outcome, I target an equilibrium interest rate in the $3-4\%$ range.[12] This is obtained with $\beta = 0.932$ for the estimated risk aversion distributions based on the PSID data, with $\beta = 0.94$ for the LN case, and with $\beta = 0.945$ for the CP case. A low value of β makes the agents less patient: by giving less weight to the future, agents consume more out of their current income and decrease their savings, preventing the interest rate to become implausibly small.

The borrowing limit b is set at its most stringent value, namely $b = 0$. This choice was mainly dictated to facilitate the comparison with previous contributions in the literature that used such a value, e.g. Aiyagari (1994) and Castaneda, Diaz-Gimenez, and Rios-Rull (2003). However, it is not clear if this value is fully satisfactory. In the data, between 6 and 15% of U.S. households have a negative net worth, as reported by Wolff (1998) and Cagetti and De Nardi (2008). This is potentially important for two reasons: 1) a less tight borrowing constraint allows people to better smooth consumption, relying on debt rather than on precautionary savings as a way to equalize the marginal utility of consumption across all possible states of the world, 2) on the one hand a less stringent borrowing constraint decreases the incentive to save, possibly reducing the upper value of the support of wealth, on the other hand it shifts mechanically the lower value of the support of wealth. By affecting the range of wealth, the borrowing constraint can have first order effects on some measures of wealth dispersion.[13]

[12] From (2), for a given labor share and capital depreciation, this is in fact equivalent to matching the capital/output ratio. This is the only wealth related target that is used in the parameterization of the model.

[13] This is one of the results found in a global sensitivity analyisis for the Aiyagari (1994) economy reported in Cozzi (2011). Incidentally, results related to different calibrations of the borrowing limit are not reported. A relatively low β makes borrowing a more attractive option, which in turn leads to wild changes in the percentage of people in debt when considering slight changes in b and β, making the comparisons among the different stochastic processes less tranparent.

5 Results

This Section presents the main results. First it is shown how introducing preference heterogeneity, in the form of non degenerate distributions of innate risk aversion, does alter considerably typical measures of wealth inequality.

[Table 4 about here]

Table 4 compares the Gini index and the Coefficient of Variation obtained in a model without preference heterogeneity (with $\gamma = 2.0$ and $\beta = 0.96$) to the inequality measures computed in the economies with preference heterogeneity.

Overall, the model economy moves from a wealth Gini index in the $0.45 - 0.54$ range (for the homogeneous preferences case) to a wealth Gini index ranging from 0.65 to 0.80, depending on the actual stochastic income process considered. Irrespective of the risk aversion distribution, the model almost matches the observed degree of wealth inequality in the U.S. with both the *FREN* and the *GRIP* income processes. The results for the other two income processes show that wealth is less concentrated in those cases.[14]

As expected, when moving from the homogeneous risk aversion set-up to the heterogeneous one, the inequality measures do increase for all endogenous variables. Consumption, income and wealth are more concentrated in the heterogeneous preference economies.

As for the other measure of wealth inequality, the Coefficient of Variation, the model is still far from matching the actual figure of 6.02 found by Diaz-Gimenez, Glover, and Rios-Rull (2011) in the SCF. The values for the two persistent income processes are in the $1.72 - 2.08$ range. The reason behind this result is that the model misses the share of wealth held by the richest households. Given that the wealthiest households in the U.S. hold approximately one third of the total wealth, this is an important finding. Preference heterogeneity alone does not fully account for the determination of the top of the wealth distribution.

To see this in more detail, Table 6 reports a set of statistics related to the wealth distribution. The five quintiles (*Q1-Q5*) and a breakdown of the top quintile are shown for both the U.S. data in 1998, 2002 and 2007 (the first three rows), and the various models. When focusing on the same income process, *GRIP*, the three risk aversion distributions (presented in the third panel) show results

[14] Notice, however, that the *GHIP* case is somewhat misleading. Unlike Guvenen (2009), I am just considering the AR(1) component of the econometric specification for labor income, that is just the restricted income profile. In order to gauge the predictions of the model for this case, I should focus only on the residual inequality, namely the one that is not accounted for by the heterogenous income profiles, which are not considered in this model.

in terms of wealth concentration and shares of wealth held by a set of quantiles that are remarkably similar. Both the LN and the CP distributions have almost identical quantiles, with the only major difference being observed for the share of wealth held by the top 1% (10.6% Vs. 12.5%). The KSS distribution differs more markedly in *Q4*, the share of wealth held by the fourth quintile (17.7%), which is well above the data ($11.2\% - 13.4\%$). The higher share for *Q4* is compensated with a lower share for *Q5*, which explains the lower Gini index found with this preference distribution.

[Table 6 about here]

Unlike the share of wealth held by the top 1%, several other features of the wealth distribution are replicated well, such as the bottom and top quintiles. The U.S. data show that the bottom three quintiles hold very little wealth. The first quintile has negative asset holdings, the second quintile has between 1.1% and 1.7% of the total wealth, while the third quintile has between 4.5% and 5.7%. The left tail of the distributions display some discrepancies with the data as well. However, these discrepancies are small. The bottom quintile is prevented to be in debt, which explains the 0% figure Vs. the negative one in the data. A similar pattern is observed for the second and third quintiles. The model predicts that relatively wealth-poor agents hold less wealth than what it is found in the data, because the bottom quintiles are dominated by risk prone agents, that save little for two different reasons. On the one hand, their willingness to face consumption profiles that are not very smooth drives their precautionary savings down. At the same time, the more risk averse agents are accumulating a lot of assets, driving up the capital supply in the economy, and reducing the rate of return of savings, making future consumption relatively more expensive than current consumption. This GE effect triggers a reduction in the interest rate that drives further down the incentives to save for agents that have a low risk aversion. These two effects combined explain why in this economy there are several agents that have little or no assets at all, as the table shows.

These underestimated bottom quintiles are compensated by overestimating the top ones: according to the model, the fourth quintile asset holdings are above the data for the KSS distribution, while for the LN and CP distributions is the fifth quintile being above the data.

5.1 Lorenz Curves

It is informative to compare the Lorenz curve related to the model without risk aversion heterogeneity to the one obtained with heterogeneous preferences.

[Figure (4) about here]

Figure (4) plots the Lorenz curve for an economy whose agents share the same risk aversion parameter $\gamma = 2.0$. Moreover, it displays the Lorenz curve for an alternative economy, which shows the same amount of uncertainty ($GRIP$), the same trading opportunities, the same baseline parameters, but risk aversion heterogeneity, with the KSS distribution.

By comparing the two Lorenz curves it is clear that the second economy displays a wealth distribution that is substantially more unequal. This is an alternative way of looking at the change in the Gini coefficient, which confirms this finding: it increases from 0.54 in the homogeneous CRRA case to 0.75 in the heterogeneous one. A similar set of comments and plots apply when considering the other two preference distributions, and the other income processes.

5.2 Robustness checks

Table 5 provides a set of robustness checks. The first three panels report the inequality measures for the economy without heterogenous preferences, when different values of the (homogenous) risk aversion parameter are assumed. The first value, $\gamma = 1.9$, corresponds to the mean risk aversion implied by the CP distribution. This is a more appropriate benchmark for the comparisons between the CP heterogenous risk aversion economy and its homogenous risk aversion counterpart, rather than the $\gamma = 2.0$ case.

[Table 5 about here]

The other two values, $\gamma = 3.3$ and $\gamma = 4.2$, correspond to the mean risk aversions implied by the LN and KSS distributions, respectively. Notice also that the results were computed using the same discount factor as in the heterogeneous preferences case. Namely, the $\gamma = 1.9$ case was solved with $\beta = 0.945$, the $\gamma = 3.3$ case with $\beta = 0.940$, and the $\gamma = 4.2$ case with $\beta = 0.932$.

It is worth stressing that all measures of inequality are substantially lower in these economies when compared to their corresponding heterogeneous preferences case, and are closer to the results found with $\gamma = 2.0$.

The second panel in Table 6 reports the quintiles for this robustness analysis. All three cases share the same features. They overestimate considerably the wealth held by the bottom three quintiles, and they undererestimate the top two quintiles. Finally, the statistic for the top 1% is less than half as much the corresponding figure obtained with preference heterogeneity.

From this analysis it seems fair to say that a simple model of uninsurable income risk and preference heterogeneity goes along way in accounting for some salient features of the wealth distribution, when relying on risk aversion distributions that have been estimated exploiting survey questions.

The results show that, under several different assumptions of the risk aversion distribution and the stochastic processes capturing the evolution of idiosyncratic risk over time, the model can match the wealth Gini index almost exactly.

At the same time, the quantitative findings show that the coefficient of variation is still far from its empirical counterpart. This result is mainly driven by the inability of the model to generate the extremely high share of wealth owned by the richest households. In the data this figure is above 30%, while the corresponding value for the model is a mere $9.0 - 12.5\%$.

5.3 Comparison of Equilibria: Homogeneous Vs. Heterogeneous Preferences

The allocations in the economies with heterogeneous preferences do not coincide with the allocations of the corresponding economy with homogeneous preferences, with γ being set to the relevant average value. A comparison of the equilibria reported in Table 7 shows that the two sets of allocations are quantitatively rather different.

[Table 7 about here]

For any income process/preferences distribution pair, the equilibrium interest rate in the heterogeneous preferences case is lower by no less than 0.35 percentage points. It follows that the capital stock, total output, and aggregate consumption are always higher in the economies with heterogeneous preferences.[15]

For the KSS distribution, output (consumption) moves from 1.061 to 1.079 (from 0.806 to 0.811) with the *FREN* specification, and from 1.070 to 1.090 (from 0.808 to 0.813) with the *GRIP* one. Overall, the increases in output are between 1.4% and 1.9%, while the increases in consumption are between 0.6% and 0.9%.[16]

5.4 Precautionary Savings

A natural application in which heterogeneous-agent models are extensively used is to measure the size of precautionary savings. It is informative to study how the heterogeneous preferences cases compare to their homogeneous preferences counterparts, which is the content of Table 8.

[15] It goes without saying that it is possible to find a representative risk preferences economy that has the same equilibrium prices and allocations. However, it is interesting to notice that the outcome of this aggregation exercise does not coincide with the economy with the average risk aversion.

[16] Tables 11 and 12 in Appendix C provide the results of the omitted income processes, which were found to have a worse fit for the wealth Gini index.

[Table 8 about here]

In the Table the value of aggregate capital is reported under the assumption of both incomplete (K^{IM}) and complete markets (K^{CM}). The capital allocations of each risk preference distribution is compared to its homogeneous case counterpart. A clear result is that models with homogeneous risk aversion underestimate the size of precautionary savings. The extent of this bias ranges between 5.7 and 9.2 percentage points, for the most persistent stochastic income processes (*FREN* and *GRIP*). A second aspect concerning precautionary savings is related to their size. Typically, in standard heterogenous agent models it is found that precautionary savings account for a small increase in the aggregate savings. This is not true for this model, in which the percentage increase of the aggregate capital stock, when moving from the complete markets economy to the incomplete markets one, is above 29.6% in 16 cases out of 24, with a maximum of 89.9%, and with only one case showing an increase of less than 10%.

5.5 Discussion

At least three aspects of the analysis call for a further discussion. The first one is related to the assumption of risk aversion being a permament feature. The second is why I consider preference heterogeneity in only one parameter, the risk aversion. The final aspect refers to risky jobs, preference for risk and sorting into careers, which is dealt with in the next section.

An infinitely lived agents model refers to dynasties. In the PSID Kimball, Sahm, and Shapiro (2009) find that risk aversion is positively correlated among different generations of the same family. However, this covariation is far from being perfect. The second remark is related to a life-cycle element. It is possible that individuals change their attitude towards risk after having gone through some specific stages in their life. Unfortunately, data limitation do not allow to capture if such dynamics are indeed taking place. Neither the HRS nor the PSID allow to single out the life-cycle component vs. a cohort one. As Kimball, Sahm, and Shapiro (2008) point out, some individuals in the HRS are asked to assess their attitudes towards risk more than once. In quite a few cases these individuals change their answers. With the available information it is hard to say whether this is a form of measurement error, or whether this reflects the fact that individuals change their attitudes towards risk in response to some economically relevant events (for example, new information on the future income streams, or aggregate shocks). However, all the people in the sample are in the later stages of their lives, that is they are in a situation where there is little uncertainty about lifetime income, at least in the form of labor earnings. This is why the measurement error approach seems to be a valid one.

As for the second aspect, the modeling choice was mainly driven by data limitation. Barsky, Juster, Kimball, and Shapiro (1997) find heterogenity not only as far as risk aversion is concerned, but also in the elasticity of intertemporal substitution, and in the rate of time preference. However, the sample size is really limited, making the estimation of the distributions for these parameters too unreliable. It goes without saying that with CRRA preferences once heterogeneity in risk aversion is allowed for, heterogeneity in the elasticity of intertemporal substitution mechanically follows. The distributions of the two parameters are just a monotone trasformation.

In our simple set-up the amount of income uncertainty from the point of view of a single agent is exogenously determined by the stochastic process for labor income. However, Guiso, Jappelli and Pistaferri (2002), Fuchs-Schundeln and Schundeln (2005), Bonin, Dohmen, Falk, Huffman, and Sunde (2007), and Schulhofer-Wohl (2011), among others, pointed out that different attitudes towards risk can also imply different career choices. High risk averse individuals can self-select into safer jobs, while low risk averse ones can be willing to take jobs with a higher variance of earnings. This endogenous sorting could imply a lower degree of inequality. However, such a research avenue poses big challenges on how to model the career specific stochastic processes for earnings. An attempt along these lines is proposed in the next section.

6 Endogenous Sorting into Risky Jobs

Part of the results obtained above could be driven by the absence of self-selection into jobs differing in their income risk. Schulhofer-Wohl (2011) exploits the question about risk tolerance in the HRS to document that less risk averse individuals are those with the largest earnings fluctuations during their working life, suggesting that preference heterogeneity may be an important factor in occupational choices and the allocation of risk. This section proposes a simple extension to deal with this issue. It is now assumed that there are two mutually exclusive careers, and that all the workers have perfect knowledge on the degree of income risk associated with these options.[17]

The workers can now self-select into different jobs, according to their characteristics. For simplicity, it is assumed that the workers make a once in a lifetime decision on their career. Upon becoming economically active, each worker chooses which career he wants to undertake, by solving the following problem:

$$\max \left\{ V^h(\varepsilon, a; \gamma), V^l(\varepsilon, a; \gamma) \right\}$$

[17] It goes without saying that this simple specification neglects considerations related to shopping for a career in the early stages of a worker's labor market experience, and to learning about the relative riskiness of a job.

where the $V^j(\varepsilon, a; \gamma)$ represent the expected lifetime utility derived when choosing career $j = h, l$. Associated with this choice there is an optimal policy function $\Phi(\varepsilon, a; \gamma)$, which is an indicator function equal to one whenever $V^h(\varepsilon, a; \gamma) \geq V^l(\varepsilon, a; \gamma)$, namely when an agent decides to work in the riskier job. Hence, in equilibrium the endogenous share of workers choosing to work in the riskier job (ϕ) is represented by:

$$\phi = \int_{\mathcal{E}\times\mathcal{A}\times\Gamma} \Phi(\varepsilon, a; \gamma) d\mu(\varepsilon, a; \gamma)$$

Since there are two markov processes for the effective units of labor, each one is going to have its own stationary distribution, and the equilibrium conditions related to the labor input have to be changed accordingly. In the steady-state the expression for L becomes:

$$L = \phi \int \varepsilon d\mu_L^h(\varepsilon) + (1-\phi) \int \varepsilon d\mu_L^l(\varepsilon)$$

where $\mu_L^j(\varepsilon)$ stands for the stationary distribution over the labor endowments implied by either markov process $j = h, l$.

6.1 Two Labor Endowment Processes

The two postulated careers are represented by two different stochastic processes, which govern the dynamics for the effective units of labor ε a worker is going to supply in the labor market. It follows that

$$\log \varepsilon' = \rho_{y,j} \log \varepsilon + \eta', \eta \sim iid\ N(0, \sigma_{y,j}^2),\ j = h, l$$

which highlights how the parameters $\rho_{y,j}$ and $\sigma_{y,j}^2$ can now differ in the two stochastic processes. It goes without saying that the riskier job ($j = h$) is such that $\sigma_{y,h}^2 > \sigma_{y,l}^2$. As for the persistence parameter, postulating an inequality is less straightforward. However, the intuition to justify the restriction $\rho_{y,h} \geq \rho_{y,l}$ goes as follows. The lower the persistence parameter, the more transitory the income shocks will be, because their effect dies out faster. Differently, more persistent processes imply that a shock of a given size will have a more long lasting effect. More risk averse individuals prefer smoother consumption profiles, hence they consider riskier a stochastic income process whose shocks have a higher variance and that are more persistent, namely shocks that cannot be easily offset by their saving behavior.

Whether these assumptions have an empirical foundation, is an open issue. However, some evidence from PSID data can be used in order to support them, together with the results reported by Schulhofer-Wohl (2011), who relies mainly on HRS data, showing that less risk averse individuals have larger income shock variances.

Although conceptually feasible, structurally estimating the four parameters characterizing the two jobs is beyond the scope of this paper. Notice that from the PSID data it is possible to track the yearly labor earnings of individuals according to their risk category. However, it is not possible to estimate exogenously two (or more) potential stochastic processes from the data and impose them in the model. The endogeneity of a person's career prevents this estimation procedure from recovering the true processes, and suggests to not using them as an input in the model. Some form of indirect inference methods would be needed to estimate the structural parameters in equilibrium, because only the actual choices are observed, and their induced volatility of earnings, with agents of the same risk aversion but with different labor income shocks and asset holdings upon labor market entry selecting different jobs. Unfortunately, the computational burden seems to be binding.[18]

A more pragmatic approach is taken instead. The baseline calibration of the two processes (ES1) is chosen in order to get a Gini index for labor earnings equal to 0.36. This value, compared to the $0.16-0.27$ range obtained in the one-job specifications used above, is consistent with the wage income concentration that is observed in the data, as reported by Heathcote, Perri and Violante (2010).

[Table 9 about here]

Table 9 reports the parameters of the four different cases which are going to be discussed, together with some statistics associated with these stochastic processes. These are labeled as ES1-ES4. Perhaps, the most interesting feature is the Gini index for labor earnings, which is now in the $0.31-0.36$ range. Mixing the two processes leads to more concentrated labor earnings.[19]

The two variances in the ES1 case are set to $\sigma^2_{y,h}=0.30$ and $\sigma^2_{y,l}=0.17$, and the two parameters of the autoregressive component are set to $\rho_{y,h}=0.988$, and $\rho_{y,l}=0.97$. Case ES2 and ES3 rely on the same persistence parameters $\rho_{y,j}$ but on different variances. ES2 keeps the same $\sigma^2_{y,l}$ as in the baseline case, but sets $\sigma^2_{y,h}=0.25$ instead. ES3 keeps the same $\sigma^2_{y,h}$ as in the baseline case, but sets $\sigma^2_{y,l}=0.10$ instead. Finally, case ES4 has the same variances $\sigma^2_{y,j}$ as case ES3, but inverts the $\rho_{y,j}$.

In the interest of space, and given that the results were similar also in the other cases, I will focus on the KSS specification for preference heterogeneity.

[18] Typically, with several parameters, slow optimization routines such as Nelder-Mead take more than 100 iterations on the parameters to converge. Faster algorithms, such as simulated annealing, might help, but would still likely require a few hundred hours to complete.

[19] An extensive sensitivity analysis was performed, with the results being quite robust. Some selected cases are reported in Figures 7 and 8 in Appendix C. Notice as well that the discount factor had to be adjusted, being equal to $\beta=0.911$ in all four cases reported here.

6.2 Wealth Inequality

The last panel in Table 5 shows the values for the wealth Gini index in the model with endogenous sorting into risky jobs. Also in this case, the model accounts for the observed concentration of wealth. The calibration matching the Gini index for labor earnings, ES1, displays a wealth Gini index of 0.75. The other three cases range from 0.75 to 0.83. As for ϕ, the endogenous share of workers selecting the risky career, it is relatively similar across specifications, ranging from 31% for the case ES1 to 35% for the ES4 one.[20]

As before, selected quantiles of the wealth distributions are shown at the bottom of Table 6. Comparing the outcomes of the models without endogenous sorting to the model with two risky jobs, it is striking how the overall patterns are quite similar. A difference worth stressing is that the ES1-ES2 specifications better capture the bottom of the distribution, getting a closer fit for the first three quintiles. The fourth quintile is still overestimated, at the expenses of the fifth one. Even though the fifth quintile holds less wealth than in the one risky job model, the top 1% has a slightly higher share of wealth, showing that the model is going in the right direction.

Since the wealth concentration in the ES31-ES4 case is above what is found in the data, it is not surprising to see that in both cases the fifth quintile holds more wealth than in the U.S., and that the bottom quintiles holds too little wealth. With such a high wealth concentration, the model shows a higher share of wealth held by the top 1%, which reaches at most 18.6%, which is still very far from what is observed in the data.

The role of precautionary savings appears to be even more important in this economy. They account for a stunning increase of the steady-state capital stock, which ranges between 96.2% and 138.0%.

A shortcoming of the model without self-selection is represented by the concentration of consumption, which is too low, never being above 0.25. Differently, the model with endogenous sorting achieves a higher consumption inequality, with the Gini index now being in the $0.3 - 0.33$ range. Also this finding is consistent with the data: Heathcote, Perri and Violante (2010) report that in the CEX, since 1990, the Gini index of consumption of non-durable goods has been above 0.3, with a maximum value above 0.32, reached in 2004 and 2005.

6.3 Empirical Evidence

In the PSID it is possible to compute the autocorrelation of labor income together with the variance of the innovations for individuals belonging to different risk aversion categories. Starting from the SRC

[20] Figure 6 in Appendix C shows the decreasing behavior of the percentage of agents choosing to work in the riskier job as a function of γ.

sample I follow a set of standard steps, outlined for example in Heathcote, Perri and Violante (2010). First, I select the PSID respondents according to the following criteria: I consider only the household head, between the age of 24 and 62. For each year $t = 1968, ..., 1996$ I run a first stage regression of labor earnings on a set of controls, $\log y_{it} = \theta_t X_{it} + \varepsilon_{it}$, with the controls X_{it} being age, education, state of residence, race, gender and number of children. With the estimated parameters $\widehat{\theta}_t$ I compute the earnings residuals $\widehat{\varepsilon}_{it}$ and estimate an AR(1) panel regression on them. The econometric model $\log \widehat{\varepsilon}_{it}^{RA} = \rho_{y,r} \log \widehat{\varepsilon}_{it-1}^{RA} + \eta_{it}^{RA}$ is estimated separately for two risk aversion categories, represented by $RA = \{h, l\}$. The breakdown in the data is chosen such that each group has approximately 50% of the respondents (1,107 individuals and 13,517 observations for the most risk averse group, and 1,117 individuals and 13,768 observations for the other group). Before reporting the results, it is worth stressing one more time that this estimation procedure does not recover the structural parameters of the model. However, they can be used as prima facie evidence supporting the parametric restrictions.

[Table 10 about here]

The first remark is that different estimators give different answers, as shown in Table 10. As for the variance of the shocks, the results are relatively stable (also with respect to the sample selection rules) and always such that $\widehat{\sigma}_{y,h}^2 > \widehat{\sigma}_{y,l}^2$. For this econometric model, OLS is inconsistent because it does not control for unobserved heterogeneity, but it is reported as a benchmark comparison: $\left(\widehat{\sigma}_{y,h}^2\right)^{POLS} = .344$ and $\left(\widehat{\sigma}_{y,l}^2\right)^{POLS} = .254$. In a dynamic panel also the LSDV estimator is inconsistent, but some Monte-Carlo studies show that in short panels it can outperform GMM based estimators in terms of efficiency. The estimates in this case are $\left(\widehat{\sigma}_{y,h}^2\right)^{LSDV} = .281$ and $\left(\widehat{\sigma}_{y,l}^2\right)^{LSDV} = .209$. The Arellano-Bond and the Blundell-Bond estimators give systematically lower variances of the idiosyncratic component of the shock, with $\left(\widehat{\sigma}_{y,h}^2\right)^{A-B} = .205$ and $\left(\widehat{\sigma}_{y,l}^2\right)^{A-B} = .154$ for the first estimator, and with $\left(\widehat{\sigma}_{y,h}^2\right)^{B-B} = .211$ and $\left(\widehat{\sigma}_{y,l}^2\right)^{B-B} = .155$ for the second one. Overall, these results are reassuring, because they confirm the assumptions used in the calibration of the model: less risk averse individuals do have larger income shock variances.

Differently, the results related to the persistence parameters tend to support the opposite inequality. The estimates are $\widehat{\rho}_{y,h}^{OLS} = .615$ and $\widehat{\rho}_{y,l}^{OLS} = .662$, $\widehat{\rho}_{y,h}^{LSDV} = .289$ and $\widehat{\rho}_{y,l}^{LSDV} = .338$, $\widehat{\rho}_{y,h}^{A-B} = .169$ and $\widehat{\rho}_{y,l}^{A-B} = .238$, $\widehat{\rho}_{y,h}^{B-B} = .196$ and $\widehat{\rho}_{y,l}^{B-B} = .244$. This issue is addressed in case ES4 and in other calibrations reported in the Appendix, which show that the results are not sensitive to this alternative assumption.

7 Conclusions

This paper contributed to the literature on wealth inequality. A model of incomplete markets and precautionary savings was extended to allow for the type of preference heterogeneity found in the PSID data, and for self-selection into risky jobs.

Does preference heterogeneity account for the U.S. wealth inequality? The findings show that the answer is not clear-cut. A result of this paper is that some forms of empirically grounded preference heterogeneity go a long way in accounting for several features of the U.S. wealth distribution. Compared to a model without preference heterogeneity, the left tail of the distribution gets closer to the little share of wealth held by the first three quintiles. At the same time, also the top quintiles are replicated relatively well. A consequence of this result is that the Gini index in the model and in the data are very close to each other.

Other features of the wealth distribution are less satisfactory. In particular, the share of wealth held by the top 1% is still far from the observed share, which is above 30%. Entrepreneurship seems still to be the candidate channel leading to such large estates, although preference heterogeneity can provide a microfoundation of why some workers self-select into entrepreneurial activities while others decide not to work as self-employed. Such a model can be considered as an alternative way of microfounding the models of entrepreneurship and wealth accumulation developed in Quadrini (2000) and Cagetti and De Nardi (2006).

Preference heterogeneity introduces another layer for heterogenous welfare effects deriving from a policy change. The results of this paper suggest that neglecting this channel can have a first order effect on the aggregate allocations as well. This points to the need for further applied research aimed at eliciting more accurately individuals' risk preferences in large longitudinal and cross-sectional data.

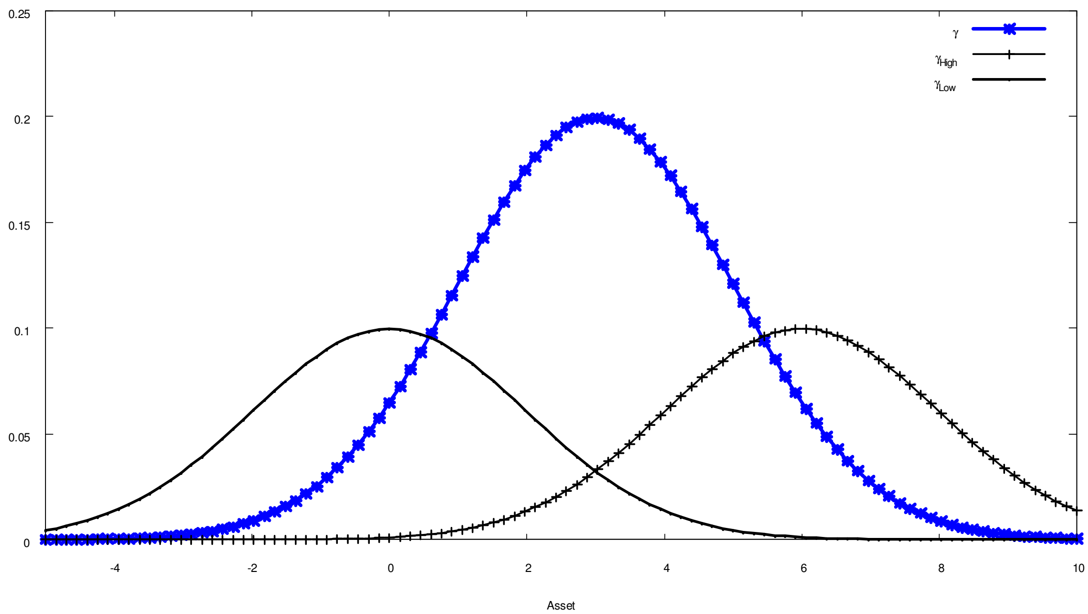

Figure 1: Heterogenous Risk Aversion and Wealth Inequality

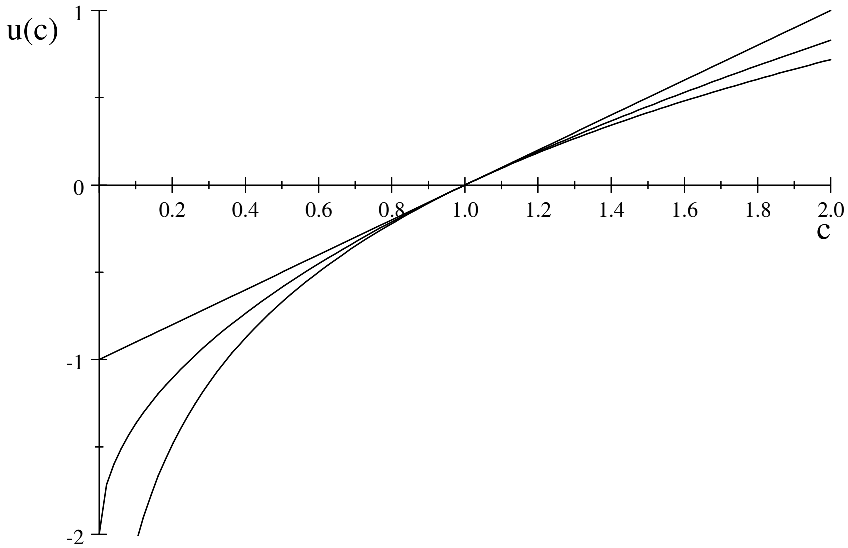

Figure 2: CRRA Utility Functions, $\gamma = \{0, 0.5, 1\}$

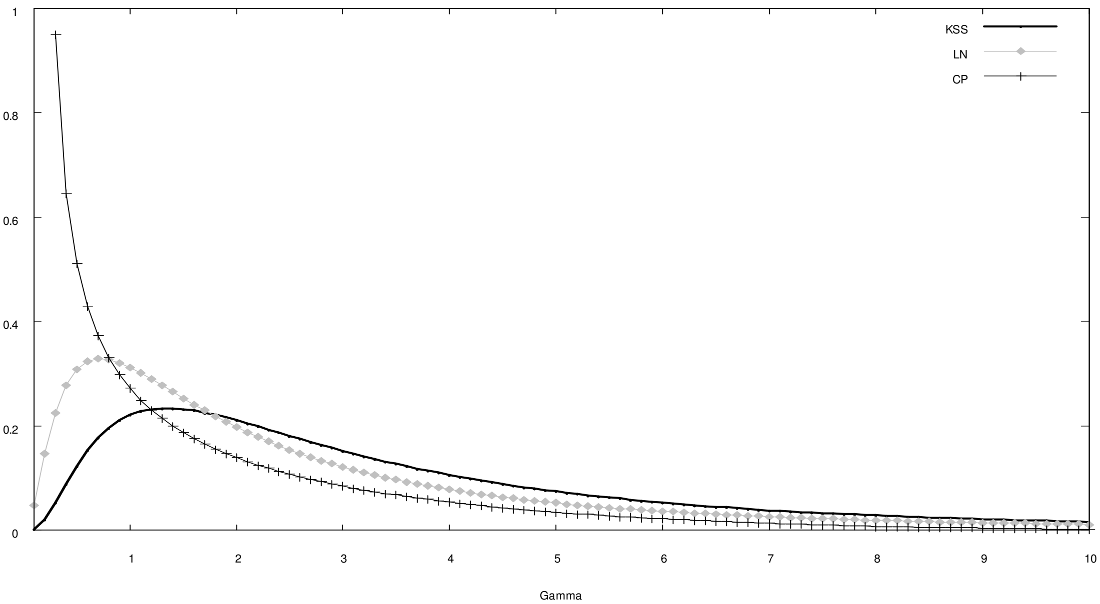

Figure 3: Preference Heterogeneity: the densities of the three cases.

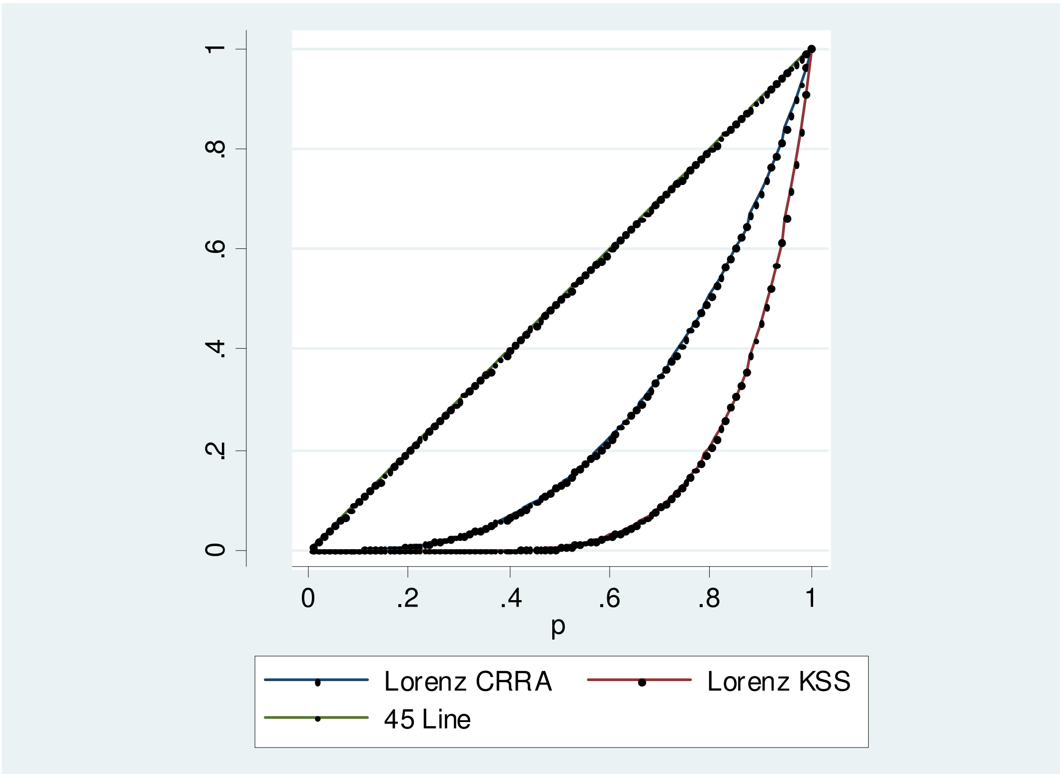

Figure 4: Lorenz Curve for Assets: Homogeneous Vs. RA Heterogeneity (GRIP)

Parameter	*Value*	*Target*
Model Period	*Yearly*	
ε - *Productivity values*	*See Table 2*	*AR(1) Stochastic Process for earnings*
γ - *CRRA*	*See Table 3*	*Risk Tolerance from PSID data*
β - *Rate of time preference*	$\{0.932, 0.94, 0.945\}$	*Annual interest rate* $\approx$ *3%* $\{KSS, LN, CP\}$
δ - *Capital depreciation rate*	0.08	*Investment/Output ratio* $\approx 25\%$
α - *Capital share*	0.36	*Capital Share of Output*
b - *Borrowing limit*	0	

Table 1: Benchmark Calibration - U.S.

Wage Process	ρ_y	σ_y	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_7	ε_8	ε_9	ε_{10}	ε_{11}
	(*Gini*)	(*CV*)	$\mu_L(\varepsilon_1)$	$\mu_L(\varepsilon_2)$	$\mu_L(\varepsilon_3)$	$\mu_L(\varepsilon_4)$	$\mu_L(\varepsilon_5)$	$\mu_L(\varepsilon_6)$	.	.	.	.	.
(1) *FREN*	.977	.12	.29	.33	.38	.43	.48	.54	.60	.67	.76	.86	1.0
French	(0.20)	(0.35)	7.16%	8.74%	9.30%	9.73%	10.01%	10.11%	.	.	.	.	.
(2) *FLIN*	.92	.21	.11	.15	.18	.23	.28	.34	.41	.50	.61	.77	1.0
Floden-Linde	(0.27)	(0.51)	2.65%	5.53%	8.71%	11.75%	13.97%	14.78%	.	.	.	.	.
(3) *GRIP*	.988	.122	.28	.33	.37	.42	.47	.53	.59	.67	.75	.86	1.0
Guvenen	(0.21)	(0.38)	8.51%	9.36%	9.21%	9.17%	9.16%	9.17%	.	.	.	.	.
(4) *GHIP*	.821	.17	.17	.21	.25	.30	.35	.41	.48	.57	.67	.81	1.0
Guvenen	(0.16)	(0.30)	0.40%	2.08%	6.11%	12.37%	18.50%	21.09%	.	.	.	.	.

Table 2: Labor Efficiency Units - Ergodic Distributions from the Tauchen Markov Chains

Case	*Distribution*	μ_γ	σ_γ
KSS (PSID Data)	$LN\left(\mu_\gamma, \sigma_\gamma^2\right)$	1.05	0.87
LN (γ^{Med}=2.0, γ^{Var}=19.7)	$LN\left(\mu_\gamma, \sigma_\gamma^2\right)$	0.69	1.01
CP (γ^{Med}=1.7, $\gamma^{0.75}$=3.0)	$Beta\left(\mu_\gamma, \sigma_\gamma\right)$	0.48	4.04

Table 3: Risk Aversion Distributions

RRA Heterogeneity:	*Case*	*Asset*	*Income*	*Consumption*
		Gini, CV	*Gini, CV*	*Gini, CV*
No - $\gamma = 2.0$:	(1) *FREN*	$0.539, 1.007$	$0.219, 0.387$	$0.182, 0.318$
	(2) *FLIN*	$0.524, 1.002$	$0.276, 0.508$	$0.201, 0.355$
	(3) *GRIP*	$0.544, 1.012$	$0.231, 0.407$	$0.195, 0.340$
	(4) *GHIP*	$0.455, 0.854$	$0.170, 0.308$	$0.113, 0.204$
Yes - *KSS*:	(1) *FREN*	$0.749, 1.720$	$0.235, 0.428$	$0.218, 0.392$
	(2) *FLIN*	$0.687, 1.496$	$0.276, 0.508$	$0.223, 0.394$
	(3) *GRIP*	$0.758, 1.760$	$0.244, 0.441$	$0.227, 0.406$
	(4) *GHIP*	$0.649, 1.379$	$0.197, 0.361$	$0.164, 0.303$
Yes - *LN*:	(1) *FREN*	$0.792, 1.954$	$0.242, 0.357$	$0.227, 0.416$
	(2) *FLIN*	$0.740, 1.729$	$0.280, 0.517$	$0.237, 0.420$
	(3) *GRIP*	$0.798, 1.990$	$0.250, 0.460$	$0.235, 0.428$
	(4) *GHIP*	$0.698, 1.587$	$0.205, 0.382$	$0.174, 0.329$
Yes - *CP*:	(1) *FREN*	$0.794, 2.037$	$0.251, 0.475$	$0.237, 0.448$
	(2) *FLIN*	$0.752, 1.850$	$0.291, 0.540$	$0.254, 0.457$
	(3) *GRIP*	$0.802, 2.083$	$0.261, 0.492$	$0.247, 0.464$
	(4) *GHIP*	$0.682, 1.622$	$0.206, 0.392$	$0.172, 0.337$

Table 4: Equilibrium - Inequality Measures: Gini Index and Coefficient of Variation

RRA Heterogeneity:	*Case*	*Asset*	*Income*	*Consumption*
		Gini, CV	*Gini, CV*	*Gini, CV*
No - $\gamma = 1.9$:	(1) *FREN*	$0.543, 1.014$	$0.227, 0.401$	$0.198, 0.348$
	(2) *FLIN*	$0.535, 1.027$	$0.280, 0.515$	$0.220, 0.391$
	(3) *GRIP*	$0.547, 1.018$	$0.239, 0.421$	$0.211, 0.369$
	(4) *GHIP*	$0.467, 0.879$	$0.176, 0.318$	$0.130, 0.234$
No - $\gamma = 3.3$:	(1) *FREN*	$0.511, 0.938$	$0.222, 0.391$	$0.189, 0.332$
	(2) *FLIN*	$0.488, 0.920$	$0.275, 0.506$	$0.201, 0.356$
	(3) *GRIP*	$0.514, 0.940$	$0.233, 0.411$	$0.202, 0.352$
	(4) *GHIP*	$0.434, 0.809$	$0.172, 0.311$	$0.124, 0.222$
No - $\gamma = 4.2$:	(1) *FREN*	$0.496, 0.904$	$0.220, 0.388$	$0.188, 0.329$
	(2) *FLIN*	$0.467, 0.875$	$0.273, 0.504$	$0.195, 0.344$
	(3) *GRIP*	$0.500, 0.908$	$0.232, 0.408$	$0.200, 0.349$
	(4) *GHIP*	$0.418, 0.775$	$0.171, 0.309$	$0.123, 0.221$
Yes - KSS:	(1) *ES1*	$0.755, 1.954$	$0.362, 0.721$	$0.327, 0.627$
	(2) *ES2*	$0.746, 1.807$	$0.333, 0.625$	$0.301, 0.550$
	(3) *ES3*	$0.834, 2.610$	$0.350, 0.799$	$0.330, 0.725$
	(4) *ES4*	$0.823, 2.451$	$0.340, 0.753$	$0.318, 0.672$

Table 5: Robustness Analysis, Equilibrium - Inequality Measures: Gini Index and C. of V.

	$Q1$	$Q2$	$Q3$	$Q4$	$Q5$	$10-5\%$	$5-1\%$	1%
U.S. - SCF07	$-.2$	1.1	4.5	11.2	83.4	11.1	26.7	33.6
U.S. - SCF98	$-.3$	1.3	5.0	12.2	81.7	11.3	23.1	34.7
U.S. - SCF92	$-.4$	1.7	5.7	13.4	79.5	12.6	23.9	29.5
CRRA=1.9	.3	4.2	13.9	28.2	53.4	13.7	13.7	4.4
CRRA=3.3	.7	5.5	15.1	28.3	50.5	13.0	12.6	4.0
CRRA=4.2	.8	6.0	15.5	28.4	49.2	12.7	12.2	3.8
KSS	.00	.07	2.9	17.7	79.3	21.0	24.9	9.0
LN	.00	.001	1.2	13.6	85.1	22.4	28.5	10.6
CP	.00	.03	1.3	13.4	85.1	21.4	28.8	12.5
ES1	.00	.03	4.0	17.9	77.8	18.2	24.3	13.5
ES2	.00	.02	4.0	18.7	77.0	19.0	23.9	11.2
ES3	.00	.001	1.4	10.8	87.8	18.1	34.2	18.6
ES4	.00	.001	1.4	11.9	86.7	18.5	32.8	16.9

Table 6: Data Vs. Equilibria - Statistics of the Wealth Distribution

Case	*RRA Heterogeneity*	r (%)	Y	I	C	K/Y
(1) *FREN*	*No - CRRA=4.2*	4.00	1.061	0.255	0.806	3.001
	Yes - KSS	3.65	1.079	0.267	0.811	3.089
(3) *GRIP*	*No - CRRA=4.2*	3.77	1.070	0.262	0.808	3.057
	Yes - KSS	3.39	1.090	0.276	0.813	3.161
(1) *FREN*	*No - CRRA=3.3*	4.04	1.059	0.253	0.805	2.990
	Yes - LN	3.62	1.080	0.268	0.812	3.098
(3) *GRIP*	*No - CRRA=3.3*	3.87	1.065	0.258	0.806	3.033
	Yes - LN	3.41	1.089	0.275	0.814	3.156
(1) *FREN*	*No - CRRA=1.9*	4.70	1.028	0.233	0.794	2.835
	Yes - CP	4.31	1.046	0.245	0.801	2.924
(3) *GRIP*	*No - CRRA=1.9*	4.61	1.029	0.235	0.793	2.855
	Yes - CP	4.19	1.049	0.248	0.800	2.954

Table 7: Homogeneous Vs. Heterogeneous Preferences: Equilibrium Allocations

	Case	K^{IM}	K^{CM}	*Prec. Saving (%)*	*Prec. Saving* (Δ)
KSS vs. CRRA=4.2:	(1) *FREN*	$3.332, 3.184$	2.178	$52.9, 46.2$	6.7
	(3) *GRIP*	$3.447, 3.271$	2.174	$58.6, 50.5$	8.1
LN vs. CRRA=3.3:	(1) *FREN*	$3.347, 3.166$	2.398	$39.6, 32.0$	7.6
	(3) *GRIP*	$3.438, 3.230$	2.393	$43.7, 34.5$	9.2
CP vs. CRRA=1.9:	(1) *FREN*	$3.057, 2.914$	2.553	$19.8, 14.1$	5.7
	(3) *GRIP*	$3.100, 2.939$	2.547	$21.7, 15.4$	6.3

Table 8: Precautionary Savings: Heterogeneous Vs. Homogeneous Preferences

W. Process	ρ_y (*Gini*)	σ_y (*CV*)	ε_1 $\mu_L(\varepsilon_1)$	ε_2 $\mu_L(\varepsilon_2)$	ε_3 $\mu_L(\varepsilon_3)$	ε_4 $\mu_L(\varepsilon_4)$	ε_5 $\mu_L(\varepsilon_5)$	ε_6 $\mu_L(\varepsilon_6)$	ε_7 .	ε_8 .	ε_9 .	ε_{10} .	ε_{11} .
(1) *ES1*	(0.357)	(0.712)											
High Risk	.988	.30	.14	.20	.28	.38	.50	.66	.88	1.16	1.57	2.16	3.15
			8.51%	9.36%	9.21%	9.17%	9.16%	9.17%	.	.	.	.	.
Low Risk	.97	.17	.27	.34	.41	.48	.57	.66	.78	.91	1.08	1.30	1.60
			6.39%	8.33%	9.31%	10.06%	10.54%	10.71%	.	.	.	.	.
(2) *ES2*	(0.325)	(0.611)											
High Risk	.988	.25	.20	.28	.36	.47	.59	.75	.94	1.19	1.53	2.00	2.73
			8.51%	9.36%	9.21%	9.17%	9.16%	9.17%	.	.	.	.	.
Low Risk	.97	.17	.31	.38	.46	.54	.64	.75	.88	1.03	1.22	1.46	1.81
			6.39%	8.33%	9.31%	10.06%	10.54%	10.71%	.	.	.	.	.
(3) *ES3*	(0.315)	(0.799)											
High Risk	.988	.30	.14	.20	.28	.38	.50	.66	.88	1.16	1.57	2.16	3.15
			8.51%	9.36%	9.21%	9.17%	9.16%	9.17%	.	.	.	.	.
Low Risk	.97	.10	.39	.45	.50	.55	.60	.66	.73	.80	.88	.98	1.11
			6.39%	8.33%	9.31%	10.06%	10.54%	10.71%	.	.	.	.	.
(4) *ES4*	(0.308)	(0.663)											
High Risk	.97	.30	.15	.21	.29	.40	.53	.70	.92	1.22	1.65	2.27	3.31
			6.39%	8.33%	9.31%	10.06%	10.54%	10.71%	.	.	.	.	.
Low Risk	.988	.10	.41	.47	.52	.58	.64	.70	.77	.84	.93	1.03	1.17
			8.51%	9.36%	9.21%	9.17%	9.16%	9.17%	.	.	.	.	.

Table 9: Labor Efficiency Units with Endogenous Sorting - Ergodic Distributions from the Two Tauchen Markov Chains

Parameter	*POLS*	*LSDV*	*A-B*	*B-B*
$\widehat{\sigma}^2_{y,h}$	.344	.281	.205	.211
$\widehat{\sigma}^2_{y,l}$	.254	.209	.154	.155
$\widehat{\rho}_{y,h}$	.615	.289	.169	.196
$\widehat{\rho}_{y,l}$	.662	.338	.238	.244

Table 10: Estimates of the Labor Earnings Shocks Variance and Persistence by two Risk Aversion Categories in the PSID (1968-1996)

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Appendix A - Computation

- All codes solving the economies and simulating samples of agents were written in the FORTRAN 95 language, relying on the Intel Fortran Compiler, build 11.1.048 (with the IMSL library). They were compiled selecting the O3 option (maximize speed), and without automatic parallelization. They were run on a 64-bit PC platform, running Windows 7 Professional Edition, with an Intel $i7-2600k$ Quad Core processor clocked at 4.8 Ghz.

- Depending on the parameters (essentially on the discount factor β, and the precision of the initial guess) without endogenous sorting the model solution took from 1 to 4 hours to complete (from 3 to 15 iterations on the interest rate are needed to find each equilibrium). With endogenous sorting the model solution took on average 3 hours.

- In the actual solution of the model I need to discretize the three continuous state variables ε, a, and γ. As for ε, I rely on Tauchen's method, which approximates the $AR(1)$ process for the efficiency units with a Markov chain. I use an eleven-state approximation. As for a, I rely on an unevenly spaced grid, with the distance between two consecutive points increasing geometrically. In order to keep the computational burden manageable, I use 151 grid points on the asset space, the lowest value being the borrowing constraint and the highest one being a value $a_{\max} > \overline{a}$ high enough for the saving functions to cut the 45 degree line ($a_{\max} = 150$). This is done to allow for a high precision of the policy rules at low values of a, that is where the change in curvature is more pronounced. As for γ, I discretize its support using 100 evenly spaced points, the lowest value being close to zero ($\gamma_{\min} = 0.001$) and the highest one being 10 ($\gamma_{\max} = 10$).

- The model is solved with a 'successive approximation' procedure on the set of value functions. I start from a set of guesses $V(\varepsilon, a; \gamma)_0$. I compute the vector of parameters Ω representing the Schumaker spline approximations of the value functions. I solve the constrained maximization problems and retrieve the policy functions, $a'(\varepsilon, a; \gamma)$. Notice that I do not restrict the agents' asset holding to belong to a discrete set. As for the approximation method, I rely on the quadratic spline approximations for the future value functions, when evaluated at the chosen saving level. I keep on iterating until a fixed point is reached, i.e. until two successive iterations satisfy:

$$\underset{a}{Sup}\left|V(\varepsilon, a; \gamma)_{n+1} - V(\varepsilon, a; \gamma)_n\right| < \epsilon, \forall\varepsilon \text{ and } \forall\gamma$$

  where ϵ is a small convergence criterion. In a previous version of the paper I solved the model with a time iteration scheme on the policy functions. For some parameter values, this method showed numerical instability when the maximum value of γ was greater than 10. With value function iteration the model can be solved more reliably with larger upper bounds for γ.

- In the model without endogenous sorting, the stationary distributions $\mu(\varepsilon, a; \gamma)$ are computed by simulating a large sample of $100,000$ individuals for $3,000$ periods, which ensure that the statistics of interest are stationary processes. With endogenous sorting, the model needs longer simulations of $20,000$ periods, with the agents re-optimizing their career choices every $2,000$ periods. As for the approximation method, I rely on a linear approximation scheme for the saving and consumption functions, for values of a falling outside the grid. Notice that I do not interpolate in the γ dimension. Some experimenting showed that a linear interpolation implied relatively large errors. Numerically, it seems more appropriate to have a large number of gridpoints, rather than a coarser grid with some interpolation scheme. It goes without saying that this impacts quite substantially the computational burden.

- The first step of the simulation is a randow draw from the risk aversion distribution for each agent, whose risk aversion type is permanent, hence it will not evolve over time. Once the risk aversion is assigned, I start simulating the sample by drawing sequences of efficiency units from the Markov chain, and compute the capital supply together with the inequality measures.

Appendix B - Solution Algorithm

The computational procedure used to solve the baseline model can be represented by the following algorithm:

- Generate a discrete grid over the CRRA space $[\gamma_{\min}, ..., \gamma_{\max}]$.
- Generate discrete grids over the asset space $[-b, ..., a_{\max}]$.
- Generate a discrete grid over the efficiency units space $[\varepsilon_{\min}, ..., \varepsilon_{\max}]$.
- Get the aggregate labor supply L.
- Guess the interest rate r_0.
- Get the capital demand k.
- Get the wage rate per efficienty units w.
- Get the saving functions $a'(\varepsilon, a; \gamma)$.
- Get the stationary distributions $\mu(\varepsilon, a; \gamma)$.
- Get the aggregate capital supply.
- Check asset market clearing; Get r_1.
- Update $r_0' = \varpi r_0 + (1 - \varpi) r_1$ (with ϖ arbitrary weight).
- Iterate until market clearing.
- Get the consumption functions $c'(\varepsilon, a; \gamma)$.
- Check the final good market clearing.

The computational procedure used to solve the two risky jobs model needs three additional steps: the computation of two job-dependent value functions $V^j(\varepsilon, a; \gamma)$, the computation of the optimal job policy functions $\Phi(\varepsilon, a; \gamma)$, and the related share of agents ϕ choosing the less risky career.

- Generate a discrete grid over the CRRA space $[\gamma_{\min}, ..., \gamma_{\max}]$.
- Generate discrete grids over the asset space $[-b, ..., a_{\max}]$.
- Generate two discrete grids over the efficiency units space $\left[\varepsilon^j_{\min}, ..., \varepsilon^j_{\max}\right]$, $j = h, l$.
- Guess the share of risky jobs ϕ_0.
- Get the aggregate labor supply L.
- Guess the interest rate r_0.
- Get the capital demand k.
- Get the wage rate per efficienty units w.
- Get the saving functions $a'_j(\varepsilon, a; \gamma)$, $j = h, l$.
- Get the job-dependent value functions $V^j(\varepsilon, a; \gamma)$.
- Get the optimal job decisions $\Phi(\varepsilon, a; \gamma)$ and ϕ_1.
- Get the stationary distributions $\mu(\varepsilon, a; \gamma)$.
- Get the aggregate capital supply.
- Check asset market clearing; Get r_1.
- Update $r'_0 = \varpi_r r_0 + (1 - \varpi_r) r_1$ (with ϖ_r an arbitrary weight).
- Update $\phi'_0 = \varpi_\phi \phi_0 + (1 - \varpi_\phi) \phi_1$ (with ϖ_ϕ an arbitrary weight).
- Iterate until market clearing.
- Get the consumption functions $c'(\varepsilon, a; \gamma)$.
- Check the final good market clearing.

Appendix C - Additional Results

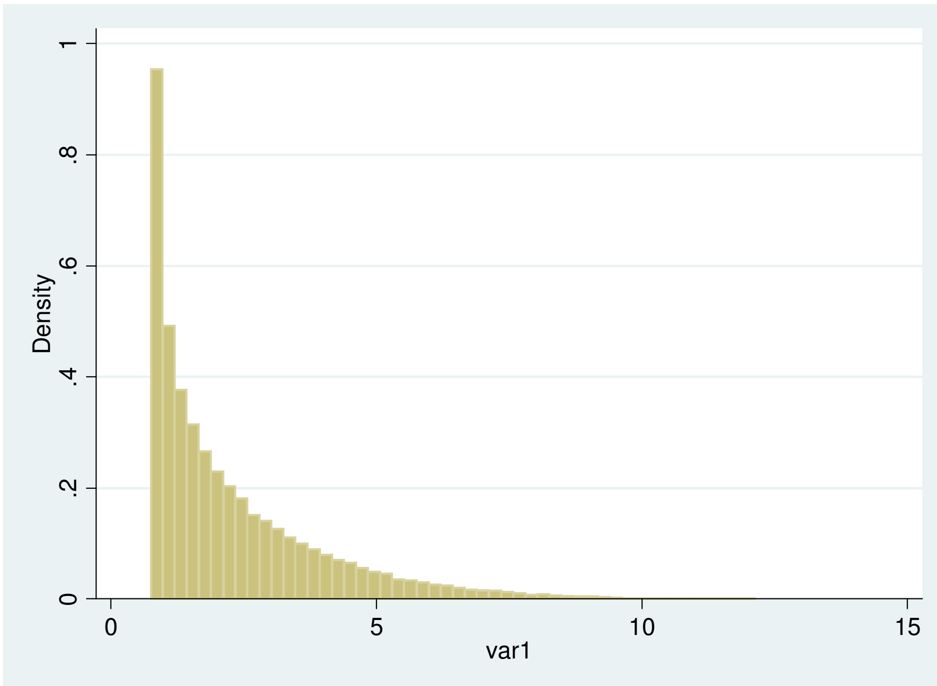

Figure 5: Risk Aversion density, CP case

Case	*RRA Heterogeneity*	r (%)	Y	I	C	K/Y
(2) *FLIN*	*No - CRRA=4.2*	1.88	0.787	0.229	0.558	3.643
	Yes - KSS	1.17	0.821	0.258	0.563	3.927
(4) *GHIP*	*No - CRRA=4.2*	4.94	0.769	0.171	0.597	2.782
	Yes - KSS	4.78	0.775	0.175	0.599	2.817
(2) *FLIN*	*No - CRRA=3.3*	2.34	0.767	0.214	0.553	3.480
	Yes - LN	2.17	0.775	0.219	0.555	3.541
(4) *GHIP*	*No - CRRA=3.3*	4.78	0.775	0.175	0.600	2.818
	Yes - LN	4.58	0.781	0.179	0.602	2.860
(2) *FLIN*	*No - CRRA=1.9*	3.70	0.716	0.176	0.539	3.076
	Yes - CP	3.25	0.732	0.187	0.544	3.201
(4) *GHIP*	*No - CRRA=1.9*	5.04	0.766	0.169	0.596	2.760
	Yes - CP	4.84	0.772	0.173	0.599	2.803

Table 11: Homogeneous Vs. Heterogeneous Preferences: Equilibrium Allocations

	Case	K^{IM}	K^{CM}	*Prec. Saving (%)*	*Prec. Saving* (Δ)
KSS vs. CRRA=4.2:	(2) *FLIN*	$2.871, 2.867$	1.449	$98.2, 97.9$	0.3
	(4) *GHIP*	$2.182, 2.140$	1.648	$32.4, 29.9$	2.5
LN vs. CRRA=3.3:	(2) *FLIN*	$2.744, 2.670$	1.595	$72.0, 67.4$	4.6
	(4) *GHIP*	$2.235, 2.183$	1.814	$23.2, 20.3$	2.9
CP vs. CRRA=1.9:	(2) *FLIN*	$2.343, 2.201$	1.698	$38.0, 29.6$	8.4
	(4) *GHIP*	$2.165, 2.114$	1.931	$12.1, 9.5$	2.6

Table 12: Precautionary Savings: Heterogeneous Vs. Homogeneous Preferences

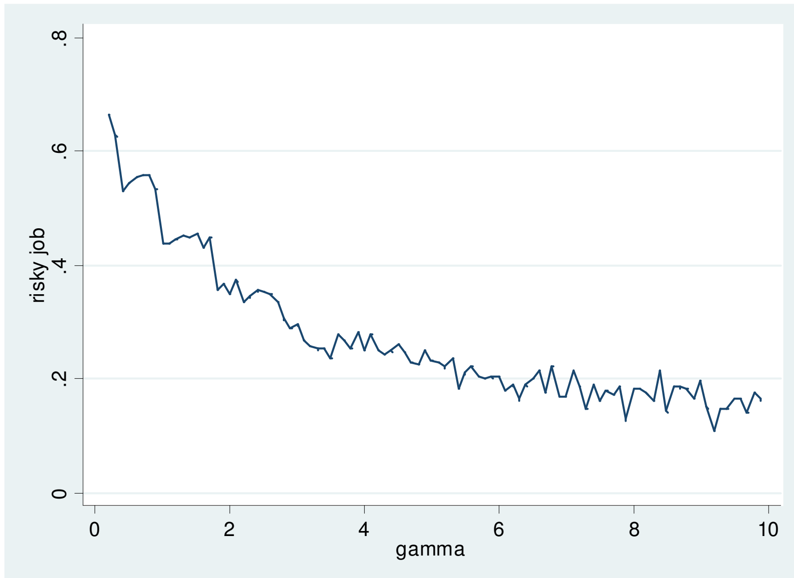

Figure 6: Percentage of agents in the riskier job, by the CRRA (Case ES1)

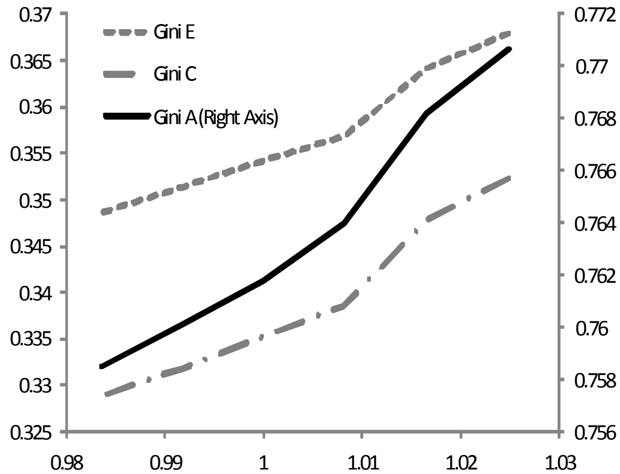

Figure 7: Robustness Analysis with respect to the income shocks persistence $\rho_{y,h}/\rho_{y,l}$ ($\rho_y^{Avg} = .975, \sigma^2_{y,h} = .30, \sigma^2_{y,h} = .17$)

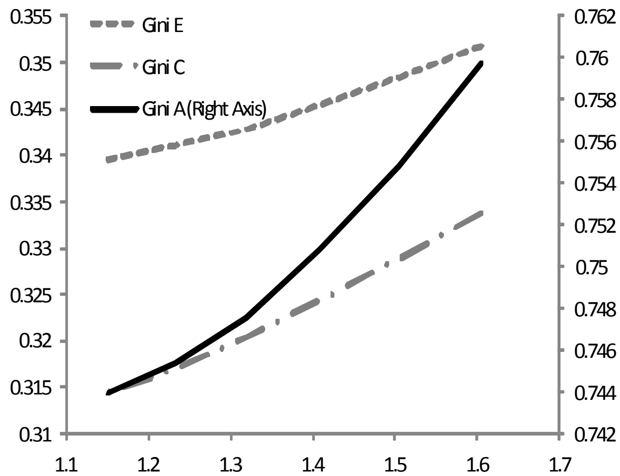

Figure 8: Robustness Analysis with respect to the income shocks variances $\sigma^2_{y,h}/\sigma^2_{y,l}$ ($\rho_y^{Avg} = .21, \rho_{y,h} = .988, \rho_{y,l} = .97$)