Job to Job Movements in A Simple Search Model

By PIETRO GARIBALDI AND ESPEN R. MOEN*

On the job search is a key feature of real life labor markets. Yet, traditional equilibrium unemployment theory has not been able to account for on-the-job search in a satisfactory manner. In this paper we present an equilibrium model which includes on-the-job search as an optimal response to search frictions and differences in firm productivity. Our model is laid out in detail in ongoing research by Garibaldi and Moen (2009).

In our model, on-the-job search is an optimal response to firm heterogeneity and search frictions in the labor market. The model has three key elements. First, it applies the competitive search equilibrium concept, initially proposed by Moen (1997). Thus, firms post wages and vacancies to minimize search and waiting costs, and the labor market is endogenously separated into submarkets. Second, firms have convex costs of maintaining vacancies (in our simulations, the number of vacancies per firm is fixed). Third, contracting between a firm and its employees is efficient, so that their joint income is maximized.

The model tends toward an equilibrium characterization in which there is a *job ladder* in the labor market. Low productivity firms pay low wages, face high turnover rates, grow slowly and hire directly from the unemployment pool. More efficient firms pay higher wages, grow more quickly and hire from the employment pool. This characterization is qualitatively consistent with a variety of stylized facts about industry dynamics and worker ows: 1) workers move from low-wage to high-wage occupations, 2) more productive firms are larger and pay higher wages than less productive firms, 3) job-to-job mobility falls with average firm size and worker tenure, 4) wages increase with firm size, and 5) wages are higher in fast-growing firms.

We also show that compared to traditional labor market models, our equilibrium model with on-the-job search delivers unexpected effects, even though it converges to traditional models as a special case (Pissarides 2000). We argue that an increase in average productivity, caused by an exogenous shift in the fraction of high-type firms in the market can actually lead to an *increase* in unemployment and a reduction in entry for a sub set of the parameter specification. Complex, albeit intuitive, composition effects between queue lenght across different submarkets rationalize these findings.

Pissarides (1994) seminal paper on on-the-job search utilizes Diamond-Mortensen-Pissarides type of matching models. The most used model of on-the-job search in empirical research is Burdett and Mortensen (1998) and its follow-ups, where firms post wages and there is no matching function. Moen and Rosen (2004) were the first to analyze competitive on-the-job search and the first to assume efficient on-the-job search. Menzio and Shi (2008), Lentz and Mortensen (2007) and Moscarini and Postel-Vinay (2009) are currently studying models of on-the-job search.

The paper proceeds as follows. Section 1 introduces the structure of the model and characterize the equilibrium. Section 2 show the non standard the effects of average productivity with a simple set of simulations.

I. The Model and Equilibrium

The labor market is populated by a measure 1 of identical workers. Individuals are risk neutral, infinitely lived, and discount the future at rate r. The technology requires an entry cost equal to K. Conditional upon entry, the firm learns its productivity, which can take two values: a low value y_1 or a high value y_2, with probabilities $1-\alpha$ and α, respectively. The productivity of a firm is fixed throughout its life. Unemployed workers have access to an income ow $y_0 < y_1$. Firms exit the market at a constant, time-independent rate δ.

Firms decide how many vacancies to post and what (net present value of) wages to attach to them. Each firm attaches the same wage to all its vacancies, but different firms of the same type may post different

* Garibaldi: Collegio Carlol Alberto, University of Torino, Via Real Collegio 30 10124 Moncalieri, pietro.garibaldi@unito.it Moen: Norwegian School of management, espen.r.moen@bi.no. We thank Dale Mortensen

wages. The maintenance cost of vacancies is given by an increasing and convex function $c(v)$. Workers exogenously leave the firm at rate s.

Search is directed. Firms post vacancies and wages to maximize expected profits. They face a relationship between the wage they set and the arrival rate of workers, which is derived from the indifference constraint of workers. Given this relationship, firms set wages so as to maximize profits.

As anticipated, we assume throughout that the firms and workers contract efficiently. In other words, the wage contract maximizes the joint income of the worker-firm pair. This simple assumption implies that a worker's on the job search behavior internalizes fully the loss of value incurred by the firm when she finds a new job. There are various wage contracts that may implement this behavior. For example, the worker pays the firm its entire NPV value up-front and then gets a wage equal to y_1. In other words, the worker buys the job from the firm and acts thereafter as a residual claimant. As an alternative contract, the worker gets a constant wage and pays a quit fee equal to the firm loss of profit if a new job is accepted (see Moen and Rosen (2004) for more examples). In any event, the wage paid to the worker in the current job does not in uence her on-the-job search behavior.[1]

A submarket is characterized by an aggregate matching functions bringing together the searching workers and vacant firms in that submarket. In equilibrium, up to three submarkets may be operating: unemployed workers searching for low-type jobs (the $_{01}$ market), workers employed in low-type firms searching for a job in a high-type firm (the $_{12}$ market), and unemployed workers searching for high-type jobs (the $_{02}$ market). As explained below, the first two submarkets are always active (attract agents), while the $_{02}$ market may or may not be active, depending on the parameter values. In all submarkets the matching technology is the same. Suppose a measure of N_{ij} workers search for a measure of V_{ij} vacancies. We assume a Cobb-Douglas matching function $x(N_{ij}, V_{ij}) = AN_{ij}^{\beta} V_{ij}^{1-\beta}$. The transition rates for workers and for firms are

$$\begin{aligned} p_{ij} &= A\theta_{ij}^{1-\beta} \\ q_{ij} &= A\theta_{ij}^{-\beta} \end{aligned}$$

where $\theta_{ij} = V_{ij}/N_{ij}$ is the labor market tightness in the market. Inverting the first of the previous conditions provide $\theta_{ij} = A^{-\frac{1}{1-\beta}} p_{ij}^{\frac{1}{1-\beta}}$ so that the transition rate for vacancies can be expressed as

$$q_{ij} = A^{\frac{\beta}{1-\beta}} p_{ij}^{-\frac{\beta}{1-\beta}} \tag{1}$$

A. Worker search

Let M_i be the expected joint income of a worker hired in a firm of type i.[2] Then

$$rM_i = y_i + (s+\delta)(M_0 - M_{ij}) + \max_j p_{ij}[W_{ij} - M_{ij}] \tag{2}$$

The first term is the ow production value created on the job. In addition, the current job can be destroyed for exogenous reasons at rate $s+\delta$. In this case the worker turns into unemployment and receives M_0 while the firm gets zero (for unemployed workers, the term is zero). Finally, the probability that the worker find a new job is p_{ij} . In this event, the worker receives a NPV wage W_{ij} while M_i is lost. Efficient on the job search implies that the workers' search so as to maximizes M_i. Since the wage paid by the firm is a pure transfer to the worker, it does not appear in the expression.

Workers employed in type 2 firms don't search, hence

$$M_2 = \frac{y_2}{r+s+\delta} \tag{3}$$

For searching workers (unemployed workers and workers employed in low-type firms) the indifference curve $p_i^w(W; M_i)$ shows combinations of p and W that provide the worker with NPV income of M_i. It follows that

$$p_i^w(W; M_i) = \frac{(r+s+\delta)M_i - y_i - (s+\delta)M_0}{W - M_i} \tag{4}$$

for $i = 0, 1$. Garibaldi and Moen (2009) show that the indifference curves only crosses once, say at $W = W'$. For wages below W', $p_0^w < p_1^w$. Hence,

[1] It follows that a worker in a low-type firm will never search for a job in another low-type firm, as these cannot offer a wage that exceeds the productivity in the current firm.

[2] We have simplified the model layout by collapsing the asset value equations for unemployed and employed workers. If $i = 0$, the worker is unemployed, and the "joint income" is the income of the worker.

if a firm advertizes a wage below W', unemployed workers accept a lower job finding rate (a lower labor market tightness) than employed workers, and the firm attracts only unemployed applicants. If the firm advertises a wage above W', the opposite happens, and the firm attracts only employed workers. In this way workers self-select into submarkets.

B. Firm search and wages

Firms decide on the number of vacancies to be posted and the wages attached to them. This only in- uences profits through future hirings, and is independent of the stock of existing workers. At any point in time, a firm maximizes the ow value of search, given by $\pi = c(v) + vq[M_j - W]$.

Suppose a firm of type j decides to search for a workers of type i. Its maximization problem then reads [3]

$$\begin{aligned} &\max_{W,v} -c(v) + vq[M_j - W]. \\ &\text{subject to} \\ q \quad &= \quad q(p_i^w(W, M_i)) \end{aligned}$$

The resulting value of p, W and π defines p_{ij}, W_{ij}, and π_{ij}. The first order conditions read

$$\begin{aligned} W_{ij} &= M_i + (M_j - M_i)\beta \\ c'(v) &= (1-\beta)(M_j - W_{ij})q(p) \end{aligned}$$

Using (2) gives

$$M_i = y_i + \beta p_{ij}(M_j - M_i) \tag{5}$$

$$c'(v) = (1-\beta)(M_j - W_{ij})q(p_{ij}) \tag{6}$$

Finally, define $\pi_j = \max \pi_{ij}$.

Since $y_1 > y_0$ and workers search equally efficiently on and off the job, the submarket $_{01}$ will attract both workers and firms. Furthermore, the $_{12}$ market will also always be open. If not, a high-type firm that opens vacancies with a wage slightly above y_1 would attract applications for all workers employed in type 1 firms. Hence q and thus also profits would be infinite, which is inconsistent with equilibrium. The $_{02}$ market may or may not be open depending on parameter values .

Finally, the expected profit of a firm of type j entering the market can be written as

$$\Pi_j = \frac{\pi_j}{r+\delta} \tag{7}$$

C. Equilibrium

Let N_i denote the measure of workers in type i firms the fraction of type j firms searching for type i workers, $\sum_{i=0}^{n} N_i = 1$. Furthermore, let $\tau \leq 1$ denote the fraction of the high-type firms searching in submarket $_{12}$ (for employed workers), and $1-\tau$ the fraction searching in market $_{02}$ (for unemployed workers). Similarly, let κ denote the fraction of unemployed workers searching for low-type firms, and $1-\kappa$ the fraction searching for high-type firms. The ow equation for N_0 is defined as

$$\begin{aligned} &N_0[\kappa p_{01} + (1-\kappa)p_{02} \\ = \quad &(s+\delta)(N_1 + N_2) \end{aligned}$$

The ow equations for N_1 and N_2 are defined analogously. Let k denote the number of firms in the economy. Labor market tightness in submarket $_{01}$ is then given by

$$\theta_{01} = k(1-\alpha)\frac{v_{01}}{\kappa N_0}$$

Labor market tightness in submarkets $_{02}$ and $_{12}$ are defined analogously.

DEFINITION 1: *The equilibrium is a vector of asset values M_0, M_1, and M_2, two fractions τ and κ, and a number k such that the following requirements are satisfied*

1. Optimal search: the asset values M_0, M_1, and M_2 are given by equations (3), (5) and (6).

2. Optimal allocation on submarkets: Either $\pi_{12} = \pi_{02} = \pi_2$ or $\kappa = \tau = 0$

3. Zero profit ex ante*: K: $(1-\alpha)\Pi_1 + \alpha\Pi_2 = K$*

4. Aggregate consistency: The ow conditions and the definitions of θ_{ij} are satisfied

D. Properties of equilibrium

An important consideration is whether the $_{02}$ market will open up (stairways to heaven), in which case we refer to a *mixed job ladder*. If the $_{02}$ market does not open up, we refer to a *pure job ladder*. As the

[3]In addition, the wage must be below (above) W' if the firm wants to attract unemployed (employed) workers. As this constraint never binds, it is ignored, see Garibaldi and Moen (2009).

next proposition shows, whether we have a mixed or pure job ladder depends on parameter values. However, the wage structure in the different submarket is always the same:

PROPOSITION 2: *a) For low values of α, all the three submarkets are active and we refer to a mixed job ladder equilibrium. For high values of α, only the $_{01}$ and the $_{12}$ markets are active and we refer to a pure job ladder equilibrium.*

b) The following is always true: $W_{01} < W_{02} < W_{12}$ and $p_{01} > p_{02} > p_{12}$

Suppose α is low, so that there are few high-type firms and many workers employed in low-type firms. By offering a wage slightly above y_1, high-type firms fill their vacancies quickly, grow quickly, and obtain a large profit. Hence they have no incentives to search for unemployed workers, and the economy is in a pure job-ladder equilibrium. As α grows, the $_{12}$ markets becomes more crowded with high-type vacancies relative to workers searching on the job, and profits fall. At some point the $_{02}$ submarket opens up, and the economy is in a mixed job-ladder equilibrium. In this kind of equilibrium, a fraction τ of the high-type firms search for unemployed workers, and a fraction κ of the unemployed workers search for high-type firms. The fractions τ and κ are determined so that high-type firms are indifferent between searching for employed and unemployed workers, while unemployed workers are indifferent between searching for high-type and low-type firms.

The concavity of the matching function implies that a high matching rate for agents on one side of the market (say firms) implies a low matching rate for agents on the other side of the market. Thus, it is efficient to let agents with a relatively low opportunity cost of waiting (employed workers and low-type firms) search for agents with a high opportunity cost of waiting (unemployed workers and high-type firms), and let the former match quickly and the latter slowly. Given the constraints imposed by the stocks of workers and firms, this is also how resources are allocated in equilibrium. In equilibrium, this is obtained by paying employed (patient) workers a high wage when matched (W_{12} is the highest wage in the economy), while the low-type (patient) firms pay a relatively low wage for workers (W_{01} is the lowest wage in the economy). The wage in the $_{02}$ submarket is intermediate.

II. The Increase in Productivity in Aggregate Labor Markets

The features of the pure and mixed job ladder equilibria can best be understood with the help of numerical simulations, obtained by a simple search routine described by Garibaldi and Moen (2009). In the specification of the model presented in this section, we assume that the convexity of the vacancy is extreme so that each firm can post at most a maximum number of vacancies υ.[4]

The main objective of the simulations is to show the mechanics of the model for different values of α. As $\overline{y} = (1-\alpha)y_1 + \alpha y_2$, an increase in α is akin to an increase in average productivity. The basic charts of the simulations are provided in Figure 1 and 2. First note that when $\alpha = 0$ or 1, the model collapses to the traditional matching model without on-the-job search (Pissarides 2000). As expected, the transition rate from unemployment to employment is higher and unemployment lower when $\alpha = 1$ than when $\alpha = 0$. (In Figure 1 unemployment falls from 0.0968 to 0.083 as α increases from 0 to 1). We refer to this as a pure *productivity effect*, and it is caused by a higher entry of firms and a higher f when output per firm is high.

For interior values, an increase in α has important *composition effects*. While the value functions increase smoothly as the economy becomes more productivity (top left panel in Figure 2), the increase in the job finding rate p_{01} in the pure job ladder is humped-shaped. For a fixed number of firms, an increase in α reduces the number of jobs available to the unemployed (who are hired in firms of type 1), and increase the jobs available to the employed (who are hired in firms of type 2). This composition effect tend to reduce the job finding rate p_{01}. The productivity effect increases the number of firms, and hence work in the opposite direction, but in the pure job ladder equilibrium it only dominates the composition effect for exceedingly low values of α. Note also that job-to-job movements, by definition equal to zero at the extremes, tend to grow naturally as the economy operates into a pure job ladder equilibrium.

[4]The rest of the parameters are as follows. The interest rate r is 0.01, the separation rate s is 0.04 while the firm exit rate is 0.02. The baseline productivity y_1 is normalized to 1 while the high type firm produtctivity is 1.08. The outside income is [0.55] and the marginal cost of vacancies is [0.2]. The matching function is Cobb Douglas with sharing parameter equal to 0.5 and constant parameter $A = 1$.

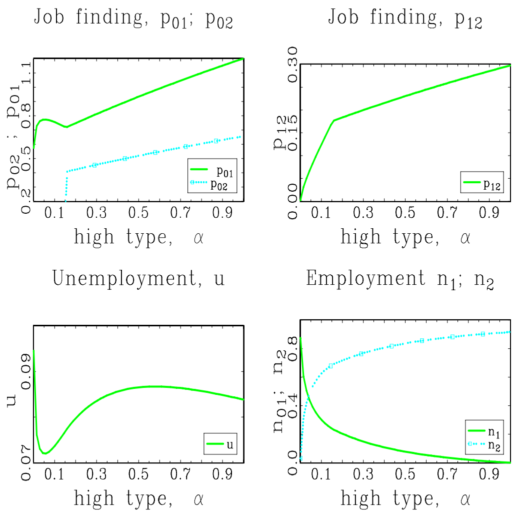

FIGURE 1. INCREASE IN AVERAGE PRODUCTIVITY, STOCKS AND JOB FINDING RATES

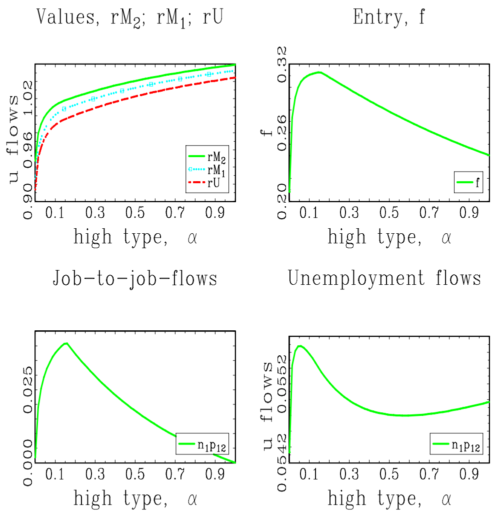

FIGURE 2. INCREASE IN AVERAGE PRODUCTIVITY, VALUE FUNCTIONS, FLOWS AND ENTRY

For higher values of α, mixed job ladder equilibrium emerges, with a different type of composition effects. In particular, the $_{02}$ submarket is characterized by lower job-finding rates. A higher α on some intervals implies larger variations in the queue lengths among unemployed workers, tending to increase unemployment. For relatively low levels of α this effect dominates the productivity effects. Eventually, as the share of high productivity firms increases toward 1, the pure productivity effects emerges and unemployment falls.

Finally, the non monotonic behavior of entry deserves comments. When α is low, the value of a high-type firm (given by 7) is extremely high since this type of firm grows so quickly. This explains the hump-shaped form of f, the number of firms in the economy.

References

Bertola, Giuseppe, and Pietro Garibaldi, (2001), Wages and the Size of Firms in Dynamic Matching Models. *Review of Economic Dynamics* 4 (2001): 335-368

Burdett, Kenneth, and Dale T. Mortensen, (1998) Wage Differentials, Employer Size, and Unemployment, *International Economic Review*, vol. 39(2), pages 257-73, May.

Garibaldi, P. and Espen R. Moen (2009), Competitive On-the-Job Search, unpublished paper.

Lentz, Rasmus, and Dale T. Mortensen (2007), Labor Market Friction, Firm Heterogeneity, and Productivity. Unpublished paper.

Jeremy Lise, Costas Meghir, and Jean-Marc Robin (2008), Matching, Sorting, and Wages. Unpublished paper.

Menzio G., and Shi S. Block Recursive Equilibria for Stochastic Models of Search on the Job.

Moen, Espen Rasmus (1997) Competitive Search Equilibrium. *Journal of Political Economy* 1997, vol 105, 385- 411

Moen Espen Rasmus, and Åsa Rosen (2004) Does poaching distort training? *Review of Economic Studies*, 2004 vol. 71, 1143-1162.

Moscarini, Giuseppe, and Fabien Postel-Vinay (2009). Non Stationary Search Equilibrium, unpublished paper

Pissarides, Chris (1994), Search Unemployment with On-the-job-search, *Review of Economic Studies*, Vol 61(3) 457-494

Pissarides, Chris (2000) *Equilibrium Unemployment Theory* Boston: MIT Press