Switching Costs and Equilibrium Prices

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Abstract

In a competitive environment, switching costs have two effects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. The conventional wisdom is that the first effect dominates the second one, so that equilibrium prices are higher the greater switching costs are. I provide sufficient conditions for the dynamic competition effect to dominate, so that switching costs lead to more competitive markets. The set of sufficient conditions includes low levels of switching costs and high values of the discount factor. I also consider a meta-game where sellers unilaterally choose the level of switching costs. I provide sufficient conditions such that this game has the nature of a prisoner's dilemma.

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An earlier draft (April 2008) circulated under the title "Small Switching Costs Lead to Lower Prices." While that title remains valid, the discovery of new results makes it incomplete.

1 Introduction

Consumers frequently must pay a cost in order to switch from their current supplier to a different supplier (Klemperer, 1995; Farrell and Klemperer, 2007). These costs motivate some interesting questions: are markets more or less competitive in the presence of switching costs? Specifically, are prices higher or lower under switching costs? How do firm profits and consumer surplus vary as switching costs increase? Is there a difference between exogenous switching costs (given by the nature of the product or transaction) and endogenous switching costs (created by sellers)?

Most of the economics literature has addressed the leading motivating questions by solving some variation of a simple two-period model.[1] The equilibrium of this game typically involves a bargain-then-ripoff pattern: in the second period, the seller takes advantage of a locked-in consumer and sets a high price (rip-off). Anticipating this second-period profit, and having to compete against rival sellers, the first-period price is correspondingly lowered (bargain).

One limitation of two-period models is that potentially they distort the relative importance of bargains and ripoffs. In particular, considering the nature of many practical applications, two-period models unrealistically create game-beginning and game-ending effects. To address this problem, I consider an infinite-period model where the state variable indicates the firm to which a given consumer is currently attached. The dynamic counterpart of the bargain-then-ripoff pattern is given by two corresponding effects on a firm's dynamic pricing incentives: the harvesting effect (firms with locked-in customers are able to price higher without losing demand) and the investment effect (firms without locked-in customers are eager to cut prices in order to attract new customers).

The harvesting and investment effects work in opposite directions in terms of market average price. Which effect dominates? Conventional wisdom and the received economics literature suggest that the harvesting effect dominates (Farrell and Klemperer, 2007). However, recent research casts doubt on this assertion (Doganoglu, 2005; Dubé, Hitsch and Rossi, 2007). In this paper, I follow this line of research. I provide sufficient conditions such that the dynamic competition effect dominates, so that switching costs lead to more competitive markets. The set of sufficient conditions includes low levels of

1. See Section 2.3.1. in Farrell and Klemperer (2007) for a survey.

switching costs and high values of the discount factor.

Some times, switching costs are given by the nature of the product or transaction. Some other times, switching costs are artificially created by sellers (e.g., by choosing rules of a frequent flyer program). If prices (and profits) are lower with switching cots, then a natural follow up question is, what level of switching costs should sellers choose if they can do so? I provide sufficient conditions such that a unilateral increase in switching costs increases a seller's payoff. Together with the previous results, this implies that the switching cost metagame has the structure of a prisoner's dilemma.

■ **Related literature.** The classic reference on infinite period competition with switching cots is Beggs and Klemperer (1992). They show that switching costs lead to higher equilibrium prices. My approach differs from theirs in two important ways. First, they assume infinite switching costs (that is, a locked-in customer never leaves its supplier). Second, unlike myself they consider the case when the seller cannot discriminate between locked-in and not locked-in consumers.[2]

In a recent paper, Dubé, Hitsch and Rossi (2007) show, by means of numerical simulations, that if switching costs are small then the investment effect dominates, that is, switching costs increase market competitiveness. I analytically solve a version of their model. Analytical solution has two advantages. First, it leads to more general results, that is, results that are not dependent on specific assumptions regarding functional forms and parameter values. Second, the process of solving the model leads to a better understanding of the mechanics underlying the result that the average market price declines when switching costs increase. It also shows why there is an important difference between small and large switching costs.

In a recent paper, Doganoglu (2005) considers the case of small switching costs and shows that, along the equilibrium path, locked-in customers switch to the rival seller with positive probability. Moreover, steady-state equilibrium prices are decreasing in switching costs. Doganoglu's (2005) approach differs from mine in various respects. He assumes uniformly distributed preferences and linear pricing strategies; by contrast, I make very mild assumptions regarding the distribution of buyer preferences and the shape of the seller's pricing strategies. Moreover, my analysis goes beyond the case of small switching costs.

2. Related papers include Farrell and Shapiro (1988), To (1995), and Padilla (1995).

2 Model

Consider an industry where two sellers compete over an infinite number of periods for sales to n infinitely lived buyers. Each buyer purchases one unit each period from one of the sellers. A buyer's valuation for seller i's good is given by z_i, which I assume is stochastic and i.i.d. across sellers and periods.[3] Moreover — and this a crucial element in the model — if the buyer previously purchased from seller j, then his utility from buying from seller i in the current period is reduced by s, the cost of switching between sellers.

In each period, sellers set prices simultaneously and then each buyer chooses one of the sellers. I assume that sellers are able to discriminate between locked-in and not locked-in buyers (that is, buyers who are locked in to the rival seller). Without further loss of generality, I hereafter focus on the sellers' competition for a particular buyer. I focus on symmetric Markov equilibria where the state indicates which seller made the sale in the previous period. I denote the seller who made a sale in the previous period (the "incumbent" seller) with the subscript 1, and the other seller (the "challenger" seller) with the subscript 0.

Symmetry implies that the buyer's continuation values from being locked in to seller i or seller j are the same. This greatly simplifies the analysis. In particular, in each period the buyer chooses the incumbent seller if and only if

$$z_1 - p_1 \geq z_0 - p_0 - s$$

Define

$$\begin{aligned} x &\equiv z_1 - z_0 \\ P &\equiv p_1 - p_0 - s \end{aligned} \tag{1}$$

In words, x is the relative preference for the incumbent's seller product, whereas P is the price difference corrected for the switching cost. It follows that the buyer chooses the incumbent if and only if $x > P$.

Define by q_1 and q_0 the probability that the buyer chooses the incumbent or the entrant, respectively. If x is distributed according to $F(x)$, then we

3. In this sense, my model differs from the literature on customer recognition, where sellers learn about their buyers' valuations. See Villas-Boas (1999, 2006), Fudenberg and Tirole (2000), Doganoglu (2005).

have

$$\begin{aligned} q_1 &= 1 - F(x) \\ q_0 &= F(x) \end{aligned}$$

I make the following assumptions regarding the c.d.f. F and the corresponding density f:

Assumption 1 *(i) $F(x)$ is continuously differentiable; (ii) $f(x) = f(-x)$; (iii) $f(x) > 0$, $\forall x$; (iv) $f(x)$ is unimodal; (v) $F(x)/f(x)$ is strictly increasing.*

In the Appendix, I present a Lemma that proves a series of properties of $F(x)$ that are derived from Assumption 1.

In this paper, I will focus on symmetric Markov equilibria. My first result shows that there exists only one such equilibrium. The proof of this and the remaining results in the paper may be found in the Appendix.

Proposition 1 *There exists a unique symmetric Markov equilibrium.*

In the next two sections, I offer two sets of sufficient conditions for the main result in the paper, namely that an increase in switching costs implies a decrease in average equilibrium price. In the next section, I consider the case of small switching costs. In Section 4, I consider the case of a high discount factor.

3 Small switching costs

My main goal is to characterize equilibrium pricing as a function of switching costs s. In this section, I consider the case of small switching costs and prove that average price is decreasing in s. Let $\bar{p}$ be the average price paid by the buyer, that is,

$$\bar{p} = q_1 \, p_1 + q_0 \, p_0.$$

Proposition 2 *If s is small, then $\bar{p}$ is decreasing in s.*

To understand the intuition for Proposition 2, it is useful to look at the sellers' first-order conditions. The incumbent seller's value function is given by

$$v_1 = \big(1 - F(P)\big)\big(p_1 + \delta \, v_1\big) + F(P) \, \delta \, v_0$$

where v_i is seller i's value. In words: with probability $1-F(P)$, the incumbent seller makes a sale. This yields a short-run profit of p_1 and a the continuation value of an incumbent, v_1. With probability $F(P)$, the incumbent loses the sale, makes zero short run profits, and earns a continuation value v_0.

Maximizing with respect to p_1, we get the incumbent seller's first-order condition:

$$p_1 = \frac{1 - F(P)}{f(P)} - \delta\, V \tag{2}$$

where $V \equiv v_1 - v_0$ is the difference, in terms of continuation value, between winning and losing the current sale. In other words, $-\delta\, V$ is the "cost," in terms of discounted continuation value, of winning the current sale.

Since $q_1 = 1 - F(P)$ and $P = p_1 - p_0 - s$, we have $\frac{d\,q_1}{d\,p_1} = f(P)$. It follows that (2) may be re-written as

$$\frac{p_1 - (-\delta\, V)}{p_1} = \frac{1}{\epsilon_1}$$

where $\epsilon_1 \equiv \frac{d\,q_1}{d\,p_1}\,\frac{p_1}{q_1}$. This is simply the "elasticity rule" of optimal pricing, with one difference: the future discounted value from winning the sale appears as a negative cost (or subsidy) on price.

We thus have two forces on optimal price, which might denoted by "harvesting" and "investing." If the seller is myopic ($\delta = 0$), then optimal price is given by the first term in the right-hand side of (2). The greater the value of s, the smaller the value of P (as shown in the proof of Proposition 2), and therefore the greater the value of p_1. We thus have harvesting, that is, a higher switching cost implies a higher price (by the incumbent seller, which is the more likely seller).

Suppose however that $\delta > 0$. Then we have a second effect, investing, which leads to lower prices. The greater the value of s, the greater the difference between being an incumbent and being an entrant, that is, the greater the value of V.

What is the relative magnitude of the harvesting and the investment effects on average price? First notice that harvesting leads to a higher price by the incumbent but lower by the entrant. If fact, by symmetry, the effects are approximately of the same absolute value when s is close to zero. This implies that, for s close to zero and in terms of average price, the harvesting effects approximately cancel out, since for $s = 0$ incumbent and challenger sell with equal probability.

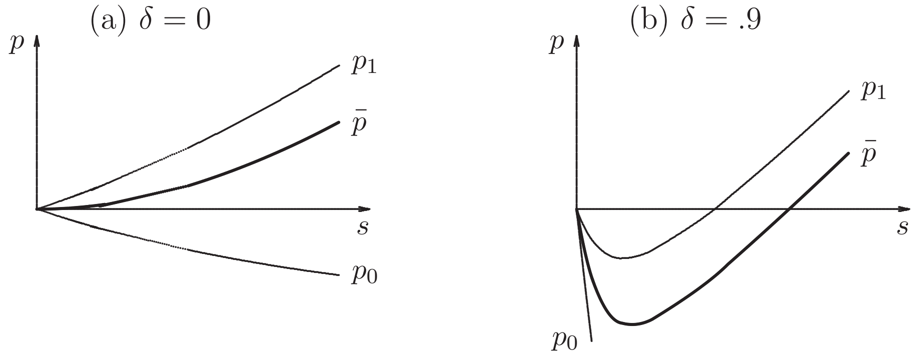

Figure 1: Switching cost and equilibrium price.

Not so with the dynamic effect. In fact, the entrant's first-order condition is given by

$$p_0 = \frac{F(P)}{f(P)} - \delta\, V$$

that is, the "subsidy" resulting from the value of winning is the same as for the incumbent. It follows that the effect on average price is unambiguously negative, and of first-order importance.

In other words, the harvesting effect is symmetric: the amount by which the incumbent increases its price is the same as the amount by which the outsider lowers its price. However, the investment effect is equal for both sellers — and negative.

Figure 1 illustrates Proposition 2. (In this and in the remaining numerical illustrations, I assume x is distributed according to a standardized normal.) On the horizontal axis, the value of switching cost varies from zero to positive values. On the vertical axis, three prices are plotted: the incumbent's price, the challenger's price, and average price. The left-hand panel shows the case when $\delta = 0$. Since there is no future, only the harvesting effect applies. The incumbent seller sets a price that is higher the higher the value of s. The challenger sets a price that is lower the higher s is. This results in an average price which is increasing in s. In other words, in a static world switching costs imply higher prices.[4] Notice however that the derivative of average price with respect to s equals zero when $s = 0$. That is, for small values of s the impact of s on $\bar{p}$ is of second-order magnitude.

4. This corresponds to the "ripoff effect" in two-period models. See Farrell and Klemperer (2007).

The investment effect, by contrast, is of first-order magnitude even for small values of s. This implies that, for a positive value of δ, the investment effects dominates the harvesting effect for small values of s. This is illustrated by the right-hand panel, where it can be seen that $\bar{p}$ is declining in s for small values of s.

Dubé, Hitsch and Rossi (2006) claim that, for various products, the value of switching cost lies in the region when the net effect of switching costs is to decrease average price. In the next section, I show that, if the discount factor is sufficiently close to one, then switching costs lead to lower prices for *any* positive value of switching costs.

4 Endogenous switching costs

Suppose that only firm i creates a switching cost. In other words, it costs s for a buyer to switch from seller i to seller j, but it costs zero for the buyer to switch from seller j to seller i. Unlike the previous sections, I now must differentiate between seller i's and seller j's prices and value functions. Moreover, I must explicitly consider the buyer's dynamic optimization problem. In fact, it is not indifferent for a buyer to be locked-in to seller i or to seller j.

How does seller i's value changes as it increases the value of s? My main result in this section addresses this issue.

Proposition 3 *If s is small, then a unilateral increase in s increases the seller's average value if and only if $\delta < \frac{1}{2}$.*

In what follows, I use the notation, for a generic variable y,

$$\hat{y} \equiv \left. \frac{d\,y}{d\,s} \right|_{s\,=\,0}$$

Note that, at $s = 0$, we have a symmetric outcome where $x_i = x_j = 0$, $u_i = u_j = u$, and $p_{1i} = p_{0j} = p_{0i} = p_{1j} = p$. Differentiating the buyer value functions with respect to s at $s = 0$, I then get

$$\hat{u}_i = \frac{1}{2}\left(\delta\,\hat{u}_i + \delta\,\hat{u}_j - \hat{p}_{1i} - \hat{p}_{0j} - 1\right)$$
$$\hat{u}_j = \frac{1}{2}\left(\delta\,\hat{u}_i + \delta\,\hat{u}_j - \hat{p}_{1j} - \hat{p}_{0i}\right)$$

This is intuitive: a buyer's expected valuation increases by the increase in expected valuation, $\delta \frac{1}{2}(\hat{u}_i + \hat{u}_j)$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}(\hat{p}_{1i} + \hat{p}_{0j})$ if the buyer is attached to seller i and $\frac{1}{2}(\hat{p}_{1j} + \hat{p}_{0i})$ if the buyer is attached to seller j. Moreover, if the buyer is attached to seller i, buyer welfare further decreases by an additional $\frac{1}{2}\, s$, the probability that an immediate switch to seller j will take place.

$$
\begin{aligned}
\hat{x}_i &= \hat{p}_{1i} - \hat{p}_{0j} - \delta\,(\hat{u}_i - \hat{u}_j) - 1 \\
\hat{x}_j &= \hat{p}_{1j} - \hat{p}_{0i} - \delta\,(\hat{u}_j - \hat{u}_i)
\end{aligned}
$$

Substituting (15) for $\hat{u}_i - \hat{u}_j$ and simplifying

$$
\begin{aligned}
\hat{x}_i &= (p_{1i} - p_{0j}) + \delta\,\frac{1}{2}\,\left(\hat{p}_{1i} + \hat{p}_{0j} - \hat{p}_{1j} - \hat{p}_{0i}\right) - \left(1 - \frac{\delta}{2}\right) \\
\hat{x}_j &= (p_{1j} - p_{0i}) + \delta\,\frac{1}{2}\,\left(\hat{p}_{1j} + \hat{p}_{0i} - \hat{p}_{1i} - \hat{p}_{0j}\right) - \frac{\delta}{2}
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
\hat{x}_i &= -\frac{2-\delta}{6} \\
\hat{x}_j &= -\frac{\delta}{6} \\
\hat{p}_{1i} &= \frac{1}{2} - \frac{\delta}{2} \\
\hat{p}_{0i} &= -\frac{\delta}{2} \\
\hat{p}_{1j} &= -\frac{\delta}{6} \\
\hat{p}_{0j} &= -\frac{1}{3} - \frac{\delta}{6}
\end{aligned}
$$

Recall that these are variations with respect to the equilibrium values at $s = 0$. The above values indicate that, if δ is small, then seller i, by creating a switching cost s, is able to increase its price when the buyer is locked-in, specifically by $\left(\frac{1}{2} - \frac{\delta}{2}\right) ds$. If the buyer is locked-in to seller j, however, then seller i must decrease its price by $\frac{\delta}{2}\, ds$. Moreover, as $\delta \to 1$, the size of the price increase is equal to the size of the price decrease.

$$\hat{v}_{1i} = \frac{2-3\,\delta}{6\,(1-\delta)}$$
$$\hat{v}_{0i} = \frac{-\delta}{6\,(1-\delta)}$$
$$\frac{1}{2}\left(\hat{v}_{0i}+\hat{v}_{1i}\right) = \frac{1-2\,\delta}{6\,(1-\delta)}$$

For seller j, we have

$$\hat{v}_{1j} = -\frac{\delta}{6\,(1-\delta)}$$
$$\hat{v}_{0j} = -\frac{2-\delta}{6\,(1-\delta)}$$
$$\frac{1}{2}\left(\hat{v}_{0j}+\hat{v}_{1j}\right) = -\frac{1}{6\,(1-\delta)}$$

In words, seller i is better off in state one if and only if $\delta < \frac{2}{3}$, and is better on average if and only if $\delta < \frac{1}{2}$. Seller j, in turn, is always worse off.

5 Hight discount factor

In the Section 3, I showed that, for any preference distribution F satisfying Assumption 1 and any positive discount factor δ, if the switching cost s is small enough then average price is decreasing in s. In this section, I provide an alternative sufficient condition for competitive switching costs. I show that, for any preference distribution F satisfying Assumption 1 and for any positive value of the switching cost s, if the discount factor is sufficiently close to 1 then average price is decreasing in s.

Proposition 4 *If δ is close to 1, then $\bar{p}$ is decreasing in s.*

Figure 2 illustrates Proposition 4. It plots average price as a function of s for various values of δ. The curve corresponding to $\delta = .9$ is identical to Figure 1. It is U shaped: for small values of s, average price is decreasing in s (Proposition 2). However, for high values of s, average price becomes increasing in s. As we consider higher values of δ, the U shape becomes more

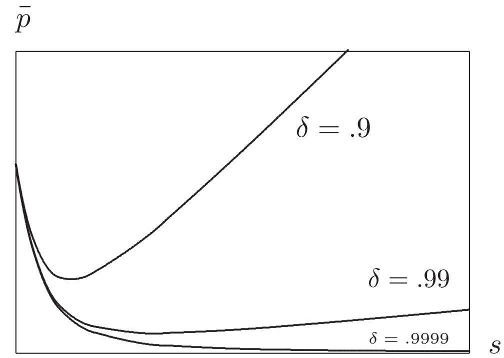

Figure 2: Switching cost and equilibrium price as $\delta \to 1$.

and more extended, so that, for a given range $[0, \bar{s}]$ of values of s, average price eventually becomes uniformly decreasing in s (Proposition 4).

I next attempt to provide an intuitive explanation for Proposition 4. In discussing Proposition 2, we saw that equilibrium prices are given by

$$\begin{aligned} p_1 &= \frac{1 - F(P)}{f(P)} - \delta\, V \\ p_0 &= \frac{F(P)}{f(P)} - \delta\, V \end{aligned} \tag{4}$$

(As I mentioned earlier, this is just the "elasticity rule" with the added element that sellers "subsidize" their cost by $\delta\, V$.) We also saw that the seller value functions are given by

$$\begin{aligned} v_1 &= \big(1 - F(P)\big)\big(p_1 + \delta\, v_1\big) + F(P)\; \delta\, v_0 \\ v_0 &= F(P)\big(p_0 + \delta\, v_1\big) + \big(1 - F(P)\big)\; \delta\, v_0 \end{aligned}$$

Substituting equilibrium prices into the value functions and simplifying we get

$$\begin{aligned} v_1 &= \frac{\big(1 - F(P)\big)^2}{f(P)} + \delta\, v_0 \\ v_0 &= \frac{F(P)^2}{f(P)} + \delta\, v_0 \end{aligned} \tag{5}$$

If $\delta = 0$, seller value is given by short-run profit, the first term on the right-hand side of the value functions. In a dynamic equilibrium, seller value is given by these short-term profits plus $\delta\, v_0$, *regardless of whether the seller wins or loses the current sale.*

This is an important point and one worth exploring in greater detail. To understand the intuition, it may be useful to think of an auction with two bidders with the same valuations. Specifically, each better gets w if he wins the auctions and l if he loses. The Nash equilibrium is for both bidders to bid $w-l$. If follows that equilibrium value is l for both bidders (winner or loser). In other words, the extra gain a bidder receives from being the winner, $w-l$, is bid away, so that a bidder can't expect more than l.

In the dynamic game at hand the analog of l is the continuation value if the seller loses the current sale, $\delta\, v_0$. So the idea is that all of the extra gain in terms of future value, $\delta\, V = \delta\,(v_1 - v_0)$, is bid away in terms of lower prices.

What does this imply in terms of equilibrium prices? From (5), we get

$$V = v_1 - v_0 = \frac{\big(1 - F(P)\big)^2}{f(P)} - \frac{F(P)^2}{f(P)} = \frac{1 - F(P)}{f(P)} - \frac{F(P)}{f(P)} \tag{6}$$

Substituting for V in (4) we get

$$\lim_{\delta\to 1} p_1 = \frac{1 - F(P)}{f(P)} - V = \frac{F(P)}{f(P)} \tag{7}$$

But, as we can see from (4), the right-hand side of (7) is simply the equilibrium value of p_0 when $\delta = 0$. In words, as the discount factor tends to 1, the incumbent's price level converges to the the entrant's static price level (i.e., when $\delta = 0$). But we know that, in a static model, increasing vertical product differentiation (in particular, increasing the switching cost) leads to a lower price by the "entrant" seller. That is, as we increase s from zero to a positive value, keeping $\delta = 0$, then the incumbent's price increases and the entrant's price decreases. If $s = 0$, then equilibrium price is the same regardless of the value of δ. Finally, putting it all together, we conclude that, as $\delta \to 1$ and $s > 0$, the high price is at the level of the lower price when $s = 0$; and so switching costs lead to lower average price.

The above argument is illustrated in Figure 3. If $s = 0$, then equilibrium price is the same for incumbent and entrant; moreover, it is independent of the value of δ. Now consider a positive switching cost, say $s = 1$. If $\delta = 0$, then we have a standard problem of vertical product differentiation. The

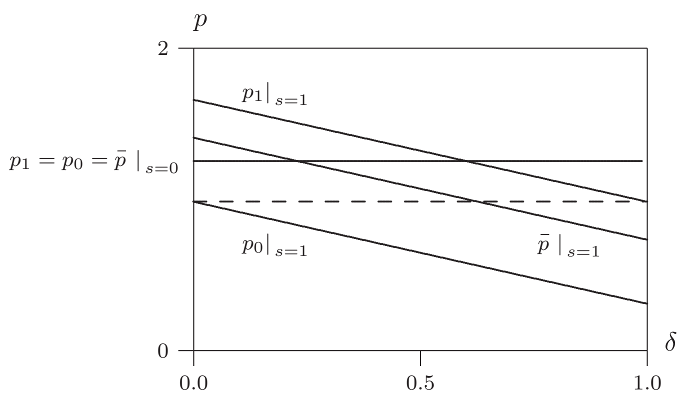

Figure 3: High price, low price, and average price as a function of the discount factor, when $s = 0$ and when $s = 1$.

incumbent's price is higher than under no switching costs, whereas the entrant's price is lower than under no switching cost. Average price increases with switching costs, for two reasons: first, the increase in the incumbent's price is greater than the decrease in the entrant's price. Second, the incumbent sells with a higher probability.

As the value of δ increases, p_1, p_0 and $\bar{p}$ decrease (linearly in δ).[5] When $\delta = 1$, p_1 is at the level of p_0 when $\delta = 0$ (by the argument presented above). We now have a series of inequalities: at $\delta = 1$ and $s = 1$, average price is lower than the high price ($\bar{p} \mid_{s=1,\delta=1} < p_1|_{s=1,\delta=1}$). This in turn is equal to the low price when $\delta = 0$ ($p_1|_{s=1,\delta=1} = p_0|_{s=1,\delta=0}$). This in turn is lower than average price when $s = 0$, regardless of the value of the discount factor ($p_0|_{s=1,\delta=0} < p_0|_{s=0}$). And so, for $\delta = 1$, average price is lower with $s = 1$ than with $s = 0$, an implication of Proposition 5 ($\bar{p} \mid_{s=1,\delta=1} < \bar{p} \mid_{s=0,\delta=1}$).

■ **Discussion.** Figure 4 summarizes the main results of Sections 3 and 5. The SE curve represents the points at which the derivative of average price with respect to switching cost is zero. At points to the SE of this curve, an increase in switching cost implies a lower average price. Propositions 2 and 4 state two important properties of this curve: points with s sufficiently small (Proposition 2) or δ sufficiently high (Proposition 4) below to region A. The

5. To understand why prices vary linearly with δ, notice that, from (6), V only depends on P. Moreover, subtracting the two equations (4), we get P as a function of s and V. It follows that the values of P and V depend on s but not on δ. Finally, from (4) p_i is linear in δ.

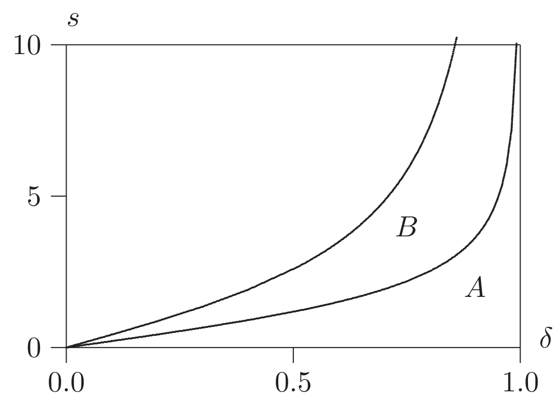

Figure 4: In region A, an increase in switching costs leads to a lower average equilibrium price. In regions A and B, average price is lower than it would be if switching costs were zero.

figure suggests that this characterization is "tight," that is, Propositions 2 and 4 describe the essential properties of the boundary of region A.

So far, I have examined how average price changes when switching costs increase. An alternative interesting comparison is between average price with $s > 0$ and average price when $s = 0$. The NW-most curve in Figure 4 depicts points such that average price is the same as when $s = 0$. For points to the SE of this curve, average price is lower with switching costs than without switching costs.

6 Conclusion

This paper contributes several points to the understanding of switching costs and market competition. First, I provide sufficient conditions such that (symmetric) switching costs lead to lower seller prices and profits. Second, I provide conditions such that a unilateral increase in switching costs increases firm profits. Taken together, these points imply that the meta game whereby firms choose their level of switching costs has the nature of a prisoner's dilemma.

Although I make only very weak assumptions regarding the nature of product differentiation, I do make some important assumptions regarding the nature of pricing and the dynamics of buyer preferences. First, as mentioned in the introduction, I assume that sellers can discriminate between buyers who are locked in and buyers who are not. If sellers cannot discriminate, then we must simultaneously consider all buyers (not just one) and the seller's optimal price will strike a balance between harvesting locked-in buyers and investing on new buyers. Beggs and Klemperer (1992) argue that the balance tends to favor higher prices than without switching costs. The contrast between my result and that of Beggs and Klemperer (1992) bears some relationship to the literature of oligopoly price discrimination (Corts, 1998). Oligopolists typically would like to commit not to price discriminate as this would soften overall price competition.

Secondly, I assume buyer preferences are i.i.d. across periods. Other models consider the possibility of serial correlation in buyer preferences. If the seller can discriminate between buyers, then we have a case of "customer recognition." Basically, conditionally on having made a sale in the previous period, a seller should expect its locked-in buyer to have a higher z than the population distribution would suggest. Villas-Boas (1999), Fudenberg and Tirole (2000), Doganoglu (2005) consider this possibility. It is not clear what the combination of switching costs and customer recognition implies for average prices.

Thirdly, I assume symmetry, both in terms of costs and in terms of buyer preferences. This assumption is not innocuous. In fact, the argument underlying Proposition 2 depends crucially on symmetry: for low values of s and δ, the increase in the incumbent's price approximately cancels the decrease in price by the entrant. If market shares are approximately 50%, then average price changes by an amount that is of second-order magnitude. However, if one of the sellers is much greater than the other one (either because it has lower costs or a better product), then the same is no longer true. In other words, in an industry with a dominant seller, switching costs are likely to increase prices and reduce buyer welfare.

Having said that, I should also add that none of my results is knife-edged. In other words, my results are based on strict inequalities. This implies that I can slightly perturb the model and still get similar qualitative results.

Appendix

The proofs of Propositions 1–4 will use repeatedly the following result, which characterizes several properties of F that follow from Assumption 1:

Lemma 1 *Under Assumption 1, the following are strictly increasing in x:*

$$\frac{F(x)^2}{f(x)}\,, \qquad \frac{F(x)-1}{f(x)}\,, \qquad \frac{2\,F(x)-1}{f(x)}\,.$$

Moreover, the following is increasing in x iff $x>0$ (and constant in x at $x=0$):

$$\frac{\big(1-F(x)\big)^2+\big(F(x)\big)^2}{f(x)}\,.$$

Proof of Lemma 1: First notice that

$$\frac{F(x)^2}{f(x)} = F(x)\,\frac{F(x)}{f(x)}.$$

Since $F(x)$ is increasing and $\frac{F(x)}{f(x)}$ is strictly increasing (by Assumption 1), it follows that the product is strictly increasing.

Next notice that, by part (ii) Assumption 1,

$$\frac{F(x)-1}{f(x)} = \frac{-F(-x)}{f(x)} = \frac{-F(-x)}{f(-x)}$$

Since $\frac{F(x)}{f(x)}$ is strictly increasing, $\frac{-F(-x)}{f(-x)}$ is strictly increasing too.

Next notice that

$$\frac{2\,F(x)-1}{f(x)} = \frac{F(x)-1}{f(x)} + \frac{F(x)}{f(x)}.$$

I have just proved that $\frac{F(x)-1}{f(x)}$ is strictly increasing. We thus has the sum of two strictly increasing functions, the result being a strictly increasing function.

Finally, taking the derivative of the fourth expression I get

$$\frac{d}{d\,x}\left(\frac{\big(1-F(x)\big)^2+\big(F(x)\big)^2}{f(x)}\right)=$$
$$=\frac{\big(-2\big(1-F(x)\big)f(x)+2\,F(x)\,f(x)\big)\,f(x)}{\big(f(x)\big)^2}-$$
$$-\frac{f'(x)\left(\big(1-F(x)\big)^2+\big(F(x)\big)^2\right)}{\big(f(x)\big)^2}$$
$$=4\left(F(x)-\frac{1}{2}\right)-f'(x)\,\xi,$$

where $\xi=\left(\big(1-F(x)\big)^2+\big(F(x)\big)^2\right)/\big(f(x)\big)^2$ is positive. The result then follows from Assumption 1. ■

Proof of Proposition 1: The seller value functions are given by

$$\begin{aligned} v_1 &= \big(1-F(P)\big)\big(p_1+\delta\,v_1\big)+F(P)\,\delta\,v_0 \\ v_0 &= F(P)\big(p_0+\delta\,v_1\big)+\big(1-F(P)\big)\,\delta\,v_0 \end{aligned} \tag{8}$$

The corresponding first-order conditions are

$$\begin{aligned} -f(P)\big(p_1+\delta\,v_1\big)+1-F(P)+f(P)\,\delta\,v_0 &= 0 \\ -f(P)\big(p_0+\delta\,v_1\big)+F(P)+f(P)\,\delta\,v_0 &= 0 \end{aligned}$$

(Recall that, from (1), $\frac{d\,P}{d\,p_1}=1$ and $\frac{d\,P}{d\,p_0}=-1$.) Solving for optimal prices, I get

$$\begin{aligned} p_1 &= \frac{1-F(P)}{f(P)}-\delta\,V \\ p_0 &= \frac{F(P)}{f(P)}-\delta\,V \end{aligned} \tag{9}$$

where

$$V\equiv v_1-v_0$$

Substituting (9) for p_1, p_0 in (8) and simplifying, I get

$$\begin{aligned} v_1 &= \frac{\big(1-F(P)\big)^2}{f(P)} + \delta\, v_0 \\ v_0 &= \frac{F(P)^2}{f(P)} + \delta\, v_0 \end{aligned}$$

It follows that

$$P = \frac{1-F(P)}{f(P)} - \frac{F(P)}{f(P)} - s = \frac{1-2\,F(P)}{f(P)} - s \tag{10}$$

$$V = \frac{\big(1-F(P)\big)^2}{f(P)} - \frac{F(P)^2}{f(P)} = \frac{1-2\,F(P)}{f(P)} \tag{11}$$

Equation (10) may be rewritten as

$$P + \frac{2\,F(P)-1}{f(P)} = -s \tag{12}$$

By Lemma 1, the left-hand side is strictly increasing in P, ranging from $-\infty$ to $+\infty$ as P itself ranges from $-\infty$ to $+\infty$. This implies there exists a unique solution P. From (11), there exists a unique V. Finally, from (9) there exist unique p_0, p_1. ■

Proof of Proposition 2: Average price is given by

$$\bar{p} \equiv q_1\, p_1 + q_0\, p_0 = \big(1-F(P)\big) p_1 + F(P)\, p_0$$

Substituting (9) for p_1, p_0 and (11) for V, and simplifying, I get

$$\begin{aligned} \bar{p} &= \big(1-F(P)\big)\left(\frac{1-F(P)}{f(P)} - \delta\, V\right) + F(P)\left(\frac{F(P)}{f(P)} - \delta\, V\right) \\ &= \frac{\big(1-F(P)\big)^2}{f(P)} + \frac{F(P)^2}{f(P)} - \delta\, V \\ &= \frac{\big(1-F(P)\big)^2 + F(P)^2}{f(P)} + \delta\left(\frac{2\,F(P)-1}{f(P)}\right) \end{aligned} \tag{13}$$

Lemma 1 implies that, at $s=0$, the first term on the right-hand side of (13) is constant in P. It also implies that the second term on the right-hand side of (13) is increasing in P. It follows that, if s is small, then $\frac{d\,\bar{p}}{d\,P} > 0$.

From (12) and the implicit function theorem, I get $\frac{d\,P}{d\,s} < 0$. Finally, if s is small then

$$\frac{d\,\bar{p}}{d\,s} = \left(\frac{d\,\bar{p}}{d\,P}\right)\left(\frac{d\,P}{d\,s}\right) < 0,$$

which concludes the proof. ■

Proof of Proposition 3: Let v_{1i} and v_{0i} be seller i's value when it is an incumbent or an entrant, respectively. Let p_{1i} and p_{0i} be seller i's price when it is an incumbent or an entrant, respectively. A similar notation applies to seller j. Let u_i be the buyer's value of being attached to seller i, measured at the beginning of the period and before learning the valuations z for that period.

The buyer, if currently locked-in to seller i, chooses seller i again if and only if

$$z_i - p_{1i} + \delta\,u_i \geq -s + z_j - p_{0j} + \delta\,u_j$$

or

$$z_i - z_j \geq p_{1i} - p_{0j} - \delta\,u_i + \delta\,u_j - s$$

If the buyer is locked-in to seller j, however, then he chooses seller j if and only if

$$z_j - z_i \geq p_{1j} - p_{0i} - \delta\,u_j + \delta\,u_i$$

Notice the asymmetry between the two cases. Define by x_i (resp. x_j) the critical level of $z_i - z_j$ (resp. $z_j - z_i$) such that the buyer will prefer to stay with his current seller, seller i (resp. j). We thus have

$$\begin{aligned} x_i &\equiv p_{1i} - p_{0j} - \delta\,u_i + \delta\,u_j - s \\ x_j &\equiv p_{1j} - p_{0i} - \delta\,u_j + \delta\,u_i \end{aligned} \tag{14}$$

Finally, knowing that $z_i - z_j$ is distributed according to $F(\cdot)$, we conclude that the incumbent seller's demand is given by $q_{1k} = 1 - F(x_k) = F(-x_k)$, $k = i, j$. Specifically,

$$\begin{aligned} q_{1i} &= 1 - F(x_i) = F\left(p_{0j} - p_{1i} + \delta\,u_i - \delta\,u_j + s\right) \\ q_{1j} &= 1 - F(x_j) = F\left(p_{0i} - p_{1j} + \delta\,u_j - \delta\,u_i\right) \end{aligned}$$

Let $\phi(\cdot)$ be the distribution of z_i (and z_j) and define

$$E(x) \equiv \int_{z-z' \geq x} z\; d\phi(z)\, d\phi(z')$$

In words, $E(x)$ is the buyer's expected valuation given that he chooses a particular seller by using the threshold x of differences in valuations (times the probability of choosing that seller). Let $e(x) \equiv \frac{d\,E(x)}{d\,x}$.

The buyer's value functions, measured *before* the buyer learns his valuations z_i, z_j, are recursively given by

$$u_i = E(x_i) + E(-x_i) + \big(1 - F(x_i)\big)\,\big(-p_{1i} + \delta\,u_i\big) + F(x_i)\,\big(-s - p_{0j} + \delta\,u_j\big)$$
$$u_j = E(x_j) + E(-x_j) + \big(1 - F(x_j)\big)\,\big(-p_{1j} + \delta\,u_j\big) + F(x_j)\,\big(-p_{0i} + \delta\,u_i\big)$$

In what follows, I use the notation, for a generic variable y,

$$\hat{y} \equiv \left.\frac{d\,y}{d\,s}\right|_{s\,=\,0}$$

Note that, at $s = 0$, we have a symmetric outcome where $x_i = x_j = 0$, $u_i = u_j = u$, and $p_{1i} = p_{0j} = p_{0i} = p_{1j} = p$. Differentiating the buyer value functions with respect to s at $s = 0$, I then get

$$\begin{aligned}
\hat{u}_i &= e(0)\,\hat{x}_i - e(0)\,\hat{x}_i + \frac{1}{2}\,\big(-\hat{p}_{1i} + \delta\,\hat{u}_i\big) - f(0)\,\hat{x}_i\,\big(-p + \delta\,u\big) + \\
&\qquad + \frac{1}{2}\,\big(-1 - \hat{p}_{0j} + \delta\,\hat{u}_j\big) + f(0)\,\hat{x}_i\,\big(-p + \delta\,u\big) \\
&= \frac{1}{2}\,\big(\delta\,\hat{u}_i + \delta\,\hat{u}_j - \hat{p}_{1i} - \hat{p}_{0j} - 1\big) \\
\hat{u}_j &= \frac{1}{2}\,\big(\delta\,\hat{u}_i + \delta\,\hat{u}_j - \hat{p}_{1j} - \hat{p}_{0i}\big)
\end{aligned}$$

Solving this system and simplifying, we get

$$\hat{u}_i - \hat{u}_j = \frac{1}{2}\,\big(\hat{p}_{1j} + \hat{p}_{0i} - \hat{p}_{1i} - \hat{p}_{0j} - 1\big) \tag{15}$$

Differentiating (14) with respect to s at $s = 0$, I get

$$\begin{aligned}
\hat{x}_i &= \hat{p}_{1i} - \hat{p}_{0j} - \delta\,(\hat{u}_i - \hat{u}_j) - 1 \\
\hat{x}_j &= \hat{p}_{1j} - \hat{p}_{0i} - \delta\,(\hat{u}_j - \hat{u}_i)
\end{aligned}$$

Substituting (15) for $\hat{u}_i - \hat{u}_j$ and simplifying

$$\begin{aligned}
\hat{x}_i &= (p_{1i} - p_{0j}) + \delta\,\frac{1}{2}\,\big(\hat{p}_{1i} + \hat{p}_{0j} - \hat{p}_{1j} - \hat{p}_{0i}\big) - \left(1 - \frac{\delta}{2}\right) \\
\hat{x}_j &= (p_{1j} - p_{0i}) + \delta\,\frac{1}{2}\,\big(\hat{p}_{1j} + \hat{p}_{0i} - \hat{p}_{1i} - \hat{p}_{0j}\big) - \frac{\delta}{2}
\end{aligned} \tag{16}$$

Seller k's value functions ($k = i, j;\ \ell \neq k$) are given by

$$\begin{aligned} v_{1k} &= \big(1 - F(x_k)\big)\,\big(p_{1k} + \delta\, v_{1k}\big) + F(x_k)\;\delta\, v_{0k} \\ v_{0k} &= F(x_\ell)\big(p_{0k} + \delta\, v_{1k}\big) + \big(1 - F(x_\ell)\big)\,\delta\, v_{0k} \end{aligned}$$

The corresponding first-order conditions are

$$\begin{aligned} -f(x_k)\big(p_{1k} + \delta\, v_{1k}\big) + 1 - F(x_k) + f(x_k)\,\delta\, v_{0k} &= 0 \\ -f(x_\ell)\big(p_{0k} + \delta\, v_{1k}\big) + F(x_\ell) + f(x_\ell)\,\delta\, v_{0k} &= 0 \end{aligned}$$

Solving for p_{1k}, p_{0k}, we get

$$\begin{aligned} p_{1k} &= \frac{1 - F(x_k)}{f(x_k)} - \delta\,(v_{1k} - v_{0k}) \\ p_{0k} &= \frac{F(x_\ell)}{f(x_\ell)} - \delta\,(v_{1k} - v_{0k}) \end{aligned} \tag{17}$$

Plugging back into the value functions I get

$$\begin{aligned} v_{1k} &= \frac{\big(1 - F(x_k)\big)^2}{f(x_k)} + \delta\, v_{0k} \\ v_{0k} &= \frac{F(x_\ell)^2}{f(x_\ell)} + \delta\, v_{0k} \end{aligned} \tag{18}$$

and so

$$v_{1k} - v_{0k} = \frac{\big(1 - F(x_k)\big)^2}{f(x_k)} - \frac{F(x_\ell)^2}{f(x_\ell)}$$

Plugging this back into the first-order conditions (17), we get

$$\begin{aligned} p_{1k} &= \frac{1 - F(x_k)}{f(x_k)} - \delta\,\frac{\big(1 - F(x_k)\big)^2}{f(x_k)} + \delta\,\frac{F(x_\ell)^2}{f(x_\ell)} \\ p_{0k} &= \frac{F(x_\ell)}{f(x_\ell)} - \delta\,\frac{\big(1 - F(x_k)\big)^2}{f(x_k)} + \delta\,\frac{F(x_\ell)^2}{f(x_\ell)} \end{aligned}$$

Differentiating with respect to s at $s = 0$, and noting that $f'(0) = 0$, I get

$$\begin{aligned} \hat{p}_{1k} &= -(1 - \delta)\,\hat{x}_k + \delta\,\hat{x}_\ell \\ \hat{p}_{0k} &= \delta\,\hat{x}_k + (1 + \delta)\,\hat{x}_\ell \end{aligned} \tag{19}$$

Equations (16) and (19) define a system of 6 equations with 6 unknowns. Its solution is given by

$$\begin{aligned}
\hat{x}_i &= -\frac{2-\delta}{6} \\
\hat{x}_j &= -\frac{\delta}{6} \\
\hat{p}_{1i} &= \frac{1}{2} - \frac{\delta}{2} \\
\hat{p}_{0i} &= -\frac{\delta}{2} \\
\hat{p}_{1j} &= -\frac{\delta}{6} \\
\hat{p}_{0j} &= -\frac{1}{3} - \frac{\delta}{6}
\end{aligned} \tag{20}$$

Differentiating the system (18), I get

$$\begin{aligned}
\hat{v}_{1k} &= -\hat{x}_k + \delta\, \hat{v}_{0k} \\
\hat{v}_{0k} &= \hat{x}_\ell + \delta\, \hat{v}_{0k}
\end{aligned}$$

Substituting (20) for x_k, x_ℓ, and simplifying, I get, for seller i

$$\begin{aligned}
\hat{v}_{1i} &= \frac{2-3\,\delta}{6\,(1-\delta)} \\
\hat{v}_{0i} &= \frac{-\delta}{6\,(1-\delta)} \\
\frac{1}{2}\left(\hat{v}_{0i} + \hat{v}_{1i}\right) &= \frac{1-2\,\delta}{6\,(1-\delta)}
\end{aligned}$$

For seller j, we have

$$\begin{aligned}
\hat{v}_{1j} &= -\frac{\delta}{6\,(1-\delta)} \\
\hat{v}_{0j} &= -\frac{2-\delta}{6\,(1-\delta)} \\
\frac{1}{2}\left(\hat{v}_{0j} + \hat{v}_{1j}\right) &= -\frac{1}{6\,(1-\delta)}
\end{aligned}$$

In words, seller i is better off in state one if and only if $\delta < \frac{2}{3}$, and is better on average if and only if $\delta < \frac{1}{2}$. ■

Proof of Proposition 4: From (13),

$$\lim_{\delta \to 1} \bar{p} = 2\,\frac{F(P)^2}{f(P)}$$

Lemma 1 then implies that, if δ is sufficiently close to 1, then $\bar{p}$ is increasing in P. From the proof of Proposition 2, P is decreasing in s. (Notice that that statement does not depend on s being small.) The result then follows by the chain rule of differentiation. ■

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