Trade, Education, and The Shrinking Middle Class*

Emily Blanchard[†] Gerald Willmann[‡]

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Preliminary
Comments Welcome

Abstract

This paper studies how the interaction between trade and educational institutions determines the distribution of human capital and income, both within and across countries. In the model, heterogenous agents self select into a continuum of occupational sectors (or tasks) depending on the wage structure and the cost of education. By exploiting the multiplicity of sectors and continuous support of possible human capital choices, we demonstrate that freer trade can induce crowding out of the middle occupations towards the skill acquisition extremes. We find that individual gains from trade may be non-monotonic in ability type, and that middle ability agents can lose from trade liberalization though aggregate gains are positive.

Keywords: Trade Policy, Skill Acquisition, Education, Income Distribution

JEL Classifications: F11, F13, F15, F16

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[†]Department of Economics, University of Virginia, P.O. Box 400182, Charlottesville, VA 22904-4182; ph. (01) 434.924.3607; blanchard@virginia.edu.

[‡]Department of Economics, Catholic University of Leuven, Naamsestraat 69, 3000 Leuven, Belgium; gerald.willmann@econ.kuleuven.be.

1 Introduction

While politicians tend to portray education as the universal panacea for rising income inequality and perceived competition with foreign exporters, there is remarkably little evidence to speak to such claims. If indeed improvements to education will improve workers' competitiveness and welfare effects of globalization, what are the potential distributional consequences of such policies? Given the structure of comparative advantage and differing tradeability of certain occupational outputs, where would educational funding best be spent – at the primary, secondary, or tertiary levels? Such questions require a careful understanding of how trade and educational institutions interact to determine individuals' skill acquisition decisions, the distribution of income, and ultimately the welfare consequences of freer trade.

This paper presents a model in which educational institutions together with trade relationships determine worker welfare and the distribution of human capital. Our approach centers on a continuum-of-sectors framework with heterogenous agents and endogenous human capital acquisition. Comparative advantage is driven by international differences in educational institutions and the resulting differential costs of skill acquisition. By exploiting the multiplicity of sectors and continuous support of possible human capital choices under our continuum set up, we explore the rich interdependence between educational institutions and trade.

The modeling strategy we propose is in contrast to most existing work in the trade literature, which treats human capital as a binary variable.[1] In these earlier models, workers are either skilled or unskilled; under such a setup, the opening of trade will induce either skill upgrading (in a country with comparative advantage in skill intensive products) or skill downgrading (in countries with comparative advantage in

[1]The important exception is the recent work by Costinot and Vogel (2008), who, as we do, allow a continuous support of possible skill levels. Workers' skill levels are taken to be exogenous in their model, however, since education is not a central issue to their study.

'basic' goods) through basic Stolper-Samuelson forces, but not both. The predictions stemming from such models is immediate and unambiguous: in developed countries (which presumably hold comparative advantage in skill intensive goods), freer trade will induce workers to move to high skill, export-oriented sectors. Accordingly, educational subsidies have the potential to smooth workers' transition to a liberalized trade environment. The opposite holds true in countries with comparative advantage in less skill intensive goods, notably developing countries. There, freer trade would increase the relative demand for unskilled workers and thus may reduce the incentives for workers to acquire human capital. By encouraging workers to remain in skill intensive sectors, educational subsidies may in fact make transition more difficult.

Although analytically parsimonious, such a simple binary approach clearly oversimplifies the process of skill acquisition and denies an important empirical regularity: that *within* countries freer trade causes some workers to 'sort down' – often into low-skill service sector jobs – while others simultaneously 'sort up' into higher skill jobs. Surely there are workers in rich countries who choose to acquire less human capital following trade liberalization, just as there are individuals in developing countries who acquire greater skill sets when trade barriers are lowered. Goos and Manning (2007) find evidence of just this. As demonstrated in Figure 1, employment growth in the United Kingdom over the last few decades has been non-monotonic, with the low and top-end segments of the job quality spectrum experiencing considerable growth, while employment in mid-range sectors has fallen. These empirical findings suggest that there may be workers in Britain who, based on their inherent ability or educational opportunities, now choose to acquire less human capital than they would have done a few decades ago, just as there are others who now choose to acquire even greater skill sets.

Whether this development is due to trade liberalization (or more generally, to globalization) or to technological change is a long-standing and yet unanswered

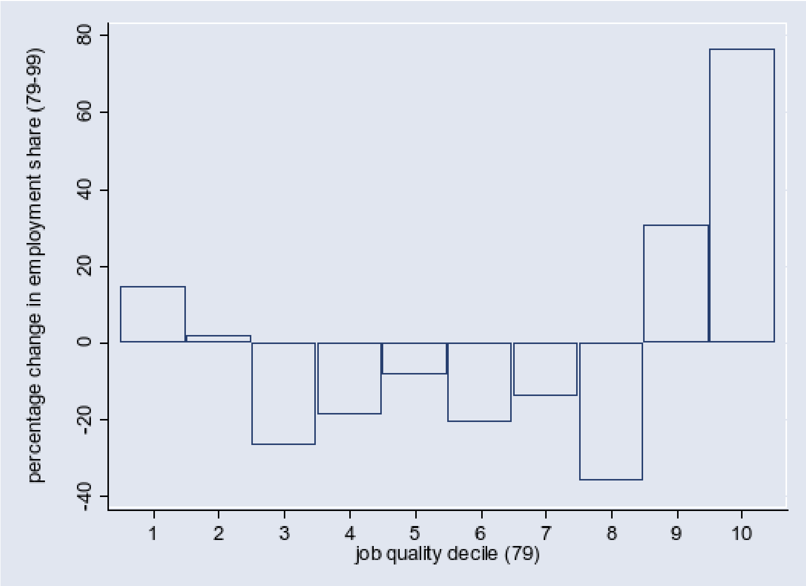

Figure 1: Non-Monotonic Changes in Employment (Goos and Manning (2007))

question. Indeed, in a recent working paper Krugman (2008) revisits the debate on the effects of trade on wage inequality and points out that the consensus that trade only plays a minor role (a position of which he was a prominent proponent just a decade ago) may need to be revised in light of current trends. He concludes that “we need a much better understanding of the increasingly fine-grained nature of international specialization and trade.” This paper contributes to precisely such a ‘better understanding’ by proposing a new modelling framework that focuses on the interplay of trade, human capital acquisition, and employment.

Our model features a continuum of heterogeneous agents who choose among a continuum of occupational sectors (or tasks), each of which requires a unique set of skills for employment. Each occupation is in turn used in the production of a particular intermediate product or service. Workers’ wages are determined by sectoral technology and intermediate good/task prices — and thus indirectly by trade policy

— while the cost of human capital acquisition is determined by individual characteristics and the structure of educational institutions. Faced with the resulting net incentive structure, agents of different inherent ability levels self-select into sectors by investing in the corresponding human capital.

Formally, we generate a one-to-one mapping from agents' inherent ability levels to their chosen occupations by assuming that the cost of skill acquisition is decreasing in an agent's ability level and increasing in the technical sophistication of the sector. Furthermore, we assume that the marginal cost of educating oneself to carry out a more sophisticated task is decreasing in ability. The negative cross partial derivative of the of the cost function ensures that the single crossing property is satisfied, such that higher ability agents self-select into more sophisticated occupations. The mapping of agents to sectors depends on wages, and thus on technology, trade, and ultimately the education differences between countries.

Trade liberalization leads to a remapping of agents to sectors, as do changes in trading partners' technologies, trade costs, or educational institutions that influence the cost of skill acquisition. The resulting shift in the demographics of skill composition can take many forms. One plausible and particularly salient scenario in line with the earlier diagram is the crowding out of the middle class towards the skill acquisition extremes, which could be brought about by increasing foreign competition in mid-technology intermediate goods/tasks (e.g., electronic components, basic machinery, or back office tasks like basic accounting or call center operation). That is, following increased competition from foreign producers in these mid-range sectors, some formerly moderate ability agents would invest more in human capital, while others optimally would invest less. While the aggregate gains from trade are positive, the distributional consequences are subtle and depend on both the relative structure of educational institutions across countries and the interaction between human capital and technology.

Our approach in this paper is motivated in part by recent work in labor economics that analyzes the diverging development of different segments of the labor market. Autor, Levy, and Murnane (2003) and Goos and Manning (2007) document that employment growth has been non-monotonic across sectors in that it is positive at the low and high ends of the labor market, but negative in the middle. While Autor and Dorn (2007) propose a simple three-sector model to explain this divergence, we provide a theoretical framework that is considerably more general and links the shift in employment and human capital to trade liberalization. The second motivation of our work, to which we alluded earlier, is the two-sector limitation of most of the trade literature that endogenizes human capital formation. This limitation, which we too are guilty of in Blanchard and Willmann (2008), leads to the artificial result that trade liberalization goes hand in hand with skill upgrading in developed economies.

The questions addressed here are also closely related to important recent advances in trade theory. Grossman and Rossi-Hansberg (2006) propose a two-sector model of trade in tasks and focus on the welfare effects of outsourcing specific tasks that can be carried out abroad. While our model is limited to a single final good, we endogenize workers' human capital decisions. In so doing, our paper offers a framework in which trade in tasks is driven by comparative advantage that arises endogenously due to international differences in educational systems. Also similar is the recent work by Jung and Mercenier (2008), who propose a model that features outsourcing of intermediates at the same time as endogenizing the human capital decision. Their approach, however, is in many respects a two sector model; as a consequence, skill upgrading in their framework is necessarily uni-directional, as in the more traditional trade literature.

The remainder of the paper is structured as follows. In Section 2, we introduce the model, analyze the effects of trade under the small country assumption, and give the equilibrium conditions for the large country case. Section 3 assumes functional forms to examine further the inner workings of the model and to present the equilibrium characteristics of a two country case with non-monotonic skill change. Section 4 summarizes and concludes.

2 The Model

The Home country is populated by a continuum of heterogeneous agents with unit mass. Individual agents differ in their inherent ability levels, a, assumed to be distributed continuously over the unit interval with cumulative distribution function $F(a)$ and corresponding density function $f(a)$. Every agent is endowed with a single unit of labor, which is supplied inelastically to the labor market.

The economy produces a single homogeneous final good, Y, using constant returns to scale technology and a continuum of intermediate tasks (or products) $j \in [0,1]$, where j may be thought of as an index of the intermediate sectors' technological sophistication. Each intermediate sector uses a specialized type of labor and produces under constant returns and perfect competition. Productivity is assumed to be the same for all workers of an acquired skill type, regardless of the agent's inherent ability.[2] The final good serves as numeraire with price denoted by $p \equiv 1$. Finally, we choose units so that the real wage in sector j, measured in units of the final good Y, is simply the trading price of the relevant intermediate good/task and is denoted by $w(j)$.

In order to supply one unit of specialized labor of type j, agents have to acquire the required skills through training and education. The cost (in units of the numeraire, Y) to agent $a \in [0,1]$ of acquiring the skills for a given sector $j \in [0,1]$ is denoted by $c(j,a) \in C^2$. We assume that the cost of skill acquisition is increasing in the

[2]We could instead build worker heterogeneity into productivity (and thus wages) rather than education costs to generate the same sorting of workers across sectors, but the exposition is somewhat cleaner with our formulation.

technological sophistication of the sector and decreasing in the ability level of the agent; further, the marginal cost of upgrading skills from one sector to the next is lower for high ability agents; finally, the cost of skill acquisition is convex across sectors for every agent. Formally:

$$\begin{aligned} \frac{\partial c(j,a)}{\partial j} &> 0, \qquad \frac{\partial c(j,a)}{\partial a} < 0 \\ \frac{\partial^2 c(j,a)}{\partial j \partial a} &< 0, \qquad \frac{\partial^2 c(j,a)}{\partial j^2} > 0. \end{aligned} \tag{2.1}$$

Additionally, for tractability, let:

$$c(j,a) \equiv h(a)g(j) \tag{2.2}$$

$$c^*(j,a) \equiv h(a)g^*(j), \tag{2.3}$$

where $h(\cdot)$ and $g(\cdot)$ are twice continuously differentiable and non-negative over the unit interval. Note that the two assumptions on the first derivatives of the cost function imply the negative cross partial of $c(j,a)$ under this specification.

Optimal Sorting and Production. Agents consume only the final good Y and have non-satiated preferences. Thus, when deciding which sector to enter, every agent a chooses j to maximize his net real wage, $w(j) - c(j,a)$. The first order condition for each individual's optimal human-capital level is then:

$$\frac{\partial c(j,a)}{\partial j} = \frac{dw(j)}{dj}, \tag{2.4}$$

Using superscript dots to denote derivatives with respect to j, the first order condition for agent a's optimal human capital decision/sectoral choice then may be rewritten:

$$\dot{c}(j,a) = \dot{w}(j) = \dot{g}(j)h(a). \tag{2.5}$$

The second order condition is satisfied as long as the wage schedule (which is exogenous in a small open economy and endogenous if the country is large or autarkic) is not more convex than the cost function; i.e. $\ddot{w}(j) \leq \ddot{c}(j,a)$. Tautologically, the second order condition must obtain for all j in any diversified equilibrium.

The first order condition in (2.5) determines the allocation of ability types to sectors. Solving yields the optimal allocation (or self-sorting) of ability levels to sectors:

$$a(j) = h^{-1}\left(\frac{\dot{w}(j)}{\dot{g}(j)}\right), \tag{2.6}$$

Note that the third inequality in (2.1) ensures that $h(\cdot)$ is invertible so that $a(j)$ is defined; from the continuity assumptions over $h(\cdot)$ and $\dot{g}(\cdot)$, $a(j)$ is continuous in j if (and only if) $\dot{w}(j)$ is also continuous. Given the assumptions in (2.1), then as long as $\dot{w}(j) > 0$ – a necessary condition for positive production[3] – the first order condition satisfies single crossing so that agents map to sectors assortatively. Moreover, we obtain strictly monotonic sorting of agents to occupations as long as the second order condition holds with strict inequality. Formally:

Lemma 2.1 *$a'(j) \geq 0$; $a'(j) > 0$ if and only if $\ddot{w}(j) < \ddot{c}(j,a(j))$.*

(Proof in appendix.) When $a(j)$ is strictly monotonic, and thus invertible, we denote the function that maps ability types to sectors by $j(a)$.

Combining this allocation of workers to sectors with the underlying distribution of ability types, $f(a)$, we then obtain the supply schedule of intermediates:

$$y^s(j) = a'(j)f(a(j)), \tag{2.7}$$

which given our one-to-one assumption for technology is simply the density of agents in each sector. Notice that the wage schedule and the cost of skill acquisition enter the intermediate supply function implicitly through $a(j)$.

[3] If $\dot{w}(j) \leq 0$, then no worker would be willing to bear the cost of education required to work in sector j since $\dot{c}(j,a) > 0 \forall a, j$.

Aggregating across intermediates, total output of the final good is given by $Y^s \equiv \psi(\vec{y})$, where $\psi(\cdot)$ denotes the constant returns technology used to produce the final good and each $y_j \equiv y_j^s + y_j^t$ includes any net imports of intermediate products ($y_j^t < 0$ represents total exports of j). We denote the unit factor demand for sector j output[4] by $x(j) \equiv x_j(\vec{w}, 1)$ and note that in general it depends on the complete wage schedule.

Equilibrium Conditions. Full employment requires that the density of agents in each sector maps to the unit mass of population; i.e.:

$$\int_0^1 a'(j)f(a(j))dj = 1 \tag{2.8}$$

Market clearing in each factor market implies, moreover, that:

$$y(j) = a'(j)f(a,j) + y_j^t = x(j)Y^s \qquad \forall j \in [0,1]. \tag{2.9}$$

The zero profit condition in aggregate good production implies that total revenue must equal total factor payments so that:

$$Y^s = \int_0^1 w(j)[a'(j)f(a(j)) + y_j^t]dj. \tag{2.10}$$

Finally consumers' balanced budget condition requires that:[5]

$$Y^d = \int_0^1 [w(j(a)) - c(a, j(a))]da, \tag{2.11}$$

where Y^d denotes aggregate consumption of the final good. Together, the system described by (2.8) - (2.11) pins down the equilibrium allocation of agents to occupational sectors, intermediate production levels, aggregate final goods output, trade in intermediates (if any), and total consumption.

[4]Recall that constant returns to scale technology implies that conditional factor demand may be written $x_j^T(\vec{w}, Y) = x_j(\vec{w}, 1)Y \equiv \arg\min_{x_j} \vec{w} \cdot \vec{x}$ s.t. $\psi(\vec{x}) \geq Y$.

[5]Tariff revenue, if appropriate, would simply be added to the right hand side of (2.11).

2.1 Partial Equilibrium: A Small Open Economy

Before moving to the full general equilibrium specification in the next section, we consider the case of a small open economy; this allows us to demonstrate the intuition underlying our main results in the simplest possible framework. In this section, we assume that all intermediates are traded under non-prohibitive trade costs so that the wage schedule for intermediate goods/tasks may be taken as fixed. We define τ_j to be one plus the ad-valorem trade cost (possibly including an import tariff/export tax) specific to sector j. Hence, for imported intermediates, $w(j) = \tau_j w^w(j)$, while for exported intermediates, $w(j) = \frac{w^w(j)}{\tau_j}$, where $w^w(j)$ denotes the world price of output j. Assume additionally that $\dot{w}(j)$ is continuous in j over the unit interval. (Continuity of the derivative wage schedule, $\dot{w}(j)$, will obtain in general equilibrium as will be clear from the next section, but for now must be assumed explicitly).

Equilibrium is best described through a pair of simple graphs. Figure 2 depicts the optimal sorting of agents to sectors as a function of the local wage schedule and the cost of education for each agent. Panel A illustrates agents' optimal sectoral choice according to the first order condition in (2.5). Notice that the second order condition requires that $\dot{w}(j)$ crosses $\dot{c}(a, j)$ from above at the optimal occupation, $j(a)$. Further, the assumption from (2.1) that $\frac{\partial \dot{c}(a,j)}{\partial a} < 0$ ensures assortive matching of agents to sectors so that $j'(a) \geq 0$. Panel B depicts the resulting mapping of agents to occupational sectors (and vice versa). Once the agent-to-occupation mapping is determined, the remaining equilibrium variables are pinned down: output of each intermediate good is given by (2.7) and aggregate final goods output then follows from $\psi(\vec{y})$ (where imports of intermediate goods are included), while consumption is determined in (2.11). The pattern of trade in intermediates is given by the vector $\vec{y}^t$ satisfying (2.9) and (2.10).

To highlight the relationship between the underlying ability distribution, $F(a)$, and the equilibrium supply of intermediate outputs, we offer a simple numerical ex-

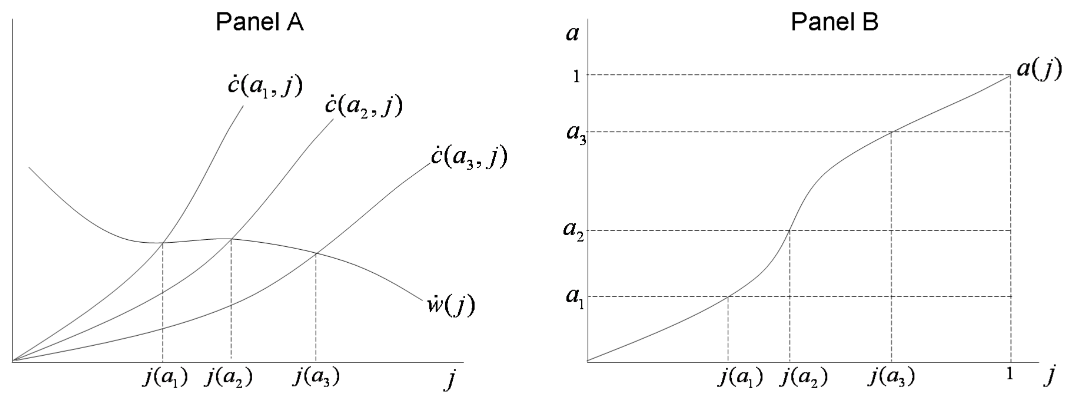

Figure 2: Optimal Sorting

ample. Given simple functional form assumptions, Figure 3 Panel A graphs the equilibrium mapping from agents to sectors. It is worth reiterating that once the intermediate wage schedule is given, this mapping is independent of the underlying distribution of ability types. Panels B and C depict the resulting intermediate supply curves, which of course do depend explicitly on the underlying ability distribution of the population. The case of a uniform distribution of ability types is given in Panel B, while the supply curve resulting from a simple linear density is depicted in Panel C. Together, the agent-to-occupation mapping function and the concomitant intermediate supply function summarize production in the economy, effectively taking the place of the familiar production possibilities frontier in a 2-good framework.

The basic small country model in hand, we can now consider the effect of an exogenous shift in the domestic price of intermediate goods/tasks caused by changes in the world economy, a decline in trading costs or tariffs, or both. First notice that if the resulting change in the wage schedule were uniform across occupations so that $\dot{w}(j)$ remained unchanged (i.e. a vertical shift in the wage schedule), there would be no impact on agents' occupational choices or aggregate output. Given that the final good is numeraire, however, such a vertical shift in the wage schedule cannot occur (since it

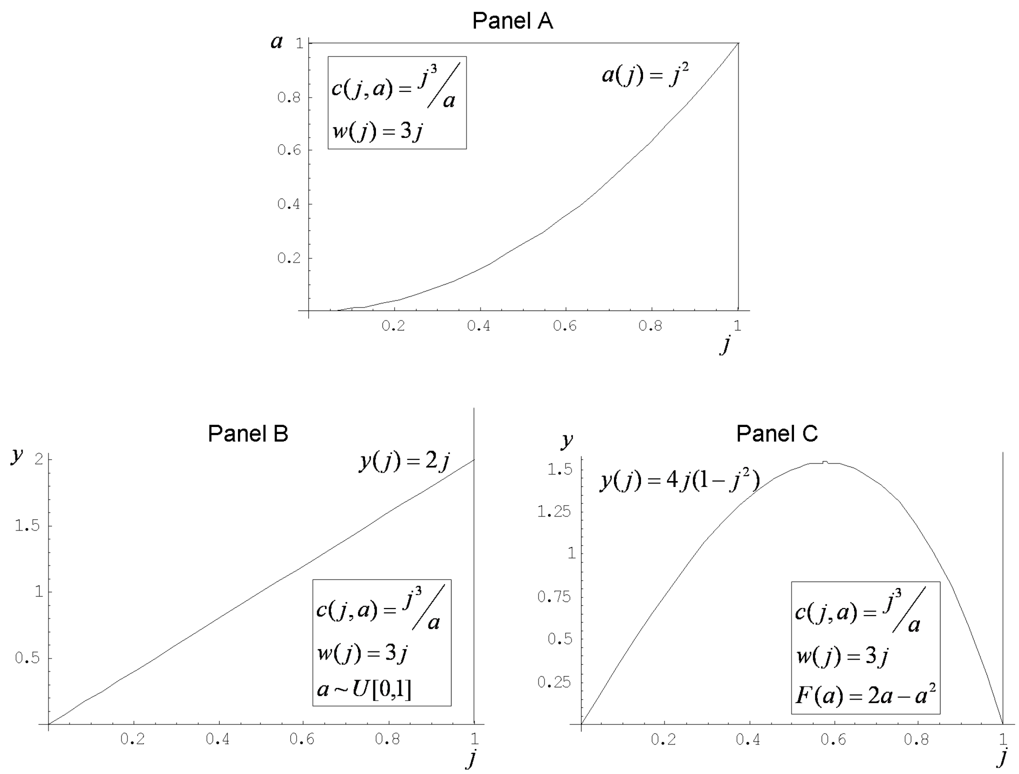

Figure 3: The Relationship Between Ability Distribution and Intermediate Supply

would imply a violation of the zero profit condition for final good production). Thus, any change in intermediate goods/task prices must be such that the wage schedule becomes steeper or flatter over at least some range of sectors, and thus that agents will adjust their skill acquisition decisions accordingly.[6] From Panel A of Figure 2, it is clear that if the new intermediate price schedule is everywhere flatter than before, so that $\dot{w}(j)$ shifts down, agents will 'sort down' to lower j occupations in response to the decreased wage premia for skill upgrading.[7] Conversely, if the new

[6]In our static model, agents' decisions necessarily reflect current prices, though in reality, one would expect sectoral adjustments to take time. To address this issue (and the related political economy questions), we plan to develop a dynamic analog to this model in a companion paper.

[7]In a dynamic framework in which agents cannot recoup the costs of over-education (in essence

wage schedule is everywhere steeper than before, agents will monotonically ‘sort up’, choosing higher skilled occupations. The interesting question is then whether (or under what conditions) non-monotonic skill change can occur within a single economy following a shift in the wage schedule.

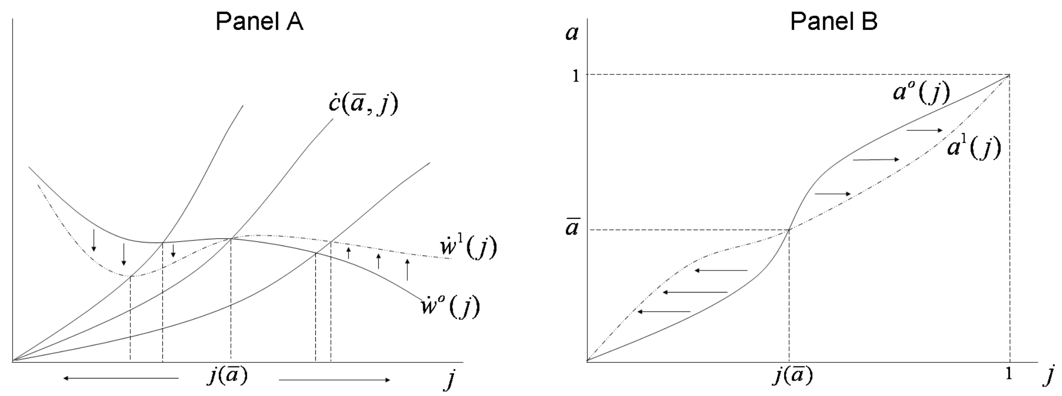

Figure 4: Non-monotonic Skill Change

Figure 4 illustrates a scenario in which the international prices or transportation/offshoring costs fall disproportionately for mid-range sectors. Starting from sector $j = 0$, a decline in the wage schedule over mid-range sectors induces an initial decline in $\dot{w}(j)$ relative to the initial derivative wage schedule; to the extent that trade costs have fallen less (or remained the same) for higher j sectors, $\dot{w}(j)$ should then eventually exceed the previous level as depicted. Here, the agent $\bar{a}$ for whom $\dot{w}(j(\bar{a}))$ remains unchanged will keep her current occupation, while agents to the left of $\bar{a}$ will sort into lower j sectors while agents to the right sort into higher j occupations.[8]

reselling their degrees), we would expect agents to remain in their same jobs or, if $\dot{w}(j) < 0$ in the relevant region, to shift into lower-skill work for which they are then overqualified.

[8]Naturally, one can envision a new derivative wage schedule that crosses the old derivative wage schedule in several places. Such a scenario is entirely plausible (for instance, there is no reason to expect changes in the transportation/offshoring costs to be consistent across sectors) and simply would result in a more complex pattern of non-monotonic skill change.

The consequence is vacating of the middle j sectors, those that require a modest skill set and offer mid-range wages. As agents move to the skill acquisition extremes, the 'middle class' occupations reduce employment and production.[9]

While the implications of changing world prices and trade costs for human capital acquisition are clear from the optimal sorting mechanism in (2.6), the individual welfare effects are necessarily more nuanced because real wages and the cost of education both move with the change in wages and the concomitant shift in human capital decisions. The net welfare effect for each individual worker will depend on the balance of these forces; it should not be surprising, then, that generalizable welfare conclusions cannot be drawn in the absence of functional form assumptions. The two country example presented in Section 3 highlights this tradeoff between shifting real wages and educational costs, calculating explicitly the individual welfare effects of trade. First, however, we lay out the basic structure of the general equilibrium framework.

2.2 General Equilibrium: A Two Country Model

To endogenize the wage schedule, we now introduce a second country, Foreign, which for the most part mirrors Home. Like Home, Foreign is assumed to have a unit mass of population with the same underlying ability distribution, $F(a)$. Home and Foreign constituents have the same non-satiated preferences over consumption of

[9]Some sectors may be abandoned completely following an increase in the tradeability or declining world prices of certain goods. The complete vacating of a range of sectors would manifest as a 'flat spot' in the mapping function $a(j)$ that arises when $\dot{w}(j)$ coincides with $\dot{c}(\tilde{a}, j)$ for some interval $[\underline{j}, \bar{j}]$ and some agent (or agents) $\tilde{a}$. For the set of sectors over which the cost-derivative and wage-derivative functions overlap, agent $\tilde{a}$ would be indifferent over sectoral choice within those bounds while all agents $a < \tilde{a}$ strictly would prefer sectors $j < \underline{j}$ while agents with ability $a > \tilde{a}$ strictly would prefer sectors indexed above $\bar{j}$. Since any given agent has zero mass, all sectors between the bounds $\underline{j}$ and $\bar{j}$ then would be effectively empty of workers.

the final good and supply their labor inelastically. Further, we assume that Home and Foreign share the same technology in producing intermediates from (trained) labor and in making the final good; this simplification allows us to focus on the role of educational institutions in driving comparative advantage.[10] Foreign country variables are identified by an asterisk, and obey the restrictions and equilibrium conditions set forward for Home.

To best illustrate the influence of trade on endogenous human capital acquisition and income, we compare autarkic equilibrium with the free trade equilibrium for both Home and Foreign. Comparative statics over changes in non-prohibitive trade costs or tariffs are instructive, but yield as much insight in the analytically more parsimonious partial equilibrium case depicted for a small open economy in Figure 4. We first define the generalized equilibrium conditions under autarky and free trade, then move to a functional form example to derive illustrative closed form results.

Autarkic equilibrium is described by the Home and Foreign wage schedules for which $y^s(j) = x(j)Y^s$, and $y^{s*}(j) = x^*(j)Y^{s*}$ respectively, for all $j \in [0, 1]$. With trade, the intermediate market clearing conditions are analogous: $y^s(j) + y^{s*}(j) = x(j)Y^s + x^*(j)Y^{s*}$ for all $j \in [0, 1]$. The appropriate set of market clearing conditions together with the full employment, zero profit, and balanced budget conditions in (2.8)-(2.11) pin down the equilibrium wage schedule and corresponding allocation of ability types to sectors in each economy and the equilibrium output and consumption levels. We denote the equilibrium autarkic wage schedule in Home by $w_A(j)$ with derivative $\dot{w}_A(j)$; the Foreign autarky equivalents are $w^*_A(j)$ and $\dot{w}^*_A(j)$. Free trade wages are denoted instead with a subscript $_{FT}$.

[10]Since technology differences are the centerpiece of the seminal Dornbusch, Fischer, and Samuelson (1977) model and its many successors, we initially silence that well understood mechanism. We plan to return to this issue by examining different intermediate good productivity in an extension of the basic model to study the interaction between technology, educational institutions, and trade.

Under autarky, Home's equilibrium wage schedule is given implicitly by:[11]

$$\dot{w}_A(j) = \dot{c}(j, a_A(j)), \text{ where } a_A(j) \text{ is such that:} \tag{2.12}$$

$$a'_A(j) f(a_A(j)) = x_A(j) Y_A^s, \text{ and} \tag{2.13}$$

$$Y_A^s = \psi(\vec{y}^s). \tag{2.14}$$

And likewise in Foreign. Thus, for a given input/factor demand structure, the more convex the cost function in j, the steeper the equilibrium wage schedule. This makes sense; to the extent that skill upgrading becomes increasingly expensive for more sophisticated (high j) sectors, the higher the incremental wage increases must be to induce workers to enter the most demanding occupations.

Under trade we have the analogous condition, where for any sector j in which both Home and Foreign produce:

$$\dot{w}_{FT}(j) = \dot{c}(j, a_{FT}(j)) = \dot{c}^*(j, a^*_{FT}(j)), \text{ where } a_{FT}(j) \text{ and } a^*_{FT}(j) \text{ are s.t.} \tag{2.15}$$

$${a_{FT}}'(j) f(a_{FT}(j)) + {a^*_{FT}}'(j) f(a^*_{FT}(j)) = x_{FT}(j) Y_{FT}^s + x^*_{FT}(j) Y_{FT}^{s*}, \tag{2.16}$$

$$Y_{FT}^s = \psi(\vec{y}), \text{ and } Y_{FT}^{s*} = \psi(\vec{y}^*), \tag{2.17}$$

$$\text{and } y_j^t = -y_j^{t*} \forall j \in [0,1]. \tag{2.18}$$

In sectors in which only one country produces, the market clearing condition is adjusted accordingly and only the producing country's cost schedule influences the wage. The equilibrium free trade wage schedule then determines the supply of intermediates, aggregate output, consumption, and the pattern of trade just as in the previous section.

In general, the market clearing conditions in (2.13) and (2.16) are characterized by a third order differential equation of the wage schedule over j.[12] Collapsing the system of market clearing conditions (of which there are an uncountable infinity) to

[11] Starting from the numeraire, $w(0) \equiv 1$, the function $\dot{w}(j)$ determines the full wage schedule, $w(j) \forall j \in [0,1]$.

[12] $a'(j)$ is a function of $\dot{w}(j)$ and $\ddot{w}(j)$ while $x(j)$ depends in general on the complete $w(j)$ schedule.

a single differential equation yields enormous returns in model tractability: namely, equilibrium properties can be summarized by the behavior of the wage schedule over $j \in [0,1]$ as in the preceding section. At the same time, however, given that the equilibrium wage schedule is a solution to a differential equation of the third order, it should not be surprising that closed form solutions prove the exception rather than the rule.

The following section describes a case in which functional form assumptions do offer analytical solutions in the general equilibrium model. In generating a set of closed form results, we highlight the role of educational institutions in determining both comparative advantage and the implications of trade for human capital acquisition, welfare, and income distribution within and across countries.

3 A General Equilibrium Example

In this section we provide a concrete example of our model that illustrates the simultaneous 'sorting up' and 'sorting down' of moderate ability agents and the negative welfare effects trade can have on the middle class. In order to make things tractable, we assume the following cost structures of education in home and foreign respectively:

$$c[j,a] = \frac{(1-a)}{a} * \frac{2j^2}{5} \tag{3.1}$$

$$c^*[j,a] = \frac{(1-a)}{a} * \frac{2j^3}{3} \tag{3.2}$$

On the factor demand side we assume Leontief production of the final good, thereby abstracting from possible substitution effects across intermediates.[13] With $\psi(\vec{y}) \equiv$

Moreover, in a model with multiple final goods in which aggregate final goods output depends on its own price (i.e. $Y(p)$), the market clearing condition would instead be given by fourth order differential equation.

[13]More generally, substitution effects would dampen the magnitude of wage schedule changes, but would not overturn our qualitative findings; the technical benefit of the Leontief assumption is that

$\min\{y_0, ..., y_1\}$, unit factor demand is simply one in each sector and country, regardless of the wage schedule; thus, $x(j) \equiv x_j(\vec{w}, 1) = x^*(j) = 1$. Following the solution procedure outlined in the previous section, we solve for the equilibrium wage schedules:

$$\dot{w}_A = \frac{4(1-j)}{5}, \tag{3.3}$$

$$\dot{w}_A^* = 2j - 2j^2, \tag{3.4}$$

$$\dot{w}_{FT} = \frac{j(2+j-10j^2) + \sqrt{j^2(4+j(4+4j(121+20j(-9+5j))))}}{10j}. \tag{3.5}$$

Where we have used the boundary condition that the wage schedule must be flat at the upper end, $\dot{w}(1) = 0$, to pin down the respective constants of integration.[14] Figure 5 shows that the slope of the equilibrium wage schedule under free trade is a weighted average of the autarky values. Note in particular that the intersection of the autarky wage slopes leads to the same value of the slope of the equilibrium wage schedule under free trade.

As discussed before, the equilibrium wage schedule implies a corresponding mapping of agents to sectors by ability level. In autarky, of course, the Leontief technology assumption implies a uniform density of workers across sectors; thus:

$$a_A(j) = a_A^*(j) = j. \tag{3.6}$$

Under free trade the sectoral mappings take the following form:

$$a_{FT}(j) = \frac{8j^2}{j(2+9j-10j^2) + \sqrt{j^2(4+j(4+j(121-20j(-9+5j))))}} \tag{3.7}$$

$$a_{FT}^*(j) = \frac{20j^3}{j(2+j+10j^2) + \sqrt{j^2(4+j(4+j(121-20j(-9+5j))))}}, \tag{3.8}$$

and are depicted in Figure 6.

$x_j(\vec{w}, 1) = 1 \forall j$ so that the intermediate market clearing condition is of only the second order (rather than third) and thus solvable.

[14]This boundary condition ensures that there is not a mass of workers clustered in sector $j = 1$.

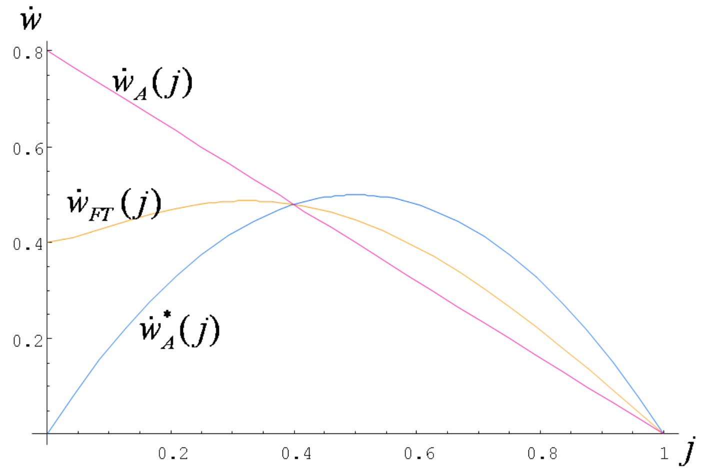

Figure 5: Slope of the Equilibrium Wage Schedules

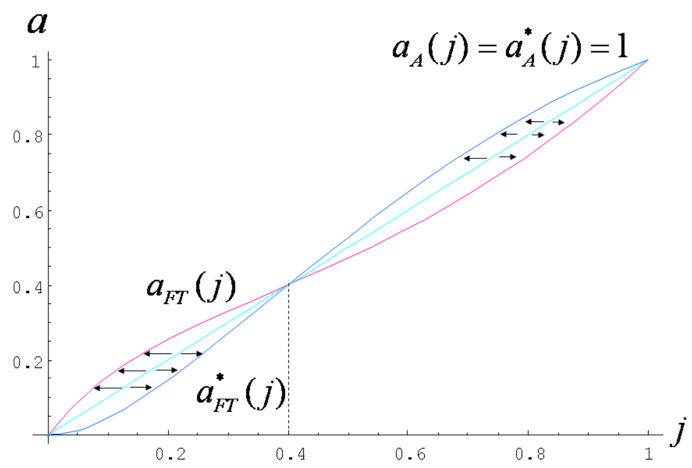

Figure 6: Mappings $a_{FT}(j)$ and $a^*_{FT}(j)$.

Figure 6 illustrates the reallocation of agents brought about by trade liberalization. Where the free trade mapping function lies above the diagonal, the corresponding ability level self-selects into a lower j sector following liberalization; i.e. agents have sorted down. Where the free trade mapping function lies below the forty-five degree line, agents self select into higher j occupations and human capital levels following the opening of trade. Overall, we see that in Home agents in the lower portion of the population distribution have shifted to lower j sectors, while agents above $a = .4$ have shifted up, thus vacating the middle j sectors toward the skill-acquisition extremes. The effects in Foreign are of course simply the reverse.

Figure 7 depicts the resulting shift in employment density across sectors, which is equivalent to the supply of each intermediate output (given our assumption that $a \sim U[0, 1]$). Again, note that the Leontief technology ensures uniform employment distribution in autarky.

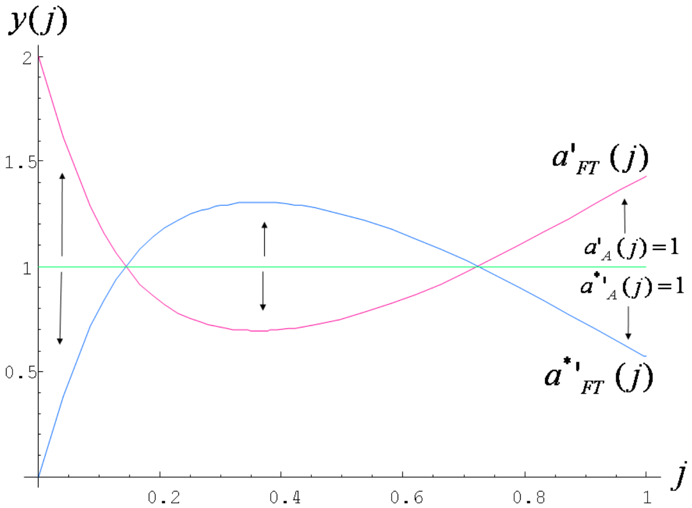

Figure 7: Employment Density by Sector: Autarky and Free Trade.

We are especially interested in the welfare effects arising from trade liberalization. To get at real welfare effects, we first determine the real wage schedules under free trade and autarky using the zero profit condition for final goods production[15] to pin down the equilibrium wage level. The equilibrium real wage in a given sector $\hat{j}$ is given by $w_0 + \int_0^{\hat{j}} \dot{w}(j)dj$, where the base wage in sector $j = 0$ is determined by $w_0 \equiv 1 - \int_0^1 \dot{w}(j)dj$. Solving, we find for low and high j sectors, the wage schedule

[15]which under the Leontief production structure and choice of Y as numeraire is simply: $1 = \int_0^1 w(j)dj$.

increases at Home and decreases in Foreign country following trade liberalization, while the converse holds for mid-range sectors.

In what follows we first focus on the welfare effects in Home. As we will see below, the effects in Foreign are virtually a mirror image. The real welfare change consists of the effect on the real wage and the change in the realized cost of education. We analyze first the effect on the real wage. The two panels in Figure 8 depict respectively the change in the real wage in sector j and the change in the real wage of agent a given her optimal sectoral choice under each trading regime.

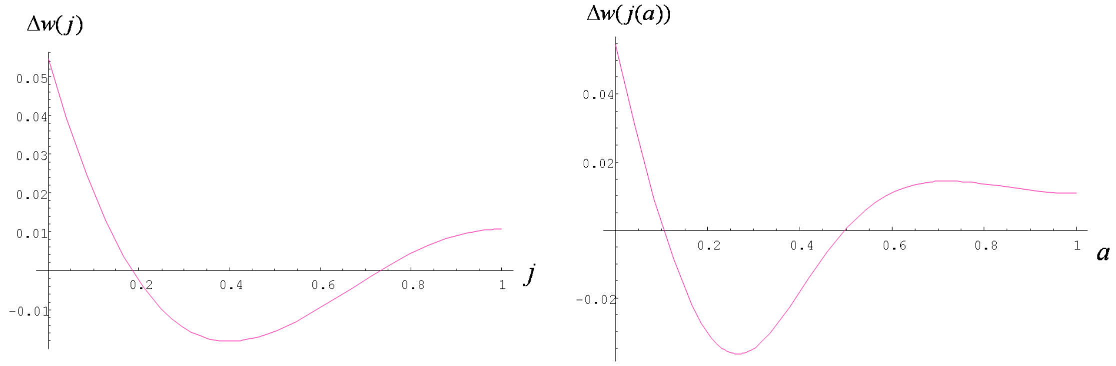

Figure 8: Effect of Trade on Wages at Home.

From the first panel in Figure 8, we see that real wages rise for the low and high j sectors, and fall for sectors j between about .2 and .7. The second panel takes into account the induced occupational shift, confirming that the change in realized real wages is non-monotonic across workers: agents with low ability earn higher real wages under trade, agents with high ability do as well, and agents in the lower-middle portion of the ability distribution see their real wages fall.

Figure 9 shows the change in the real cost of education across workers. Remembering that agents in the lower forty percent of the ability distribution sort down while agents in the upper part of the distribution sort up, it is obvious that the real cost of education should decrease for the left portion of the distribution and increase for the right.

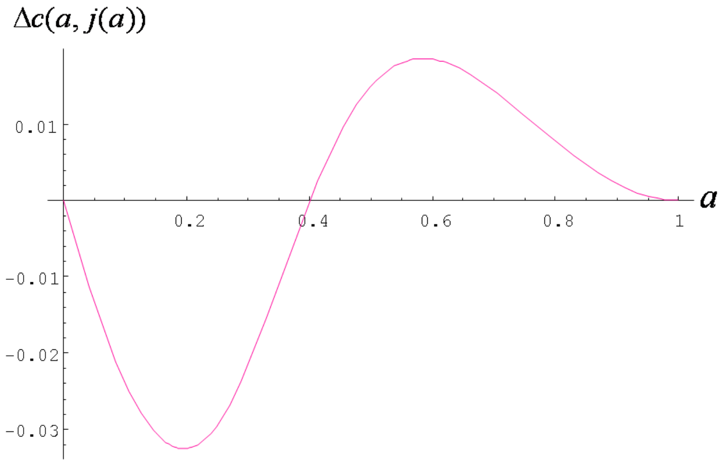

Figure 9: Change in the Home Real Cost of Education across Workers

Figure 10 shows the net welfare change for Home's population. Combined with the effect on the real wage, the adjustment for the changing cost of education has shifted the identity of the 'biggest loser' to the right. Indeed, consider the plight of the agent $a = .6$. Although her real wage has increased, the increased cost of education required to achieve the higher paying job more than offsets the wage gain so that the net welfare change is negative. Conversely, agent $a = .2$ suffers a substantial real wage loss yet enjoys a modest net welfare improvement due to his now lower cost of education. A crucial caveat to this second statement is that lower costs of education cannot be recovered if they are sunk. In a dynamic framework with unanticipated trade shocks, we therefore would expect to see the burden of increased costs of (potentially mid-career) education manifest in net welfare changes, while education savings would not be realized for the older generations.

Turning now to the Foreign country, we see that the net welfare effects are a mirror image of what happens at home. The three panels in Figure 11 depict the change in the real wage per agent a, the change in the real cost of education, and the

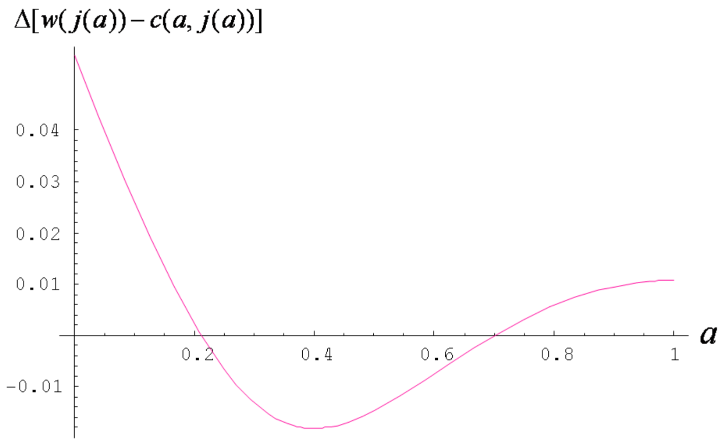

Figure 10: Net Welfare Effect of Trade in the Home Country.

net welfare effect in the foreign country. In contrast to Home, we find that the real wage increases most for Foreign agents in the middle of the income distribution while falling at the distribution extremes. At the same time, the cost of education rises for the lower forty percent of the distribution of workers and fall for the remainder. The net welfare effect of trade is shown in the final panel of Figure 11, where we see that middle ability agents gain from trade, while the highest and lowest ends of the population distribution lose.

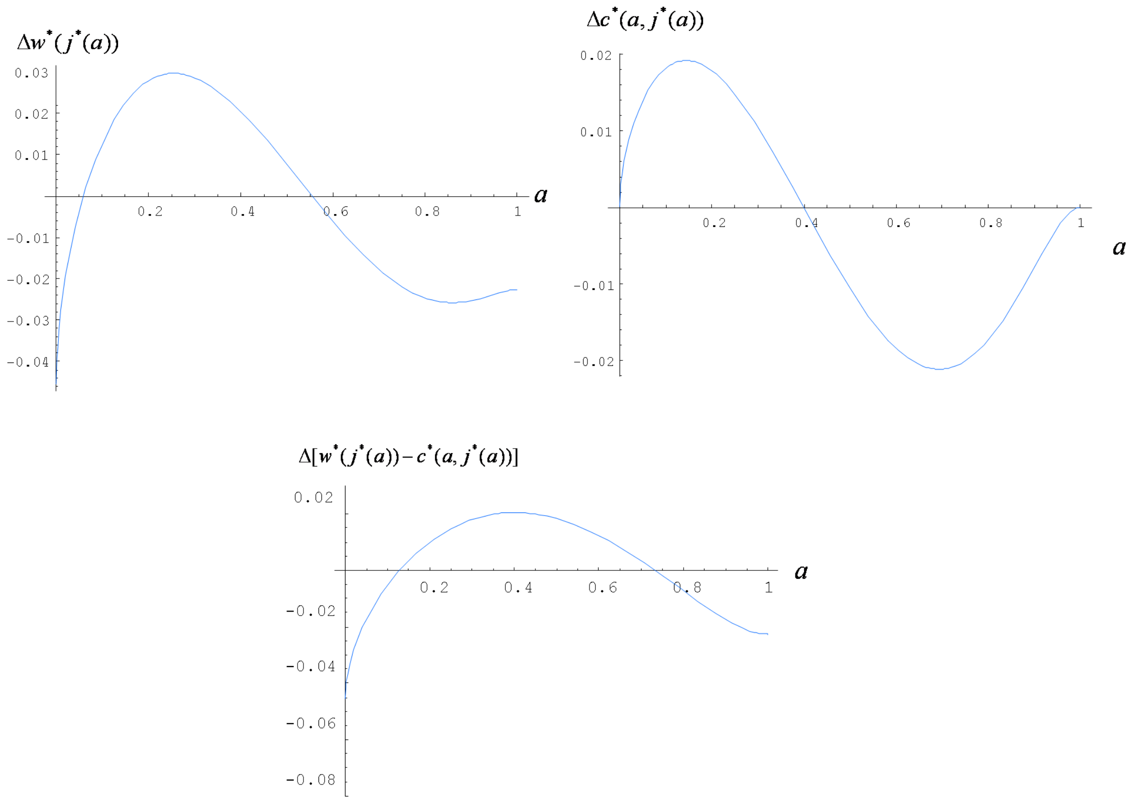

Figure 11: Effect of Trade on Real Wages, Education Costs, and Welfare in Foreign

To summarize the results from this general equilbrium example, we depict both the Home and Foreign net welfare changes by worker in Figure 12. While in the home country it is the medium ability agents who suffer, their foreign counterparts are the main beneficiaries of trade liberalization, together with high and low ability agents at Home. Integrating the net real effects in each country confirms that there are positive gains from trade for both countries.

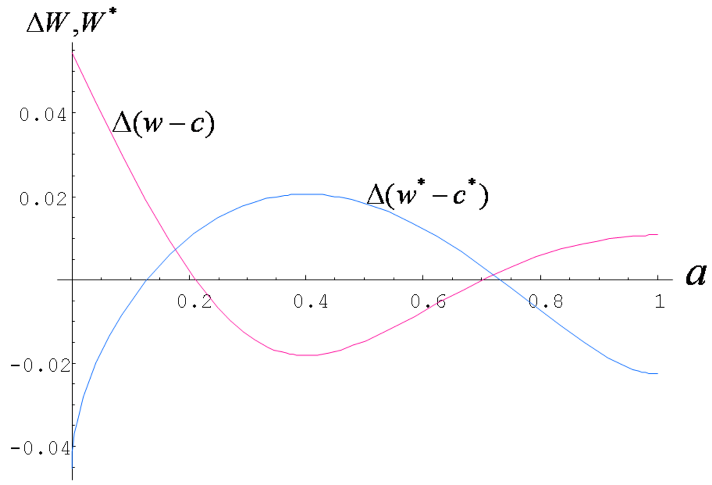

Figure 12: Opposing Individual Welfare Effect of Trade in Home and Foreign.

4 Conclusion

In this paper we develop a model of trade and education that allows for differentiated effects of trade liberalization on skill acquisition. Agents of different ability levels self-select into sectors by acquiring the specific education necessary to work in a particular sector, or perform a particular task. This mapping of ability level to sectors depends on the wage or price schedule and hence on a country's openness to trade. We show how changes in the price schedule affect this mapping and lead agents to sort up and down simultaneously. If a country's educational cost structure is less convex than that of its trading partners, then low ability agents sort down and higher ability agents sort up, and we obtain a 'vacating of the middle' with corresponding negative welfare effects for the middle class. This result provides one possible explanation for the current public concern over the negative effects of globalization on the middle class.

Our framework can shed light on the potential differential impacts of strengthening educational institutions. Government subsidies to education or similar institutional improvements that decrease the cost of skill acquisition over some ranges of sectors or for certain agents would impact the distribution of human capital decisions and thus the pattern of trade and comparative advantage, aggregate social welfare, and intra-national income distribution. In more general terms, the model developed here provides a novel reason for trade. By abstracting from differences in technology or preferences, we show how differences in educational institutions endogenously give rise to comparative advantage and hence trade.

In future work, we intend to use the framework developed here to analyze the effects of a differentiated educational policy that focuses on primary, secondary, or tertiary education. Uniform subsidies to education across the board are hardly the optimal policy recommendation resulting from our model. Perhaps highly targeted educational policies such as Brazil's are not as irrational and driven by specific interests as might seem at first sight. In addition, our model is well suited to study the effects of educational migration, i.e. the phenomenon recently documented in Blanchard, Bound, and Turner (2008) that students acquire education in another country and then either stay or return, which is of particular relevance for developing countries. In a somewhat more technical extension, we plan to explore systematically the nature of the interaction of technological changes with trade and education, in an effort to inform an empirical strategy for identifying the welfare effects of trade apart from technological innovation (while still recognizing the endogeneity of worker's human capital decisions). Finally, we intend a simplified version of this model as the building block for a dynamic endogenous trade policy model along the lines of our previous work in Blanchard and Willmann (2008).

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A1 Appendix

A1.1 Proof of Lemma 2.1

Taking the derivative of $a(j)$ with respect to j yields:

$$a'(j) = h^{-1'}\left(\frac{\dot{w}}{\dot{g}}\right)\left[\frac{\ddot{w}\dot{g} - \dot{w}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.1}$$

Substituting from the first order condition in (2.5):

$$a'(j) = h^{-1'}\left(\frac{\dot{w}}{\dot{g}}\right)\left[\frac{\ddot{w}\dot{g} - f(a)\dot{g}\ddot{g}}{\dot{g}^2}\right]. \tag{A1.2}$$

Then, from the definition of the cost function:

$$a'(j) = h^{-1'}\left(\frac{\dot{w}}{\dot{g}}\right)\left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2}\right] \geq 0, \tag{A1.3}$$

using the second order condition ($\ddot{c} \geq \ddot{w}$) and the assumptions on the cost function in (2.1), which imply that $\dot{g} > 0$ and $h^{-1'}(x) < 0$ iff $x > 0$ (recall that $\dot{w} \geq 0 \forall j$). Finally, if $\dot{w} > 0$, then

$$a'(j) = \underbrace{h^{-1'}\left(\frac{\dot{w}}{\dot{g}}\right)}_{(-)}\left[\frac{\dot{g}(\ddot{w} - \ddot{c})}{\dot{g}^2}\right] > 0 \iff \ddot{c} > \ddot{w}. \diamond \tag{A1.4}$$