

Inequality and a Repeated Joint Project*

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Abstract

Agents voluntarily contribute to an infinitely repeated joint project. We investigate the conditions for cooperation to be a renegotiation-proof and coalition-proof equilibrium before examining the influence of output share inequality on the sustainability of cooperation. When shares are not equally distributed, cooperation requires agents to be more patient than under perfect equality. Beyond a certain degree of share inequality, full efficiency cannot be reached without redistribution. This model also explains the coexistence of one cooperating and one free-riding coalition. In this case, increasing inequality can have a positive or negative impact on the aggregate level of effort (or social surplus).

1 Introduction

Agents take part in a joint project to which they voluntarily contribute efforts. The output is distributed among the players according to their share, which can be their relative wealth. In this kind of setting, we know that, if the game is played once, the first-best optimum will be impossible to sustain¹, as deviation is a dominant strategy and the aggregate level of Nash equilibrium contributions is suboptimal. In contrast,

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¹It can if all the shares are concentrated into the hands of a single individual and efforts are perfectly substitutable.

the Folk theorem states that, provided agents are not too impatient, new and more efficient equilibria can be reached thanks to the repetition of the game. In this paper, we investigate to what extent inequality of shares influences first-best sustainability in an infinitely repeated game. *Does introducing share inequality render cooperation more difficult to support?*

There exist numerous examples of voluntarily provided joint projects in the real world. One can think of an irrigation scheme where individuals have to put in efforts for building and/or maintaining the infrastructure. In that case, the level of their contributions to this collective action will depend on their benefit, which is directly related to the amount of land they cultivate. The same logic applies to many other cases: voluntary provision to local public goods, collective problems in management of environmental resources (forests, fisheries, pest and weed control), cooperatives, financial lobbying, defence alliances, etc. Industrial Organization also encompasses related issues, i.e. tacit collusion, moral hazard in teams or principal-agent model of team production.

Although most of the literature on these issues uses static game formalizations, these situations are often better described as repeated games. In many cases, agents interact repeatedly and have to find ways to sustain cooperation between them, not knowing when this game is going to end. Our infinitely repeated game framework therefore seems realistic.

In one-shot games, Olson (1965), followed by several authors, argued that the effect of inequality was positive on collective action. He claimed that if one single (or a few) agent has a great interest in the collective action, the good is more likely to be provided even if this user is the only one to bear its cost. Olson brought up the two following insights: that contributions are positively related to wealth and that great inequality implies a great likelihood of success of collective action. These results however rest upon several unstated hypotheses: perfect substitutability of efforts, single interaction, Nash behavior and identical marginal costs. Moreover, several papers show that the conditions for Olson's latter result to remain valid are quite demanding. This is put forward, among others, in empirical studies by Bardhan (2000) and Dayton-Johnson (2000) but also put in perspective in more theoretical papers such as Bardhan, Ghatak, Karaivanov (2007) and Ray, Baland, Dagnelie (2007).

As for now, the effect of inequality on efficiency or the aggregate level of effort seems to be ambiguous, depending on the cost function (Banerjee et al., 2006). In an empirical paper on collective action in Pakistani communities, Khwaja (2006) finds a U-shaped relationship between inequality in the distribution of project returns or land ownership and maintenance, the aggregate level of efforts. It also appears that, in many cases, departing from an equalitarian distribution will harm collective action or efficiency, by increasing the poor's incentives to free-ride or by allowing the rich and powerful to take over rents - as in the case of the Maharashtra sugar cooperatives (Banerjee et al.,

2001). Deepening inequality can therefore increase or decrease further the incentives of the beneficiaries from the redistribution to cooperate².

With the prospect of future interactions with the same people³ arises the possibility of punishing undesired actions, which is conducive to cooperative outcomes. A few papers address the question of inequality and efficiency in dynamic or repeated games. The former refers to a paper by Tarui (2007) investigating the influence of inequality in productivity, access to markets and credit into a dynamic intergenerational game of common property resource use⁴. According to the punishment used and the harvest sharing rule, Tarui shows that first-best sustainability may or not be affected by an increase of inequality.

As to Bardhan and Singh (2005), they explore the influence of wealth inequality on cooperation, sustained by trigger strategies, i.e. Nash reversion. In their model, agents are endowed with private capital which enters, with a complementary input, a constant return to scale Cobb-Douglas production function. To produce this complementary input, agents have to choose between a status quo technology which guarantees some level of output and a cooperative technology the fruits of which can be captured by one or more deviating players. They establish that, in this setting, inequality can affect cooperation and that redistribution can improve the welfare of the rich thanks to the greater possibility of cooperation.

This paper also relates to Itaya and Yamada (2003) who investigate the impact of income inequality on a repeated game of private provision of public goods with two players and renegotiation-proof equilibria. They point out the negative effect of inequality on first-best sustainability.

This research is also close in spirit to Vasconcelos (2005), a paper on tacit collusion in quantity setting supergames with asymmetric costs. In both our papers, inequality decreases the scope of cooperation sustainability, by increasing the discount factor of interest. Since Vasconcelos' market shares are allocated according to the firms' production capacity (which affects marginal costs), he shows that the smallest firms are more prone to deviate from the collusive agreement, which fits our framework. We mainly differ in our punishment strategies. As he uses optimal penal codes à la Abreu (the stick and the carrot, 1986, 1988) to sustain cooperation, while punishing, the largest firms have the greatest incentives to deviate from the punishment path⁵. We resort to renegotiation-

²A comprehensive discussion on this issue can be found in Baland and Platteau (2003) from p. 161.

³This renders our framework different from reputation matching games à la Kandori (1992) where relationships are infrequent and "agents change their partners over time and dishonest behaviour against one partner causes sanctions by other members in the society".

⁴Rendering our model dynamic would only make, at each period, the rich, richer and the poor, poorer. It would then lead to a situation where only one player would be rich enough for cooperation to be more desirable than deviation. No one would therefore produce the efficient level of effort.

⁵Abreu proposes symmetric punishments during which everyone suffers. Large firms, having a large

proof punishments which are known to substantially reduce the sustainability of full collusion in competition à la Bertrand or Cournot (Farrell, 2000).⁶

A common and easy solution considered in the literature for sustaining cooperation is Nash reversion (Friedman, 1971) consisting in a permanent return to the Nash equilibrium after a single deviation. We will use it as a benchmark for our analysis. However, one of the main drawbacks of Nash reversion is that, without being the harshest punishment, the punisher suffers from giving a punishment. As stated by Bernheim and Ray (1989) and Farrell and Maskin (1989), who introduced the concept of renegotiation-proofness in the literature⁷, this renders the threat not credible. Everybody indeed anticipates that, ex post, the punishers and the punished will be tempted to renegotiate. Furthermore, after a single deviation, all the agents are stuck for ever in a pareto dominated equilibrium. It would be hard to believe that, in a repeated setting, agents fail to exploit the existing opportunities to reach the pareto frontier. Actually, it is rarely observed on the field as stated in Tarui (2007). The latter quotes Ostrom (1990) who argues that many commons overcame occasional deviations. This indirectly supports the evidence that the punishments are only temporary, allowing a return to cooperation.

This confirms the need to elaborate punishments allowing a return to cooperation after the punishment phase. In this research, we extend the concept of renegotiation-proofness from 2 to n players, taking into account credible deviations⁸ of coalitions to the prisoner's dilemma with continuous strategies, which, to our knowledge, is a novelty.

We also introduce a new explanation as to why inequality can result in suboptimal outcomes. We indeed show that the introduction of inequality under Nash reversion or punishments resisting to renegotiation and deviation by credible coalitions is detrimental to cooperation. Compared to perfect share equality, agents have to be more patient to cooperate in the presence of inequality of shares.

In Section 2, we present our simple model offering a binary choice to the agents: cooperating or deviating from the socially optimal contribution. Nash reversion is addressed in Section 3. We propose, in Section 4, a renegotiation-proof and coalition-proof punishment scheme and investigate the influence of share inequality on the limit dis-

market share, suffer more from the price decrease following the breach in the collusive agreement.

⁶Farrell (2000) shows that the renegotiation-proofness requirement makes full collusion impossible to sustain in symmetric Bertrand games when $n > 3$ and when $n > 9$ in symmetric Cournot games.

⁷While their equilibrium concept is fairly similar, the former refer to 'collective dynamic inconsistency' and the latter to 'weak renegotiation-proofness'. Readers interested in this topic could also see the works of van Damme (1989) and Asheim.

⁸The concept we use is coalition proofness which implies that we restrict ourselves to pareto efficiency within the class of self-enforcing agreements (Bernheim et al, 1987). An agreement is self-enforcing if no subcoalition taking the action of others as given can deviate in a way that makes all of its members better off. (Bernheim et al., 1987) It means that an agreement is not self-enforcing and hence not credible if it is profitable for one member to deviate from this agreement. In this paper we focus on credible deviations.

count factor. We characterize, in Section 5, the lowest share compatible with generalized cooperation, discuss the coexistence of one free-riding and one cooperating coalition and how redistribution can increase or decrease the total amount of effort put in the project. We also examine how redistributing shares from the rich to the poor players can improve the welfare of everybody. Eventually, before concluding in Section 7 as to the negative influence of wealth inequality in this setting, we introduce, in Section 6 outside options. The proofs are collected in an Appendix.

2 Repeated Joint Production with Shares

A group of n agents decide to produce jointly and repeatedly a particular output. All the i agents are identical except for, λ_i , their share in output which can also be a measure of their relative wealth. λ is the vector of shares, $[\lambda_1, \lambda_2, \dots, \lambda_n]$, the sum of which equals 1. Note that $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$. All the agents exhibit the same degree of impatience and therefore have the same discount factor, δ .⁹

The collective output is, as in the standard pure public good model, the sum of the (nonnegative) efforts, e_i , of all the agents taking part in the project. This modelling of the output implies that efforts are perfect substitutes what, with respect to efficiency, favours an unequal distribution of shares¹⁰. A straightforward illustration of this would be to consider a single agent concentrating all the incentives ($\lambda_i = 1$) to produce the efficient effort. In this case, the first-best optimum is produced at each stage of the game while using a Leontieff production function, where efforts are perfect complements, would result in a suboptimal production of effort.

Regarding the cost function, even though it looks simple and elegant to introduce an isoelastic convex cost of effort¹¹ it would be at the cost of imposing an assumption of strict equality of marginal costs of effort. This would imply that the efficient level of effort is the same for each agent whatever her endowment¹² which could be suspected of driving the results. We therefore adapted the cost expression so that each agent putting effort in the project has to undergo an isoelastic convex cost expressed in terms of her share, $\frac{e_i^\gamma}{\gamma \lambda_i}$ with $\gamma \geq 2$ ¹³, taking into account, for instance, the imperfections of the credit

⁹If one introduces inequality in the distribution of discount factors assuming that the poor are less patient than the rich cooperation is even harder to sustain and our conclusions easier to reach.

¹⁰For a discussion on the influence of inequality on joint projects when efforts are not perfect substitutes in a one-shot game, see Ray, Baland and Dagnelie (2007).

¹¹This same exercise was run with an isoelastic convex cost e_i^γ with $\gamma > 1$ in a previous version of the paper (Dagnelie, 2007). Results are equivalent.

¹²It means that it would be rapidly difficult for the poor to sustain cooperation since they would have to put in a very high level of effort (the same as for the rich) but would earn only a small share of the total output (contrarily to the rich).

¹³If $\gamma < 2$, a strong inequality implies that the very rich players prefer generalized deviation to general-

markets¹⁴. Since the marginal cost of effort is decreasing in the share endowment, the efficient level of effort is smaller for the poor which makes it easier for them to cooperate. Even though releasing this assumption of equality of marginal costs renders our results harder to achieve, both modelings produce similar results. γ is also present at the denominator for analytical tractability, without loss of generality.

Collective action individual payoffs, π_i , have therefore the following form:

$$\pi_i = \lambda_i \sum_j e_j - \frac{e_i^\gamma}{\gamma \lambda_i} \quad (1)$$

Once a game is repeated, many Nash equilibria arise. In this research, we focus on two of them: the good outcome (π_i^C), which is pareto efficient in the class of subgame perfect equilibria, and the bad one, the Nash equilibrium of the stage game (π_i^N). In most of the paper, we consider that the reservation utility of the agents equals zero, we remove this assumption in Section 6. Maximizing the social surplus provides us with the optimal, cooperative level of effort, e_i^C , equal to $\lambda_i^{\frac{1}{\gamma-1}}$, which depends on the distribution of shares.

If this game is played once, then every agent is going to maximize her own payoff, given the efforts contributed by the other agents.

$$\pi_i^N \equiv \max_{e_i} \lambda_i (e_i + \sum_{j \neq i} e_j) - \frac{e_i^\gamma}{\gamma \lambda_i} \quad (2)$$

In this case where everyone deviates from the cooperative level of effort and produces $e_i^N \equiv \lambda_i^{\frac{2}{\gamma-1}}$, the outcome is known to be pareto dominated by the first-best optimum¹⁵.

The game described above is similar to a prisoner's dilemma with n agents and continuous strategies. Inequality creates tensions in the sense that the premium from deviating from cooperation is, proportionally to their endowment, higher for the poor which therefore renders deviation more attractive to them. Hence we can introduce the following lemma.

LEMMA 1 *The agent who benefits most from deviating, relatively to one's share, is always the one with the lowest share.*

ized cooperation and at $\gamma = 1$, surplus maximization is unbounded.

¹⁴What we have in mind is restricted access to credit for the poor.

¹⁵It means that $e_i^N = e_i^C \lambda_i^{\frac{1}{\gamma-1}}$ with $e_i^N < e_i^C$ as, if $\lambda_i < 1$, $\lambda_i^{\frac{1}{\gamma-1}} < 1$ for all γ while $\pi_i^N = \lambda_i (\sum_{k \in n} \lambda_k^{\frac{2}{\gamma-1}}) - \gamma^{-1} \lambda_i^{\frac{\gamma+1}{\gamma-1}}$.

When we introduce share inequality, the cooperation payoff decreases faster than the deviation payoff which makes deviation more profitable for the poor. This tends to confirm Olson's hypothesis as to the exploitation by the poor.

Since we consider deviations of coalitions and simultaneous unilateral deviations of independent agents, it is possible that, in the latter case, $\pi_i^{C^*} < \pi_i^C$ - where π^{C^*} is the deviation profit¹⁶ which depends on the endowment (and the number) of the deviating players and π^C is the cooperation profit¹⁷ - deviating from the cooperation effort is not profitable.¹⁸ While cooperating produces a big surplus, if too few cooperate and too many deviate, the small surplus is divided among too many deviating players for deviating to remain profitable.¹⁹

3 Nash Reversion

In this section, we will temporarily not address the problems posed by renegotiation and consider the following punishment: Once a coalition has deviated, everybody produces the Nash level of effort for ever. For cooperation to be sustainable, the discount factor must respect the following condition:

$$(1 - \delta_N)\pi_i^{C^*} + \delta_N\pi_i^N < \pi_i^C \Rightarrow \delta_N > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^{C^*} - \pi_i^N}$$

If $\frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^{C^*} - \pi_i^N} < 1$, it is theoretically possible to support cooperation. As long as players have a discount factor greater than δ_N and smaller than 1, Nash reversion can be used as a threat against deviation. Hence, the necessary condition for cooperation to become sustainable is :

$$\pi_i^C > \pi_i^N \equiv \lambda_i \left[\sum_{j \in n} \lambda_j^{\frac{1}{\gamma-1}} (1 - \lambda_j^{\frac{1}{\gamma-1}}) \right] - \gamma^{-1} \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}}) > 0 \quad (3)$$

The bigger the difference $\pi_i^C - \pi_i^N$, the smaller δ_N and therefore the easier cooperation can be sustained.

¹⁶Note that the exponent * always refers to a deviation from what precedes. $\pi_i^{C^*} = \lambda_i (\sum_{j \in n_C} \lambda_j^{\frac{1}{\gamma-1}} + \sum_{k \in n_D} \lambda_k^{\frac{2}{\gamma-1}}) - \gamma^{-1} \lambda_i^{\frac{\gamma+1}{\gamma-1}}$, where $n - n_D = n_C$ cooperate but n_D deviate.

¹⁷ $\pi_i^C = \lambda_i (\sum_{j \in n} \lambda_j^{\frac{1}{\gamma-1}}) - \gamma^{-1} \lambda_i^{\frac{1}{\gamma-1}}$

¹⁸When $\lambda_i = n^{-1}$, deviation is profitable as long as $n_D < \frac{n+1}{2}$.

¹⁹These instances of non profitable deviation are more easily deterred with Nash reversion and a renegotiation-proof and coalition-proof punishment.

One can see that, if the distribution of shares is perfectly equal, as long as the agents have a discount factor smaller than 1 and greater than δ_N , cooperation can always be sustained by Nash reversion. The condition in equation (3) is indeed always verified and becomes :

$$\gamma > \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \quad (4)$$

As cooperation can always be sustained with Nash reversion when the distribution is equalitarian (and $\delta_N < \delta < 1$) and since deviation is a dominant strategy, even when the game is repeated, for the agents for which inequality renders $\pi_i^C < \pi_i^N$, one can expect a negative effect of inequality on cooperation. Following the introduction of inequality, the agents losing from the redistribution of shares, i.e. $\lambda_i < n^{-1}$, have to be more patient than before not to enter a deviation phase. All this means that increasing share inequality makes cooperation harder and harder to sustain - as the limit discount factor, δ_N , rises - up to a point where inequality is such that, whatever the impatience degree of the agent with a low share, generalized cooperation is not possible any more. This enables us to put forward the following proposition.

PROPOSITION 1 *Introducing inequality among agents renders the condition to sustain cooperation with Nash reversion more difficult to fulfill.*

4 A Renegotiation-Proof and Coalition-Proof Equilibrium

As mentioned before, Nash reversion suffers from several flaws. We therefore want to devise an equilibrium concept resisting to renegotiation and deviation of credible coalitions. We also want this equilibrium to enable returning to cooperation after the punishment phase.

As suggested by van Damme (1989) for the 2 players prisoner's dilemma with discrete strategies, cooperation can be sustained if after player 1's deviation, player 2 is allowed to deviate (produce the Nash level of effort) until player 1 cooperates²⁰. Extending renegotiation-proofness from 2 to n players requires taking account of deviations of coalitions. Nonetheless, the original concept with 2 players fits in with the framework set out below.

van Damme's idea, exposed above, is used to devise our punishment scheme. After a deviation of a coalition of one or more players, the cooperative players enter a punishment phase during which they produce a retaliation quantity of effort (R_i). They keep playing this level of effort as long as the cheaters - the ones who deviated in the normal phase - have not played the punishment level of effort (P_i). During this punishment,

²⁰If player 2 does not cooperate any more, both players are stuck in the pareto dominated equilibrium.

everybody has the incentives to play accordingly to the scheme - what ensures subgame perfection - and the punishers get at least as much as when everybody cooperates and produces the first-best optimum - what guarantees renegotiation-proofness.

We can restrict the length of the punishment phase thanks to the following lemma.

LEMMA 2 *Aiming at the smallest discount factor compatible with a renegotiation-proof and coalition-proof punishment limits the length of the punishment phase to one period.*

A multi-period punishment would have two effects, ex ante, it would make deviation less attractive but as the punishment is harsher, ex post, it would increase the incentives to deviate from the punishment. As will become clearer below, there is a trade-off between these two effects. In this particular case, the latter effect would be dominating as the punishment level of effort is high enough for the punishers to be willing to punish, given the requirement of renegotiation-proofness.

For a punishment to be *renegotiation-proof* - in equilibrium no continuation payoff being pareto dominated by another continuation payoff (Farrell and Maskin, 1989) - and *coalition-proof* - restricting to pareto efficiency within the class of self-enforcing agreements (Bernheim et al, 1987) -, the following 5 conditions must be met.

Condition 1 *Ex ante, the punishment must be such that deviations are deterred.*

$$(1 - \delta)\pi_i^{C^*} + \delta(1 - \delta)\pi_i^P + \delta^2\pi_i^C < \pi_i^C$$

$$\Rightarrow \delta_{XA} > \frac{\pi_i^{C^*} - \pi_i^C}{\pi_i^C - \pi_i^P} \quad (5)$$

δ_{XA} must be smaller than 1 which implies that $2\pi_i^C - \pi_i^{C^*} - \pi_i^P > 0$, with π_i^P being the payoff obtained by an agent during her punishment. Meeting this condition will prevent a subcoalition of players to alternate between deviating and being punished every other period. If the punishment effort is fixed at the level of the cooperative effort, this condition is always verified as $2\pi_i^C - \pi_i^{C^*} - \pi_i^P > 0$ boils down to equation (4). Hence we know that, if the punishment effort to put in is greater than during cooperation, δ_{XA} decreases.

Condition 2 *Ex post, the punishment must be such that deviations from the punishment are deterred.*

$$(1 - \delta)\pi_i^P + \delta\pi_i^C > (1 - \delta)\pi_i^{P^*} + \delta(1 - \delta)\pi_i^{P'} + \delta^2\pi_i^C$$

$$\Rightarrow \delta_{XP} > \frac{\pi_i^{P^*} - \pi_i^P}{\pi_i^C - \pi_i^{P'}} \quad (6)$$

$\pi_i^{P^*}$ is the payoff obtained by the subcoalition of punished agents who deviate from undergoing the punishment. $\pi_i^{P'}$ is the payoff obtained by the same subcoalition of agents

when accepting the punishment (while all the other agents, including those who had accepted the punishment the previous period, produce the retaliation level of effort). The basic idea is that the payoff of the punished must be greater when conforming to their punishment than when deviating. When $n_{D^*} = 1$, the number of punished players deviating from their punishment equals 1, δ_{XP} reaches its minimum which therefore makes Condition 2 the most easily fulfilled. The benefit from deviating from the punishment is by far outweighed by the burden of the penalty for which the single deviator has to compensate all the cooperating players. If the size of the deviating subcoalition rises, $\pi_i^{P^*}$ decreases whereas $\pi_i^{P'}$ rises much faster which means that δ_{XP} also rises. It is therefore expected that $\arg \max_{n_{D^*} \in (1, n_D)} \delta_{XP} = n_D$.

The following two conditions ensure that all the punishers are willing to conform to the punishment phase.

Condition 3 *The payoff of the punishers must be greater when conforming than when deviating from punishing and then conforming, i.e. $\pi_i^{1/P} > \pi_i^{1/P^*}$.*

If $\pi_i^{1/P}$ and π_i^{1/P^*} represent respectively the payoff from punishing and from deviating from giving the punishment, we get:

$$(1 - \delta)\pi_i^{1/P} + \delta\pi_i^C > (1 - \delta)\pi_i^{1/P^*} + \delta\pi_i^C \Rightarrow \pi_i^{1/P} > \pi_i^{1/P^*}$$

In case of perfect equality of wealth among agents, we have:

$$\gamma \frac{n_{C^*}}{n} (R - R^*) > (R^\gamma - R^{*\gamma}) \quad (7)$$

The intuition behind this condition is that it could more interesting, for the punishing players, to skip the punishment phase and go back directly to cooperation. We check which values of R and R^* are compatible with a renegotiation and coalition-proof punishment. R^* being the level of effort put in when deviating from giving a punishment. Equation (7) must hold for $n_{C^*} = 1$, while if it does not hold when $n_{C^*} > 1$ we have to ensure that all these deviating coalitions are not credible. This is done in Condition 4.

One could imagine that the simplest form of punishment would be for the deviators to put in a level of effort giving at least the cooperative payoff to the punishers while the latter do not produce. However, as long as $R < e_i^N$, for very high values of γ , it could be interesting for one punisher to deviate and produce the cooperative level of effort²¹. Once R is fixed at the Nash level of effort, e_i^N , it is never interesting for one punisher²²

²¹As showed by $\lim_{\gamma \rightarrow +\infty} \frac{\gamma}{n} (\beta n^{\frac{-1}{\gamma-1}} - 1) - (\beta^\gamma n^{\frac{-\gamma}{\gamma-1}} - 1) < 0$ when $\beta < 1$. For the particular case of $R = 0$, it is easy to check that the condition $R^* > (\frac{\gamma}{n})^{\frac{1}{\gamma-1}}$ is not satisfied for many values of γ, n .

²²A coalition of $n_{C^*} > 1$ players could be tempted to deviate from giving the punishment. This prevents this scheme to be strong Nash.

to deviate from the punishment scheme and it is easy to prove that:

$$\frac{\partial \delta_{XP}}{\partial n_{D^*}} > 0 \quad (8)$$

Hence, we have to focus on the case where Condition 2 is the hardest to satisfy, i.e. $n_{D^*} = n_D$, which gives $\delta_{XP} > \frac{\pi_i^N - \pi_i^P}{\pi_i^C - \pi_i^P}$. As it is easy to see that $\frac{\partial \delta_{XP}}{\partial \pi_i^P} < 0$ ²³, we know that the minimum of δ_{XP} is reached when the punishment is also fixed at its minimum.

All that has been said so far enables us to point out that π_i^N is again a focal point. If the cooperative agents return to putting in the Nash effort, the threat point of the game is infinite repetition of the Nash equilibrium. The deviators can indeed renege for ever on the punishment and produce the Nash effort. We therefore know that $\pi_i^C > \pi_i^N$ is again to satisfy.

For this setting to be coalition-proof, we must now ensure that no deviating coalition of punishers is credible which implies that no one should have one's interest in further deviating from the deviating coalition. We hence turn to the next condition.

Condition 4 *For the equilibrium to be coalition-proof, no deviation of punishers should be credible, i.e. $\pi_i^{1/P^*} < \pi_i^{1/P^{**}}$.*

If $\lambda_i = \frac{1}{n}$ and $n_{C^{**}} = 1$ ²⁴, $\pi_i^{1/P^*} < \pi_i^{1/P^{**}}$ is equivalent to:

$$\frac{\gamma}{n} < \frac{(1 - n^{\frac{-\gamma}{\gamma-1}})}{(1 - n^{\frac{-1}{\gamma-1}})} \quad (9)$$

For all γ and n , this condition is fulfilled which means it is always more interesting for one player to further deviate from the deviating coalition by producing the Nash level of effort²⁵. Condition 4 is always met and no deviation of punishers is therefore credible.

It is to be noted that, if all the punished agents conform to the punishment phase and produce a high level of effort, no coalition of punishers is willing to deviate. A fortiori, if only a subcoalition of punished players deviates from the punishment phase - and therefore produces less -, no subcoalition of punishers is tempted to skip this phase. Given the punished and the punishers have opposite duties and interests, no deviation of mixed coalitions is credible. This ensures that our punishment scheme is immune to deviation from coalitions of punishers and/or punished players and guarantees sub-game perfection.

²³ $\frac{\partial \delta_{XP}}{\partial \pi_i^P} = \frac{\pi_i^N - \pi_i^C}{(\pi_i^C - \pi_i^P)^2}$ which is negative as long as $\pi_i^C > \pi_i^N$.

²⁴ One player is enough to show that coalitions are not credible.

²⁵ It is not surprising as e_i^N is the best response of player i , being the solution to equation (2).

The following condition certifies that the punishers will not renegotiate the punishment scheme.

Condition 5 *The payoff from punishing a deviating coalition must be greater or equal than the payoff from generalized cooperation, i.e. $\pi_i^{1/P} \geq \pi_i^C$.*

It means that, if $\lambda_i = \frac{1}{n}$,

$$\underline{P} \geq n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - n_C - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_D} \quad (10)$$

Now that the conditions to satisfy are stated, we want to select the punishment producing the lowest discount factor compatible with generalized cooperation, $\underline{\delta}$. We have therefore to fix the effort level corresponding to the punishment, P , so that the couple $(\delta_{XA}, \delta_{XP})$ is the lowest possible and, in any case, smaller than one. As our punishment scheme has to simultaneously respect the conditions expressed in equations (5) and (6), we have to find the $\max(\delta_{XA}, \delta_{XP})$. In order to pick the punishment corresponding to the highest degree of impatience, we have to determine P such that we get $\underline{\delta} \equiv \min \max(\delta_{XA}, \delta_{XP})$.

Comparing δ_{XA} and δ_{XP} boils down to comparing $\pi_i^{C^*} - \pi_i^C$ and $\pi_i^N - \pi_i^P$, in the limit case where $n_{D^*} = n_D$, since the denominator of these ratios is the same. After simplification,

$$\pi_i^{C^*} - \pi_i^C \leq \pi_i^N - \pi_i^P \quad (11)$$

becomes with an equalitarian distribution of shares:

$$-\gamma^{-1} n^{\frac{-1}{\gamma-1}} \left[(n^{\frac{1}{\gamma-1}} P)^\gamma - 1 \right] + n_D n^{\frac{-\gamma}{\gamma-1}} (n^{\frac{1}{\gamma-1}} P - 1) \leq 0 \quad (12)$$

The root of interest in equation (12) is $P = e^C$. Hence we know that when $P = e^C$, $\underline{\delta} = \delta_{XA} = \delta_{XP}$. We also know that as soon as $P > e^C$, (10) must be binding, hence we have to minimize $\pi_i^N - \pi_i^P$ (as $\delta_{XA} \leq \delta_{XP}$) and thus P to get $\underline{\delta}$. It means that the minimal punishment compatible with renegotiation-proofness and coalition-proofness is \underline{P} as long as it is greater than the cooperative level of effort. As long as the parameters of equation (10) produce a $\underline{P} < e^C$, resorting to such a punishment effort will not prevent deviations. The penalty incurred by the deviators would be too small to deter them from alternating between deviating and being punished. In this case, the punishment must be to put in the cooperative level of effort. On the other hand, if $P < \underline{P}$, the punishment is not renegotiation-proof.

We have therefore to find $P \equiv \max(e^C, \underline{P})$ and turn to the following equation.

$$\underline{P} \geq e^C \equiv \frac{n_C}{n} \gamma \geq \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \quad (13)$$

The cases where equation (13) is not verified, with $\gamma \geq 2$, is when $n_D \geq \frac{n}{2}$. In these cases, \underline{P} is too small a punishment and the punished have to put in the cooperative level of effort, exactly as in the two players game put forward by van Damme (1989).

Considering we took into account the different conditions imposed by our punishment scheme when $\lambda_i = n^{-1}$, we are equipped with the parameters of our punishment, P and R . As δ_{XP} must be smaller than 1 for cooperation to be a renegotiation-proof and coalition-proof equilibrium, π_i^N must be smaller than π_i^C . It is the same binding constraint as with Nash reversion and we know from equation (4) that it is always true. This allows us to put forward the following proposition:

PROPOSITION 2 *As long as $\delta_{XP} < \delta < 1$, $\lambda_i = n^{-1}$ and the game is infinitely repeated²⁶, it is possible to sustain cooperation with a renegotiation-proof and coalition-proof punishment.*

So far, we have shown that, under a perfectly equalitarian distribution of shares, it is possible to use a renegotiation-proof and coalition-proof punishment scheme to sustain generalized cooperation. One can remark that, with γ known and a perfectly observable and certain output, this equilibrium requires particularly little information. It can be completely decentralized as, after deviation, the punished and the punishers know exactly which level of effort to provide.

We now want to investigate how introducing share inequality influences the way the first-best optimum can be supported. The punishment is, as expected, very similar to the case of perfect share equality. If we introduce inequality, $\pi_i^{C*} - \pi_i^C \leq \pi_i^N - \pi_i^P$ becomes:

$$\lambda_i \left[\sum_{k \in n_D} (P_k - \lambda_k^{\frac{1}{\gamma-1}}) \right] - \gamma^{-1} \lambda_i^{-1} \left(P_i^\gamma - \lambda_i^{\frac{\gamma}{\gamma-1}} \right) \leq 0 \quad (14)$$

It is easy to see that for this condition to be met, the punishment effort has to be greater or equal than the cooperative level of effort, i.e. $P_i \geq \lambda_i^{\frac{1}{\gamma-1}}$. It means that equation (10) becomes:

$$\underline{P}_k \geq \frac{\lambda_k}{\sum_{k \in n_D} \lambda_k} \left[\sum_{i \in n} \lambda_i^{\frac{1}{\gamma-1}} - \sum_{j \in n_C} \lambda_j^{\frac{2}{\gamma-1}} - \gamma^{-1} \frac{\sum_{j \in n_C} \lambda_j^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_j^{\frac{\gamma}{\gamma-1}})}{n_C} \right]$$

While, as each punisher must receive $\pi_i^{1/P} = \pi_i^C$, all the deviators have to put in \underline{P}_k so that:

$$\pi_i^{1/P} = \lambda_i \left(\sum_{j \in n_C} \lambda_j^{\frac{2}{\gamma-1}} + \sum_{k \in n_D} \underline{P}_k \right) - \gamma^{-1} \lambda_i^{\frac{\gamma+1}{\gamma-1}}$$

²⁶It remains true if the game is finitely repeated and the agents do not know when the game ends.

where $\forall j \in n_C : \sum_{k \in n_D} P_k = \sum_{i \in n} \lambda_i^{\frac{1}{\gamma-1}} - (\sum_{j \in n_C} \lambda_j^{\frac{2}{\gamma-1}}) - \gamma^{-1} \lambda_j^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_j^{\frac{\gamma}{\gamma-1}})$.

We showed that, under a perfectly equalitarian distribution of shares, our punishment scheme prevents all the agents from deviating from cooperation. We also know that inequality is detrimental to cooperation since it is possible that deviation becomes a dominant strategy for poorly endowed agents - i.e. if $\pi_i^C < \pi_i^N$. However, even in less extreme cases, we can state the following proposition.

PROPOSITION 3 *After introducing inequality, the agents losing from the change in the distribution of shares have to be more patient than before to produce the efficient level of effort when the punishment is renegotiation-proof and coalition-proof.*

As in the case of Nash reversion, introducing inequality of shares renders first-best efficiency more difficult to support with a renegotiation-proof and coalition-proof punishment.

Now that the minimal discount factor is characterized in two different punishment schemes, we can compare them. It is easy to show that $\delta_N \leq \delta_{XP}$ as, after rearranging and symplifying, we get equation (11).

5 Redistribution and Cooperation

In this section, we try to characterize the lower bound of theoretical cooperation, i.e. when δ tends towards 1. Note again that the further $(\pi_i^C - \pi_i^N)$ is from 0, the lower δ , which increases the scope for cooperation. We then investigate the issue of redistribution.

5.1 Characterization of the lowest share compatible with generalized cooperation

As the utmost condition to satisfy for cooperation to be sustainable is $\pi_i^C > \pi_i^N$, it is possible to partially characterize the lowest share compatible with generalized cooperation, λ_{min} . To see this, let us define several distributions of shares:

$$\begin{aligned} \underline{\lambda} &\equiv \underline{\lambda}_1 = \dots = \lambda_{n-1} < \lambda_n \\ \bar{\lambda} &\equiv \bar{\lambda}_1 = \dots = \lambda_{n-2} < \lambda_{n-1} < \lambda_n \\ \hat{\lambda} &\equiv \hat{\lambda}_1 = \dots = \lambda_{n-2} < \lambda_{n-1} = \lambda_n \\ \bar{\lambda} &\equiv \bar{\lambda}_1 < \lambda_2 = \dots = \lambda_n \end{aligned}$$

The lowest share compatible with cooperation depends on the convexity parameter of the cost term, γ . Therefore, we get:

$$\lambda_{min} \geq \begin{cases} \bar{\lambda}_1 & \text{if } 2 \leq \gamma \leq 4 \\ \bar{\lambda}_1 \text{ or } \underline{\lambda}_1 & \text{if } \gamma > 4 \end{cases} \quad (15)$$

When $\gamma > 4$, the second derivative of the function $\lambda_i^{\frac{1}{\gamma-1}} - \lambda_i^{\frac{2}{\gamma-1}}$ is either positive or negative. Hence the latter function is respectively convex or concave which determines the form of the shares distribution giving the lowest share compatible with generalized cooperation. A high γ favours convexity and therefore inequality while a high n is in favour of concavity and equality of shares.

5.2 Redistribution

We know that, whatever the punishment strategy we use, if $\pi_k^C < \pi_k^N$, the best strategy for player k is to deviate at each period of the game. If the rich players know that some people are too poor to cooperate (and hence do not punish them), the first-best optimum cannot be attained any more but cooperation can still be sustained for the subset of players whose cooperation profit is greater than the Nash profit.

Observation 1 *If a subset of agents do not cooperate because of a low λ_i ²⁷ (such that $\pi_k^C < \pi_k^N$), a low-producing (putting in the Nash level of effort) and a high-producing (providing the efficient level of effort) coalition can coexist.*

If we restrict cooperation to the well endowed agents, equation (3), $\pi_i^C > \pi_i^N$, becomes

$$\lambda_i \left[\sum_{j \in n_C} \lambda_j^{\frac{1}{\gamma-1}} (1 - \lambda_j^{\frac{1}{\gamma-1}}) \right] - \gamma^{-1} \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}}) > 0 \quad (16)$$

This inequality is never verified if $n_C = 1$.²⁸ It means that, if more than one agent gets a positive share, producing the first-best level of effort requires at least two agents to get a big enough share²⁹, i.e. a share such that $\pi_i^C > \pi_i^N$. The number of people whose shares add up to a low λ_{inflim} does not influence cooperation. At the same time, there can coexist one cooperating and one free-riding coalition.

As we know that cooperators put in an effort of $\lambda_j^{\frac{1}{\gamma-1}}$ and deviators contribute $\lambda_k^{\frac{2}{\gamma-1}}$, the total level of effort put in the project is $\sum_i e_i = \sum_{j \in n_C} \lambda_j^{\frac{1}{\gamma-1}} + \sum_{k \in n_D} \lambda_k^{\frac{2}{\gamma-1}}$. It is therefore possible to compare the different distributions with respect to the total level of effort contributed to the project³⁰.

²⁷ $\sum_i \lambda_k = \lambda_{inflim} \Rightarrow \sum_{i \neq k} \lambda_i = 1 - \lambda_{inflim}$

²⁸The maximum of equation (16) with 1 player is at $\lambda_i = 1$ and equals 0. It means that as long as $\lambda_i < 1$ the former expression is negative.

²⁹It implies that, if all the agents but one have a share such that $\pi_i^C < \pi_i^N$, the richest player (whose $\lambda_n < 1$) will produce the deviating level of effort.

³⁰The same exercise done on the social surplus, $\sum_{j \in n_C} \lambda_j^{\frac{1}{\gamma-1}} (1 - \gamma^{-1}) + \sum_{k \in n_D} \lambda_k^{\frac{2}{\gamma-1}} (1 - \gamma^{-1} \lambda_k)$, produces equivalent results.

The only case of unambiguous effect of disequalizing redistribution³¹ is when it lowers the share of a poor player to the benefit of another poor player to the extent that the latter becomes rich enough to cooperate. In this case higher inequality increases the total sum of contributions. In all the other instances of redistribution, the effect on the aggregate level of effort depends on γ which determines the concavity or convexity of the function. Regarding redistribution from/to a deviator to/from a cooperator, its effects are symmetrical. Redistribution from a deviator to a cooperator (from a cooperator to a deviator) decreases (increases) the aggregate level of effort, if $\gamma \geq 3$. When $\gamma < 3$ there exists a $\tilde{\gamma}$ where redistribution is neutral as to the aggregate level of contributions while below this cutoff value $\tilde{\gamma}$ increasing inequality will raise the total level of efforts. As to the redistribution between deviators, the pivotal value of γ is 3. At 3, redistribution does not influence the amount of efforts provided in the project. Below (above) 3, every disequalizing redistribution among the poor increases (decreases) the aggregate level of effort. We therefore show that in some cases increasing inequality can have a positive effect on the amount contributed. This means we can observe a U-shaped relationship between inequality and the aggregate level of effort.

We now want to investigate whether it would be profitable for the rich agents to redistribute part of their share to the less endowed so that the latter can afford to cooperate. If the agents are perfectly patient, once the poor get a share such that their cooperation profit is at least equal to their deviation profit, they are expected to produce the efficient level of effort. The benefits from cooperation are such that, in some cases, they outweigh the loss of welfare undergone by the rich agents redistributing part of their share.

Let us first examine the extreme case where only one individual is rich enough for cooperation to be attractive. In this case, all produce the deviation level of effort. There are therefore large potential gains from cooperation. Simulations on the following system of equations³² show us that there exists a range for $\gamma \in [\underline{\gamma}, \tilde{\gamma}]$ where redistributing so that each agent gets a big enough share to cooperate is always profitable, whatever the initial distribution of shares. This range increases with the number of players, while it is empty when $n = 2$. It does not mean however that redistribution is not profitable for values of γ outside this range.

$$\begin{cases} \pi_{\tilde{n}}^C \geq \pi_n^N \text{ with agent } \tilde{n} \text{ being } n \text{ after redistributing part of her share.} \\ \pi_{min}^C = \pi_{min}^N \text{ with agent } min \text{ owning a share equalizing both payoffs.} \end{cases}$$

Redistribution is obviously also conceivable in share distributions where (not generalized) cooperation already exists. In these cases, the payoff from cooperation after

³¹To the exception of $\gamma = 2$, where redistribution is neutral, disequalizing redistributions between cooperators always lower the aggregate level of effort.

³²The detail of this system is presented in Appendix B.

redistribution (hence with a larger coalition of cooperative players) must be at least as large as the payoff from cooperation before redistribution for all the players on the giving side of the scheme. Once again we assume that the poorly endowed agents get a share such that they are indifferent between generalized cooperation and generalized deviation.³³

As in the case of weakest-link public goods where the aggregator index used to describe the production function is the minimum of all the contributions³⁴, the repetition of the game could induce rich players to redistribute part of their share to render cooperation attractive for the poor. Note that it would not be dictated by altruistic considerations but would pertain to a payoff maximizing behaviour of the richest agents.

6 Outside Option

So far we have constrained the players to produce either a cooperative or a deviation level of effort while preventing them from opting out of the game. This could be the right way to model many joint projects in real life where, once the decision of joining the project has been made, it is not possible to go astern. Nevertheless, the alternative is relevant for other types of project, all the more since we focus on projects with perfect substitutability of efforts. In the presence of outside options, it makes sense to consider that, to get the same quantity of goods as jointly produced by n agents, working solo would require more effort and therefore be costlier than being involved in a joint project. In the case of irrigation schemes, once the decision to participate has been made, the irrigation infrastructure is built. It then becomes difficult to opt out and, unless one is ready to sustain an important cost, the agent's choice is limited to produce the cooperative or the deviation level of maintenance effort. Likewise, regarding tacit collusion in competition à la Cournot, firms, once in the market, produce either the collusive or the Nash quantity. As to defence alliances, countries can decide to leave a coalition, in which case they will probably have to increase their level of effort to benefit from the same level of protection.

Introducing outside options changes the conditions to be fulfilled for cooperation to be sustainable. In the case of Nash reversion, it is necessary to take into account the possibility that the profit from working solo (π_i^S) is higher than the profit from generalized deviation (π_i^N) in which case the condition to be respected becomes:

$$(1 - \delta)\pi^{C^*} + \delta\pi^S < \pi^C \Rightarrow \delta > \frac{\pi^{C^*} - \pi^C}{\pi^{C^*} - \pi^S}$$

³³The first derivative of the cooperation payoff after redistribution with respect to the redistributed amount being negative.

³⁴See Vicary (1990) for redistributions and weakest-link technology.

Deviating from cooperation then leaving the collective action for working on one's own must indeed be less profitable than producing the cooperative level of effort in the joint project. The ultimate condition for cooperation to be fulfilled is therefore:

$$\pi^C > \max(\pi^N, \pi^S)$$

One can immediately realize that, if $\pi_i^S > \pi_i^N$, the discount factor is higher than without exit option. Outside options can therefore, as expected, decrease the scope for cooperation.

As to renegotiation-proofness where we have to ensure that $\delta = \min \max(\delta_{XA}, \delta_{XP})$, we have to take into account an additional condition. As the punishment lasts one period and allows a return to cooperation, the payoff received on the punishment path must be higher than the payoff obtained while working solo.

$$(1 - \delta)\pi^P + \delta\pi^C > \pi^S \Rightarrow \delta_S > \frac{\pi^S - \pi^P}{\pi^C - \pi^P}$$

As in the case of Nash reversion, the ultimate condition to be respected, while δ_{XA} and δ_{XP} must be simultaneously smaller than 1, becomes:

$$\pi^C > \max(\pi^N, \pi^S)$$

Obviously, as long as the outside option gives nothing more advantageous than the joint project, working solo is not even considered. Once the solo payoff is higher than the payoff from generalized deviation, deviating then working solo becomes a conceivable option which is not chosen unless the player is too impatient.

To summarize the different possibilities created by the introduction of an outside option, let us present the following tables. Note that the outside option is not specified and could differ according to the agent's share (or other factors absent from this model). The first table typically corresponds to the situation of the rich or to an equal to moderately unequal distribution of shares where, for all, the cooperation profit is greater than the Nash profit.

	π_i^{C*}	>	π_i^C	>	π_i^N
π_i^S	\uparrow π_i^S		\uparrow $(1 - \delta)\pi_i^{C*} + \delta\pi_i^S$		\uparrow π_i^C if $\underline{\delta}_S < \delta$
					\uparrow π_i^C if $\underline{\delta} < \delta$

According to the level of their solo payoff (π_i^S), agents make different decisions as to their participation in the project. If the payoff from working alone is even greater than deviating from the project for agent i , the latter is going to work individually and will never participate in the project. If the solo payoff is greater than the cooperation payoff

but smaller than the payoff from deviating, one can expect this agent to deviate in the first period of her participation in the collective action and then to opt out and work solo. In this case, it is likely that the cooperative players decide, if possible, to restrict access to the project.³⁵ While if working solo is less profitable than cooperating but more than generalized deviation, as long as the player's discount factor is greater than the limit discount factor taking account of the outside option, the player cooperates. That, she will also do if working solo is less interesting than generalized deviation. Whatever, the discount factor in this latter case, she takes part in the joint project. Whether she cooperates or deviates will then depend on the player's and the limit discount factors.

It is interesting to note that once some players opt out, the condition for cooperation to be sustainable among the remaining players boils down to equation (16). Even if the payoff of the rich decreases when poor and thus deviators leave the joint project, the departure of the latter does not render cooperation harder to sustain for the rich. Outside options might therefore increase the scope for feasible redistribution. It could indeed be interesting for the rich to redistribute so that the poor get $\pi_i^N > \pi_i^S$ and, even though deviating, keep participating in the project³⁶.

The second table exhibits the case where there is enough inequality for the Nash profit, typically of the poorly endowed agents, to be greater than the cooperation profit.

	π_i^{C*}	$>$	π_i^N	$>$	π_i^C
π_i^S	\uparrow π_i^S		\uparrow $(1 - \delta)\pi_i^{C*} + \delta\pi_i^S$		\uparrow $(1 - \delta)\pi_i^{C*} + \delta\pi_i^N$

In those cases where cooperation is never a conceivable option, the agent either works directly on her own if her solo payoff is greater than her deviation payoff or first deviates then leaves the joint project if her solo payoff is comprised between her deviation and Nash payoff. Eventually, if the payoff from generalized deviation is greater than the solo payoff, at each period of the game, this player is going to produce the deviation level of effort.

Again, in some cases, the rich players knowing the situation of the poorest could let them deviate while a subcoalition could keep cooperating.

³⁵They could also introduce a membership fee or a commitment device for discouraging such strategic behaviours.

³⁶A larger redistribution such that $\pi_i^C > \pi_i^N$ for all i is also to be considered in this case.

7 Conclusion

We showed that, in this particular model, cooperation can be supported under Nash reversion or a renegotiation-proof and coalition-proof punishment. We also demonstrated that introducing inequality of shares among players increases the discount factor compatible with sustainable cooperation, reducing the scope for cooperation. Once inequality has been introduced, the poorer agents involved in the repeated project have to be more patient than before to keep cooperating. Hence, we established that inequality is, in this game, detrimental to generalized cooperation, the efficient outcome. In this respect, this paper puts forward new insights as to the relationship between inequality and efficiency.³⁷

Inequality can be such that some agents cannot afford to produce the efficient level of effort while others may keep cooperating. Our model can therefore also explain the coexistence of well endowed players providing a high level of effort and poor agents only putting in the Nash level of effort. This, in a way, complies with Olson's hypothesis that contributions are positively related to wealth.

Comparing several distributions of share where cooperators and deviators coexist shows that increasing share inequality can have a positive or negative impact on the aggregate level of effort (or social surplus) depending on the cost parameter, γ , (and the number of cooperators). Hence, share inequality can have a U-shaped relationship with the aggregate level of effort (or social surplus), as regularly seen in case studies.

We also mentioned that, in some cases, it can be profitable for the rich agents to redistribute part of their share to the poor players so that the latter can afford to cooperate at each period of the game. Repetition of the game can therefore enlarge the scope for redistribution.

Eventually, we put forward that outside options could restrict the scope for cooperation and increase the scope for redistribution.

APPENDIX

³⁷Several other explanations have been proposed so far in the vast literature exploring the link between inequality and efficiency. Missing or imperfect capital markets have been shown to prevent poor individuals to develop their full potential leading to inefficiency from the viewpoint of the social surplus (Loury, 1981). Inequality implying high redistribution would according to Alesina and Rodrik, 1994, Persson and Tabellini, 1994, generate inefficiency through tax induced distortions on resource allocation. As modelled by Esteban and Ray (2006), agents lobbying for government support put in effort along two dimensions: productivity of the project and wealth. This blurs the signal received by the government which leads it to make inefficient decisions in the presence of wealth inequality.

A Proofs

Proof of Lemma 1:

The relative premium from deviating in terms of her own share is the following:

$$\frac{\pi_i^{C^*} - \pi_i^C}{\lambda_i} = \left[\sum_{n_D} \lambda^{\frac{1}{\gamma-1}} (\lambda^{\frac{1}{\gamma-1}} - 1) \right] - \gamma^{-1} \lambda^{\frac{2-\gamma}{\gamma-1}} (\lambda^{\frac{\gamma}{\gamma-1}} - 1)$$

If the derivative with respect to the share is negative, a wealth decrease renders the deviation more attractive.

$$\frac{\partial(\pi_i^{C^*} - \pi_i^C)/\lambda_i}{\partial \lambda_i} \equiv \frac{1}{\gamma-1} \lambda_i^{\frac{-\gamma+2}{\gamma-1}} \left[(\lambda_i^{\frac{1}{\gamma-1}} - 1) + \frac{2-\gamma}{\gamma} (\lambda_i^{-1} - \lambda_i^{\frac{1}{\gamma-1}}) \right]$$

As the derivative is always negative when $\gamma \geq 2$, the relative premium from deviating rises when the share declines. ■

Proof of Equation (4)

After rearranging equation 4, we get:

$$(\gamma - 1) - n^{\frac{-1}{\gamma-1}} (\gamma - n^{-1}) > 0$$

Taking alternatively the limit of γ towards 2 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

Proof of Proposition 1

We are going to compare two discount factors compatible with cooperation, first under a perfectly equalitarian distribution of shares, then after introducing a disequalizing change in the distribution. As discount factors and ease to sustain cooperation vary in opposite directions, we are done if we can prove that introducing inequality in the distribution of shares makes the discount factor rise.

As long as $\delta_N < 1$, it is theoretically possible to sustain cooperation. We know that the bigger $\pi_i^C - \pi_i^N$, the lower δ_N .

$$\pi_i^C - \pi_i^N = \lambda_i \left[\left(\sum \lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}}) \right) - \gamma^{-1} \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \gamma^{\frac{\gamma}{\gamma-1}}) \right]$$

As $\frac{\partial(\pi^C - \pi^N)}{\partial \lambda_i} > 0$, a decrease in λ_i lowers the gain from cooperation and therefore increases the incentives to deviate and the limit discount factor, δ_N . ■

Proof of Lemma 2

Let us assume the length of the punishment is t periods with $t \in [1, \dots, T]$. If we compare the different δ corresponding to equations (5) and (6), we get:

- ex ante, no one wishes to deviate

$$\sum_{t=1}^T \delta_{XA}^t > \frac{\pi^{C^*} - \pi^C}{\pi^C - \pi^P} \Rightarrow \delta_{XA-1} > \dots > \delta_{XA-T}$$

- The payoff of the punished must be greater when conforming to their punishment than when deviating.

$$\delta_{XP-t} > \left(\frac{\pi^{P^*} - \pi^P}{\pi^C - \pi^P} \right)^{\frac{1}{t}} \Rightarrow \delta_{XP-1} < \dots < \delta_{XP-T}$$

To get the equilibrium compatible with the biggest impatience of the agents, we have to find the lowest δ , $\underline{\delta} \equiv \min \max(\delta_{XA-t}, \delta_{XP-t})$ with t being the number of periods of the punishment.

If $\delta_{XA-1} \leq \delta_{XP-1}$, we are done as this couple would be smaller than any other one in the following general ordering: $\delta_{XA-t} < \dots < \delta_{XA-1} \leq \delta_{XP-1} < \dots < \delta_{XP-t}$.

It therefore boils down to prove that $\pi^{C^*} - \pi^C \leq \pi^{P^*} - \pi^P$ which is the case when $P \geq e_i^C$ as explained after equation (12). ■

Proof of Equation (8)

Taking into account that $\delta_{XP} < 1$ and that $\pi^C - \pi^{P'} > 0$, and rearranging, we get $\pi^{P^*} + \pi^{P'} - \pi^C - \pi^P < 0$. Our strategy is to focus on the terms varying with n_{D^*} to get the simplest expression of the derivative.

Considering the punishment level of effort of equation (10), we get respectively for P and P' :

$$\begin{aligned} P &= n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - (n - n_D) - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_D} \\ P' &= n^{\frac{-2}{\gamma-1}} \left[n^{\frac{\gamma}{\gamma-1}} - (n - n_{D^*}) - \gamma^{-1}(n^{\frac{\gamma}{\gamma-1}} - 1) \right] \frac{1}{n_{D^*}} \end{aligned}$$

Deriving $\pi^{P^*} + \pi^{P'} - \pi^C - \pi^P$ with respect to n_{D^*} , we get:

$$n^{-1} \left[-P + P' + n_{D^*} \frac{\partial P'}{\partial n_{D^*}} - n^2 P'^{\gamma-1} \frac{\partial P'}{\partial n_{D^*}} \right]$$

Substituting in the latter expression $\frac{\partial P'}{\partial n_{D^*}} = \frac{1}{n_{D^*}} [-P' + n^{\frac{-2}{\gamma-1}}]$, we have the following sum which we can sign:

$$n^{-1} \left[-(P - n^{\frac{-2}{\gamma-1}}) + \frac{n^2}{n_{D^*}} P'^{\gamma-1} (P' - n^{\frac{-2}{\gamma-1}}) \right]$$

It is easy to prove that the second term is greater than the first one if $n_{D^*} = 1$.

As we know that $P \leq P'$ with equality when $n_D = n_{D^*}$, we are done if we can show

that the derivative is positive when all the punished deviate from the prescribed punishment. In this case, the derivative becomes:

$$n^{-1} \left[\left(P - n^{\frac{-2}{\gamma-1}} \right) \left(P^{\gamma-1} \frac{n^2}{n_D} - 1 \right) \right]$$

Regarding the first difference, $P - n^{\frac{-2}{\gamma-1}}$, the condition for it to be positive boils down to:

$$\gamma > \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}}$$

which we know is always true.

While the second difference, $P^{\gamma-1} \frac{n^2}{n_D} - 1$, is positive if $n^{\frac{\gamma}{\gamma-1}} - n - n^{\frac{\gamma}{\gamma-1}} + n_D - \gamma^{-1} (n^{\frac{\gamma}{\gamma-1}} - 1) > 0$. Since the higher bound on n_D is $\frac{n}{2}$ and the lower bound on γ is 2, it is always respected.

Those two differences being positive, we know that $\frac{\partial \delta_{XP}}{\partial n_{D^*}} > 0$. ■

Proof of Equation (9)

The scheme of this proof follows the proof of equation 4, what gives:

$$(\gamma - n) - n^{\frac{-1}{\gamma-1}} (\gamma - 1) > 0$$

Taking alternatively the limit of γ towards 2 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

Proof of Proposition 3

As, in the limit case where $n_{D^*} = n_D$, the condition for cooperation to be sustainable becomes $\pi_i^C > \pi_i^N$, the proof of Proposition 3 boils down to the proof of Proposition 1. ■

Proof of Equation (15):

As the condition for one agent to cooperate is $\pi_i^C - \pi_i^N > 0$, the bigger the positive difference, the easier the cooperation. We can therefore compare equation (3)³⁸, with two different distributions of λ to see which one will be more favourable to cooperation. If λ_1 is compatible with cooperation such that equation (3) is verified then all the 'less compatible with cooperation' distributions of shares will produce a smaller result.

This being said, we can compare the different distributions of shares.

³⁸ $\pi_i^C > \pi_i^N \equiv \sum \lambda_i^{\frac{1}{\gamma-1}} (1 - \lambda_i^{\frac{1}{\gamma-1}}) - \lambda_i^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_i^{\frac{\gamma}{\gamma-1}}) > 0$

We compare equation (3) with $\underline{\lambda}$ and $\tilde{\lambda}$:

$$\begin{aligned} & (n-x)(\lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}}) + x \left[\left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{2}{\gamma-1}} \right] - \gamma^{-1} \lambda_1^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_1^{\frac{\gamma}{\gamma-1}}) \geq \\ & (n-x-1)(\lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}}) + (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} \\ & + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] - \gamma^{-1} \lambda_1^{\frac{2-\gamma}{\gamma-1}} (1 - \lambda_1^{\frac{\gamma}{\gamma-1}}) \end{aligned}$$

It simplifies to:

$$\begin{aligned} & \lambda_1^{\frac{1}{\gamma-1}} - \lambda_1^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1}{x} \right)^{\frac{2}{\gamma-1}} \right] \geq \\ & (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] \end{aligned}$$

As the second derivative of $\lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}})$ with respect to λ is equal to:

$$\frac{1}{(x-1)^2} \lambda^{\frac{3-2\gamma}{\gamma-1}} [2 - \gamma - 2(3-\gamma)\lambda^{\frac{1}{\gamma-1}}]$$

this function is concave if $2 - \gamma - 2(3-\gamma)\lambda^{\frac{1}{\gamma-1}} < 0$ (convex if > 0).

As $\lambda_1 < \lambda_2 < \dots < \lambda_n \Rightarrow \lambda_1 < \frac{1}{n}$ we get:

$$\frac{1 - (n-x)\lambda_1}{x} > \frac{1 - (n-x)\lambda_1 - \varepsilon}{x} > \lambda_1$$

This point and the concavity of the function $\lambda^{\frac{1}{\gamma-1}} (1 - \lambda^{\frac{1}{\gamma-1}})$ when $2 \leq \gamma \leq 4$ allow us to state that, if $x = 1$, $\underline{\lambda}_1 < \tilde{\lambda}_1$. If ε grows and becomes $= \frac{1-n\lambda_1}{x+1}$,

$$\begin{aligned} & (\lambda_1 + \varepsilon)^{\frac{1}{\gamma-1}} - (\lambda_1 + \varepsilon)^{\frac{2}{\gamma-1}} + x \left[\left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-x)\lambda_1 - \varepsilon}{x} \right)^{\frac{2}{\gamma-1}} \right] = \\ & (x+1) \left[\left(\frac{1-(n-(x+1))\lambda_1}{x+1} \right)^{\frac{1}{\gamma-1}} - \left(\frac{1-(n-(x+1))\lambda_1}{x+1} \right)^{\frac{2}{\gamma-1}} \right] \end{aligned}$$

The same reasoning shows therefore that $\underline{\lambda}_1 < \tilde{\lambda}_1 < \hat{\lambda}_1$. Then applying the same reasoning with $x = [2, \dots, n-1]$ gives our result when $2 \leq \gamma \leq 4$.

The reverse is true when $\gamma > 4$, and the function $\lambda^{\frac{1}{\gamma-1}} - \lambda^{\frac{2}{\gamma-1}}$ is convex, this being dependent on the shares (and therefore the number of players). ■

B Systems of equations

Extreme case:

$$\lambda_{\bar{n}} \left[\lambda_{\bar{n}}^{\frac{1}{\gamma-1}} + (n-1) \lambda_{min}^{\frac{1}{\gamma-1}} \right] - \gamma^{-1} \lambda_{\bar{n}}^{\frac{1}{\gamma-1}} \geq \lambda_n \left(\sum_{k \in n} \lambda_k^{\frac{2}{\gamma-1}} \right) - \gamma^{-1} \lambda_n^{\frac{\gamma+1}{\gamma-1}}$$

$$\lambda_{min} \left[\lambda_{\bar{n}}^{\frac{1}{\gamma-1}} (1 - \lambda_{\bar{n}}^{\frac{1}{\gamma-1}}) + (n-1) \lambda_{min}^{\frac{1}{\gamma-1}} (1 - \lambda_{min}^{\frac{1}{\gamma-1}}) \right] - \gamma^{-1} \lambda_{min}^{\frac{1}{\gamma-1}} (1 - \lambda_{min}^{\frac{\gamma}{\gamma-1}}) = 0$$

Note that $\lambda_{\bar{n}} = \lambda_n - \varepsilon$ and $\lambda_{min} = \lambda_i + \alpha_i \varepsilon$ (where $\alpha_i = \frac{\lambda_{min} - \lambda_i}{\sum_{k \in n_D} \lambda_{min} - \lambda_k}$) equalizes the payoffs from generalized cooperation and from generalized deviation.

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