# Setting weights in multidimensional indices of well-being and deprivation*

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Preliminary version

December 19, 2008

**Abstract**

Multidimensional indices of well-being and deprivation have become increasingly popular, both in the theoretical and in the policy-oriented literature. By now, there is a wide range of methods to construct multidimensional well-being indices, differing in the way they transform, aggregate and weight the relevant dimensions. We use a unifying framework that allows us to compare the different approaches and to analyze the specific role of the dimension weights in each of them. In interplay with the choices on the transformation and aggregation, the weights play a crucial role in determining the trade-offs between the dimensions. Setting weights is hence inherently a delicate matter, reflecting important value judgements about the exact notion of well-being. From this perspective, we critically survey six methods that are proposed in the literature to set the weights.

Keywords: Weights, Multidimensional Well-being index, Multidimensional deprivation index

*Work in progress. Comments and suggestions are most welcome. This draft has been prepared as background paper for the OPHI-workshop on Weighting Dimensions (Oxford, 26-27 May 2008). We thank Tony Atkinson, Sabina Alkire, James Foster as well as participants at the OPHI workshop for valuable comments and remarks. The usual disclaimer applies.

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# 1 Introduction

The notion that well-being is inherently multidimensioned has by now become well-established in the theoretical and policy-oriented literature. First, rooted in a tradition going back to Aristotle, philosophers such as Rawls (1971), Dworkin (2000), Sen (1985), Nussbaum (2000), or Townsend (1979) have advocated a multidimensional perspective on the good life and well-being, exposing the deficiencies of a sole focus on income as indicator of well-being. Second, in a large survey the World Bank collected the voices of more than 60,000 poor women and men from 60 countries, to understand poverty from the perspective of the poor themselves. One of the main conclusions of the survey is that for the poor, well-being and deprivation are multidimensional with both material and psychological dimensions (Narayan 2000).[1]

When it comes to operationalizing the multidimensional approaches, one quickly runs into the crucial problem of how to describe individuals' multidimensional well-being by one single index.[2] This is the so-called *indexing problem* (Rawls 1971, p. 80).[3] In the literature this problem has been taken up in two different branches from a slightly different and complementary perspective. First, from an operational perspective rooted in measurement theory, the approach has been to provide clear and relatively simple guidelines on how to construct composite or social indicators in many fields, including well-being. The most popular example of such a well-being composite indicator is probably the Human Development Index (HDI), used to compare the performance of countries in terms of their combined achievements in income –as command over resources–, health and education.[4] The second approach uses a social choice perspective to define measures

[1] A third justification could come from the rapidly emerging literature on the determinants of happiness and life-satisfaction in psychology and economics. There exists by now a broad consensus that people's happiness is affected by many aspects of life such as their health, employment, marital status, and material resources. For an overview of this booming literature on happiness, see Kahneman and Krueger (2006). In his paper, Schokkaert (2007) deals with the links between the happiness and the capability literature, proposed by Sen (1985) and Nussbaum (2000).

[2] One could reasonable argue that there is no need to aggregate dimensions of well-being into a single index, and thus full cardinalization is unwarranted. Two alternatives are generally proposed. The first involves considering each dimension at a time, the item-by-item approach. This might a valid approach in the case of, for instance, a social planner interested in how the policies in each separate area are performing and there is no need to arrive to *one* ranking (see, for instance, the proposal by Atkinson *et al.* (2004) on indicators for social inclusion in the European Union to assess the performance of Member States in the Joint Report on Social Inclusion). However, if she were interested instead in how her '*citizens*' are performing in each of the dimension of well-being, the item-by-item approach is rather inappropriate as it is oblivious to the possible dependence between these dimensions. In other words, she would equally value a society where all the wealth and all the health is concentrated in a group of people and one where the rich in income are poor in health and vice-versa. The second approach uses dominance criteria based on the multidimensional distribution function (see for instance, Atkinson and Bourguignon (1982) and Muller and Trannoy (2004) for a theoretical formulation and Duclos *et al.* (2006), Crawford (1999), and Gravel *et al.* (2005) for statistical formulation and applications) which requires minimal agreement of the characteristics of the underlying welfare function. In many situations, especially when one needs to rank more than two states, dominance conditions are not able to provide a complete ranking of distributions. Though incomplete rankings can be to some extent be informative, an analyst or policy maker is often more satisfied with a complete ranking.

[3] Define 'indexing problem'.

[4] Many international institutions are active in using and promoting the use of composite indicators - see Nardo *et al.* (2005, 2005) for a survey of some alternative composite indicators. To give one example, the reader is referred to the detailed information server on composite indicators hosted by the European

of multidimensional welfare, inequality or poverty and puts the emphasis on the measurability and comparability of the different dimensions, and on desirable properties of the obtained indices.[5] The results of this second strand are mainly theoretical, although they are increasingly empirically applied.[6] In this paper, we survey the different approaches to deal with the indexing problem, particularly focussing on the question how to weight the different dimensions of well-being. Thereby we combine the insights from both the theoretical and operational branches of the literature.

To order and compare the plethora of existing well-being indices from the literature, the paper starts by proposing a unifying framework in section 2. This framework reduces the differences between the well-being indices to differences in the chosen transformations of the original variables, the aggregation function and the weighting scheme for the dimensions. The focus of this paper is on the weights. In section 3 we analyze the meaning of the weights within the proposed unifying framework. Together with the choices about transformation and aggregation, the weights will be shown to play a crucial role in the imposed trade-offs between the dimensions. Inescapably, the weights reflect important value judgements about the (vague) notion of well-being. Researchers should therefore be as clear as possible about how the weights are set. As Anand and Sen argued:

> "Since any choice of weights should be open to questioning and debating in public discussions, it is crucial that the judgments that are implicit in such weighting be made as clear and comprehensible as possible and thus be open to public scrutiny" (Anand & Sen 1997, p. 6)

Section 4 critically surveys six proposed methods to set the weights in multidimensional measures of well-being: equal weighting, frequency based weighting, most favorable weighting, multivariate statistical weighting, regression based weighting and normative weighting. We argue that whether the weights are set reasonably should be judged upon the acceptability of the implicitly imposed trade-offs by them. Section 5 concludes.

## 2 A unifying framework

Let us assume that agreement has been reached on the domains of well-being that are relevant for the assessment of persons' standard of life and, moreover, that the achievement in all of these dimensions can be measured in a interpersonal comparable way. Let $x_j$ denote the achievement of an individual on dimension $j = 1, ..., q$ and let the well-being vector $\underline{x} = (x_1, ..., x_q) \in \mathbb{R}^q_+$ summarize these achievements across all dimensions.

Commission on http://composite-indicators.jrc.ec.europa.eu/.

[5]See Weymark (2006) or Maasoumi (1999) for an overview of the literature on multidimensional inequality and Bourguignon and Chakravarty (2003) for a survey of the multidimensional poverty or deprivation literature.

[6]See Justino (2005) for an overview.

The indexing problem can be described as the search for an appropriate *well-being index* $W$, that maps the well-being bundle $\underline{x}$ on the real line, so that it can be naturally ordered and used to assess the position of any two persons and the distance between them.[7]

the unit of Inescapably, the choice of a specific well-being index entails important value judgements about the meaning of well-being and about the respective contribution of its components. In the present paper we confine ourselves to the following wide class of well-being indices:

$$W(\underline{x}|\beta) = \left[\frac{w_1 I_1(x_1)^\beta + ... + w_q I_q(x_q)^\beta}{w_1 + ... + w_q}\right]^{1/\beta}. \qquad (1)$$

The individual well-being index $W(\underline{x}|\beta)$ is defined as a weighted mean of order $\beta$ of the transformed achievements $I_j(x_j)$.[8] The dimension-weights $w_1, \ldots w_q$ are all non-negative, and are often assumed to sum up to one so that the denominator of expression (1) drops out. The interpretation of these weights and how to set them is the topic of this paper. Before turning to the weights, though, we discuss briefly the other two components of the well-being index, that is, the transformation functions $I_j(.)$ and the parameter $\beta$.

Appropriate transformation functions for well-being indices should satisfy at least two criteria. First, since the achievements $x_j$ are often measured in different measurement units –such as income in pounds and health in years –, they need to be transformed or standardized to a common basis before they can be sensibly aggregated. Transformation functions typically make the achievements scale independent. Common examples include the z-score and the ratio to the mean standardisations. Second, if the original distribution is skewed the transformation functions should avoid that excessive relative importance is given to outliers or extreme values. One example of such transformation is the logarithmic transform. [9]

Expression (1) can also be used to construct an index of multidimensional poverty

[7]The unit of analysis considered in this section is the *individual*, though one could use the present framework for other relevant units, such as countries, regions or districts. If the unit of analysis is the individual but the assessment is done at a higher level, say the country of these individuals, then this section can be seen as the first step into the full aggregation from the space of individuals/attributes to a real number. The second step would be the aggregation across individuals. See Decancq and Lugo (2008) and Dutta *et al.* (2008) on the order of aggregation of multidimensional distributions of well-being and deprivation.

[8]Blackorby and Donaldson (1982) provide an axiomatic characterization of the weighted mean of order $\beta$. In the literature on multidimensional inequality, Maasoumi (1986) provides an information-theoretic justification of this class of well-being indices. Further, it belongs to the wider class of well-being indices proposed by Bourguignon (1999) while it is similar to Foster *et al* (2005)'s proposal for a distribution-sensitive measure of human development. Decancq and Lugo (2008) axiomatize it as part of a multidimensional Gini measure and Decancq et al. (2007) have used it to analyze the trend in multidimensional global inequality. Furthermore, in the related literature on the measurement of multidimensional poverty and deprivation, this class of indices has been suggested by Anand and Sen (1997) and is a special case of the class proposed by Bourguignon and Chakravarty (2003).

[9]In the context of building an index of deprivation for small areas, Noble *et al.* (2006, 2008) include two additional criteria for standardization: first, it should imply an appropriate degree of substitutability or cancelation between domains, second, it should facilitate the identification of the most deprived.

or deprivation. In this setting, the function $I_j(.)$ is defined as a negative function of $x_j$ and transforms the achievement in dimension $j$ into the shortfall in that dimension. These transformation functions typically request also a dimension-specific poverty-line $z_j$ to be defined. Let us call these transformation function $D_j(x_j|z_j)$. The most widely used transformation for poverty measure is the relative shortfall defined as

$$D_j(x_j|z_j) = \frac{z_j - x_j}{z_j}. \tag{2}$$

We can then defined a generalized class of deprivation indices $D(\underline{x}|\beta)$ as follows:

$$D(\underline{x}|\beta) = \left[\frac{w_1 D_1(x_1|z_1)^\beta + ... + w_q D_q(x_q|z_q)^\beta}{w_1 + ... + w_q}\right]^{1/\beta}, \tag{3}$$

Table 1 in the Appendix surveys some widely used transformation functions in the literature. In this paper, we do not prioritize one transformation method over another, but limit ourselves to presenting them while highlighting the crucial role they play on the interpretation of relative weights -as shown in the next section. We refer the interested reader to Jacobs *et al.* (2004) and Nardo *et al.* (2005) for an extensive survey of the alternative transformation methods and their properties. In general, we see that transformation functions used to construct a well-being index are increasing in $x_j$, whereas the transformation functions used to construct an index of deprivation are decreasing in the achievements.

The second component in expression (1) is the parameter $\beta$. One useful interpretation of $\beta$ is related to the elasticity of substitution between transformed achievements of well-being, $\sigma$ where $\sigma = \frac{1}{1-\beta}$. The smaller the $\beta$, the smaller the allowed substitutability between dimensions, that is, the more one has to give up of one attribute to get an extra unit (of transformed achievement) of a second attribute while keeping the level of well-being constant. Generally, for $\beta \leq 1$ the well-being index is a weakly concave function, which reflects a preference for well-being bundles that are more equally distributed. When expression (1) is used to describe multidimensional deprivation, a restriction of $\beta \geq 1$ seems more appropriate.

For $\beta = 1$, the weighted mean of order $\beta$ is reduced to the standard weighted arithmetic mean of the following form:

$$W(\underline{x}|1) = \frac{w_1 I_1(x_1) + \cdots + w_q I_q(x_q)}{w_1 + \cdots + w_q}. \tag{4}$$

Due to its simplicity and clarity of procedure, expression (4) is used frequently to construct composite indices.[10] However, the consequence of setting $\beta = 1$ might not always be desirable. The elasticity of substitution between (transformed) achievements is infinite and dimensions are perfect substitutes, meaning that there is a fixed rate at which

[10] See Jacobs *et al.* (2004) and Nardo *et al.* (2005).

transformed attributes can be exchanged which is constant for all possible levels of all attributes. One could argue that the amount of money needed to compensate for a less year of life should be quite different whether the person is in her youth or at the end of a normal life. In other words, it might be desirable to allow for the rate of substitution between dimensions to vary depending on their levels. An even stronger argument is made by the Human Development Report (2005).

> "Losses in human welfare linked to life expectancy, for example, cannot be compensated for by gains in other areas such as income or education" *Human Development Report (2005)*

At the same time, however, the leading index of the Human Development Report – the Human Development Index– makes use of a linear aggregation assuming *perfect* substitutability between the transformed achievements.[11]

Two other equally simple choices -though not as widely used as the previous one- are worth exploring which capture the feeling of the previous two arguments. The first option is to set $\beta = 0$ which makes the well-being index a weighted geometric mean,

$$W(\underline{x}|0) = I_1(x_1)^{w_1/(w_1+\cdots+w_q)} * \cdots * I_q(x_q)^{w_q/(w_1+\cdots+w_q)}. \tag{5}$$

The well-being index in (5) has unit elasticity of substitution between all pairs of dimensions. This means that a one percent decrease in one of the dimensions can be compensated by a one percent increase in another dimension. Note that this formulation allows for the rate of substitution between transformed attributes to change as the levels of achievements vary. As argued above, this characteristic of index (5) might be considered sensible and desirable both in theory and in practice. Figure 2 shows in a two dimension space (health and income) the lines where the level of well-being is maintained constant, for different options of $\beta$ (for clarity of exposition of the graph we assume $I_j(x_j) = x_j$).

We can express (5) by its ordinarily equivalent logarithmic version so that it becomes a weighted arithmetic mean of the logs of the transformed achievements.

[11] Note that the rate at which dimensions are traded off, measured in its original units -and not transformed, is constant (though not perfect) between the pair health-education, and non constant for health-income and education income pairs. Due to the log transformation employed for per capita GDP, the tradeoff between, say, per capita GDP and life expectancy depends also on the level of income the country achieves. In particular, the amount of money required to compensate for a less year of life expectancy is increasing in income; for a rich country such as Belgium an extra year of life expectancy is valued at nearly 7,000 US$ (in PPP terms) which for a relative poor country, such as Cote d'Ivoire this is merely 300 US$. Therefore, contrary to the claim above the Human Development Index does indeed allow for compensation between dimensions, even when this compensation might vary across levels. In the area of poverty and deprivation, the UNDP suggest two Human Poverty Indices (HPI) setting $\beta = 3$. "This gives an elasticity of substitution of 1/2 and places greater weight on those dimensions in which the deprivation is larger" (Anand and Sen 1997, p. 16). For very high levels of any one attribute the compensation necessary for decreases in that attribute tend to very a very large number -depending on the extent of deprivation- but still it allows for some substitutability.

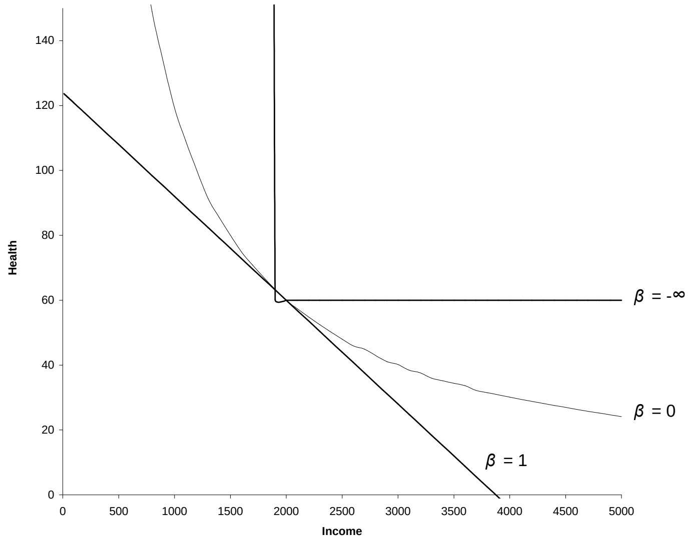


Figure 1: Iso- well-being curves for $\beta = 0, 1, -\infty$

$$W(\underline{x}|0) = \frac{w_1}{w_1 + \cdots + w_q} \ln \left(I_1(x_1)\right) + \cdots + \frac{w_q}{w_1 + \cdots + w_q} \ln \left(I_q(x_q)\right). \tag{6}$$

Note if in addition we set the the identity function as transformation, $I_j(x_j) = x_j$, this expression becomes equivalent to choosing $\beta = 0$ and $I_j(x_j) = \ln(x_j)$ for all $j$. While one could interpret it either way, we prefer to refer to it as $W(\underline{x}|0)$ emphasizing the curvature of the iso-well-being lines as shown in the previous figure.

A potentially crucial problem of setting $\beta = 0$ is that when a person has no achievement in one of the (transformed) dimensions the overall well-being index will be insensitive to the achievements in the other dimensions.

A second alternative is to let $\beta$ go to $-\infty(+\infty)$ where the elasticity of substitution becomes 0, and the well-being index becomes the minimum (maximum) of the transformed achievements across the dimensions,

$$W(\underline{x}| - \infty) = min[I_1(x_1), ..., I_q(x_q)]. \tag{7}$$

In this extreme case, there is no substitution between dimensions possible, which seems to reflect better the philosophy of the above UNDP Report quote.

We assumed a common degree of substitution for all pairs of dimension. For example, if income, health and education are taken as the components of well-being, all the above indices assume that the rate at which income and health are substituted is the same as that between income and education or health and education. We chose to do so for simplicity of exposition. Nonetheless, this might not be always a sensible assumption to make. One alternative is to use a nested approach where, first, several subsets of dimensions are aggregated using expression (1) with each subset having a different $\beta$ and, second, these subsets are combined using again the same expression.[12]

The choice of the substitutability parameter $\beta$ is intimately linked to the choice of the transformation function $I_j(.)$. In an interesting paper, Ebert and Welsch (2004) investigate the extent to which the ordering of the well-being bundles is invariant to the choice of the specific transformation function. Building on results from social choice theory, they conclude that the multiplicative aggregation ($\beta = 0$) is the only aggregation form that makes the ordering of the well-being bundles robust to the choice of a *dimension-specific* ratio scale transformation (rows 5-7 in table 1). Since well-being indices typically aggregate very different dimensions, a dimension-specific transformation is most often needed. Instead, the more general form defined in (1) is robust only to transformations involving the *same rescaling across all dimensions*, that is, attributes are being divided or multiplied by the same factor.[13] In other words, apart from some very restricted choices for $\beta$ and $I_j(.)$, the decision on the transformation function to use typically affects the ordering of the bundles. The lesson is not necessarily that we should restrict the transformation functions and $\beta$ to those case, but rather that this decision is to be handled with care and, preferably, guided by either a theoretical framework or empirical observations (or both) about the true meaning of well-being.

In sum, the framework proposed reduces the decisions to be made to three: the value of the parameter $\beta$, the transformation functions $I_1(.), \ldots, I_q(.)$ and the weights $w_1, \ldots, w_q$. Table 2 gives an overview of the common choices made in the literature with respect to these decisions.[14] These choices reflect alternative viewpoints on the meaning of the notion well-being and will potentially have a non-trivial impact on the resulting ordering of bundles.[15] In the next section we go deeper into the meaning of the weights. Before we will do so, we introduce an example to illustrate the effect that the choice of the parameters in expression (1) has on the ordering of persons in terms of their well-being.

We compare the well-being of two persons –Ann and Bob– in two dimensions –income and health– denoted $y$ and $h$ respectively, with the former being measured in dollars and the latter in expected years of life. Ann is healthier than Bob, her life

[12]Another alternative is to allow the substitutability parameter $\beta$ to be a function of the achievements, as in Bourguignon and Chakravarty (2003).

[13]See Ebert and Welsch (2004) and the reference therein for more details. Note that Ebert and Welsch's argument implies that indices such as the Human Development Index -which uses dimension-specific linear transformations- or the Index of Multiple Deprivation -which relies on an exponential transformation- are not meaningful.

[14]The table includes some indices used in practice, such as the HDI, and studies that provide empirical applications, while it leaves out studies that are solely theoretical.

[15]A striking example can be found in the work by Becker, *et al.* (1987). The authors studied the quality of life in 329 metropolitan areas of the U.S. by ordering them according to standard variables such as quality of climate, health, security, economical performance. The authors find that, depending on the weighting scheme chosen, there were 134 cities that could be ranked first, and 150 cities that could be rank last. Moreover, there were 59 cities that could be rated either first or last, using the same data, but by selecting alternative weighting schemes. Based on this example, Diener and Suh (1997) conclude that a procedure for resolving how to weight the dimensions is lacking.

expectancy is 90 years whereas his is only 50 years. But Bob is richer; he has an income of $2,000, whereas she earns only $1,000. We use expression (1) to evaluate who is better off of the two. Figures 1 to 3 depict the position of Ann and Bob in the income-health space, and the iso-well-being curves connecting all the points leading to the same level of the well-being index for alternative definitions of the index.

We define a benchmark case as follows: the dimension-weights are set equal ($w_y = w_h = \frac{1}{2}$), transformed attributes are perfectly substitutable ($\beta = 1$), and the achievements are rescaled by the median achievement ($I_y = \frac{x_y}{Me_y}$ for $y$ and $I_h = \frac{x_h}{Me_h}$ for $h$), where $Me_y = \$2,500$ and $Me_h = 80$ years. The benchmark iso-well-being curves are represented in the graphs by the dotted lines. In figure 2 Ann's bundle lies on a higher iso-well-being line than Bob's, which means that, according to this particular index, she is better off than Bob. Let us now look at three alternative parameter choices. First, we increase the relative weight assigned to income so that $w_y = \frac{3}{4}$ and $w_h = \frac{1}{4}$. The corresponding iso-well-being curves are represented in figure 1 by the solid lines. The iso-well-being curves are steeper than the benchmark case and Bob is now considered to be better off than Ann, hence reversing the ordering between them.

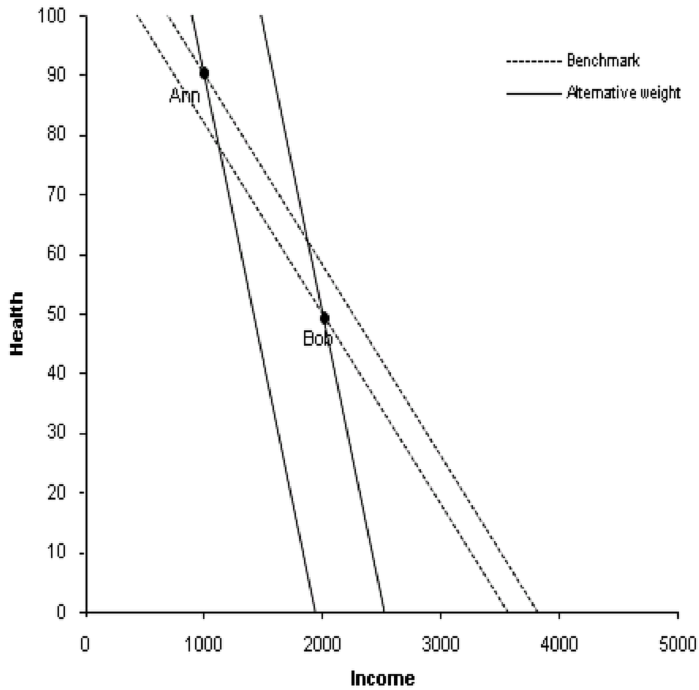


Figure 2: Iso- well-being curves. Alternative dimension-weights.

In the second case –figure 2– we use a different transformation function. Let us assume that the achievements of other individuals in the society have deteriorated, leading to a drop of the median achievement to an income of $1,000 (instead of $2,500) and a life expectancy of 60 years (instead of 80 years). In the new situation is represented by the solid line where Bob turns out to be better off than Ann.

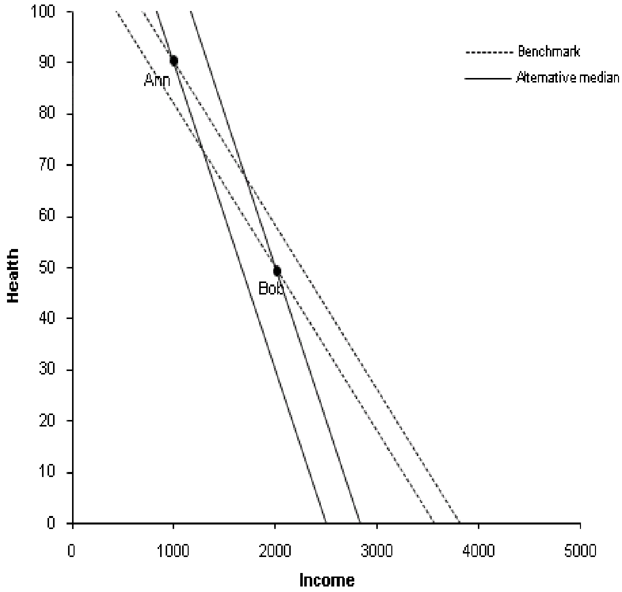


Figure 3: Iso- well-being curves. Alternative transformation (median).

Finally, in figure 2 we use a lower degree of substitutability between dimensions $\beta = 0.1$, in place of the previous from $\beta = 1$. The previously linear iso-well-being curves become now convex. This means that the rate at which one can substitute income for health is no longer constant across the well-being space. Compared to Bob, Ann -relatively rich in health- would be willing to give up many more years of life in the future for an extra dollar today. On this new assessment of their well-being, averages are preferred to extreme cases, which positions Bob at a higher well-being curve than Ann.

These stylized examples illustrate that the ordering of the well-being bundles can be very sensitive to the choice of the parameters. The lesson is that one should be careful when deciding about the parameters. In the following section we go deeper into the meaning of the parameters, in general, and of the weights, in particular.

# 3 What do weights mean?

A straightforward way to look at the meaning of the weights within the framework of the previous section, is to study some of the properties of the well-being index in terms how it reacts to changes in the parameters and the achievements in the different dimensions. We do this by analyzing the partial derivatives and the corresponding marginal rate of substitution, and by looking specifically at the role played therein by the weights. (See also Anand and Sen (1997) for a similar approach).

The derivative of the well-being index $W(.)$ with respect the weight of dimension

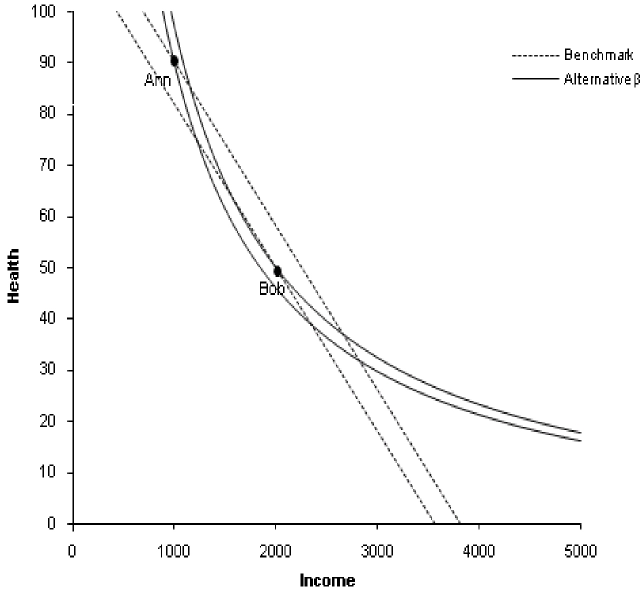


Figure 4: Iso- well-being curves. Alternative $\beta$.

$j$ will tell us how the well-being index reacts to small changes in $w_j$, while keeping all other parameters and achievements constant.

**Proposition 1.** *For all well-being indices defined by expression (1), it holds that:*

$$\frac{\partial W(\underline{x}|\beta)}{\partial w_j} = \frac{\left[I_j(x_j)^{\beta} - W(\underline{x}|\beta)^{\beta}\right]}{\beta\left[w_1 + ... + w_q\right] W(\underline{x}|\beta)^{\beta-1}}. \tag{8}$$

*Proof.* The proof uses some straightforward algebraic manipulations and is extended to the appendix, together with all other proofs of this section. □

Irrespective of the selected $\beta$, an increase in the weight of dimension $j$ leads to an increase the well-being index if the transformed achievement in dimension $j$ is larger than the total well-being, in other words if the individual is performing relatively well in dimension $j$. Indeed, it is intuitive that increasing the weight of a dimension on which the individual performs well leads to an increase in overall well-being, whereas increasing the weight of a dimension on which the individual performs relatively weak leads to a decrease in her well-being.

Expression (8) also provides interesting insights into the cases when the obtained well-being index $W(.)$ is *in*sensitive to the exact choice of the weight $w_j$. This happens when $I_j(x_j)^{\beta} = W(X)^{\beta}$ for all $j$, in other words when the transformed achievements are very alike across dimensions, or the correlation between them is sufficiently high.

Intuitively, one expects changes in dimensions with a higher weight to have more impact on total well-being, than dimensions with a lower weight. In a recent paper, Chowdhury and Squire (2006) write:

> "The ideal approach would presumably involve using as weights the impact of each component on the ultimate objective ...". Chowdhury and Squire (2006, p. 762)

We, therefore, investigate the first derivative of the well-being index with respect the achievement in dimension $j$ itself. This derivative captures how the well-being index reacts to small changes of the achievement in a given dimension, keeping all the other variables and parameters constant.

**Proposition 2.** *For all well-being indices defined by expression (1), it holds that:*

$$\frac{\partial W(\underline{x}|\beta)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q} I_j'(x_j) \left[\frac{W(\underline{x}|\beta)}{I_j(x_j)}\right]^{1-\beta}, \tag{9}$$

*where* $I_j' = \frac{\partial I_j(x_j)}{\partial x_j}$.

The impact of a small change of the achievements of dimension $j$ on total well-being depends on three terms. The first term is the relative dimension-specific weight. As one might expect, the larger the relative weight of a certain dimension, the larger the impact of a small change in the achievement of that dimension. Secondly, the impact depends on the derivative of the transformation curve. The larger this derivative, in other words, the steeper the transformation curve, the larger the effect of a small increase in the achievement on the transformed achievement and hence on total well-being. For multidimensional well-being indices, the derivative tends to be positive, whereas it is negative for multidimensional poverty indices. Finally, the effect depends on the ratio $\frac{W(X)}{I_j(x_j)}$ to the power $1 - \beta$. For values of $\beta \leq 1$, if the person performs worse in that dimension than in the overall well-being, an increase in achievement of such dimension will have a positive effect on overall well-being. Note the the parameter $\beta$ offers an instrument to increase the relative impact of a dimension on total well-being. The lower the $\beta$ the more sensitive the index is to weak performing dimensions. A policy maker seeking to maximize the well-being $W(X)$ will spend more effort on the relatively weak performing dimensions if $\beta \leq 1$, leading to a more equalized development across dimensions.

For the simple additive well-being indices ($\beta = 1$), the term between square brackets in expression (9) drops out, and the effect of a small change of one of the achievements only depends on its relative weight and the steepness of the transformation function, that is,

$$\frac{\partial W(\underline{x}|1)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q} I_j'(x_j). \tag{10}$$

If, moreover, the transformation function is the identity function ($I_j(x_j) = x_j$) the impact of a small change in achievement $j$ is only determined by the weight of dimension $j$, i.e. $\frac{w_j}{w_1+...+w_q}$. Therefore, for this specific choice of parameters, the relative weight of a dimension captures the impact of small change in the achievement of that dimension. The total well-being is indeed more sensitive to changes in a dimension with larger weight.

If instead $\beta = 0$ the effect of a marginal increase in dimension $j$ on the well-being index is defined as

$$\frac{\partial W(\underline{x}|1)}{\partial x_j} \quad = \quad \frac{w_j}{w_1 + ... + w_q} I'_j(x_j) \frac{W(\underline{x}|0)}{I_j(x_j)}, \tag{11}$$

where the effect depends not only on the weight but, crucially, on the level at which the change has taken place. This seems to be a reasonable feature. Using the same example as in the previous section, an extra year of life for a young person might be valued differently to an additional year given to the elder.

An alternative but related meaning of the weights is as substitution rates between two dimensions –$y$ and $h$– denoted $MRS_{yh}$.[16] Let us reconsider the previous example of Anne and Bob where dimension $h$ represents health and dimension $y$ is income. The marginal rate of substitution between these dimensions is the amount of health an individual would like to gain if she were to sacrifice one unit of income, while maintaining the same level of well-being. In graphical terms, the $MRS_{yh}$ reflects the slope of the iso-well-being curves and is formally defined as:

$$MRS_{yh} = -\frac{dx_h}{dx_y} = -\frac{\partial W(\underline{x}|\beta)}{\partial x_y} \Big/ \frac{\partial W(\underline{x}|\beta)}{\partial x_h}. \tag{12}$$

By substituting expression (9) into (12) we obtain the following expression:

**Proposition 3.** *For all well-being indices defined by expression (1), it holds that:*

$$MRS_{yh} = -\frac{w_y}{w_h} \frac{I'_y(x_y)}{I'_h(x_h)} \left[\frac{I_h(x_h)}{I_y(x_y)}\right]^{1-\beta}. \tag{13}$$

The marginal rate of substitution between dimension income and health also consists of three parts. We will relate each of these components to the cases illustrated in the previous section in figures 1 to 3. The first component is the ratio of the dimension-specific weights $w_y/w_h$. The larger the weight assigned to income the more the years of life (health) that the person needs to gain to compensate for the loss of one dollar of income. Going back to figure 2, the new income-weight $w_y$ is increased, leading to a larger ratio $w_y/w_h$, a larger $MRS_{yh}$ and a steeper iso-well-being curve. The second part of expression (13) is the ratio of the derivatives of the transformation functions of dimension $h$ and $y$. The steeper the transformation function of income, $x_y$ –or equally,

[16] See, for instance, Munda and Nardo, (2005).

the flatter the transformation function of health, $x_h$– the larger the amount of dimension $h$ necessary to compensate for the loss in $x_y$. In figure 2, the deteriorated medians of the society lead to a larger ratio $I_y'(x_y)/I_h'(x_h)$,and hence to a steeper iso-well-being curve.[17] Finally, the marginal rate of substitution depends on the ratio of the transformed achievements to the power $1-\beta$. For $\beta < 1$, the amount of dimension $h$ needed to compensate for the loss in dimension $y$ is greater, the smaller the original achievement in dimension $y$. This makes sense; achievements are more valuable as they become more scarce. In figure 2 using the alternative iso-well-being curves, the poorer the person the steeper the iso-well-being curve becomes. Anne should be given more health than Brian to compensate for a unit decrease in income.

In the linear case ($\beta = 1$), the trade-off is assumed constant at all levels of achievements.

$$MRS_{yh} = -\frac{w_y}{w_h}\frac{I_y'(x_y)}{I_h'(x_h)}. \tag{14}$$

If, in addition, the ratio of the derivatives of the transformation functions is unity, the marginal rate of substitution between two dimensions is uniquely defined by the ratio of their weights.

$$MRS_{yh} = \frac{w_y}{w_h}. \tag{15}$$

In the geometric case, instead, the trade-off between dimensions depends on the levels where the exchange is taking place. Specifically,

$$MRS_{yh} = -\frac{w_y}{w_h}\frac{I_y'(x_y)}{I_h'(x_h)}\frac{I_h(x_h)}{I_y(x_y)}. \tag{16}$$

When the transformation is the identity, the marginal rate of substitution between health and income results

$$MRS_{yh} = -\frac{w_y}{w_h}\frac{x_h}{x_y}. \tag{17}$$

In short, the analysis of some properties of the general class of the well-being index proposed in expression (1), gives us three lessons. First, that the higher the weight the better the person will perform if she fares better in that dimension than in the others. Second, in close interplay with the other parameters, the dimension-weights determines the contribution of that dimension to total well-being. Finally, dimension-weights form part of the trade-off between attributes and can be interpreted directly as *the* trade-offs only under certain assumptions –perfect substitutability between dimensions and no transformation of the original variables. In the next section we survey some procedures to set the weights from this perspective.

[17] To be precise, the ratio raise from $\frac{I_y'(x_y)}{I_h'(x_h)} = \frac{1/Me_y}{1/Me_h} = \frac{Me_h}{Me_y} = 80/2500 = 0.032$ to $60/1000 = 0.06$.

## 4 How can we set weights in a reasonable way?

In the previous section we concluded that whether the weights are set reasonably or not, can and should be evaluated based on the trade-offs they imply between the dimensions of well-being. In this section we survey from this perspective some of the most commonly used methods to set the weights in practice.[18]

### 4.1 Equal weights

The most commonly used approach to weighting in multidimensional indices of well-being has been equal weighting.[19] Despite its popularity, equal weighting is far from uncontroversial. Chowdhury and Squire refer to equal weighting as "obviously convenient but also universally considered to be wrong." (Chowdhury & Squire 2006, p. 762).

Equal weighting has often been defended from an *agnostic viewpoint*, by its simplicity or indeed from the recognition that all indicators are equally important.[20] As an example of the agnostic viewpoint, Mayer and Jencks defend equal weighting by remarking that: "ideally we would have liked to weight ten hardships according to their relative importance in the eyes of legislators and the general public, but we have no reliable basis for doing this" (Mayer & Jencks 1989, p. 96).

However, there is a fallacy in setting the weights equally motivated from an agnostic viewpoint. As has been shown in the previous section, there is no escape from the fact that the weights reflect an important aspect of the trade-offs between the dimensions. As any other weighting scheme, the equal weighting scheme implies in interplay with choices about the transformation and substitutability specific trade-offs between the dimensions, that can and should be made explicit, and might be considered reasonable or not. In a paper on the HDI, Ravallion (1997) looks at the implied marginal rates of substitution in the HDI and finds that: "The HDIs implicit monetary valuation of an extra year of life rises from a remarkably low level in poor countries to a very high level in rich ones. In terms of both absolute dollar values and the rate of GDP growth needed to make up for lower longevity, the construction of the HDI assumes that life is far less valuable in poor countries than in rich ones; indeed, it would be nearly impossible for a rich country to make up for even one year less of life on average through economic growth,

[18] An alternative method, not reviewed here, would be to use market or personalized prices as weights, so that the well-being index (with identity transformations and $\beta = 1$) coincides with the individual's expenditures. Srinivasan (1994) advocates such an approach. Smeeding *et al.* (1993) present a related approach, where non-monetary dimensions are given valued (for instance, education and health services are imputed global values based on the amount the government spends on them) and added to the current expenditure. On the other hand, as stated by Foster and Sen (1997), prices do not exist for many relevant dimensions of well-being and are in general inappropriate for well-being comparisons, a task for which they are not constructed.

[19] Examples are the Human Development Index, the Human Poverty Indices, the Commitment to Development Index (Roodman 2007), the English Index of Local Conditions (Department of Environment, 1994), and the Townsend Material Deprivation Score (Townsend, Phillimore & Beattie 1988), among others.

[20] Strictly speaking, equal weighting assigns zero weight to all dimensions of well-being ignored by the index.

but relatively easy for a poor country" He concludes: "The value judgements underlying these trade-offs built into the HDI are not made explicit, and they are questionable." (Ravallion 1997, p. 633).[21]

In sum, researchers that would like to avoid the hazardous question of how to set the weights, and therefore choose for equal weighting, should be aware that the equal weighting scheme is actually a weighting scheme as any other without specific normative attractiveness, and just as any other weighting scheme it implies trade-offs that might be reasonable or not.[22]

## 4.2 Data-driven weighting schemes

Many methods to obtain dimension weights rely in some way or another on the data at hand. We compare four approaches and will criticize them on similar grounds.

### 4.2.1 Frequency-based weights

One method to determine the weights is to set them in terms of the proportion of the population suffering deprivation in that dimension. Two different approaches can be found in the literature, taking quite opposite perspectives. First, in the context of multidimensional deprivation measurement, Desai and Shah (1988) and Cerioli and Zani (1990) argue that the smaller the proportion of individuals with a certain deprivation, the higher should be the weight, on the grounds that a hardship shared by few has more impact than one shared by many.[23] On the other hand, in their work on well-being indices, Osberg and Sharpe (2002) make use of frequency-based weights to aggregate subcomponents of the economic security component whereby smaller weight is given to dimensions with a smaller proportion of the population at risk.[24]

A related way of setting the weights is based on the quality of the data. Jacobs et al. (2004) suggest to give less weight to those variables where data problems exist or with large amounts of missing values. The advantage is that the reliability of the well-being index can be improved by giving more weight to good quality data.

Apart from the apparent disagreement how the weights should depend on the relative proportions, the fundamental question seems to be *why* the weights and the implied trade-offs should depend on the relative proportion achieved by the population or on the data quality.

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[21] Decanq *et al.* (2007) make a similar point, based on the most recent calculation method of the Human Development Index.

[22] One could use the transformation functions to obtain reasonable trade-offs while keeping the weights equal across dimensions. But this seems rather to turn the problem on its head and might even defeat the purpose of simplifying the index if the resulting transformation functions were less than simple.

[23] In a recent paper, Brandolini (2007) points out that when applying Desai and Shah's weighting formula to Italian data, he comes to a rather questionable and unbalanced weighting scheme.

[24] The index of Economic Well-being proposed by Osberg and Sharpe aggregate four components of wellbeing (consumption, accumulation, income distribution, and economic security) using equal weights but frequency weights to aggregate indicators within the last component.

### 4.2.2 Most favorable weights

When applying the same weighting scheme to all individuals, some of them might feel that the evaluation of their well-being is submitted to someone elses perspective on what well-being exactly is.[25] Therefore, a researcher might want to give all individuals the "benefit of the doubt" and select for each individual the most favorable weighting scheme. This method has originally be proposed for evaluating macro-economic performance (Melyn & Moesen 1991) and has recently been used in the construction of composite indicators (Despotis 2005, Mahlberg & Obersteiner 2001). The weights are individual-specific and endogenously determined such that they maximize the obtained well-being of the individual.[26] The highest relative weights are given to those dimensions on which the individual performs best. To avoid that all weight is given to one dimension (the best dimension of the individual), extra constraints can be imposed upon the weights assuring that minimal weight is given to each dimension of well-being.

Drawbacks of this approach can be the following: First, since every individual has her own weighting scheme, the comparison of well-being levels across individuals is not straightforward.[27] Second, the obtained results depend highly on the exact formulation of the technical constraints chosen by the analyst, making it a less transparent procedure. Finally, and most importantly, there is no guarantee that the most favorable weights lead to reasonable trade-offs between the dimensions. There seems to be no *a priori* reason, why a certain dimension on which the individual performs relatively well should have a larger impact on total well-being, *because* the individual performs well on that dimension.

### 4.2.3 Statistical weights

There are two sets of techniques that are employed to choose weights for multidimensional indices: descriptive and explanatory models.[28]

The first approach relies on multivariate statistical methods to summarize the data. The most commonly used techniques are based on principal components (Klasen 2000, Noorbaksh 1998) and cluster analysis (Hirschberg, Maasoumi & Slottje 1991). The use of these statistical techniques is motivated by a concern for the so-called problem of

[25] In recent social choice literature, the question of *whose* preferences or ideas about trade-offs should matter has been taken up under the label of the "indexing dilemma" –see Fleurbaey (2007). In his elegant paper, Fleurbaey investigates the apparent impossibility of finding a weighting scheme that is individual specific and, at the same time, is not susceptible to the critiques formulated against welfarism, as expressed by Sen (1985). He argues in favor of a way out the impossibility based on an approach that takes information on the individual iso-well-being curves into account.

[26] In case $\beta = 1$, this problem reduces to linear programming problem, see Cherchye *et al.* (2006) for technical details. The authors provide an overview composite indicators that set the weights based on this most favorable weighting scheme, which is an application of so-called Data Envelopment Analysis. Mahlberg and Obersteiner (2001) proposes a specific application to the Human Development Index, and Ramos and Silber (2005) compare the approach to alternative ones. See also Despotis (2005).

[27] Despotis (2005) propose a way of using the individual most favourable weights to construct a common weighting structure. This is done minimizing the distance between the individual (country)-specific weight and the global weights.

[28] For a detailed overview of the statistical properties of some methods to set the weights based on multivariate statistics, we refer the reader to Kirshnakumar and Nadar (2008).

*double counting.* In many empirical applications the dimensions of well-being are found to be strongly correlated.[29] Loosely speaking, most multivariate statistical techniques adjust for the correlation between indicators by either choosing the dimensions that are not correlated or by adjusting the weights so that correlated dimensions get less weight (Nardo, Saisana, Saltelli & Taranto 2005). For instance, in principal component analysis, a given set of dimensions is transformed into an equal number of mutually uncorrelated linear combinations of dimensions. One can compute the proportion of the variance explained by each linear combination. If a small group of those linear combinations can explain a large proportion of the variance, then the information contained by the initial dimensions is largely contained in the small group of combinations that are, by definition, uncorrelated and which solves the double counting problem. The two most commonly used methods to obtain weights from the linear combinations, is to use either the principal component that explains the largest proportion of the variance, or to use a weighted average of all the linear combinations.

The second approach, sometimes known as *latent variable models* is an explanatory approach that assumes that some observed variables (dimensions) are dependent on a certain number of unobserved latent variables (Krishnakumar & Nadar 2008). Factor analysis is possibly the simplest case of latent variable model, imposing that the observed dimensions are in fact different manifestations of the latent component, called factor. In the context of well-being and deprivation indices, factor analysis have been widely employed (Maasoumi & Nickelsburg 1988, Schokkaert & Van Ootegem 1990, Nolan & Whelan 1996, Noble, McLennan, Wilkinson, Whitworth, Barnes & Dibben 2008). More advanced latent models include other exogenous variables that also might influence the latent variable but are not part of the selected set of dimensions used to construct the index. In this line, Multiple Indicator and Multiple causes model (MIMIC) and structural equation model (SEM) have been proposed to construct multidimensional indices, particularly among those supporting the capability approach (Di Tommaso 2006, Kuklys 2005, Krishnakumar 2007, Krishnakumar & Ballon 2007).

There are, however, some drawbacks to these multivariate statistical approaches. First, the obtained linear combinations of dimensions might be hard to interpret as a facet of human well-being (Srinivasan 1994). Additionally, the derivation of weights through principal component or latent variables models is, by no means, straightforward and hence it lacks transparency, which makes these technique less attractive as a method to informing local and international policy makers(de Kruijk & Rutten 2007). Most crucially, statistical approaches can lead to normatively inappropriate results. For instance,

[29] For instance, Srinivasan (1994) reports a correlation coefficient of about 0.8 between the dimensions of the Human Development Index. Whether double counting is really a problem, is open for discussion. One could argue that the correlation between the dimensions in a society reflects an important aspect of the real situation and as such it should be included, not eliminated from the analysis. The pluralistic egalitarian notion of Walzer (1981), for instance, considers that the correlation between the dimensions is one of the most essential characteristics of the society. From that perspective, correcting for correlation between the dimension might be completely inappropriate.

in the construction of the Environmental Sustainability Index, the principal component method was found to assign negative weights to some sub-indicators (World Economic Forum, 2002). We obtain the same result when we apply this method to the example of Ann and Bob.[30] Moreover, there is a priori no reason to believe that statistical weights are in accord with people's perceptions about priorities and relative importance of each dimension (de Kruijk & Rutten 2007). Along that line, Brandolini (2007) warns the reader that "we should be cautious in entrusting a mathematical algorithm with a fundamentally normative task". Multivariate statistical techniques, especially principal component analysis, are developed to summarize the data in a *statistically* reasonable and parsimonious way. As such, they can be useful to aggregate indicators *within* dimensions. But this is quite a different task to setting weights that are *normatively* reasonable.

### 4.2.4 Regression based weights

The final approach to set the weights based on data is to estimate the coefficients $\alpha_j = \alpha_1, \ldots, \alpha_q$ of the following equation:

$$Y_i = \alpha_1 I_1(x_{1i}) + ... + \alpha_q I_q(x_{qi}) + \varepsilon_i, \tag{18}$$

where $Y_i$ is some output variable capturing the well-being of individual $i$. This expression shows great similarity with the linear well-being index as defined in expression (2), with the role played by the coefficients $\alpha_j$ corresponding to the weights $w_j$. The main problem to operationalize this approach to find a reasonable $Y_i$ for every individual, approximating her well-being.

In a recent paper, Schokkaert (2007) proposes to rely on the emerging measures of life satisfaction as proxy for individual well-being. He writes "On the one hand, the robust statistical relationship between functionings and life satisfaction may provide useful information on the relative weights to be given to the various dimensions in the calculation of individual living standards. On the other hand, from a non-welfarist point of view we do not want idiosyncratic individual factors to wipe out the effects of conditions of material deprivation, linked, for example, to unemployment or job satisfaction" (Schokkaert 2007, p.423). Schokkaert proposes an approach in which individual life satisfaction is used as lefthand-side variable in expression (18), and where all idiosyncratic individual factors are set at their mean value for the population to take account of the second point. His approach can be made more flexible by allowing for non-linearities in expression (18) or for the coefficients $\alpha_j$ to vary across different groups in the population.

In general the regression based weights have the drawback that if the well-being could be measured in an appropriate way by the single variable $Y_i$, there would be not need to construct a well-being index in the first place. A less critical concern is that

[30] The weights $w_h$, $w_y$ obtained by principal component of the transformed data (z-scores) are respectively -0.7071 and 0.7071.

the the coefficients of $\alpha_j$ might suffer from the problem of multicollinearity were the dimensions of well-being be strongly correlated.

Data-driven approaches offer an interesting way to obtain weights for multidimensional well-being indices. There is an expanding literature proposing their use and perfecting the methods so that they are more than just data summarizing techniques. Two points of caution are in order. First, these methods generally assume a linear form $\beta = 1$ which implies perfect substitutability between all pair of dimensions at all levels. Some of the techniques could overcome this problem straightforwardly. Second, an inconvenient property of the weights obtained by statistical techniques is that are sensitive to adding new observations to the data-set (Nardo, Saisana, Saltelli, Tarantola, Hoffman & Giovannini 2005). In other words, given that weights are data-specific they can change from one point in time to the next, and from country to country, which makes any meaningful comparison of situations problematic.

### 4.3 Normative weights

From the previous section on the meaning of the weights we recall that the weights are crucial in determining the trade-offs between the dimensions of well-being. A third approach is to obtain more normatively inspired weights. Unfortunately, there are very few guidelines in the ethical or philosophical literature on how the obtain reasonable trade-offs between dimensions of well-being. Fleurbaey (2008) states: “One can of course invoke the ethical preferences of the observer and ask her, for instance, how she trades the suicide rate off against the literacy rate, but there is little philosophical or economic theory that gives us clues about how to form such preferences.” (Fleurbaey 2008, p. 21).

One approach would be to ask all individuals in the society how they personally would trade-off the different dimensions, and then aggregate these opinions somehow. In practice, however, asking all individuals in a society might not be feasible, therefore one often relies on the preferences of a limited group of people that are thought to represent, to some extent, the rest of the society.[31] Generally, four sets of groups are considered: a random sample of the population, ‘experts’ from the academic and international organization communities, and policy makers -usually deciding where and how to spend resources- and groups of representatives of different sections of society.

In the literature, there exist some methods to elicit the preferred trade-offs between the dimensions of the (representative group of) individuals. A first method is to survey directly how the individuals would trade off different dimensions of well-being. Similar approaches have been used in health economics to obtain an estimation how much health gain one is prepared to sacrifice for a reduction in health inequality (see for instance Shah

[31] However, public opinion polls have been used in problems of eliciting the public concerns about environmental issues. In that way the concern the public opinion attaches to the different environmental subindicators is determined. Parker (1991 p.95-98) advocates such an approach: “public opinion polls have been extensively employed for many years for many purposes, including the setting of weights”.

et al. 2001 and Jacobs et al. 2004). An example within the well-being index literature is give by de Kruijk and Rutten (2007). The authors uses the Maldivean household survey –Poverty and Vulnerability Assessment 1997/98 and 2005– where (randomly sampled) respondents are asked to rank living standard dimensions according to their relative importance in determining the overall standard of living and deprivation. The dimension weight for each individual is then computed as a function of the number of dimensions considered and the specific ranking of that dimension.[32] Finally, the paper uses the average weight to compute the individual specific human vulnerability index, so that every respondent is assigned the mean priority weight of the country. In an interesting paper, Chowdury and Squire (2006) use electronic surveys to elicit weighting schemes to assess whether the equal weighting scheme of the Human Development Index had support from the 'expert community', understood as development researchers throughout the world placed in academic institutions. Each person was asked to weight each component of the HDI from 0 to 10 in order of importance, and the average of these weighting schemes was considered. Interestingly enough, they find that the average weighting scheme does not statistically differ from the present equal weighting scheme. Survey methods are in practice, a voting mechanism. When the question is simple enough and the options are limited, standard voting procedures can also be applied. Survey methods have been used to collect preferences of representative individuals of the society and so called-experts, and less frequently, policy-makers.[33]

A second and related method is to use budget allocation. The members of the representative group are asked to distribute a budget of points to a number of dimensions, paying more for those dimensions whose importance they want to stress. Moldan and Billharz (1991) report a case study in which 400 German experts were asked to allocate a budget to a set of environmental indicators related to air pollution, leading to very consistent results, where experts came form very different social backgrounds. Budget allocation mechanism belongs to a more general class of participatory planning.

A third method is the analytic hierarchy process. This has been proposed by Saaty (1987) originating from multi-attribute decision making. In this procedure, all members of the representative group are asked to compare pairwise the dimensions by asking the

[32]Specifically, the weight for dimension $j$ is determined by

$$w_j = \frac{1 + n_d - r_j}{\sum_{j=1}^{n_d} 1 + n_d - r_j},$$

where $n_d$ is the number of dimensions and $r_j$ is the ranking of dimension $j$ with value 1 if it is the most important, 2 if it is second most important dimension, and so on.

[33]Another example of elicitation of expert preferences is given by Carlucci and Pissani (1995). The authors make use of the Delphi methodology, where participants ('experts') are asked to give values to complete predefined lotteries. In this way, they are able to elicit not only dimension weights but also some measures of interdependence between the dimensions, in the eyes of the participants. This method seems particularly applicable when the participants are literate enough to understand the questions possed and give a meaningful cardinal answer.

question: “Which of the two is the more important? - and by how much?”. The strength of the preference is expressed on a semantic scale of 1-9. These comparisons result in a comparison matrix from which the relative weights can be calculated using an eigenvector technique (see Nardo et al (2005) and the references therein for a detailed treatment).

The main source of concern with participatory methods relates to the selection of participants, a concern that holds true for any of the above mentioned sets of groups (experts, representative individuals, and policy-makers), and any technique employed. Selection of participants can be biased -some groups being under-represented- or simply uninformed, and hence the resulting weighting scheme will be skewed. A second problem is that, even when the selection is bias-free, the participatory technique may lead to unrepresentative preferences were the process be subject of pressures from power groups and vested interests. A final critique is that participatory approaches in general can lead to some sort of paternalism. Although to a certain degree, the extent of paternalism can be handled with appropriate selection of participants and elicitation mechanism, normative participatory approaches rely on imposing some people’s ideas of how important each dimension is to other individuals. Hence, the critique of paternalism prevails.

Although normative approaches have the disadvantages mentioned above, they are in nature closer related to the meaning of the weights as trade-offs, and as such they can be expected to lead to more reasonable results.

After having surveyed these six methods, a final remark is in place. Researchers might find it difficult to pinpoint a unique weighting scheme, whereas they might find it easier to obtain ”ranges” in which reasonable values of weights can be found. Foster and Sen (1997, p. 206) state that while ”the possibility of arriving at a unique set of weights is rather unlikely, that uniqueness is not really necessary to make agreed judgements in many situations”. Such an approach of working with ranges of weights, rather than exact values has the advantage of allowing for some degree of agnosticism. However, that agnosticism comes at a price: an approach based on ranges of weights, is likely to lead to a partial ordering of the well-being bundles. How incomplete the ordering becomes, or how many bundles will become incomparable, depends on the allowed width of the ranges and the correlation between the achievements of the individuals across the dimensions. The stronger the correlation between the dimensions, the less important the exact specification of the weights. A sensitivity analysis for alternative weighting schemes can be very helpful in determining how robust the well-being index and the implied ordering of the bundles is for alternative weighting schemes.[34] Although it is clear that a sensitivity analysis can never answer the question on how to set weights in

[34]For example, in the context of the measurement of multidimensional global welfare, Decancq and Ooghe (2008) propose a normative framework in which they carry out a sensitivity analysis for all possible weighting schemes. They find that the obtained trend in increasing welfare is robust for almost all weighting schemes, except for the one giving almost all weight to life-expectancy. Foster *et al* (2008) propose a way to easily test the robustness of weights. They apply a rank-robustness technique to assess Human Development Index weights.

a reasonable way, it might give an idea how important the answer is for the obtained results and how much room there is for agnosticism, concerning the weights.

# 5 Conclusion

In this paper, we surveyed different approaches for setting weights in multidimensional indices. We provided a general framework where most methods fit in. This framework allowed us to understand the meaning of weights as crucial factors determining the trade-off between dimensions. Dimension-weights are, however, not the only component determining this trade-off. The form of the transformation of the original variables into commensurable units and the parameter of substitution between dimensions also play an important role. However, these components are more often than not ignored in the literature.

We reviewed six approaches used to set dimension weights, highlighting their advantages and drawbacks. Ultimately, the definite test for any weighting scheme should be in terms of its reasonability in terms of implied trade-offs between the dimensions. As long as there is no widely accepted theoretical framework how to set these trade-offs, the researcher has no choice than to rely on her common sense and to be very cautious in interpreting the obtained orderings of the well-being bundles. In all cases, robustness tests to determine whether results are driven solely by the specific value of weights selected, should be called upon.

In terms of the specific approaches surveyed we conclude the following. First, equal weights are in no sense neutral and can be questioned on ethical grounds and do not make explicit the underlying assumptions. Second, data-driven methods are useful when aggregating indicators within a given dimension as by definition, they help reducing the dimensionality of the data-set at hand. Finally, normative-based methods seem more appropriate to aggregate across dimensions. Within those, participatory approaches are a promising route but still too many things left subject to the democracy and efficiency of the process -more work on this is needed.

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## Appendix

Starting from expression (1):

$$W(X|\beta) = \left[\frac{w_1\,[I_1(x_1)]^\beta + ... + w_q\,[I_q(x_q)]^\beta}{w_1 + ... + w_q}\right]^{\frac{1}{\beta}} \qquad \text{for } \beta \neq 0 \qquad (19)$$

$$W(X|0) = I_1(x_1)^{w_1/(w_1+\cdots+w_q)} * \cdots * I_q(x_q)^{w_q/(w_1+\cdots+w_q)} \qquad \text{for } \beta = 0 \qquad (20)$$

For proposition 1:

$$\begin{aligned}
\frac{\partial W(X|\beta)}{\partial w_j} &= \frac{1}{\beta}\left[w_1\,[I_1(x_1)]^\beta + ... + w_q\,[I_q(x_q)]^\beta\right]^{\frac{1}{\beta}-1}[I_j(x_j)]^\beta\,[w_1 + ... + w_q]^{-\frac{1}{\beta}} \\
&\quad -\frac{1}{\beta}\,[w_1 + ... + w_q]^{-\frac{1}{\beta}-1}\left[w_1\,[I_1(x_1)]^\beta + ... + w_q\,[I_q(x_q)]^\beta\right]^{\frac{1}{\beta}} \\
\frac{\partial W(X|\beta)}{\partial w_j} &= \frac{1}{\beta}\left[w_1\,[I_1(x_1)]^\beta + ... + w_q\,[I_q(x_q)]^\beta\right]^{\frac{1-\beta}{\beta}}[w_1 + ... + w_q]^{-\frac{1}{\beta}} * \\
&\quad \left[[I_j(x_j)]^\beta - W(X)^\beta\right]
\end{aligned}$$

$$\frac{\partial W(X|\beta)}{\partial w_j} = \frac{\left[I_j(x_j)^\beta - W(X)^\beta\right]}{\beta\,[w_1 + ... + w_q]\,W(X)^{\beta-1}} \qquad (21)$$

When $\beta = 0$

$$\frac{\partial W(X|0)}{\partial w_j} = I_1(x_1)^{w_1/(w_1+\cdots+w_q)} * ... * I_q(x_q)^{w_1/(w_1+\cdots+w_q)}\left(\frac{1}{w_1 + ... + w_q} - \frac{w_j}{(w_1 + ... + w_q)^2}\right)$$

$$\frac{\partial W(X|0)}{\partial w_j} = \frac{W(X/0)}{w_1 + ... + w_q}\left(1 - \frac{w_j}{w_1 + ... + w_q}\right) \qquad (22)$$

If also $\sum_j^q w)j = 1$ then

$$\frac{\partial W(X|0)}{\partial w_j} = W(X/0)\,(1 - w_j)$$

For proposition 2:

$$\frac{\partial W(X|\beta)}{\partial x_j} = \left[\frac{w_1\,[I_1(x_1)]^\beta + ... + w_q\,[I_q(x_q)]^\beta}{w_1 + ... + w_q}\right]^{\frac{1-\beta}{\beta}}\frac{w_j}{w_1 + ... + w_q}\,[I_j(x_j)]^{\beta-1}\,I'_j(x_j)$$

$$\frac{\partial W(X|\beta)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q}I'_j(x_j)\left[\frac{I_j(x_j)}{W(X|\beta)}\right]^{\beta-1} \qquad (23)$$

For $\beta = 0$

$$\frac{\partial W(X|0)}{\partial x_j} = \frac{w_j}{w_1 + ... + w_q} I'_j(x_j) \left[ \frac{W(X|\beta)}{I_j(x_j)} \right]$$

For proposition 3:

$$MRS_{yh} = \frac{\frac{\partial W(X|\beta)}{\partial x_y}}{\frac{\partial W(X|\beta)}{\partial x_h}} = \frac{\left[ \frac{w_1 [I_1(x_1)]^\beta + ... + w_q [I_q(x_q)]^\beta}{w_1 + ... + w_q} \right]^{\frac{1-\beta}{\beta}} \frac{w_y}{w_1 + ... + w_q} [I_y(x_y)]^{\beta - 1} I'_y(x_y)}{\left[ \frac{w_1 [I_1(x_1)]^\beta + ... + w_q [I_q(x_q)]^\beta}{w_1 + ... + w_q} \right]^{\frac{1-\beta}{\beta}} \frac{w_h}{w_1 + ... + w_q} [I_h(x_h)]^{\beta - 1} I'_h(x_h)}$$

$$MRS_{yh} = \frac{\frac{\partial W(X|\beta)}{\partial x_y}}{\frac{\partial W(X|\beta)}{\partial x_h}} = \frac{w_y}{w_h} \frac{I'_y(x_y)}{I'_h(x_h)} \left[ \frac{I_y(x_y)}{I_h(x_h)} \right]^{\beta - 1} \tag{24}$$

For $\beta = 0$

$$MRS_{yh} = \frac{\frac{\partial W(X|\beta)}{\partial x_y}}{\frac{\partial W(X|\beta)}{\partial x_h}} = \frac{w_y}{w_h} \frac{I'_y(x_y)}{I'_h(x_h)} \frac{I_h(x_h)}{I_y(x_y)}$$

## Tables

Table 1: Transformation functions

| Transformation method | Definition | Characteristics | Examples |
|---|---|---|---|
| Identity | $I(.) = x_j$ | | |
| *Linear scale transformations* | | | |
| - z-scores | $I_j = \frac{x_j - \bar{x}_j}{sd(\bar{x}_j)}$ | Imposes a standard normal distribution.<br>It produces negative values.<br>Extreme values are given a large weight. | Environment Sustainability Index |
| - by range | $I_j = \frac{x_j - \min(x_j)}{\max(x_j) - \min(x_j)}$ | Robust to outliers | Human Development Index<br>Index of Economic Well-Being |
| - shortfall | $I_j = \frac{z_j - x_j}{z_j}$ | where $z_j$ is the dimension-specific threshold | Human Poverty Index |
| *Ratio scale transformations* | | | |
| - over time | $I_j = \frac{x_j^t - x_j^{t-1}}{x_j^t}$ | Only feasible with longitudinal data | Economic Sentiment Indicator |
| - by mean<br>(or relative distance from) | $I_j = \frac{x_j}{\bar{x}_j}$ | Not robust to outliers | |
| - by best performer | $I_j = \frac{x_j}{\max(x_j)}$ | Not robust to outliers | Environment Policy Index |
| *Increasing transformations* | | | |
| Logarithmic | $I_j = \ln(x_j)$ | Coefficients are interpreted as elasticities.<br>Higher weight to changes at the bottom | |
| Exponential and rank<br>(or relative distance from) | $R = 23 * \ln\{1R_j * (1e^{100/23})\}$ | where $R_j$ is the rank in each $x_j$ | Multiple Deprivation Index |
| *Ordinal transformations* | | | |
| Ranking | $I_j = rank(x_j)$ | Uses ordinal information only,<br>hence discarding all level information | Medicare Health Care Index |
| *Other transformations* | | | |
| Number of indicators<br>above/below mean | $I_j = \frac{x_j}{\bar{x}_j} - (1+p)$ | $p$ arbitrary threshold above/below the mean | |
| Scoring | $I_j = score(x_j)$ | Discards information when original<br>variable is continuous | |

Table 2: Selection of composite well-being indices

| Composite Index | Dimensions | Transformation $I_j(x_j)$ | $\beta$ | Dimensions weights $w_j$ |
|---|---|---|---|---|
| Human Development Index | Income, education, health | linear scale - range<br>increasing - log(pcGDP) | 1 | equal weight |
| Human Poverty Index - 1 | rates of survival, education, economic perf | identity | 3 | equal weight |
| Human Poverty Index - 2 | (rates for) health, knowledge,<br>economic, social exclusion | identity | 3 | equal weight |
| Gender-related development<br>index | $GDI = \left[\sum_{j=1}^{4} w_j E(x_j)\right]$<br>$E(x_j) = \left[fI(x_{jf})^{\beta} + fI(x_{jm})^{\beta}\right]^{1/\beta}$<br>$x_{jf}$: achievement of female ($m$: male)<br>$f$: female population share ($m$: male) | linear scale - range | -1 | equal weight<br>gender frequency weight |
| Index of Multiple Deprivations | 7 domains<br>38 indicators | increasing - exponential | 1 | participatory for domains<br>factor analysis and equal<br>weight for indicators |
| Index of Economic Well-Being<br>Osberg and Sharp (2002) | 4 domains: consumption, accumulation<br>income distribution, economic security<br>16 indicators | linear scale - range | 1 | equal weights for domains<br>frequency weights w/in security<br>arbitrary/equal weights for others |

Table 3: Selection of underlying well-being indices in multidimensional inequality and poverty measures with empirical application

| Composite Index | Dimensions | Transformation $I_j(x_j)$ | $\beta$ | Dimensions weights $w_j$ |
|---|---|---|---|---|
| Brandolini (2007) | | | [1,2,5, 10,100,500] | range [0,1] |
| De Kruijk (2007) | 12 dimensions indicators | linear scale - shortfall | 1 | normative priority weights |
| Decancq, Decoster, and Schokkaert (2007) | income, health, education | linear scale - range<br>ratio scale - by mean<br>linear scale - z-score<br>increasing - logarithm | [-5,1] | equal and principal component weights |
| Desai and Shah (1988) | family composition, education, income, wealth, region, health, place of birth | linear scale - shortfall | 1 | frequency weights |
| Despotis (2005) | income, education, health | linear scale - range | 1 | most favourable weights |
| Deutsch and Silber (2005) | | | 1 | equal and frequency weights |
| Justino (2005) | income/expenditure, education, health | linear scale - range | [-1/3, 1] | equal and range weights |
| Kirshnakumar (2007) | education, health, social participation (9 indicators) | identity | 1 | latent variable model |
| Klasen (2000) | 14 indicators | scoring (1,5) | 1 | principal component analysis and equal weights |
| Lugo (2007) | income, health, education | linear scale - range | [-20,1] | equal weights |
| Nilsson (2007) | expenditure, land holding, education, health | linear scale - range | [-20,1] | equal weights |
| Maasoumi and Lugo (2008) | expenditure, health, education | identity<br>linear scale - shortfall | [-3, 1] | equal weights |
| Mahlberg and Obersteiner (2001) | income, education, health | linear scale - range | 1 | most favourable weights |
| Schokkaert, Fleurbaey, Decancq (2008) | income, health, housing employment | identity | 1 | regression-based weights |