Supplemental Appendix
Attitudes towards success and failure

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Ordering the Remaining Attitudes

An analogous exercise to that of ordering failure avoidance can be conducted for all the remaining attitude. Much of the reasoning above carries through, *mutatis mutandis*, to the definitions and results of these attitudes, as we now discuss.

Concerning success attachment, the first requirement will be that the more success-attaching agent will have a smaller set of lotteries that he regards as net *failures*, in the sense of being worse than the certain x_0. In the continuous case, this would be identical to saying that there is a larger set of lotteries that he regards as net successes, and so the first requirement is simply the reverse of that for failure avoidance. But, as discussed above, for the discontinuous case it is important to account for the existence of certainty equivalents, and hence the notion of *net failure* is adequately captured by a set, $\mathcal{F}_i(x,x')$, whose definition is specular to $\mathcal{S}_i(x,x')$ above:

$$\text{(C1)} \qquad \mathcal{F}_i(x,x') := cl\left\{p \in \Delta(x,x') : CE_i(p) \text{ exists and } CE_i(p) < x_0\right\}$$

Similarly, we define the set $RL_i(x,x')$ of lotteries over which success attachment is *not* manifested, symmetrically to the $RA_i(x,x')$ sets above:

$$\text{(C2)} \qquad RL_i(x,x') := cl\left\{p \in \Delta(x,x') : CE_i(p) \text{ exists and } CE_i(p) > Ep\right\}.$$

The ranking over success attachment is thus defined as follows:

DEFINITION 7: *Let preferences $\succsim_1$ and $\succsim_2$ both satisfy the conditions in Def.3 with respect to the same $x_0 \in \mathbb{R}$. Then, $\succsim_1$ displays (weakly)* more success attachment than *$\succsim_2$ if there exist $x_f, x_s : x_f < x_0 < x_s : \forall x' \in (x_0, x_s]$, $\exists \underline{x} \in [x_f, x_0)$ such that, for each $x \in [\underline{x}, x_0)$, both the following conditions are satisfied: (i) $\mathcal{F}_1(x,x') \subseteq \mathcal{F}_2(x,x')$, and (ii) $RL_1(x,x') \subseteq RL_2(x,x')$.*

Analogous of Theorems 5 and 6 hold for this definition too. Here we only reproduce the statement of the differentiable case, which is easier to read and most useful in applications:

THEOREM 7 (Success Attachment: Interpersonal Comparisons): *Suppose that $(\succsim_i)_{i=1,2}$ are such that $Du_i^- > 0$ and $Du_i^+ < \infty$ and u_i is twice differentiable in some left- and right-neighborhoods of x_0. Then: $\succsim_1$ displays* more success avoidance than *$\succsim_2$* ***only if*** *one of the following applies:*

1) $\frac{K_1}{Du_1^-} > \frac{K_2}{Du_2^-}$,

2) $\frac{K_1}{Du_1^-} = \frac{K_2}{Du_2^-} > 0$ *and* $\frac{Du_1^+}{Du_1^-} \geq \frac{Du_2^+}{Du_2^-}$,

3) $\frac{K_1}{Du_1^-} = \frac{K_2}{Du_2^-} = 0$, $\frac{Du_1^+}{Du_1^-} \geq \frac{Du_2^+}{Du_2^-}$ *and* $\frac{D^2u_1^+}{Du_1^- - Du_1^+} \geq \frac{D^2u_2^+}{Du_2^- - Du_2^+}$.[1]

These conditions are also ***sufficient*** *if all the inequalities hold strictly.*

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[1] Note that, given the restrictions imposed by Theorem 2, both the numerators and the denominators on both sides of the latter inequality are negative.

For the remaining two attitudes, Success Seeking and Failure Resignation, things are simpler, due to the fact they only admit a continuous representation, and hence the certainty equivalent existence requirement in the definitions of the $\mathcal{S}$, $\mathcal{F}$, RA and RL sets are moot. As a consequence, the $\mathcal{F}$ and RL sets are, respectively, the complements of the $\mathcal{S}$ and RA sets, and hence $\mathcal{F}_1 \subseteq \mathcal{F}_2$ if and only if $\mathcal{S}_2 \subseteq \mathcal{S}_1$, and $RA_1 \subseteq RA_2$ if and only if $RL_2 \subseteq RL_1$. The definitions of the orderings for these two attidues therefore may be equivalently expressed in several ways.

DEFINITION 8: *Let preferences $\succsim_1$ and $\succsim_2$ both satisfy the conditions in Def.4 with respect to the same $x_0 \in \mathbb{R}$. Then, $\succsim_1$ displays (weakly)* more failure resignation than *$\succsim_2$ if thre exist $x_f, x_s : x_f < x_0 < x_s$ s.t. $\forall x \in [x_f, x_0)$, $\exists \bar{x} \in (x_0, x_s]$ such that, for each $x' \in (x_0, \bar{x}]$, both the following conditions are satisfied: (i) $\mathcal{F}_1(x, x') \subseteq \mathcal{F}_2(x, x')$, and (ii) $RA_1(x, x') \subseteq RA_2(x, x')$.*

DEFINITION 9: *Let preferences $\succsim_1$ and $\succsim_2$ both satisfy the conditions in Def.3 with respect to the same $x_0 \in \mathbb{R}$. Then, $\succsim_1$ displays (weakly)* more success seeking than *$\succsim_2$ if there exist $x_f, x_s : x_f < x_0 < x_s$ s.t.: $\forall x' \in (x_0, x_s]$, $\exists \underline{x} \in [x_f, x_0)$ such that, for each $x \in [\underline{x}, x_0)$, both the following conditions are satisfied: (i) $\mathcal{S}_1(x, x') \subseteq \mathcal{S}_2(x, x')$, and (ii) $RL_1(x, x') \subseteq RL_2(x, x')$.*

The next results provide characterize these orderings in the space of utility representations. They are completely analogous to the previous two theorems, with the only difference that they only account for the continuous case, and hence $K_i = 0$ for both agents:

THEOREM 8 (Failure Resignation: Interpersonal Comparisons): *Suppose that $(\succsim_i)_{i=1,2}$ are such that $Du_i^- > 0$ and $Du_i^+ < \infty$ and u_i is twice differentiable in some left- and right-neighborhoods of x_0. Then: $\succsim_1$ displays* more success avoidance than *$\succsim_2$ **only if** both (i) $\frac{Du_1^+}{Du_1^-} \geq \frac{Du_2^+}{Du_2^-}$ and (ii) $\frac{D^2u_1^-}{Du_1^- - Du_1^+} \geq \frac{D^2u_2^-}{Du_2^- - Du_2^+}$. These conditions are also **sufficient** if all the inequalities hold strictly.*

THEOREM 9 (Success Seeking: Interpersonal Comparisons): *Suppose that $(\succsim_i)_{i=1,2}$ are such that $Du_i^- < \infty$ and $Du_i^+ > 0$ and u_i is twice differentiable in some left- and right-neighborhoods of x_0. Then: $\succsim_1$ displays* more success seeking than *$\succsim_2$ **only if** both (i) $\frac{Du_1^-}{Du_1^+} \geq \frac{Du_2^-}{Du_2^+}$ and (ii) $\frac{D^2u_1^+}{Du_1^- - Du_1^+} \geq \frac{D^2u_2^+}{Du_2^- - Du_2^+}$. These conditions are also **sufficient** if all the inequalities hold strictly.*

Interpersonal Comparisons: A Tight Characterization

The next result provides a tight characterization of the ranking of agents' failure avoidance (as per Def. 6), in terms of the key elements in the main representation theorem:

THEOREM 10: *Let preferences $\succsim_1$ and $\succsim_2$ both satisfy the conditions in Def. 2 with respect to the same $x_0 \in \mathbb{R}$. Then, $\succsim_1$ displays* more failure avoidance than *$\succsim_2$ if and only if there exists $\underline{x} < x_0$, such that $\forall x \in (\underline{x}, x_0)$, there exists $\bar{x} > x_0$, s.t., for all $x' \in (x_0, \bar{x})$, one of the following applies:*

1) $K_1 > 0$ *and* $\frac{K_1}{m_1(x')} - \frac{K_2}{m_2(x')} > \left[\frac{m_2(x)}{m_2(x')} - \frac{m_1(x)}{m_1(x')}\right](x_0 - x)$.

2) $K_1 = K_2 = 0$, $\frac{m_1(x)}{m_1(x')} > \frac{m_2(x)}{m_2(x')}$ *and*

$$\text{(D1)} \qquad \frac{m_1(E\hat{p}_1(x,x')) - m_1(x)}{m_2(E\hat{p}_1(x,x')) - \beta m_2(x)} > \frac{m_1(E\hat{p}_1(x,x')) - m_1(x')}{m_2(E\hat{p}_1(x,x')) - m_2(x')} + \left[1 - \beta(x,x')\right]\gamma(x,x'),$$

where $\beta = \frac{x_0 - x}{x' - x}$, *and* $\gamma(x,x') = \frac{m_1(E\hat{p}_1(x,x'))m_1(x)m_2(x') - m_2(x)m_1(x'^2)}{m_1(x')(m_2(E\hat{p}_1(x,x')) - m_2(x'))(m_2(E\hat{p}_1(x,x')) - \beta(x,x')m_2(x))}$.

PROOF:

Lemma 10 above proves part 1 of the theorem, while Lemma 4, together with Lemma 6 (concerning the $\bar{p}_i$ ranking, noting that for $K_1 = K_2 = 0$, the expression in the lemma reduces to $\frac{m_1(x)}{m_1(x')} > \frac{m_2(x)}{m_2(x')}$) and Lemma 14 (concerning the $\hat{p}_i$ ranking) prove part 2 of the theorem.

Lemma 15: If both u_1 and u_2 are continuous, $\hat{p}_1(x,x') > \hat{p}_2(x,x')$ if and only if

$$\frac{m_1(y)-m_1(x)}{m_2(y)-\beta m_2(x)} > \frac{m_1(y)-m_1(x')}{m_2(y)-m_2(x')} + (1-\beta)\gamma(x,x',y), \tag{E1}$$

where $\gamma(x,x',y) = \frac{m_1(y)m_1(x)m_2(x')-m_2(x)m_1(x'^2)}{m_1(x')(m_2(y)-m_2(x'))(m_2(y)-\beta m_2(x))}$.

PROOF:

Let $\beta := \frac{x_0-x}{x'-x}$, and note that $\beta \in (0,1)$ and $\beta \to 1$ as $x' \to x_0$. Also let $y = E\hat{p}_1$, and note that $y \to x_0$ as $x' \to x_0$ (these facts will be useful in the lemmas that follow). Then, from Lemma 13, we have that:

$$\left(\frac{x_0-y}{x'-y}\right)\left(\frac{(x'-x)}{(x_0-x)}\right) = \frac{m_1(x)-m_1(x')}{\beta m_1(y)-m_1(x')}. \tag{E2}$$

Substituting this notation in the condition of Lemma 14, and particularly using eq. (E2), we obtain

$$\frac{m_1(y)-m_1(x)}{m_1(x')} - \frac{m_2(y)-m_2(x)}{m_2(x')} > \left(\frac{m_1(y)}{m_1(x')} - \frac{m_2(y)}{m_2(x')}\right)\left(1-\frac{m_1(x)-m_1(x')}{\beta m_1(y)-m_1(x')}\right). \tag{E3}$$

Next, re-arrange (E3) to:

$$\left(\frac{m_1(y)}{m_1(x')} - \frac{m_2(y)}{m_2(x')}\right) - \left(\frac{m_1(x)}{m_1(x')} - \frac{m_2(x)}{m_2(x')}\right) > \left(\frac{m_1(y)}{m_1(x')} - \frac{m_2(y)}{m_2(x')}\right)\left(1-\frac{m_1(x)-m_1(x')}{\beta m_1(y)-m_1(x')}\right)$$

$$\Longleftrightarrow$$

$$\left(\frac{m_1(y)}{m_1(x')} - \frac{m_2(y)}{m_2(x')}\right)\left(\frac{m_1(x)-m_1(x')}{\beta m_1(y)-m_1(x')}\right) > \left(\frac{m_1(x)}{m_1(x')} - \frac{m_2(x)}{m_2(x')}\right)$$

$$\Longleftrightarrow$$

$$\frac{1}{m_1(x')}\left(\frac{m_1(y)(m_1(x)-m_1(x'))}{\beta m_1(y)-m_1(x')} - m_1(x)\right) > \frac{1}{m_2(x')}\left(\frac{m_2(y)(m_1(x)-m_1(x'))}{\beta m_1(y)-m_1(x')} - m_2(x)\right)$$

$$\Longleftrightarrow$$

$$\frac{m_1(y)(m_1(x)-m_1(x'))-m_1(x)(\beta m_1(y)-m_1(x'))}{m_1(x')(\beta m_1(y)-m_1(x'))} > \frac{m_2(y)(m_1(x)-m_1(x'))-m_2(x)(\beta m_1(y)-m_1(x'))}{m_2(x')(\beta m_1(y)-m_1(x'))}$$

$$\Longleftrightarrow$$

$$\frac{m_1(y)\left((1-\beta)m_1(x)-m_1(x')\right)+m_1(x)m_1(x')}{m_1(x')} > \frac{m_2(y)(m_1(x)-m_1(x'))-m_2(x)(\beta m_1(y)-m_1(x'))}{m_2(x')}$$

$$\Longleftrightarrow$$

$$\frac{m_1(x')(m_1(x)-m_1(y))+(1-\beta)m_1(y)m_1(x)}{m_1(x')} > \frac{m_2(y)(m_1(x)-m_1(x'))-m_2(x)(\beta m_1(y)-m_1(x'))}{m_2(x')}$$

$$\Longleftrightarrow$$

$$m_1(x)-m_1(y)+\frac{(1-\beta)m_1(y)m_1(x)}{m_1(x')} > \frac{m_2(y)(m_1(x)-m_1(x'))+m_2(x)m_1(x')}{m_2(x')} - \frac{\beta m_2(x)m_1(y)}{m_2(x')}$$

$$\Longleftrightarrow$$

$$m_1(y)\left(\frac{\beta m_2(x)}{m_2(x')}-1\right)+m_1(x)>\frac{m_2(y)(m_1(x)-m_1(x'))+m_2(x)m_1(x')}{m_2(x')}-\frac{(1-\beta)m_1(y)m_1(x)}{m_1(x')}$$

$$\Longleftrightarrow$$

$$\frac{m_1(y)\left(\beta m_2(x)-m_2(x')\right)+m_1(x)m_2(x')}{m_2(x')}>\frac{m_2(y)(m_1(x)-m_1(x'))+m_2(x)m_1(x')-\frac{(1-\beta)m_1(x)m_1(y)m_2(x')}{m_1(x')}}{m_2(x')}$$

and re-arranging further, which we obtain the following (details are in the online appendix):

(E4)

$$m_1(y)\left(\beta m_2(x)-m_2(x')\right)-m_1(x')m_2(x)+\frac{(1-\beta)m_1(x)m_1(y)m_2(x')}{m_1(x')}>m_2(y)(m_1(x)-m_1(x'))-m_1(x)m_2(x').$$

rearranging now Equation (C15) (and writing γ rather than $\gamma(x,x',y)$, we have:

$$m_1(y)m_2(y)-m_1(y)m_2(x')-m_1(x)m_2(y)+m_1(x)m_2(x')>$$
$$m_1(y)m_2(y)-\beta m_1(y)m_2(x)-m_1(x')m_2(y)+\beta m_1(x')m_2(x)-\gamma(1-\beta)\left(m_2(y)-m_2(x')\right)(m_2(y)-\beta m_2(x))$$
$$\Longleftrightarrow$$
$$m_1(y)\left(\beta m_2(x)-m_2(x')\right)-\beta m_1(x')m_2(x)>$$
$$m_2(y)\left(m_1(x)-m_1(x')\right)-m_1(x)m_2(x')-\gamma(1-\beta)(m_2(y)-m_2(x'))(m_2(y)-\beta m_2(x)).$$

Using that $-\beta m_1(x')m_2(x)=(1-\beta)m_1(x')m_2(x)-m_1(x')m_2(x)$, we obtain:

$$m_1(y)\left(\beta m_2(x)-m_2(x')\right)-m_1(x')m_2(x)+\left[\gamma(1-\beta)(m_2(y)-m_2(x'))(m_2(y)-\beta m_2(x))+(1-\beta)m_1(x')m_2(x)\right]>$$

(E5)

$$m_2(y)\left(m_1(x)-m_1(x')\right)-m_1(x)m_2(x').$$

For Inequality E4 to hold if and only Inequality E5 holds, it must be that:

$$\gamma(1-\beta)(m_2(y)-m_2(x'))(m_2(y)-\beta m_2(x))+(1-\beta)m_1(x')m_2(x)=\frac{(1-\beta)m_1(x)m_1(y)m_2(x')}{m_1(x')}$$

(E6)
$$\Longleftrightarrow$$

(E7)
$$\gamma=\frac{\frac{m_1(y)m_1(x)m_2(x)}{m_1(x')}-m_1(x')m_2(x)}{(m_2(y)-m_2(x'))(m_2(y)-\beta m_2(x))}$$

(E8)
$$\Longleftrightarrow$$

(E9)
$$\gamma=\frac{m_1(y)m_1(x)m_2(x')-m_2(x)m_1(x'^2)}{m_1(x')\left(m_2(y)-m_2(x')\right)\left(m_2(y)-\beta m_2(x)\right)},$$

which concludes the proof of the lemma. QED.