Supplemental Appendix for

"Soviet Mathematics and Economic Theory in the Past Century: A Historical Reappraisal"

Ivan Boldyrev
Radboud University

Soviet Mathematical Economics: A Very Short Introduction (With Some Illustrations)

The Appendix takes a closer look at Soviet mathematical economics (rather than mathematics; see also Introduction). It elaborates on its contexts and discusses in more detail some exemplary contributions, focusing predominantly on the postwar years.

The term "mathematical economics" is not very specific as most postwar economics was mathematized. Perhaps one could say that some areas of economic theory are primarily motivated by the interest to solve a mathematical problem, and as such could be called "mathematical economics." In the Soviet context, however, the term does have a more specific meaning. It captures any work that was informed by mathematical techniques and thus deviated from what was largely taught and researched in the economic departments within the system of higher education or the Soviet Academy of Sciences.

Joseph Stalin was responsible not only for the repression of many economists, including those associated with the Conjuncture Institute (see Introduction), but also for instilling an overarching mistrust toward quantitative reasoning in the social sciences. For decades, this sort of research was ideologically suspect. Only after Stalin's death in 1953, with the softening of political regime (the epoch of the "Thaw"), could the discussion around applying mathematical methods in economics be re-opened (Ellman 1973; Gerovitch 2002; Leeds 2016; Boldyrev and Kirtchik 2017). The first study program was organized in 1959 at the economics department of Leningrad University, where Kantorovich was active. In 1960, a similar program opened at Moscow State University (MSU), initiated by Nemchinov. In 1963, the Central Economic-Mathematical Institute (CEMI) was created. It became the major mathematical economics research institution in the Soviet Union. In 1964, with the help of Girsanov (see Section 4.2) and others, a separate division of economic cybernetics was created at MSU, subsequently emulated by other universities across the whole country. The year 1965 saw the launch of the first professional journal, *Economics and Mathematical Methods*, institutionalizing the field. Formally, this research was relegated to the separate disciplines like "mathematical methods" and "economic cybernetics," with no claims to become "economic theory." This defined the subaltern status of the field.

Note that most of Soviet science was being done not at the universities (although many scholars had regular teaching responsibilities) but in the research centers associated with the Academy of Sciences. Typically, the research was published in the home journals of the relevant institutes. Some of it was translated, but not widely read. Soviet scholars almost never submitted their work to international economic journals.

There were two important social tendencies that defined the context for mathematical economics in the USSR. First, quite generally, mathematics and other less "controversial" technical disciplines attracted talented people (whatever definition of talent one adopts), who otherwise would have pursued a career in the humanities or in the social sciences. The fields were seen as too ideological and did not facilitate open and critical conversation. Mathematical economics, on the contrary, provided a space that allowed for some more freedom to discuss alternative forms of economic organization. The second tendency was the infamous academic antisemitism, for which, sadly, various mathematicians, including Pontryagin, bear some responsibility. From the mid-1960s to the mid-1980s, an academic career in mathematics, especially at the major centers, like Steklov Mathematical Institute and the *mekhmat* of Moscow University, became difficult for Jews. They were discriminated against and had lots of troubles in getting into a PhD programme, receiving academic appointments, and traveling to international conferences. While the antisemitic discrimination has been well documented (see the materials in Shifman 2005), there does not seem to be any systematic studies of the general tendency to avoid humanities and social sciences. Nonetheless, it repeatedly pops up in the memoirs of mathematicians.

For example, consider Anatole Katok, a mathematician of a Jewish origin, who was, before his emigration in 1978, a researcher at CEMI. His biography illustrates *both* tendencies. Katok

> explained that his choice of mathematics as a vocation was influenced by the relative freedom mathematicians enjoyed because their discipline was least affected and controlled by ideological impositions […] From the late 1960s antisemitism and suppression of liberal thought grew at Moscow State University, and almost no Jews were accepted as students or faculty. So, Katok instead assumed an appointment at the […] CEMI […], which allowed him to combine work on mathematical problems in economics, if any, with research in pure mathematics (Hasselblatt 2019: 711).

The new community of mathematical economists integrated scholars of the older generation. Thus, the former members of the Conjuncture Institute who survived the repressions were able to return to research. These included the statistician Yakov Gerchuk, or the statistician and cliometrician Albert Vainshtein, as well as other mathematically minded economists of the previous generations, such as Alexander Lur'e and Viktor Novozhilov. All these individuals played some role in creating a research environment that should have been different from the vacuum of the previous decades.

The work of Alexander Konüs (1895-1990), another surviving economist, is of particular interest to illustrate how huge this vacuum was. Konüs became famous for his early study on the cost-of-living index (Konüs 1924/1939).[1] His work was probably the most fruitful collaboration of an economist and a mathematician in the Soviet Union between the wars. Konüs worked with the Moscow University geometer Sergei Byushgens. Their paper (Konüs and Byushgens 1926) was rediscovered in the 1970s by W. E. Diewert (1976), a Canadian economist of Russian origin, who studied the language and could read their text in Russian. It has been recently translated into English. In the preface to the translation, Diewert and Zelenyuk (2023) write that the paper contained several important theoretical results. For example, the early demonstration of microeconomic duality theory makes the paper not only "a landmark in the history of index number theory," but also an important document in the history of microeconomics.

CEMI enjoyed relative intellectual autonomy, providing a home for many scholars who would otherwise have had troubles getting academic jobs. Its director Nikolai Fedorenko promoted CEMI as an important organizational project and was for years defending, not without success, both the institute and the specific academic program CEMI suggested for policymakers. This program came to be associated with the so-called SOFE (system of optimal functioning of the economy), a multi-level

---

[1] The paper was known to econometricians of that time via references, but not in full. It was Henry Schultz, American demand theorist and statistician, who, in the mid 1930s, organized the translation of the paper he could not read, used it in his teaching, and suggested publishing it in *Econometrica*. (Schultz also initiated the publication of Slutzky (1937)).

system of models that was supposed to inform planning.[2] Although the relations between CEMI and Gosplan, the Soviet Planning Committee, were strained, the two organizations did manage to collaborate, and CEMI proposals informed some planning initiatives. Perhaps the most important bridging project was that of ASPR (*Automatized System of Planning Calculations*), which sought to integrate insights from input-output and optimization models into the planning process (Urinson 1986). During the *perestroika* (1985-1991) and the first years of economic reforms, or, as Makarov (1988: 459) put in at the 100th AEA meeting, during "revolutionary changes [,] a transition from an excessively stable and rigid economic system to one which is much more flexible," the economists affiliated with or coming from CEMI turned out to be influential economic experts and policymakers.

Theoretical and applied work on economic modeling was also done at the Gosplan research institute, at the Chief Computer Center of the Academy of Sciences,[3] and at the Institute of Control Problems. Important institutions outside Moscow were located in Novosibirsk.[4] Kantorovich was working there in the 1960s, but after he left for Moscow, many mathematicians and mathematical economists, including Vladimir Bulavskii, Valeri Marakulin, Leonid Polishchuk, Alexander Rubinov, Gennadii Rubinstein, and Valeri Vasil'ev, continued this research. In Kyiv, the Institute of Cybernetics of the Ukrainian Academy of Sciences was another important center.

Although international collaborations with Soviet economists were very limited, there were some exceptions. For example, Martin Weitzman co-authored papers with Soviet authors, frequently visited the USSR, and generally belonged to those few American mainstream economists who managed to maintain long-term contacts with the Soviet mathematical economics community.[5] Other major economists belonging to this group were Tjalling Koopmans (Düppe 2016) and David Gale.

In the beginning of the 1980s, Yuri Yermoliev, an operations researcher and expert in stochastic programming from the Kyiv Institute of Cybernetics, and another Soviet Ukrainian mathematician, Yuri Kaniovski, collaborated with W. Brian Arthur at the International Institute for Applied Systems Analysis (IIASA) in Austria, while Arthur was doing his now famous research on increasing returns (published later as Arthur 1989). This collaboration–initiated at the institution created precisely to further academic exchange across the Iron Curtain (Rindzevičiūtė 2016)–illustrates the potential carried by the international division of academic labor between Western economics and Soviet mathematics.

What kind of research prevailed in Soviet mathematical economics? One cannot possibly do justice to all the relevant developments over several decades. However, a bird's-eye view can provide a general idea of what the field was about.

It would be fair to say that a lot of intellectual effort, beginning from the end of the 1950s, was directed at input-output modeling. This technique was connected both with the idea of improving economic calculation, considered as a basic rationale behind the "mathematical methods," and with the promise of practical application.

---

[2] The first formulations were given by Volkonskii (1967) and Katseneligenboigen et al. (1969), see the history in Ericson 2019.

[3] There, the department of mathematical economics was founded in 1968 by Nikita Moiseev. On Moiseev see Rindzevičiūtė (2016).

[4] The Institute of Economics and Industrial Engineering and the Mathematical Economics division at the Institute of Mathematics were both of the Siberian Division of the Academy of Sciences.

[5] For example, Kantorovich's (1965) book was used in Weitzman's 1974 MIT course on core micro theory https://www.irwincollier.com/m-i-t-core-micro-theory-resource-allocation-price-system-weitzman-1974/

Although the practical results were reported to be quite modest (Tretyakova and Birman 1976), input-output techniques became the standard element in the university curricula in "economic cybernetics" and in applied research. Along with input-output, many mathematical economists were busy developing the efficiency criteria for investment–something not easy in the context where openly discussing capital theory was risky. Another CEMI economist, Boris Mikhalevski, explored repressed inflation. But studying the actual state of the Soviet economy was also subject to censorship.

At CEMI, apart from the work that promised a practical import, a lot of attention was paid to improving optimization techniques (see the overview in Polyak 2002). CEMI served as a temporary home for some bright mathematicians, including Eugene Dynkin, Gennadi Henkin, Anatole Katok, and Boris Mityagin.[6] Some of them influenced this research agenda and contributed to defining standards of rigor in theoretical modeling. Overall, Soviet operations researchers made significant contributions, as evidenced by the work of David Yudin, Arkadi Nemirowski, Yuri Nesterov, and Leonid Khachiyan, who received multiple international awards.[7]

In the beginning of the 1960s, Kantorovich's student in Novosibirsk, Valery Makarov, became interested in optimal growth theory and general equilibrium analysis (Makarov and Rubinov 1973/1977, see the review by Gale 1978). Makarov moved to Moscow in 1985 to become the director of CEMI. The Novosibirsk scholars did mathematically sophisticated work on optimization, general equilibrium, and cooperative games. At CEMI, Arkin and Evstigneev (1987) reformulated optimal growth theory in stochastic terms, a contribution that played some role in this literature (see Brock and Dechert 2010 for an overview).

Victor Polterovich, another CEMI economist, was particularly active as a general equilibrium theorist. He formulated a criterion for the monotonicity of aggregate demand functions (Mitiushin and Polterovich 1978); provided a version of a synthesis between optimal growth and general equilibrium theory (Polterovich 1983); and used disequilibrium modeling to understand the realities of shortage (Polterovich 1990; 1993). In fact, disequilibrium analysis in the USSR was pioneered by Emmanuil Braverman (1972), who was working in the influential group led by Mark Aizerman at the Institute of Control Problems. Since the 1970s, the group did research on voting and abstract choice theory.[8] The Aizerman group was, perhaps, one of the most internationalized research collectives in Soviet mathematical economics, hosting regular seminars and being visited by major social choice theorists and game theorists of the time.

When asked in the 1980s, some of these protagonists said that at the time, mathematics was perceived as the way to improve the general culture of economic planning (Katseneligenboigen 1981) and to infuse Soviet economics with some rationality (Volkonskii 1989). Indeed, mathematical formalisms, once accepted, helped further the discussion – at least academically. Overall, the work of Soviet mathematical economists remained quite abstract and remote from actual

[6] On Dynkin's contributions and on the links with further literature, see Evstigneev (2000).

[7] In 1982, Yudin, Nemiroski and Khachiyan received Fulkerson Prize in discrete mathematics, Nemirovski and Nesterov received Dantzig Prize in mathematical programming (in 1991 and 2000, respectively), and the John von Neumann Prize (in 2003 and 2009), the major award in operations research.

[8] On the career of Polterovich and his work in demand theory and disequilibrium modeling, see Boldyrev and Kirtchik (2014), Boldyrev (2023); Braverman's work is covered in Kirtchik (2019); on Aizerman and his group (in particular, Andrei Malishevski and Fuad Aleskerov) see Boldyrev (2020).

applications.[9] But sometimes it was brilliant work, still present in today's formal economic theory. In this short overview, I have tried to demonstrate that exploring the intellectual legacy of this diverse field alongside the experiences of Soviet mathematical economists and the context of their work is an instructive endeavor in the history of economic ideas.

Arkin, Vadim, and Igor Egstineev. 1987. *Stochastic Models of Control and Economic Dynamics*. New York: Academic Press

Arthur, W. Brian. 1989. "Competing Technologies, Increasing Returns, and Lock-In by Historical Events." *The Economic Journal* 99(394): 116–31.

Boldyrev, Ivan. 2023. "Victor Polterovich." In *The New Palgrave Dictionary of Economics*, edited by Matias Vernengo, Esteban Perez Caldentey, and Barkley J. Rosser, Jr. London: Palgrave Macmillan. https://doi.org/10.1057/978-1-349-95121-5_3142-1

Boldyrev, Ivan and Olessia Kirtchik. 2014. "General Equilibrium Theory Behind the Iron Curtain: The Case of Victor Polterovich." *History of Political Economy* 46(3): 435-61.

Boldyrev, Ivan and Olessia Kirtchik. 2017. "The Cultures of Mathematical Economics in the Postwar Soviet Union: More than a Method, Less than a Discipline." *Studies in History and Philosophy of Science Part A* 63(6): 1–10.

Braverman, Emmanuil. 1972. "Model of Production with Disequilibrium Prices." (In Russian.) *Ekonomika i Matematicheskie Metody* 8: 40-64.

Brock, William A. and W. Davis Dechert. 2010. "Growth Models, Multisector." In *Economic Growth. The New Palgrave Economics Collection*, edited by Steven N. Durlauf and Lawrence E. Blume, 127-32. London: Palgrave Macmillan, London.

Diewert, W. Erwin. 1976. "Exact and Superlative Index Numbers." *Journal of Econometrics* 4(2): 115-45.

Diewert, W. Erwin and Valentin Zelenyuk. 2023. „On the Problem of the Purchasing Power of Money by A. A. Konüs and S. S. Byushgens: Translation and Commentary." *Journal of Productivity Analysis*. https://doi.org/10.1007/s11123-023-00696-x

Düppe, Till. 2016. "Koopmans in the Soviet Union: A Travel Report of the Summer of 1965." *Journal of the History of Economic Thought* 38(1): 81–104.

[9] Yuri Kabanov, currently a scholar in mathematical finance (see also 4.2), worked in the CEMI lab headed by Vadim Arkin and focused on stochastic calculus. In a characteristic episode, he recalls having encountered a paper by Merton (1975). 'I asked Vadim [Arkin] […]: "What is a portfolio?" He said: "Yura, forget about it. […] In capitalist countries, there are portfolios, in socialist economies, it is not an important issue, you cannot be promoted at our institute [if you focus on that] […] one should do something more applied, more economic"' (Kabanov 2010). Indeed, in a country without financial markets, portfolio theory was entirely impractical.

Ellman, Michael. 1973. *Planning problems in the USSR: The contribution of mathematical economics to their solution 1960-1971*. Cambridge: Cambridge University Press.

Evstigneev, Igor V. 2000. “Dynkin’s Work in Mathematical Economics.” In: *Selected papers of E. B. Dynkin with Commentary*, edited by Alexander A. Yushkevich, Gary M. Seitz, Arkady L. Onishchik, 793-5. Providence, RI: AMS, and Cambridge, MA: International Press.

Gale, David. 1978. “Review of: *Mathematical Theory of Economic Dynamics and Equilibria*, by V. L. Makarov and A. M. Rubinov.” *Bulletin of the American Mathematical Society* 84: 665-71.

Gerovitch, Slava. 2002. *From Newspeak to Cyberspeak: A History of Soviet Cybernetics*. Cambridge, MA: MIT Press.

Hasselblatt, Boris. 2019. “Anatole Katok - A Half-Century of Dynamics.” *Notices of the AMS* 66(5): 708-19.

Kabanov, Yuri. 2010. “Mathematical Finance and Mathematics from Finance. A talk at the International Symposium *Visions in Stochastics (Leaders and their Pupils)* Moscow, November 1, 2010. https://www.mathnet.ru/php/presentation.phtml?option_lang=eng&presentid=2554

Katseneligenboigen, Aron I. 1981. Interview Conducted by Eugene Dynkin with Aron Katsenelinboigen on July 5, 1981. https://hdl.handle.net/1813/17453

Katsenelinboigen, Aron I., Iosif L. Lakhman, Yuri V. Ovsienko. 1969. *Optimality and Commodity-Money Relations*. (In Russian.) Moscow: Nauka.

Kirtchik, Olessia. 2019. “From Pattern Recognition to Economic Disequilibrium: Emmanuil Braverman’s Theory of Control of the Soviet Economy.” *History of Political Economy* 51 (S1): 180–203.

Konüs, Alexander A. 1924/1939. “The Problem of the True Index of the Cost of Living.” *The Economic Bulletin of the Institute of Economic Conjuncture* 9-10 (36-37): 64-71, Engl. translation in *Econometrica* 7: 10-29.

Konüs, Alexander A., and Sergei S. Byushgens. 1926. “On the Problem of the Purchasing Power of Money.” (In Russian.) *Voprosi Konyunkturi* 2(1): 151-72.

Leeds, Adam. 2016. “Dreams in Cybernetic Fugue: Cold War Technoscience, the Intelligentsia, and the Birth of Soviet Mathematical Economics.” *Historical Studies in the Natural Sciences* 46(5): 633–68.

Makarov, Valery L. 1988. “On the Strategy for Implementing Economic Reform in the USSR.” *American Economic Review* 78(2): 457–60.

Makarov, Valery, and Alexander Rubinov. 1973/1977. *Mathematical Theory of Economic Dynamics and Equilibria* (In Russian.) Moscow: Nauka. Engl. translation: New York, Heidelberg, Berlin: Springer.

Merton, Robert C. 1975. "Optimum Consumption and Portfolio Rules in a Continuous-Time Model." In *Stochastic Optimization Models in Finance*, edited by William T. Ziemba and Raymond G. Vickson, 621-61. New York: Academic Press.

Mitiushin, Leonid, and Victor Polterovich. 1978. Criterion for Monotonicity of Demand Functions." (In Russian.) *Ekonomika i Matematicheskie Metody]* 14(1): 122–8.

Polterovich, Victor M. 1983. "Equilibrium Trajectories of Economic Growth." *Econometrica* 51(3): 693–729.

Polterovich, Victor M. 1990. *Economic Equilibrium and Economic Mechanism*. (In Russian.) Moscow: Nauka.

Polterovich, Victor M. 1993. "Rationing, Queues, and Black Markets." *Econometrica* 61(1): 1–28.

Rindzevičiūtė, Egle. 2016. *The Power of Systems: How Policy Sciences Opened Up the Cold War World*. Ithaca and London: Cornell University Press.

Shifman, Mikhail, ed. 2005. *You Failed Your Math Test, Comrade Einstein: Adventures and Misadventures of Young Mathematicians*. Singapore: World Scientific.

Tretyakova, Albina, and Igor Birman. 1976. "Input-output Analysis in the USSR." *Soviet Studies* 28(2): 157-86.

Urinson, Yakov. M. 1986. *Perfecting the Technology of National Economic Planning* (In Russian.) Moscow: Ekonomika.

Volkonskii, Viktor. 1989. Interview conducted by Eugene Dynkin with Viktor Volkonsky, September 16, 1989. http://dynkincollection.library.cornell.edu https://ecommons.cornell.edu/items/f90fdfec-602a-47f1-a887-9630ef5baf9b