Online Appendix
Aiming for the Goal: Contribution Dynamics of Crowdfunding

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PROOFS

B1. General properties of PT assessments and PT equilibria

PROPERTIES OF PT ASSESSMENTS

In this section, we present some properties of PT assessments and the induced probability of success $\pi^\Delta(N,D,u)$ that we will use for the construction of PT equilibria.

Lemma 1. *Consider a PT assessment with donation threshold $D_*^\Delta(N,u)$. If the campaign reaches a state (N,D,u) with $D < D_*^\Delta(N,u+\Delta)$, it has failed with probability one.*

Proof. Assume that a state $(N_t, D_t, T-(t+\Delta))$ with $D_t < D_*^\Delta(N_t, T-t)$ is reached. Then $D_t = w$, because the donor is playing a PT strategy and $w < D_*^\Delta(N_t, T-t')$ for all $t' \geq t$ by Condition i) in Definition 2 of PT assessments. Thus, $N_{t'} = N_t$ for all $t' > t$, given the investor strategy in Equation PT-investor. All in all, $(N_{t'}, D_{t'}) = (N_t, w)$ for all $t' > t$, where $N_t p + w < N_t p + D_*^\Delta(N_t, T-t) < N_t p + G - (N_t+1)p < G$. This concludes the proof. ■

Lemma 1 implies that beliefs in a PT assessment are consistent and that the induced probability of success π^Δ can be written in a recursive manner, as we show in Lemma 2. We also derive some other properties of π^Δ. For the proof, we use that for a PT assessment, cumulative donations at time t must satisfy

$$D_t = \max_{t' \leq t} \min\{D_*^\Delta(N_{t'}, T-t'), w\}. \tag{B1}$$

Lemma 2. *A PT assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ with donation threshold $D_*^\Delta(N,u)$ satisfies the following properties:*

i) Beliefs F^Δ are consistent with the strategies b^Δ, D_+^Δ;

ii) The induced probability $\pi^\Delta(N,D,u)$ satisfies the following:

- *$N+1 \geq \underline{M}(D)$ if and only if $\pi^\Delta(N,D,u) = 1$;*
- *If $N+1 < \underline{M}(D)$ and $D \geq D_*^\Delta(N,u+\Delta)$, then $\pi^\Delta(N,D,0) = \frac{1-F_0(G-p(N+1))}{1-F_0(D)}$,*

and for $u > 0$,

$$\pi^{\Delta}(N,D,u) = \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{u}{\Delta}}(1-\Delta\lambda)^{i-1}\Delta\lambda \\ \pi^{\Delta}\Big(N+1,\ \max\big\{D, D_*^{\Delta}\big(N+1, u-(i-1)\Delta\big)\big\},\ u-i\Delta\Big) \\ \mathbb{1}\Big(W \geq D_*^{\Delta}\big(N+1, u-(i-1)\Delta\big)\Big) \\ +(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W \geq G-(N+1)p)\,\Big|\, W \geq D\Bigg];$$

- *If $N+1 < \underline{M}(D)$ and $D < D_*^{\Delta}(N, u+\Delta)$, $\pi^{\Delta}(N,D,0) = 0$, and for $u > 0$,*

$$\pi^{\Delta}(N,D,u) = \mathbb{P}(D \geq \max_{\substack{N<N'\leq \underline{M}(D) \\ \tau^u_{N'-N}<T}} D_*^{\Delta}(N', T-\tau^u_{N'-N})), \tag{B2}$$

where τ^u_n is the time of the n-th arrival after time $t = T-u$.[16]

iii) $\pi^{\Delta}(N,D,u)$ is continuous and strictly increasing in D for $G-(N+1)p \geq D \geq D_*^{\Delta}(N, u+\Delta)$, and $\pi^{\Delta}(N,D,u)$ is weakly increasing in D otherwise;

iv) $\pi^{\Delta}(N,D,u) \leq \pi^{\Delta}(N+1,D,u-\Delta) \leq \pi^{\Delta}(N+1,D,u)$, and $\pi^{\Delta}(N,D,u)$ is strictly increasing in N, u, if $0 < \pi^{\Delta}(N,D,u) < 1$.

Proof. i) Consider an investor in an on-path state (N,D,u). By (B1) this state is reached with zero probability by donors with $w < D$, and if $D < D_*^{\Delta}(N, u+\Delta)$, then $D = w$. Further, if $D \geq D_*^{\Delta}(N, u+\Delta)$, any donor with $w \geq D$ must have followed the same donation strategy on any equilibrium path history that led to (N,D,u). Hence, by Bayes' rule, the distribution of donor types in state (N,D,u) is a truncation of F_0 at D.

ii) For $N+1 \geq \underline{M}(D)$, $\pi^{\Delta}(N,D,u) = 1$ as the goal is reached if the $(N+1)$th investor pledges. For $N+1 < \underline{M}(D)$, absent additional donations, at least one more investor must arrive for the project to reach the goal G after the $(N+1)$th investor pledges, because $D_*^{\Delta}(N,u) < G-(N+1)p$, so $\pi^{\Delta}(N,D,u) < 1$. The probability of success must satisfy the following recursive property: First, $\pi^{\Delta}(N,D,0) = \frac{1-F_0(G-(N+1)p)}{1-F_0(D)}\mathbb{1}(D \geq D_*^{\Delta}(N,\Delta))$,

[16] Note that $\pi^{\Delta}(N,D,u)$ is defined even if the corresponding purchase is not consistent with the investor strategy. If $D < D_*^{\Delta}(N, u+\Delta)$ and the investor pledges, this deviation is not observed by an investor in period $u' < u$. Thus, she pledges if $D \geq D_*^{\Delta}(N+1, u'+\Delta)$. The probability is with respect to the random arrival time $\tau^u_{N'-N}$.

given F^Δ defined in Equation PT-belief. For $u > 0$ and $D \geq D_*^\Delta(N, u+\Delta)$,

$$\begin{aligned}
&\pi^\Delta(N,D,u) = \\
&\mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\frac{u}{\Delta}} \underbrace{(1-\Delta\lambda)^{i-1}\Delta\lambda}_{\substack{\text{next investor arrives}\\ \text{at } u-i\Delta}}\, \underbrace{\pi^\Delta\big(N+1,\ \max\{D, D_*^\Delta(N+1, u-(i-1)\Delta)\},\ u-i\Delta\big)}_{\text{probability of success if the } N+2\text{nd investor pledges}} \\
&\underbrace{\mathbb{1}\big(W \geq D_*^\Delta(N+1, u-(i-1)\Delta)\big)}_{\text{wealth exceeds donation threshold}} + \underbrace{(1-\Delta\lambda)^{u/\Delta}}_{\text{no investor}}\mathbb{1}(W \geq G-(N+1)p) \,\Big|\, \underbrace{W \geq D}_{\substack{\text{beliefs are}\\ \text{truncation of}\\ F_0 \text{ at } D}}\Bigg],
\end{aligned}$$

because by Lemma 1 the campaign fails with probability one if $W < D_*^\Delta(N+1, u-(i-1)\Delta)$.

For $D < D_*^\Delta(N, u+\Delta)$, the investor believes that $W = D$ with probability one. Hence, in the last period ($u = 0$), the campaign cannot succeed, since $D_*^\Delta(N, u+\Delta) < G-(N+1)p$ even if the $N+1$th investor pledges. If $u > 0$ and the $N+1$th investor pledges, then a subsequent investor arriving in state (N', D, u') with $N' \geq N+1$ and $u' < u$ pledges if $D \geq D_*^\Delta(N', u'+\Delta)$.

iii) We first show that $\pi^\Delta(N, D, u)$ is strictly increasing and continuous in D for $D_*^\Delta(N, u+\Delta) \leq D \leq G-(N+1)p$ by induction in u.

Induction start ($u = 0$): $\pi^\Delta(N, D, 0) = \frac{1-F_0(G-(N+1)p)}{1-F_0(D)}\mathbb{1}(D \geq D_*^\Delta(N, \Delta))$ is continuous and strictly increasing in D for $D_*^\Delta(N,\Delta) \leq D \leq G-(N+1)p$.

Induction hypothesis for u: $\pi^\Delta(N, D, u)$ is continuous and strictly increasing in D for $D_*^\Delta(N, u+\Delta) \leq D \leq G-(N+1)p$.

Induction step ($u \rightsquigarrow u+\Delta$): For $D_*^\Delta(N, u+2\Delta) \leq D \leq G-(N+1)p$ we have by ii)

$$\begin{aligned}
&\pi^\Delta(N, D, u+\Delta) = \\
&\sum_{i=1}^{\frac{u+\Delta}{\Delta}} (1-\Delta\lambda)^{i-1}\Delta\lambda\, \pi^\Delta\big(N+1, \max\{D, D_*^\Delta(N+1, u+\Delta-(i-1)\Delta)\}, u+\Delta-i\Delta\big)\cdot \\
&\frac{1-F_0(\max\{D, D_*^\Delta(N+1, u+\Delta-(i-1)\Delta)\})}{1-F_0(D)} + (1-\Delta\lambda)^{u/\Delta}\frac{1-F_0(G-(N+1)p)}{1-F_0(D)},
\end{aligned}$$

which is continuous in D by the induction hypothesis because $D_*^\Delta(N+1, u+\Delta-(i-1)\Delta) \leq \max\{D, D_*^\Delta(N+1, u+\Delta-(i-1)\Delta)\} \leq G-(N+1)p$ and also strictly increasing because $\frac{1-F_0(\max\{D, D_*^\Delta(N+1, u+\Delta-(i-1)\Delta)\})}{1-F_0(D)}$ is equal to 1 if $D \geq D_*^\Delta(N+1, u+\Delta-(i-1)\Delta)$ and $\frac{1}{1-F_0(D)}$ is strictly increasing in D.

Finally, if $D > G-(N+1)p$, then $\pi^\Delta(N, D, u) = 1$, and if $D < D_*^\Delta(N, u+\Delta)$, then it follows that $\pi^\Delta(N, D, u)$ is weakly increasing in D directly from (B2).

iv) By Condition i) in Definition 2 of PT assessments, $D_*^\Delta(N, u) \geq D_*^\Delta(N+1, u-\Delta) \geq D_*^\Delta(N+1, u)$. Hence, a donor w, who can incentivize the next investor to pledge in a state (N, D, u), can incentivize the next investor to pledge in state $(N+1, D, u-\Delta)$ in the next period. Thus, more future investors are incentivized to pledge after state

$(N+1, D, u-\Delta)$ than after (N, D, u), so $\pi^\Delta(N+1, D, u-\Delta) \geq \pi^\Delta(N, D, u)$. Similarly, a donor w, who can incentivize the next investor to pledge in a state $(N+1, D, u-\Delta)$, can incentivize the next investor to pledge in state $(N+1, D, u)$ in the period before. Thus, more future investors are incentivized to pledge after state $(N+1, D, u)$ than after $(N+1, D, u-\Delta)$, so $\pi^\Delta(N+1, D, u) \geq \pi^\Delta(N+1, D, u-\Delta)$.

Next, we show by induction in N that if $0 < \pi^\Delta(N, D, u) < 1$, then $\pi^\Delta(N+1, D, u) > \pi^\Delta(N, D, u)$. To this end, note that for $N+1 < \underline{M}(D)$ and $D \geq D_*^\Delta(N, u+\Delta)$ we can write by ii) for $u > 0$

$$\begin{aligned}\pi^\Delta(N, D, u) \quad = \quad & \mathbb{E}\Big[\Big(\Delta\lambda\pi^\Delta\big(N+1, \max\{D, D_*^\Delta(N+1, u)\}, u-\Delta\big) + \\ & (1-\Delta\lambda)\pi^\Delta(N, \max\{D, D_*^\Delta(N+1, u)\}, u-\Delta)\Big) \\ & \mathbb{1}\big(W \geq D_*^\Delta(N+1, u)\big)\Big| W \geq D\Big]\end{aligned}$$

because if no investor arrives in period $u-\Delta$, then the probability of success is the same as if the investor in period u arrived a period later but with a new donation threshold, i.e., it is $\pi^\Delta(N, \max\{D, D_*^\Delta(N+1, u)\}, u-\Delta)$.
Induction start ($N = \underline{M}(D)-1$): $\pi^\Delta(N+1, D, u) = 1 > \pi^\Delta(N, D, u)$.
Induction hypothesis for $N < \underline{M}(D)-1$: Assume $\pi^\Delta(N+1, D, u) > \pi^\Delta(N, D, u)$ if $0 < \pi^\Delta(N, D, u) < 1$.
Induction step ($N \rightsquigarrow N-1$): Let $0 < \pi^\Delta(N-1, D, u) < 1$. If $D \geq D_*^\Delta(N, u+\Delta)$, then

$$\begin{aligned}\pi^\Delta(N, D, u) = \mathbb{E}\Big[\Big(\Delta\lambda \; & \underbrace{\pi^\Delta\big(N+1, \max\{D, D_*^\Delta(N+1, u)\}, u-\Delta\big)}_{\substack{> \pi^\Delta(N, D, u-\Delta) \text{ by induction hypothesis} \\ \text{and monotonicity in } D}} + \\ (1-\Delta\lambda) & \underbrace{\pi^\Delta(N, \max\{D, D_*^\Delta(N, u)\}, u-\Delta)}_{\substack{> \pi^\Delta(N, D, u-\Delta) \\ \text{by monotonicity in } D}}\Big)\mathbb{1}\big(W \geq D_*^\Delta(N+1, u)\big)\Big| W \geq D\Big] \\ > \pi^\Delta(N, D, u-\Delta) & \, \mathbb{P}(W \geq \underbrace{D_*^\Delta(N+1, u)}_{< D_*^\Delta(N, u)} | W \geq D) \geq \pi^\Delta(N-1, D, u)\end{aligned} \tag{B3}$$

because $\mathbb{P}(W \geq D_*^\Delta(N, u) | W \geq D) = 1$ for $D \geq D_*^\Delta(N, u)$. If $D < D_*^\Delta(N, u+\Delta)$, then for $\pi^\Delta(N-1, D, u) > 0$,

$$\pi^\Delta(N, D, u) = \mathbb{P}(D \geq \underbrace{\max_{\substack{N < N' \leq \underline{M}(D) \\ \tau^u_{N'-N} < T}} D_*^\Delta(N', T - \tau^u_{N'-N}))}_{< \max_{\substack{N-1 < N' \leq \underline{M}(D) \\ \tau^u_{N'-N+1} < T}} D_*^\Delta(N', T - \tau^u_{N'-N+1})} > \pi^\Delta(N-1, D, u).$$

Finally, we consider strict monotonicity in u. Consider $N+1 < \underline{M}(D)$. If $D \geq$

$D_*^\Delta(N, u+\Delta)$, then (B3) implies $\pi^\Delta(N, D, u) > \pi^\Delta(N, D, u-\Delta)$, where we use the strict monotonicity of π^Δ in N. If $D < D_*^\Delta(N, u+\Delta)$, then since $\tau^{u-\Delta}_{N'-N}$ and $\tau^{u}_{N'-N}+1$ are equally distributed by the Markov property, and since $D_*^\Delta(N,u)$ is decreasing u for $D_*^\Delta(N,u) > 0$,

$$\mathbb{P}(D \geq \max_{\substack{N<N'\leq \underline{M}(D)\\ \tau^{u-\Delta}_{N'-N}<T}} D_*^\Delta(N', T-\tau^{u}_{N'-N})) > \mathbb{P}(D \geq \max_{\substack{N<N'\leq \underline{M}(D)\\ \tau^{u}_{N'-N}<T}} D_*^\Delta(N', T-\tau^{u-\Delta}_{N'-N})).$$

Hence, $\pi^\Delta(N, D, u) > \pi^\Delta(N, D, u-\Delta)$ as long as $\pi^\Delta(N, D, u) \in (0,1)$. ■

For the construction of the donation thresholds, it is useful to consider the auxiliary probability of success in a state (N, D, u) if the investor believed that donor wealth was distributed according to F_0 truncated at D for all D:

$$\text{(B4)} \quad \begin{aligned} \tilde{\pi}^\Delta(N, D, u) \quad := \quad & \sum_{i=1}^{\frac{u}{\Delta}} (1-\Delta\lambda)^{i-1}\Delta\lambda \, \frac{1-F_0(\max\{D, D_*^\Delta(N+1, u-(i-1)\Delta)\})}{1-F_0(D)} \\ & \pi^\Delta\Big(N+1, \max\{D, D_*^\Delta(N+1, u-(i-1)\Delta)\}, u-i\Delta\Big) \\ & + (1-\Delta\lambda)^{u/\Delta}\frac{1-F_0(G-(N+1)p)}{1-F_0(D)}. \end{aligned}$$

The following is a corollary of Lemma 2. We use it in the proof of Proposition 1 to define the donation threshold $\underline{D}(N,u)$.

Corollary 1. *The auxiliary probability of success $\tilde{\pi}^\Delta(N, D, u)$ is continuous and (strictly) increasing in D (as long as $\tilde{\pi}^\Delta(N, D, u) \in (0,1)$).*

Finally, Lemma 3 shows that the donor strategy specified in any PT assessment is a best response to the specified investor strategy.

Lemma 3. *For any PT assessment with donation threshold $D_*^\Delta(N,u)$, the donor PT strategy is a best response to the investor strategy.*

Proof. We argue by backwards induction in t.
Induction start ($t = T$): First, consider histories in the last period $h_T^{D,\Delta}$ with cumulative contributions N_T and $D_{T-\Delta}$. Ignoring the constraint imposed by previous donations, the donor would want to donate $\min\{w, G-N_T p\}$, because he would want to give just enough for the campaign to succeed without exceeding his valuation. However, the donor cannot take out funds. Thus, a cumulative donation of $\max\{D_{T-\Delta}, \min\{w, G-N_T p\}\}$ is a best response. Hence, in all histories that correspond to a state $(N, D, 0)$, a Markov strategy of $\tilde{D}_+^\Delta(h_T^{D,\Delta}; w) = D_+^\Delta(N_T, D_{T-\Delta}, 0; w) = \max\{D_{T-\Delta}, \min\{w, G-N_T p\}\}$ is optimal.
Induction hypothesis for $s \geq t$: Next, we assume that for all $s \geq t$ and all $h_s^{D,\Delta}$ with corresponding cumulative contributions N_s and $D_{s-\Delta}$, the donor payoff is maximized by $\tilde{D}_+^\Delta(h_s^{D,\Delta}; w) = D_+^\Delta(N_s, D_{s-\Delta}, T-s; w) = \max\{D_{s-\Delta}, \min\{w, D_*^\Delta(N_s, T-s)\}\}$.
Induction step ($t \rightsquigarrow t-\Delta$): Consider an arbitrary donor strategy $\tilde{D}_+^\Delta$ where for all $s \geq t$, $\tilde{D}_+^\Delta(h_s^{D,\Delta}; w) = D_+^\Delta(N_s, D_{s-\Delta}, T-s; w) = \max\{D_{s-\Delta}, \min\{w, D_*^\Delta(N_s, T-

$s)\}\}$. Consider an on-path history $h_{t-\Delta}^{D,\Delta}$ with corresponding cumulative contributions $N_{t-\Delta}, D_{t-2\Delta}$ and a donor valuation $w \geq \max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$ such that

$$\tilde{D}_+^\Delta\left(h_{t-\Delta}^{D,\Delta}; w\right) < D_*^\Delta(N_{t-\Delta}, T-(t-\Delta)).$$

According to the PT assessment, if an investor arrives in period t, the investor does not pledge. Since $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta)) < D_*^\Delta(N_{t-\Delta}, u)$ for all $u < T-(t-\Delta)$, the donor needs to donate at least $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$ in order to make a future investor pledge and to prevent the campaign from failing. Furthermore, $D_*^\Delta(N_{t-\Delta}, u) > D_*^\Delta(N', u)$ for all $N' > N_{t-\Delta}$. Hence, a donor with valuation w is strictly better off by donating $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$ after history $h_{t-\Delta}^{D,\Delta}$, so an optimal donor strategy must be to give at least $D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))$. Similarly, monotonicity of D_*^Δ in N, u implies that it cannot be optimal that the donor gives more than $\max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$. If $w < \max\{D_{t-2\Delta}, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}$, the campaign succeeds with probability zero, because cumulative donations are below w. Thus, a best-response donor strategy is given by

$$\tilde{D}_+^\Delta\left(h_{t-\Delta}^{D,\Delta}; w\right) = \max\{D_{t-2\Delta}, \min\{w, D_*^\Delta(N_{t-\Delta}, T-(t-\Delta))\}\}.$$

■

PROPERTIES OF PT EQUILIBRIA

Recall that a PT equilibrium is a PT assessment $(b^\Delta, D_+^\Delta, F^\Delta)$ such that given the induced probability of success $\pi^\Delta(\mathbf{x})$, we have buyer optimality: $\pi(\mathbf{x}) > \frac{v_0}{v-p} \Rightarrow b^\Delta(\mathbf{x}) = 1$ and $\pi(\mathbf{x}) < \frac{v_0}{v-p} \Rightarrow b^\Delta(\mathbf{x}) = 0$. Donor optimality is guaranteed automatically by Lemma 3. The buyer optimality condition allows us to define cutoff times $\xi_j^\Delta(w)$ as in Equation CT for each j, w, with $j \leq \underline{M}(w)$. We can show that $\xi_j^\Delta(w)$ is monotone in j.

Lemma 4. *In any PT equilibrium, the cutoff time $\xi_j^\Delta(w)$ is strictly increasing in j.*

Proof. By Lemma 2 iv), we have for $j' > j$ that if $\pi^\Delta(\underline{M}(w)-j', w, u) \geq \frac{v_0}{v-p}$, then $\pi^\Delta(\underline{M}(w)-j, w, u-\Delta) \geq \pi^\Delta(\underline{M}(w)-j', w, u) \geq \frac{v_0}{v-p}$, so

$$\pi^\Delta(\underline{M}(w)-j, w, \xi_{j'}^\Delta(w)-\Delta) \geq \pi^\Delta(\underline{M}(w)-j', w, \xi_{j'}^\Delta(w)) \geq \frac{v_0}{v-p}.$$

Hence, $\xi_j^\Delta(w) \leq \xi_{j'}^\Delta(w)-\Delta < \xi_{j'}^\Delta(w)$. ■

As a result, in a PT equilibrium, after $\xi_j^\Delta(w)$ is reached, no buyer pledges, i.e.,

$$\text{(B5)}\qquad \begin{cases} \pi^\Delta\left(\underline{M}(w)-j, w, u\right) \geq \frac{v_0}{v-p} \text{ for } u \geq \xi_j^\Delta(w) \\ \pi^\Delta\left(\underline{M}(w)-j, w, u\right) < \frac{v_0}{v-p} \text{ for } u < \xi_j^\Delta(w) \end{cases}$$

$\xi^{\Delta}_{j}(w) > \xi^{\Delta}_{j'}(w)$ for any $j > j'$ if $\xi^{\Delta}_{j}(w) > 0$. Furthermore, $w \geq D^{\Delta}_{*}(N, u+\Delta) \Leftrightarrow u \geq \xi^{\Delta}_{\underline{M}(w)-N}(w)$. This allows us to rewrite the probability of success in a different way. For $N < \underline{M}(D)-1$ and $u > 0$, the probability of success is given by

$$
\text{(B6)}\qquad
\begin{aligned}
\pi^{\Delta}(N,D,u) = \quad & \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\max\{(u-\xi^{\Delta}_{\underline{M}(W)-(N+1)}(W))/\Delta,0\}} (1-\Delta\lambda)^{i-1}\Delta\lambda \\
& \pi^{\Delta}\big(N+1,\, \max\{D, D^{\Delta}_{*}(N+1, u-\Delta(i-1))\},\, u-\Delta i\big) \\
& +(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W \geq G-(N+1)p)\Big| W \geq D\Bigg]
\end{aligned}
$$

if $D \geq D^{\Delta}_{*}(N, u+\Delta)$. If $D < D^{\Delta}_{*}(N, u+\Delta)$, $\pi^{\Delta}(N,D,u) < \frac{v_0}{v-p}$.

B2. Proof of Proposition 1 (Success-Maximizing Equilibrium)

In Subsection B.B2, we first construct a PT equilibrium. Subsection B.B2 states that the limit of these equilibria as $\Delta \to 0$ exists and is as specified in Proposition 1. The limit is formally derived in the Online Appendix. Finally, in Subsection B.B2, we show that for any $\Delta > 0$, the constructed equilibrium maximizes the probability of success and that the outcomes of any sequence of success-maximizing PBE converge to the same limit.

CONSTRUCTION OF A PT EQUILIBRIUM

The following lemma specifies a PT equilibrium with a donation threshold that makes the next investor just indifferent between pledging and not.

Lemma 5 (Success-maximizing equilibrium)**.** *Given any $\Delta > 0$, there exists a PT equilibrium $(b^{\Delta}, D^{\Delta}_{+}, F^{\Delta})$ with donation threshold $\underline{D}^{\Delta}(N,u)$ and induced probability of success $\pi^{\Delta}(\mathbf{x})$, $\mathbf{x} \in \mathbb{X}^{\Delta}$ such that for $u > 0$*

$$
\begin{cases}
\underline{D}^{\Delta}(N,u) = 0 & \text{if } \pi^{\Delta}(N,0,u-\Delta) > \frac{v_0}{v-p}, \\
\pi^{\Delta}\big(N, \underline{D}^{\Delta}(N,u), u-\Delta\big) = \frac{v_0}{v-p} & \text{if } \pi^{\Delta}(N,0,u-\Delta) \leq \frac{v_0}{v-p}.
\end{cases}
$$

We denote by $\underline{\pi}(N,D,u)$ the corresponding probability of success from the investor's perspective in state (N,D,u) if the investor contributes.

Proof. We construct the equilibrium strategies and beliefs for every state (N,D,u) by induction in $j = \underline{M}(D)-N$. In order to define the donation threshold $\underline{D}(N,u)$ such that investors are indifferent between pledging and not, we need to know the probability of success $\pi^{\Delta}(N,D,u)$ induced by the assessment for arbitrary D. We tackle this issue by constructing a sequence of PT assessments $(b^{\Delta}_{j}, D^{\Delta}_{+,j}, F^{\Delta}_{j})$ for $j = 1,\ldots,M_0 = \underline{M}(0)$ such that $(b^{\Delta}_{M_0}, D^{\Delta}_{+,M_0}, F^{\Delta}_{M_0})$ is a PBE and satisfies the properties in Lemma 5. We start with an arbitrary PT assessment $(b^{\Delta}_{1}, D^{\Delta}_{+,1}, F^{\Delta}_{1})$. The induction hypothesis assumes that for each $1 \leq j' \leq j-1$ there is a PT assessment $(b^{\Delta}_{j'}, D^{\Delta}_{+,j'}, F^{\Delta}_{j'})$

Table B1—: List of Notation

Notation	Description
$(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$	assessment in the j-th induction step
$\xi_j^\Delta(w)$	time threshold defined for all w in the j-th induction step
$D_{*,j}^\Delta(N,u)$	donation threshold corresponding to $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$
$\underline{D}^\Delta(N,u)$	donation threshold that is defined inductively for $N = M_0 - j$, and $N < M_0 - j$ and $u \le \xi_j^\Delta(G-(N+j)p)$

such that in states (N, D, u) with $\underline{M}(D) - N \le j'$ investor strategies are optimal, i.e., in the continuation games after such states, the assessment specifies a PBE. Donor strategies are automatically optimal in a PT assessment, by Lemma 3. Then, in the induction step $j-1 \rightsquigarrow j$, we construct a PT assessment $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$ such that for states (N, D, u) with $\underline{M}(D) - N \le j$, investor strategies are optimal, and

$$\left.\begin{aligned} b_j^\Delta(N,D,u) &= b_{j-1}^\Delta(N,D,u),\\ D_{+,j}^\Delta(N,D,u) &= D_{+,j-1}^\Delta(N,D,u),\\ F_j^\Delta(N,D,u) &= F_{j-1}^\Delta(N,D,u), \end{aligned}\right\} \text{ for all states } (N,D,u) \text{ with } \underline{M}(D)-N \le j-1,$$

which implies that for the corresponding probabilities of success we have

$$\pi_j^\Delta(N,D,u) = \pi_{j-1}^\Delta(N,D,u) \text{ for } \underline{M}(D) - N \le j-1.$$

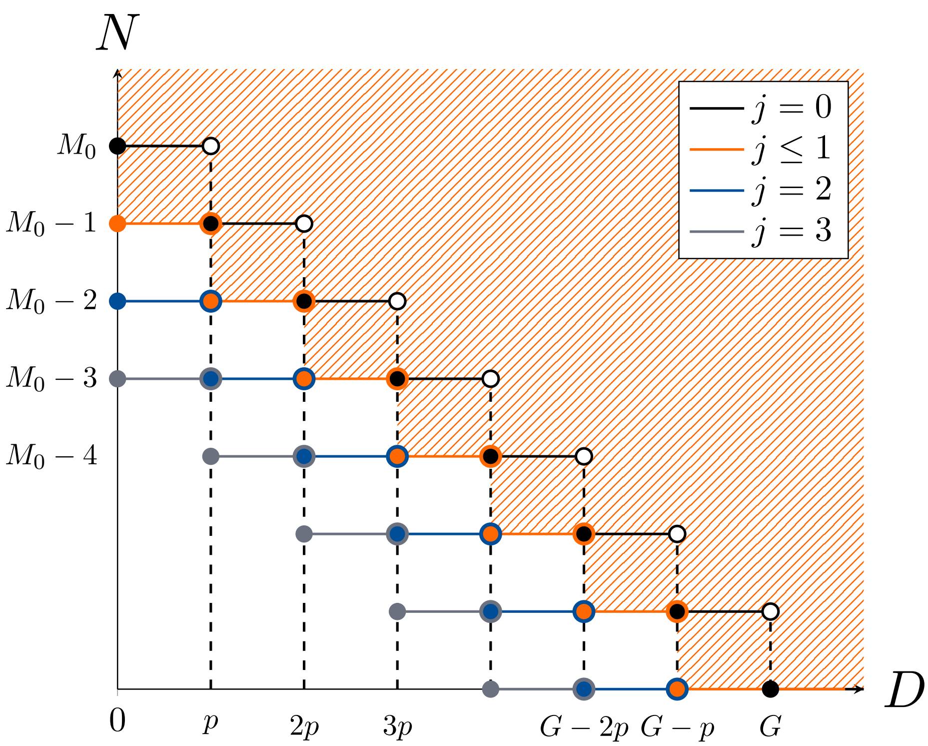
Figure B1 depicts pairs of (N, D) such that $j = \underline{M}(D) - N$ for $j = 0, 2, 3$ and the shaded region including the orange line captures all $j \le 1$, which is our induction start for the equilibrium construction. The induction ends at $j = M_0$, when the entire state space is covered. Importantly, if the game is in state (N, D, u), then N and D increase only in the continuation game, i.e., j is decreasing over time.

While we denote by $D_{*,j}^\Delta(N,u)$ the donation threshold corresponding to $(b_j^\Delta, D_{+,j}^\Delta, F_j^\Delta)$, we also construct $\xi_j^\Delta(\cdot)$ and parts of the threshold function $\underline{D}^\Delta(N,u)$ in each step. In particular, in step j, we define $\underline{D}^\Delta(N,u)$ for (N,u) such that $N = M_0 - j$, or such that $N < M_0 - j$ and $u \le \xi_j^\Delta(G-(N+j)p)$. After the last step ($j = M_0$), $\underline{D}^\Delta(N,u)$ is defined for all N and u and $\underline{D}^\Delta(N,u) = D_{*,M_0}^\Delta(N,u)$. Figure B2 illustrates this construction schematically. For a cleaner illustration that avoids drawing step functions, we assume $\Delta \to 0$ in this figure.

Finally, Table B1 summarizes the relevant notation.

(a) Induction start ($j \le 1 \Leftrightarrow D \ge G - (N+1)p$): We set $(b_1^\Delta, D_{+,1}^\Delta, F_1^\Delta)$ to be an arbitrary PT assessment (which trivially exists). Further, for $j \le 1$, we set $\xi_j^\Delta(w) := 0$ for all w, which is consistent with Equation CT. We also set $\underline{D}(N,u) := 0$ for $N \ge M_0 - 1$. Finally, consider states (N, D, u) with $\underline{M}(D) - N \le 1$. The probability of success is $\pi_1^\Delta(N,D,u) = 1$, so it is a best response for investors to pledge. Trivially, $\pi_1^\Delta(N,D,u)$

Figure B1. : Schematic illustration of induction in $j = \underline{M}(D) - N$



Note: The figure depicts pairs of (N, D) such that $j = \underline{M}(D) - N$ for $j = 0, j \leq 1, j = 2, j = 3$. The induction start considers states $j \leq 1$, and each $j > 1$ corresponds to one induction step.

is weakly increasing in N, D, u for $D \geq G - (N+1)p$.

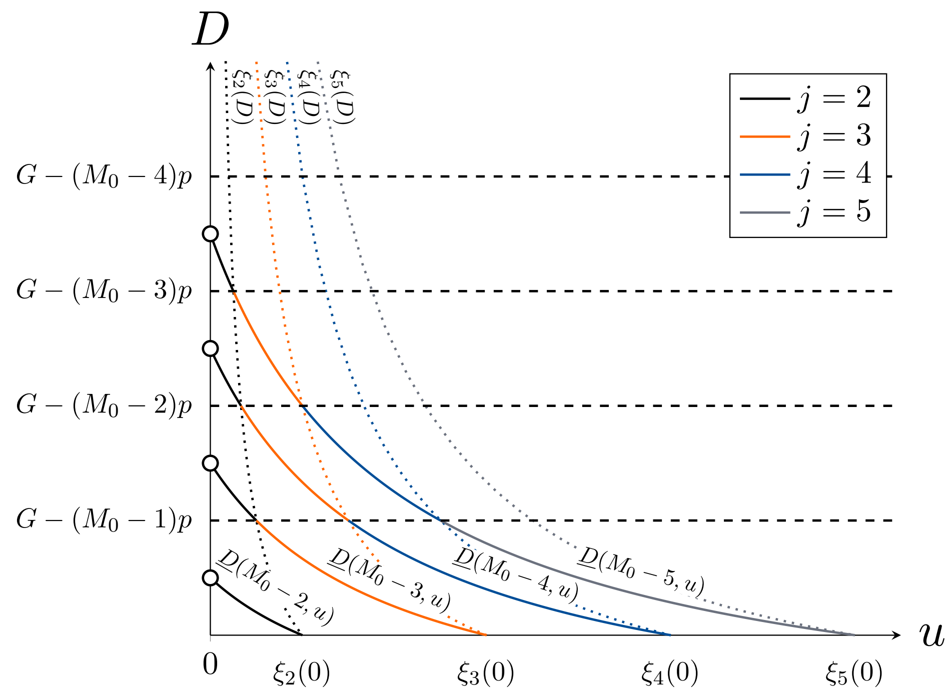
(b) Induction hypothesis ($j' \leq j-1$): For the induction hypothesis, we suppose that we have constructed PT assessments $(b^{\Delta}_{j'}, D^{\Delta}_{+,j'}, F^{\Delta}_{j'})$ with a donation threshold $D^{\Delta}_{*,j'}(N, u)$ for $j' = 1, \ldots, j-1$ with the following properties:

i) **Time threshold $\xi^{\Delta}_{j'}(w)$:** For $w < G - (j'-1)p$, we define $\xi^{\Delta}_{j'}(w)$ by (B5). For $w \geq G - (j'-1)p$, we set $\xi^{\Delta}_{j'}(w) = 0$. $\xi^{\Delta}_{j'}(w) > \xi^{\Delta}_{j'-1}(w)$ if $\xi^{\Delta}_{j'}(w) > 0$.

ii) **Donation threshold $\underline{D}^{\Delta}(N, u)$:** Donation threshold $\underline{D}^{\Delta}(N, u)$ is defined for (N, u) such that either $N \geq M_0 - (j-1)$ or $N < M_0 - (j-1)$ and $u \leq \xi^{\Delta}_{j-1}(G - (N + j - 1)p)$. For (N, u) with $N \leq M_0 - j'$ and $u \leq \xi^{\Delta}_{j'}(G - (N + j')p)$,

(B7)
$$\pi^{\Delta}_{j-1}\big(N,\, \underline{D}^{\Delta}(N, u),\, u - \Delta\big) = \frac{v_0}{v - p}.$$

Figure B2. : Schematic illustration of construction of $\underline{D}^{\Delta}(N,u)$ and $\xi_j(D)$ (for small Δ)



Note: The figure depicts the donation thresholds $\underline{D}(M_0-j,u)$ as a function of u in the limit $\Delta\to 0$. In step j, the portion between $\xi_{j-1}(G-(N+j-1)p)$ and $\xi_j(G-(N+j)p)$ of each $\underline{D}(N,u)$ is constructed.

Note that in that case, $\underline{D}^{\Delta}(N,u)<G-(N+1)p$. For $N=M_0-j'$, $u>\xi^{\Delta}_{j'}(0)$, $\underline{D}^{\Delta}(N,u)=0$. $\underline{D}^{\Delta}(N,u)$ is strictly decreasing in N, u when it satisfies (B7).

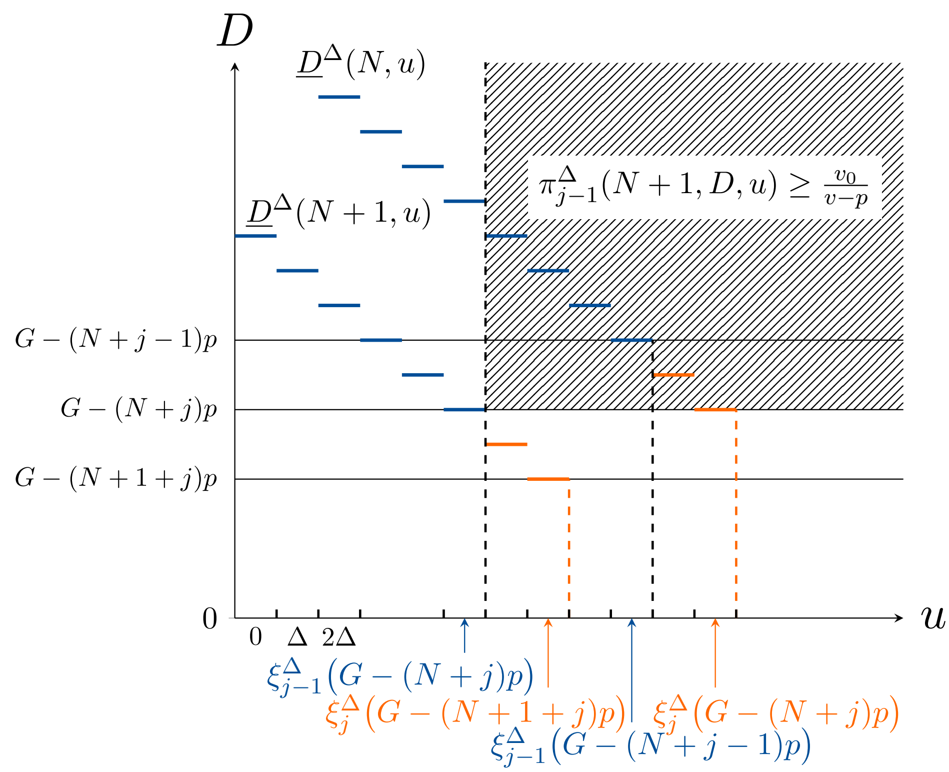
In Figure B3, the blue step functions represent the portion of $\underline{D}^{\Delta}$ at N and $N+1$ that are defined in the induction hypothesis, and black dotted lines show the corresponding $\xi^{\Delta}_{j-1}\big(G-(N+j-1)\big)$ and $\xi^{\Delta}_{j-1}\big(G-(N+1+j-1)p\big)=\xi^{\Delta}_{j-1}\big(G-(N+j)p\big)$.

iii) **PT assessment:** Here $(b^{\Delta}_{j'},D^{\Delta}_{+,j'},F^{\Delta}_{j'})$ are PT assessments (as in Definition 2) with donation thresholds $D^{\Delta}_{*,j'}(N,u)$ satisfying

$$D^{\Delta}_{*,j'}(N,u)=\underline{D}^{\Delta}(N,u)\qquad \text{for } u\le \xi^{\Delta}_{j'}(G-(N+j')p),\text{ and}$$
$$\text{for } N=M_0-j',\, u>\xi^{\Delta}_{j'}(0).$$

iv) **Probability of success:** For all $N\ge \underline{M}(D)-j'$, $\pi^{\Delta}_{j'}(N,D,u)$ satisfies (B6) if $D\ge D^{\Delta}_{*,j'}(N,u+\Delta)$ and $\pi^{\Delta}_{j'}(N,D,u)<\frac{v_0}{v-p}$ if $D<D^{\Delta}_{*,j'}(N,u+\Delta)$.

Figure B3. : Schematic illustration of construction of $\underline{D}^{\Delta}(N,u)$ for N and $N+1$



Note: The figure depicts the donation thresholds for cumulative purchases N and $N+1$ with $N < M_0 - j$. In step $j-1$ only the blue portion of $\underline{D}^{\Delta}$ is constructed, while in step j the orange portion is added. For example, we construct $\underline{D}^{\Delta}(N+1,u)$ for $u \leq \xi^{\Delta}_{j-1}(G-(N+1+j-1)p)$ in step $j-1$ and extend it to $u \leq \xi^{\Delta}_{j-1}(G-(N+1+j)p)$ in step j. With $D \geq G-(N+j)p$, and $N+1$ purchases, the campaign is active until $\xi^{\Delta}_{j-1}\big(G-(N+j)p\big)+\Delta$ or longer, even if no additional donations are being made (shaded area). For such states, strategies of the assessment $(b^{\Delta}_{j-1}, D^{\Delta}_{+,j-1}, F^{\Delta}_{j-1})$ are not optimal and π^{Δ}_{j-1} might not be increasing and continuous in D. We only assume $\pi^{\Delta}_{j-1} \geq 0$. Hence, the donation threshold cannot be constructed for $(N+1,u)$ with $u > \xi^{\Delta}_{j-1}(G-(N+j)p)$ in step $j-1$.

Note that by monotonicity of $\pi^{\Delta}_{j'}(N,D,u)$ in N,u (Lemma 2 iii),

$$\pi^{\Delta}_{j'}(N,D,u) \geq \frac{v_0}{v-p} \qquad \text{for } u > \xi^{\Delta}_{j'}(G-(N+j')p),\ D \geq G-(N+j')p.$$

This is illustrated in Figure B3 in the shaded area. Similarly, it implies that $u \leq \xi^{\Delta}_{j'}(D) \Leftrightarrow D < D^{\Delta}_{*,j'}(\underline{M}(D)-j', u+\Delta) = \underline{D}^{\Delta}(\underline{M}(D)-j', u+\Delta)$.

v) **Best response:** For the PT assessments $(b^{\Delta}_{j'}, D^{\Delta}_{+,j'}, F^{\Delta}_{j'})$, investors best respond by pledging if and only if $D \geq D^{\Delta}_{*,j'}(N,u+\Delta)$ in states all (N,D,u) with $N \geq \underline{M}(D)-j$.

(c) Induction step ($j-1 \rightsquigarrow j$, $j \geq 2$): In this step, we assume the induction hypothesis (b) is true and construct a PT assessment $(b^{\Delta}_{j}, D^{\Delta}_{+,j}, F^{\Delta}_{j})$ such that the same statements are true for states (N,D,u) with $N = \underline{M}(D)-j$, i.e., $G-(N+j)p \leq D < G-\big(N+(j-1)\big)p$.

i) **Time threshold $\xi^{\Delta}_{j}(w)$:** First, note that for $w \geq G-(N+j)p$, there is a $j' \leq j-1$ such that $\underline{M}(w)-j' = N+1$. Then, we know by the induction hypothesis that

$$\begin{cases} \text{for } u' < \xi^{\Delta}_{j'}(w): & w < D^{\Delta}_{*,j-1}(N+1,u') = \underline{D}^{\Delta}(N+1,u') \\ \text{for } \xi^{\Delta}_{j'}(w) \leq u' \leq \xi^{\Delta}_{j-1}\big(G-(N+j)p\big): & w \geq \underline{D}^{\Delta}(N+1,u') = D^{\Delta}_{*,j-1}(N+1,u') \\ \text{for } u' > \xi^{\Delta}_{j-1}\big(G-(N+j)p\big) & w \geq \underline{D}^{\Delta}\Big(N+1, \xi^{\Delta}_{j-1}\big(G-(N+j)p\big)\Big) \\ & > D^{\Delta}_{*,j-1}(N+1,u') \end{cases}$$

Hence, $w \geq D^{\Delta}_{*,j-1}(N+1,u') \Leftrightarrow u' \geq \xi^{\Delta}_{j'}(w)$. Therefore, letting $\tilde{\pi}^{\Delta}_{j-1}(N,D,u)$ be the auxiliary probability corresponding to the assessment $(b^{\Delta}_{j-1}, D^{\Delta}_{+,j-1}, F^{\Delta}_{j-1})$ as defined in Equation B4, we can write

$$\begin{aligned} \tilde{\pi}^{\Delta}_{j-1}(N,D,u) = \quad & \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\max\{(u-\xi^{\Delta}_{\underline{M}(W)-(N+1)}(W))/\Delta,0\}} (1-\Delta\lambda)^{i-1}\Delta\lambda \\ & \pi^{\Delta}_{j-1}\big(N+1, \max\{D, D^{\Delta}_{*,j-1}(N+1,u-\Delta(i-1))\}, u-\Delta i\big) \\ & +(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W \geq G-(N+1)p)\Big| W \geq D\Bigg]. \end{aligned}$$

Next, note that the above also implies that for $u-i\Delta < \xi^{\Delta}_{j-1}(D)$, then $D < D^{\Delta}_{*,j-1}(N+1, u-(i-1)\Delta) = \underline{D}^{\Delta}(N+1,u-(i-1)\Delta)$ and for $u-i\Delta \geq \xi^{\Delta}_{j-1}(D)$, $D \geq D^{\Delta}_{*,j-1}(N+1,u-(i-1)\Delta)$. Hence, for $j = \underline{M}(D)-N$

$$\begin{aligned} & \tilde{\pi}^{\Delta}_{j-1}(N,D,u) = \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\max\{(u-\xi^{\Delta}_{\underline{M}(W)-(N+1)}(W))/\Delta,0\}} (1-\Delta\lambda)^{i-1}\Delta\lambda \\ & \Big(\pi^{\Delta}_{j-1}\big(N+1,D,u-\Delta i\big)\mathbb{1}\big(u-\Delta i \geq \xi^{\Delta}_{j-1}(D)\big) + \tfrac{v_0}{v-p}\mathbb{1}\big(u-\Delta i < \xi^{\Delta}_{j-1}(D)\big)\Big) \\ & +(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W \geq G-(N+1)p)\Big| W \geq D\Bigg]. \end{aligned}$$

This expression depends only on $\xi^{\Delta}_{j'}(\cdot)$, $j' \leq j-1$, and $\pi^{\Delta}_{j-1}(N+1,D,u')$, where $\underline{M}(D)–(N+1) \leq j–1$, which are defined in the induction hypothesis. Since $\pi_{j-1}(N,D,u)$ is strictly increasing in u and $\pi^{\Delta}_{j-1}\big(N+1,D,u-\Delta i\big) \geq \frac{v_0}{v-p}$ for $u-\Delta i \geq \xi^{\Delta}_{j-1}(D)$, $\tilde{\pi}_{j-1}(N,D,u) < 1$ is strictly increasing in u. Hence, for any $j \leq \underline{M}(D)$ there is a unique

$$\xi^{\Delta}_{j}(D) = \arg\min\left\{u \middle| \tilde{\pi}^{\Delta}_{j-1}\big(\underline{M}(D)-j,D,u\big) \geq \frac{v_0}{v-p}\right\}.$$

Recall that $\pi^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)=\tilde{\pi}^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)$ for $D\ge D^{\Delta}_{*,j-1}(\underline{M}(D)-(j-1),u+\Delta)$, and by the induction hypothesis, $\pi^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)<\frac{v_0}{v-p}$ and $\tilde{\pi}^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)<\frac{v_0}{v-p}$ for $D<D^{\Delta}_{*,j-1}(\underline{M}(D)-(j-1),u+\Delta)$. Hence,

$$\begin{cases} \tilde{\pi}^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)\ge\frac{v_0}{v-p} \text{ for } u\ge\xi^{\Delta}_{j-1}(D) \\ \tilde{\pi}^{\Delta}_{j-1}\big(\underline{M}(D)-(j-1),D,u\big)<\frac{v_0}{v-p} \text{ for } u<\xi^{\Delta}_{j-1}(D) \end{cases},$$

and we have $\xi^{\Delta}_{j}(w)>\xi^{\Delta}_{j-1}(w)$ if $\xi^{\Delta}_{j}(w)>0$.

ii) **Donation threshold $\underline{D}^{\Delta}(N,u)$:** Since $(b^{\Delta}_{j-1},D^{\Delta}_{+,j-1},F^{\Delta}_{j-1})$ is a PT assessment by the induction hypothesis, $\tilde{\pi}^{\Delta}_{j-1}\big(N,D,u\big)$ is strictly increasing in D by Corollary 1. For $N\ge M_0-j$ and $u<\xi^{\Delta}_{j}(G-(N+j)p)$, we define $\underline{D}^{\Delta}(N,u+\Delta)$ to be the unique value satisfying

$$\tilde{\pi}^{\Delta}_{j-1}\big(N,\underline{D}^{\Delta}(N,u+\Delta),u\big)=\frac{v_0}{v-p},$$

which must also be satisfied for $u<\xi_{j'}(G-(N+j')p)$, $N\ge M_0-j', j'\le j-1$ by the induction hypothesis ii). Since $\tilde{\pi}^{\Delta}_{j-1}$ is increasing in N,D, and u, $\underline{D}^{\Delta}$ is decreasing in N and u. Further, for $N=M_0-j$, we set $\underline{D}^{\Delta}(N,u+\Delta)=0$ for $u>\xi^{\Delta}_{j}(0)$.

iii) **PT assessment:** We set

$$\begin{aligned} D^{\Delta}_{*,j}(N,u):=\underline{D}^{\Delta}(N,u) \qquad & \text{for } u\le\xi^{\Delta}_{j}\big(G-(N+j)p\big), \text{ and} \\ & \text{for } N=M_0-j,\, u>\xi^{\Delta}_{j}(0), \end{aligned}$$

and, otherwise, define $D^{\Delta}_{*,j}(N,u)$ arbitrarily so that it is overall decreasing in N and u. This defines a PT assessment $(b^{\Delta}_{j},D^{\Delta}_{+,j},F^{\Delta}_{j})$. Note that $(b^{\Delta}_{j},D^{\Delta}_{+,j},F^{\Delta}_{j})=(b^{\Delta}_{j-1},D^{\Delta}_{+,j-1},F^{\Delta}_{j-1})$ for states (N,D,u) with $\underline{M}(D)-N\le j-1$ because for all such states $D^{\Delta}_{*,j-1}(N,u)=D^{\Delta}_{*,j}(N,u)$.

iv) **Probability of success:** The corresponding probability of success has the following properties:

- First, $\pi^{\Delta}_{j}(N,D,u)=\pi^{\Delta}_{j-1}(N,D,u)$ for $\underline{M}(D)-N\le j-1$ by definition of the corresponding donation thresholds, because $(b^{\Delta}_{j},D^{\Delta}_{+,j},F^{\Delta}_{j})=(b^{\Delta}_{j-1},D^{\Delta}_{+,j-1},F^{\Delta}_{j-1})$ for these states and all states (N',D',u') with $N'\ge N$, $D'\ge D$ that can be reached in a continuation game, as they satisfy $\underline{M}(D')-N'\le j-1$.

- For $D\ge D^{\Delta}_{*,j}(N,u+\Delta)$, $\pi^{\Delta}_{j}(N,D,u)=\tilde{\pi}^{\Delta}_{j-1}(N,D,u)$ by Lemma 2 ii), and for $D<D^{\Delta}_{*,j}(N,u+\Delta)$, $\pi^{\Delta}_{j}(N,D,u)<\frac{v_0}{v-p}$ and $\tilde{\pi}^{\Delta}_{j-1}(N,D,u)<\frac{v_0}{v-p}$ by monotonicity of the probabilities in D. Hence, $\xi^{\Delta}_{j}(D)$ satisfies (B5). Further, this implies that $\pi^{\Delta}_{j}(N,D,u)$ is strictly increasing in u for $D\ge D^{\Delta}_{*,j}(N,u+\Delta)$, $N+1<\underline{M}(D)$. Otherwise, $\pi^{\Delta}_{j}(N,D,u)=1$ or $\pi^{\Delta}_{j}(N,D,u)$ is given by (B2), which is strictly increasing in u or equal to zero.

v) **Best response:** It is immediate from the construction and because π_j^Δ is increasing in D that for all (N, D, u) with $N \geq \underline{M}(D) - j$, $\pi_j^\Delta(N, D, u) \geq \frac{v_0}{v-p}$ if and only if $D \geq D_{*,j}^\Delta(N, u+\Delta)$. ■

TAKING THE CONTINUOUS TIME LIMIT

The following lemma implies Proposition 1 ii):

Lemma 6 (Success-maximizing equilibrium limit)**.** ***i)*** *The pointwise limit of the donation threshold* $\underline{D}(N, u) := \lim_{\Delta\to 0} \underline{D}^\Delta(N, \lceil \frac{u}{\Delta} \rceil \Delta)$ *exists, where* $\lceil \frac{u}{\Delta} \rceil \Delta$ *is the smallest multiple of* Δ *that is larger than* u*. Further, for any* $\mathbf{x} = (N, D, u)$ *the following pointwise limits exist:*

$$
\begin{aligned}
b(\mathbf{x}) &:= \lim_{\Delta\to 0} b^\Delta\big(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\big), \quad & D_+(\mathbf{x}; w) &:= \lim_{\Delta\to 0} D_+^\Delta(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta; w), \\
\xi_j(w) &:= \lim_{\Delta\to 0} \xi_j^\Delta(w), & F(w; \mathbf{x}) &:= \lim_{\Delta\to 0} F^\Delta\Big(w; \big(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\big)\Big)
\end{aligned}
\tag{B8}
$$

Finally,

$$
\pi(N, D, u) := \lim_{\Delta\to 0} \pi^\Delta(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta) \text{ uniformly in } u \text{ and } D.
\tag{B9}
$$

ii) *Proposition 1 ii) holds for this limit.*

The proof of this lemma is in the Online Appendix.

OPTIMALITY OF CONSTRUCTED EQUILIBRIUM

PROOF OUTLINE:

Next, we show that the equilibrium constructed in Section B.B2 maximizes the probability of success and that for any success-maximizing sequence of PT equilibria, the outcome converges pointwise to the same limit as specified in Proposition 1. The proof proceeds in four steps. In **Step 1**, we formulate a relaxed version of the success-maximization problem. In **Step 2**, we solve the relaxed problem. In **Step 3** we show that the outcome of the solution is attained by the equilibrium constructed in Section B.B2. In **Step 4** we show convergence as $\Delta \to 0$.

The key idea of the proof stems from the observation that the donor will always donate enough to reach the goal at the deadline if it is needed and feasible. Hence, to maximize the probability of success, the exact amount the donor donates during the campaign before the deadline is not important as long as investors keep pledging. To find the PBE outcomes that maximize the probability of success, we consider reduced histories that ignore donation amounts and keep track of only whether a donation incentivizes the next potential investor to pledge or not. This idea allows us to recast the success-maximization problem into one in which we choose probabilities of reaching these reduced histories, rather than choosing over the set of PBEs.

Proof:
Step 1: The relaxed success-maximization problem

Consider a particular assessment $\left(\tilde{D}_{+}^{\Delta}, \tilde{b}^{\Delta}, \tilde{F}^{\Delta}\right)$. Given this assessment, any investor history $h_t^{B,\Delta} = \prod_{s\in\mathbb{T}^{\Delta}, s\le t} \left(N_{s-\Delta}, D_{s-\Delta}\right)$ corresponds to a reduced investor history

$$\tilde{h}_t^B := \prod_{s\in\mathbb{T}^{\Delta}, s\le t} \left(N_{s-\Delta}, b_{s-\Delta}\right), \quad \text{where} \quad b_{s-\Delta} := \tilde{b}^{\Delta}\left(\prod_{s'\in\mathbb{T}^{\Delta}, s'\le s} (N_{s'-\Delta}, D_{s'-\Delta})\right),$$

so that instead of recording the donation $D_{s-\Delta}$, the history records the probability $b_{s-\Delta} \in [0,1]$ with which an investor arriving in period s pledges on observing cumulative donation amount $D_{s-\Delta}$, and the entire history of donations and pledges. We omit the Δ-superscipts for the reduced histories, to simplify notation. Let $\mathscr{R}_{\tilde{b}^{\Delta}}$ be the mapping so that

$$\mathscr{R}_{\tilde{b}^{\Delta}} : h_t^{B,\Delta} \mapsto \tilde{h}_t^B$$

as defined above. We will use this mapping in the proof of Proposition 3.

In a platform-optimal equilibrium, the investor always pledges when she is indifferent between pledging and not pledging, so henceforth we assume $b_{s-\Delta} \in \{0,1\}$. Let the set of such reduced investor histories in period t be $\tilde{\mathscr{H}}_t^B$. Further, let us denote the corresponding set of reduced donor histories in period t by

$$\tilde{\mathscr{H}}_t^D := \left\{ \tilde{h}_t^D = \left(\tilde{h}_t^B, N_t\right) \,\middle|\, \tilde{h}_t^B \in \tilde{\mathscr{H}}_t^B,\ N_t \in \{N_{t-\Delta}, N_{t-\Delta}+1\} \right\}.$$

The assessment, the arrival process, and distributions of donor valuation define a probability measure $\mathbb{P}$ on the space of outcomes $\prod_{t\in\mathbb{T}^{\Delta}} (N_t, D_t)$ and hence on $\tilde{\mathscr{H}}_t^B$ and $\tilde{\mathscr{H}}_t^D$. Given this probability space, we define the following probabilities:

i) $\kappa(\tilde{h}_t^B; w)$ is the probability that $\tilde{h}_t^B \in \tilde{\mathscr{H}}_t^B$ is reached if the donor's valuation is w;

ii) $\mathbb{P}(\tilde{h}_t^D; w)$ is the probability that $\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D$ is reached if the donor's valuation is w.

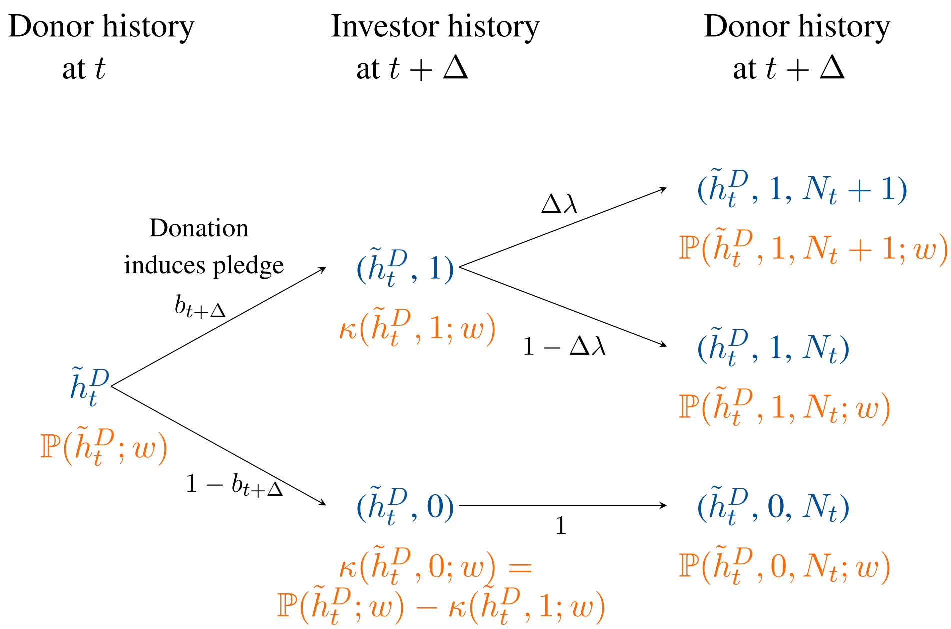
Note that this implies that for each w and $t \in \mathbb{T}^{\Delta}$, we have

$$\sum_{\tilde{h}_t^B \in \tilde{\mathscr{H}}_t^B} \kappa(\tilde{h}_t^B; w) = \sum_{\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D} \mathbb{P}(\tilde{h}_t^D; w) = 1 \text{ and } \mathbb{P}(\tilde{h}_t^D; w) = \kappa(\tilde{h}_t^D, 1; w) + \kappa(\tilde{h}_t^D, 0; w),$$

and, in particular,

$$\kappa(\tilde{h}_t^D, 1; w) \le \mathbb{P}(\tilde{h}_t^D; w) \text{ for all } \tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D. \tag{$\mathbb{P}$}$$

Figure B4. : Transitions between reduced histories



Note: The blue brackets represent reduced histories, and the orange expressions below, the probability of reaching the corresponding reduced history.

Further, the following intertemporal link between reduced histories must hold:

$$(\mathbb{P}-t) \qquad \left.\begin{array}{lcl} \mathbb{P}(\tilde{h}_t^D, 1, N_t+1; w) & = & \Delta\lambda\, \kappa(\tilde{h}_t^D, 1; w) \\ \mathbb{P}(\tilde{h}_t^D, 1, N_t; w) & = & (1-\Delta\lambda)\, \kappa(\tilde{h}_t^D, 1; w) \\ \mathbb{P}(\tilde{h}_t^D, 0, N_t; w) & = & \mathbb{P}(\tilde{h}_t^D; w) - \kappa(\tilde{h}_t^D, 1; w) \end{array}\right\} \text{ for all } \tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D.$$

The reduced histories and probabilities are illustrated in Figure B4. The probabilities of reaching investor histories after which an investor pledges uniquely determine all other probabilities, so we define

$$\tilde{\mathscr{H}}_t^1 := \left\{ \tilde{h}_t^B = \left(\tilde{h}_{t-\Delta}^D, 1\right) \,\middle|\, \tilde{h}_{t-\Delta}^D \in \tilde{\mathscr{H}}_{t-\Delta}^D \right\} \subset \tilde{\mathscr{H}}_t^B.$$

Formally, $\mathbb{P}(0; w) = 1$ and the sequence $\boldsymbol{\kappa}_\Delta(0; w) := \left((\kappa(\tilde{h}_t^B; w))_{\tilde{h}_t^B \in \tilde{\mathscr{H}}_t^1} \right)_{t \geq \Delta}$ uniquely define $\left((\mathbb{P}(\tilde{h}_t^D; w))_{\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D} \right)_{t \geq 0}$ and $\left((\kappa(\tilde{h}_t^D, 0; w))_{\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^B} \right)_{t \geq 0}$. Thus, $(\boldsymbol{\kappa}_\Delta(0; w))_{w \in [0,\infty)}$ determines the outcome of the game and will be the choice variable in the relaxed problem. In order to be able to formulate investor IC constraints after reaching an arbitrary donor history $\tilde{h}^D_{t-\Delta}$, we define *continuation donor histories* at times $t' \geq t$ by

$$\mathscr{H}^D_{t'}\left(\tilde{h}^D_{t-\Delta}\right) := \left\{\tilde{h}^D_{t'} \in \tilde{\mathscr{H}}^D_{t'} : \text{ the first entries of } \tilde{h}^D_{t'} \text{ are } \tilde{h}^D_{t-\Delta}\right\}.$$

The problem of maximizing the probability of success can be written as

$$\begin{aligned}\max_{(\boldsymbol{\kappa}_\Delta(0;w))_{w\in[0,\infty]}} \sum_{\tilde{h}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}} \quad & \Delta\lambda\, \mathbb{E}^{F_0}\left[\kappa(\tilde{h}^D_{T-\Delta},1;W)\mathbb{1}\left(G-(N_{T-\Delta}+1)p\leq W\right)\right]+ \\ & (1-\Delta\lambda)\,\mathbb{E}^{F_0}\left[\kappa(\tilde{h}^D_{T-\Delta},1;W)\mathbb{1}\left(G-N_{T-\Delta}p\leq W\right)\right]+ \\ & \mathbb{E}^{F_0}\left[\left(\mathbb{P}(\tilde{h}^D_{T-\Delta};W)-\kappa(\tilde{h}^D_{T-\Delta},1;W)\right)\mathbb{1}\left(G-N_{T-\Delta}p\leq W\right)\right],\end{aligned}$$

subject to $\mathbb{P}(0;w)=1$, Equation $\mathbb{P}$, Equation $\mathbb{P}-t$, and for all $\tilde{h}^D_t \in \tilde{\mathscr{H}}^D_t$, $t \in \mathbb{T}^\Delta$, $N_t \in \mathbb{N}$, $w \in [0,\infty)$,

$$\text{(Investor IC)} \qquad \frac{\displaystyle\int \overbrace{q_{t+\Delta}(\tilde{h}^D_t,1,N_{t-\Delta}+1;W)}^{\substack{\text{prob. of success if}\\ \text{period-}t\text{ investor pledges}}}\,dF_0(W)}{\displaystyle\int \kappa(\tilde{h}^D_t,1;W)\,dF_0(W)} \geq \frac{v_0}{v-p},$$

where the unconditional probability of success if a period-t investor pledges after history $\tilde{h}^D_t$ is given by

$$\begin{aligned}q_{t+\Delta}(\tilde{h}^D_{t+\Delta};w) = \sum_{\tilde{h}^D_{T-\Delta}\in\tilde{\mathscr{H}}^D_{T-\Delta}(\tilde{h}^D_{t+\Delta})} \quad & \Delta\lambda\,\kappa(\tilde{h}^D_{T-\Delta},1;w)\mathbb{1}\left(G-(N_{T-\Delta}+1)p\leq w\right)+ \\ & (1-\Delta\lambda)\,\kappa(\tilde{h}^D_{T-\Delta},1;w)\mathbb{1}\left(G-N_{T-\Delta}p\leq w\right)+ \\ & \left(\mathbb{P}(\tilde{h}^D_{T-\Delta};w)-\kappa(\tilde{h}^D_{T-\Delta},1;w)\right)\mathbb{1}\left(G-N_{T-\Delta}p\leq w\right)].\end{aligned}$$

This is a relaxed problem because the vectors $(\boldsymbol{\kappa}_\Delta(0;w))_{w\in[0,\infty)}$ that satisfy the above constraints do not necessarily correspond to a PBE. Further, we are ignoring donor incentives by considering reduced histories.

Finally, note that for a PT equilibrium, it must be that for any investor history $(\tilde{h}^D_{t-\Delta},1)\in\tilde{\mathscr{H}}^1_t$ there exists $\tilde{D}^*\left(\left(\mathbb{P}(\tilde{h}^D_{t-\Delta};w)\right)_w\right)\geq 0$ such that

$$\text{(PT-}\kappa\text{)} \qquad \kappa(\tilde{h}^D_{t-\Delta},1;w) = \begin{cases} \mathbb{P}(\tilde{h}^D_{t-\Delta};w) & \text{for } w \geq \tilde{D}^*\left((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\right) \\ 0 & \text{otherwise}\end{cases}.$$

Step 2: Solution to the relaxed problem

In the following, we show any solution satisfies Equation PT-κ. Such $\boldsymbol{\kappa}_\Delta$ with $\tilde{D}^*\left((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\right)=$

$\underline{W}\big((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\big)$ where

$$(\underline{W}) \qquad \begin{aligned}&\underline{W}\big((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\big):=\\ &\min\big\{\underline{w}\big|(\text{Investor IC) is satisfied for } \kappa(\tilde{h}^D_{t-\Delta},1;w)=\mathbb{P}(\tilde{h}^D_{t-\Delta};w)\mathbb{1}(w\geq \underline{w})\big\}\end{aligned}$$

is always a solution. We set $\underline{W}\big((\mathbb{P}(\tilde{h}^D_t;w))_w\big)=\infty$ if the set on the right-hand side is empty. Further, to establish uniqueness in the limit $\Delta\to 0$, we show that for any solution satisfying Equation PT-κ, it must be that

$$(\tilde{D}^*) \qquad \begin{aligned}&\tilde{D}^*\big((\mathbb{P}\big(\tilde{h}^D_{t-\Delta};w)\big)_w\big)\in\\ &\left[\underline{W}\big((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\big),\ \max\Big\{G-\big(N_{t-\Delta}+\tfrac{T-(t-\Delta)}{\Delta}\big)p,\underline{W}\big((\mathbb{P}(\tilde{h}^D_{t-\Delta};w))_w\big)\Big\}\right],\end{aligned}$$

where $G-\big(N_{t-\Delta}+\frac{T-(t-\Delta)}{\Delta}\big)p$ is the amount that the donor needs to donate even if an investor arrives and pledges in every future period. Note that as $\Delta\to 0$, $G-\big(N_t+\frac{T-t}{\Delta}\big)p\to-\infty$.

We show that the solution must satisfy Equation PT-κ with $(\tilde{D}^*)$ by contradiction. Consider an arbitrary solution $\boldsymbol{\kappa}^*_\Delta$ and corresponding $\mathbb{P}^*$ such that there is at least one history in which it does not satisfy Equation PT-κ with $(\tilde{D}^*)$. Consider the latest period $\bar{t}$ in time after which Equation PT-κ with $(\tilde{D}^*)$ is satisfied for all histories, and consider a period $\bar{t}-\Delta$ history $\tilde{h}^D_{\bar{t}-\Delta}$ such that $\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)$ does not satisfy Equation PT-κ with $(\tilde{D}^*)$. Then, the probability of success conditional on reaching history $\tilde{h}^B_{\bar{t}}=(\tilde{h}^D_{\bar{t}-\Delta},1)$ given by $\frac{q_t(\tilde{h}^D_{\bar{t}-\Delta},1,N_{t-\Delta}+1;w)}{\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)}$ is increasing in w and is independent of the choice of $\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)$. Let

$$c(\tilde{h}^D_{\bar{t}-\Delta}):=\int \kappa^*(\tilde{h}^D_{\bar{t}-\Delta},1;W)\,dF_0(W).$$

We now construct a $\boldsymbol{\kappa}'_\Delta$ such that the objective function is higher than with $\boldsymbol{\kappa}^*_\Delta$ while keeping $\int \kappa'(\tilde{h}^D_{\bar{t}-\Delta},1;W)\,dF_0(W)\leq c(\tilde{h}^D_{\bar{t}-\Delta})$ in all histories. To this end, let $\underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta})$ be the uniquely defined by[17]

$$\int_{\underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta})}^{\infty}\mathbb{P}^*(\tilde{h}^D_{\bar{t}-\Delta};W)\,dF_0(W)=c(\tilde{h}^D_{\bar{t}-\Delta}).$$

Since $\frac{q_t(\hat{h}^D_{\bar{t}-\Delta},1,N_{t-\Delta}+1;w)}{\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)}$ is increasing in w, $\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)=\mathbb{P}^*(\tilde{h}^D_{\bar{t}-\Delta};w)\mathbb{1}(w\geq \underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta}))$ satisfies Equation Investor IC. We set $\kappa'(\hat{h}^D_t;w):=\kappa^*(\hat{h}^D_t;w)$ for all histories $\hat{h}^D_t$ at

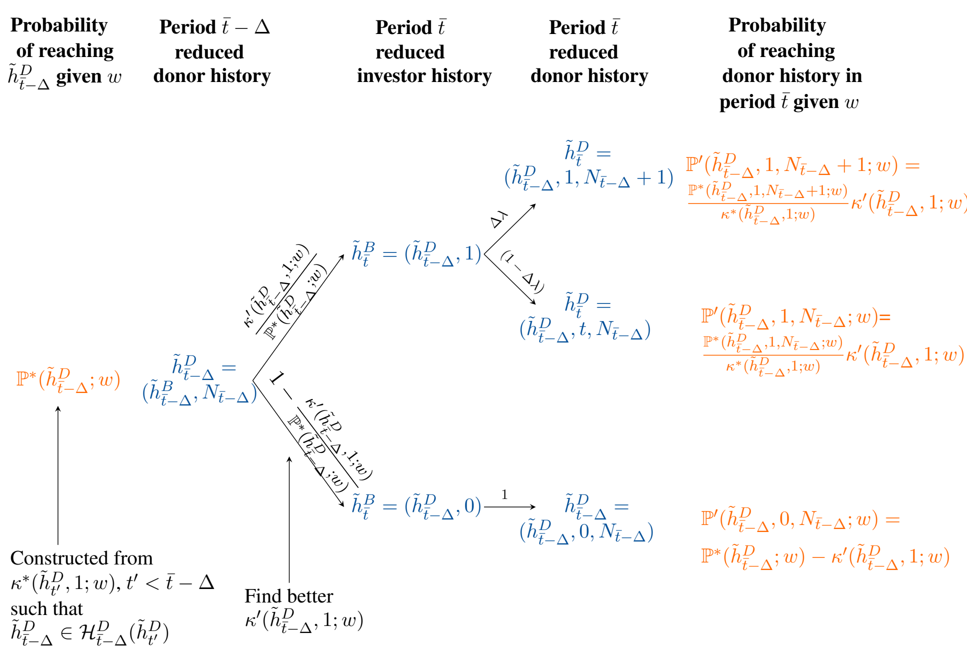
[17]Uniqueness follows because for all $t\geq\bar{t}$, $\kappa(\tilde{h}^D_{\bar{t}},1;w)$ satisfies Equation PT-κ.

$t < \bar{t} - \Delta$ and all histories $\hat{h}_t^D \notin \tilde{\mathscr{H}}_t^1(\tilde{h}_{\bar{t}-\Delta}^D)$, $t \geq \bar{t} - \Delta$. Further, let

$$\kappa'(\tilde{h}_{\bar{t}-\Delta}^D, 1; w) := \begin{cases} \mathbb{P}^*(\tilde{h}_{\bar{t}-\Delta}^D; w) & \text{for } w \geq \underline{W}_c(\tilde{h}_{\bar{t}-\Delta}^D) \\ 0 & \text{otherwise,} \end{cases}$$

and for histories $\hat{h}_t \in \tilde{\mathscr{H}}_t^1(\tilde{h}_{\bar{t}-\Delta}^D)$ where $t > \bar{t} - \Delta$, we set $\kappa'(\hat{h}_t^D; w) := \mathbb{P}'(\hat{h}_t^D; w)\frac{\kappa^*(\hat{h}_t^D; w)}{\mathbb{P}^*(\hat{h}_t^D; w)}$ so that all constraints remain satisfied and the transition probabilities remain unchanged. Figure B5 illustrates the transitions. In the objective function, this $\boldsymbol{\kappa}'$ achieves

Figure B5. : Schematic illustration of transition probabilities



Note: The blue brackets represent reduced histories, and the orange expressions, the probability of reaching the corresponding reduced histories given a realized w.

states with higher $N_{T-\Delta}$ more frequently, so $\boldsymbol{\kappa}'$ yields strictly higher profits than $\boldsymbol{\kappa}^*$ does. Thus, any solution $\boldsymbol{\kappa}_\Delta^*$ must satisfy Equation PT-κ with $(\tilde{D}^*)$ almost surely.

Step 3: Implementation by equilibrium

Finally, we show that the optimal solution is achieved by the PBE constructed in Proposition 1. To this end, it is useful to write the probability of success for donor type w after a history $\tilde{h}_{t-\Delta}^D$ recursively as a function of $\boldsymbol{\kappa}_t\left(\tilde{h}_{t-\Delta}^D; w\right)$ and $\mathbb{P}\left(\tilde{h}_{t-\Delta}^D; w\right) >$

0:
(W-Π)

$$\Pi_{t-\Delta}\left(\boldsymbol{\kappa}_t\left(\tilde{h}^D_{t-\Delta};w\right),\mathbb{P}\left(\tilde{h}^D_{t-\Delta};w\right);w\right)=$$

$$\underbrace{\Delta\lambda}_{\text{arrival}}\underbrace{\frac{\kappa\left(\tilde{h}^D_{t-\Delta},1;w\right)}{\mathbb{P}\left(\tilde{h}^D_{t-\Delta};w\right)}}_{\text{investor pledges}}\Pi_t\left(\boldsymbol{\kappa}_{t+\Delta}(\tilde{h}^D_{t-\Delta},1,N_{t-\Delta}+1;w),\mathbb{P}(\tilde{h}^D_{t-\Delta},1,N_{t-\Delta}+1;w);w\right)$$

$$+\underbrace{(1-\Delta\lambda)}_{\text{no arrival}}\underbrace{\frac{\kappa\left(\tilde{h}^D_{t-\Delta},1;w\right)}{\mathbb{P}\left(\tilde{h}^D_{t-\Delta};w\right)}}_{\text{investor pledges}}\Pi_t\left(\boldsymbol{\kappa}_{t+\Delta}(\tilde{h}^D_{t-\Delta},1,N_{t-\Delta};w),\mathbb{P}(\tilde{h}^D_{t-\Delta},1,N_{t-\Delta};w);w\right)$$

$$+\underbrace{\left(1-\frac{\kappa(\tilde{h}^D_{t-\Delta},1;w)}{\mathbb{P}\left(\tilde{h}^D_{t-\Delta};w\right)}\right)}_{\text{investor does not pledge}}\Pi_t\left(\boldsymbol{\kappa}_{t+\Delta}(\tilde{h}^D_{t-\Delta},0,N_{t-\Delta};w),\mathbb{P}(\tilde{h}^D_{t-\Delta},0,N_{t-\Delta};w);w\right),$$

and for $\mathbb{P}(\tilde{h}^D_{t-\Delta};w)=0$, we set $\Pi_{t-\Delta}(\boldsymbol{\kappa}_t(\tilde{h}^D_{t-\Delta};w),\mathbb{P}(\tilde{h}^D_{t-\Delta};w);w)=0$ without loss. Then, we can write the Investor IC constraint as follows:
(Investor IC')

$$\frac{\displaystyle\int \overbrace{\kappa(\tilde{h}^B_t;W)}^{\substack{\text{prob. of}\\ \text{reaching } \tilde{\mathbf{h}}^B_t}}\overbrace{\Pi_t(\boldsymbol{\kappa}_{t+\Delta}(\tilde{h}^B_t,N_{t-\Delta}+1;W),\mathbb{P}(\tilde{h}^B_t;W);W)}^{\substack{\text{prob. of success if}\\ \text{period-}t\text{ investor pledges}}}\,dF_0(W)}{\displaystyle\int \kappa(\tilde{h}^B_t;W)\,dF_0(W)}\geq\frac{v_0}{v-p}.$$

Consider the PT equilibrium $(D^\Delta_+,b^\Delta,(F^\Delta(\cdot|\mathbf{x}))_\mathbf{x})$ from the proof of Proposition 1. This assessment induces a probability measure $\mathbb{P}$ on outcomes and a corresponding system of probabilities $\kappa(\tilde{h}^D_t,1;w)$ and $\mathbb{P}(\tilde{h}^D_t,;w)$ over reduced histories, as defined in **Step 1**. Consider any on-path investor history in the last period $h^{B,\Delta}_T=\prod_{s\in\mathbb{T}^\Delta,s\leq T}(N_{s-\Delta},D_{s-\Delta})$. The PBE specifies that investors pledge if and only if the probability of success is at least $\frac{v_0}{v-p}$. In addition, in the preceding period, unless success is already guaranteed, donors with $w\geq\underline{D}^\Delta(N_{T-\Delta},\Delta)$ donate $\max\{D_{T-2\Delta},\underline{D}^\Delta(N_{T-\Delta},\Delta)\}$. This makes the next investor just indifferent between pledging and not pledging if such a donation amount exists and $\underline{D}^\Delta(N_{T-\Delta},\Delta)=\overline{W}$ otherwise.

Therefore, for any on-path history $h^{D,\Delta}_{T-\Delta}=\left(\prod_{s\in\mathbb{T}^\Delta,s\leq T-\Delta}(N_{T-\Delta},D_{s-\Delta}),N_{T-\Delta}\right)$, the induced probabilities over reduced histories satisfy

$$\kappa(\tilde{h}^D_{T-\Delta},1;w)=\mathbb{P}(\tilde{h}^D_{T-\Delta};w)\text{ if and only if }w\geq\underline{D}^\Delta(N_{T-\Delta},\Delta).$$

Notice that since $\underline{D}^\Delta(N_{T-\Delta},\Delta)$ is calculated using the indifference condition for investors, $\pi^\Delta(N,D,u)$ is increasing in D, and F^Δ is a truncation given by Equation PT-belief, this $\underline{D}^{\Delta}(N_{T-\Delta}, \Delta)$ is exactly $\underline{W}((\mathbb{P}(\tilde{h}^{D}_{T-\Delta}; w))_w, N_{T-\Delta})$ defined in Equation $\underline{W}$ in the solution to the relaxed problem when we write the expression for the indifference condition as in Equation Investor IC'. Analogous arguments apply to any history $h_t^{D,\Delta} = \left(\prod_{s\in\mathbb{T}^{\Delta}, s\le t} (N_{s-\Delta}, D_{s-\Delta}), N_t \right)$. Therefore, the PBE assessment from the proof of Proposition 1 induces exactly $(\boldsymbol{\kappa}^*_{\Delta}(0; w))_w$, and it achieves the optimum in the relaxed problem. Hence, $(\boldsymbol{\kappa}^*_{\Delta}(0; w))_w$ is platform-optimal in the full class of PBEs.

Step 4: Uniqueness of limits

We have shown in **Step 2** that solutions to the reduced problem satisfy Equation PT-κ with Equation $\tilde{D}^*$. Now, for a given t if Δ is sufficiently small, then $G - \left(N + \frac{T-t}{\Delta}\right)p < 0$, so any sequence of outcomes converges pointwise to the equilibrium outcome attained by the Markov equilibrium constructed in **Step 1**.

B3. Proof of Proposition 2 (Success-Minimizing Equilibrium)

First, in Section B.B3, we characterize a PT equilibrium for each Δ. Then, in Section B.B3, we show that the limit of these equilibria as $\Delta \to 0$ exists and is as specified in Proposition 2. Finally, in Section B.B3 we establish that this PBE minimizes the probability of success.

CHARACTERIZATION OF PT EQUILIBRIUM

Lemma 7 (Success-minimizing equilibrium)**.** *Given any* $\Delta > 0$*, a PT assessment* $(b^{\Delta}, D_+^{\Delta}, F^{\Delta})$ *with donation threshold* $\overline{D}^{\Delta}(N, u) \in [0, G-(N+1)p)$ *constitutes a PT equilibrium.*

We denote by $\overline{\pi}(N, D, u)$ the corresponding probability of success from the investor's perspective in state (N, D, u) if the investor contributes.

Proof. Note that the donation threshold is well-defined in Section II.D (unlike in the construction of the success-maximizing equilibrium): $\overline{D}^{\Delta}(N, u+\Delta) := \max\{G-(j-1)p - Np, 0\}$ for $u \in \left(\overline{\xi}^{\Delta}_{j-1}, \overline{\xi}^{\Delta}_{j}\right]$. This defines strategies and beliefs of the PT assessment. It is immediate that $\overline{D}^{\Delta}(N, u)$ is strictly decreasing in N and u as long as $\overline{D}^{\Delta}(N, u) > 0$, weakly decreasing otherwise, $\overline{D}^{\Delta}(N, u) \in [0, G-(N+1)p)$, and $\overline{D}^{\Delta}(N, u) = 0$ for $(N+1)p \ge G$. It only remains to show that the investor strategies are optimal in every state (N, D, u), since the donor is best-responding by Lemma 3. We show this by induction in $j = \underline{M}(D) - N$ and for each j by backward-induction in u.

(a) Induction start ($j \le 1 \Leftrightarrow D \ge G-(N+1)p$): For $N \ge \underline{M}(D) - 1$, the campaign is either already successful or an investor can complete the campaign. Hence $\pi^{\Delta}(N, D, u) = 1$ and $b^{\Delta}(N, D, u) = 1$ for all $u \in \mathbb{U}^{\Delta}$, and $D \in [0, \overline{W}]$ in any equilibrium. Note that $\overline{\xi}^{\Delta}_1 = 0$ and $D_+^{\Delta}(N, D, u; w) = D$.

(b) Induction hypothesis ($j' \le j-1$): Assume that we have shown that the above strategy profiles are best responses for investors for all states (N, D, u) with $N = \underline{M}(D) - j'$ with $j' \le j-1$.

(c) Induction step ($j-1 \rightsquigarrow j,\ j \geq 2$): Consider an investor in state (N, D, u) with $N = \underline{M}(D) - j$. If $D < \overline{D}^{\Delta}(N, u+\Delta)$, then $u < \overline{\xi}^{\Delta}_{j}$, and the belief system dictates that an investor assigns a probability of success equal to

$$\pi^{\Delta}(\underline{M}(D)-j, D, u) \quad = \quad \mathbb{P}(\tau^{u}_{1} \leq T - \overline{\xi}^{\Delta}_{j-1}, \ldots, \tau^{u}_{j-2} \leq T - \overline{\xi}^{\Delta}_{2}, \tau^{u}_{j-1} \leq T) < \frac{v_0}{v-p},$$

where τ^{u}_{i} is the arrival time of the i-th investor after period u. The inequality follows directly from the definition of $\overline{\xi}^{\Delta}_{j}$. Hence, $b^{\Delta}(\underline{M}(D)-j, D, u) = 0$ is optimal for the investor.

If $D \geq \overline{D}^{\Delta}(N, u+\Delta)$, then $u \geq \overline{\xi}^{\Delta}_{j}$; by the induction hypothesis, we have

$$\begin{aligned}\pi^{\Delta}(N, D, u) = \ & \mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\max\{(u-\overline{\xi}^{\Delta}_{\underline{M}(W)-(N+1)}(W))/\Delta, 0\}} (1-\Delta\lambda)^{i-1}\Delta\lambda \\ & \pi^{\Delta}(N+1, \max\{D, \overline{D}^{\Delta}(N+1, u-\Delta(i-1))\}, u-\Delta i) \\ & +(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W \geq G-(N+1)p)\Big| W \geq D\Bigg] \\ & > \mathbb{P}(\tau^{u}_{1} \leq T - \overline{\xi}^{\Delta}_{j-1}, \ldots, \tau^{u}_{j-2} \leq T - \overline{\xi}^{\Delta}_{2}, \tau^{u}_{j-1} \leq T) \geq \tfrac{v_0}{v-p},\end{aligned}$$

where the last inequality follows because $u \geq \overline{\xi}^{\Delta}_{j}$ and the definition of $\overline{\xi}^{\Delta}_{j}$ via Proposition 1. Hence, indeed $b^{\Delta}(\underline{M}(D)-j, D, u) = 1$. ■

Taking the continuous time limit

We know from Proposition 1 that the pointwise limits $\bar{\xi}_{j} := \lim_{\Delta\to 0} \bar{\xi}^{\Delta}_{j}$ and

$$\overline{D}(N, u) := \lim_{\Delta\to 0} \overline{D}^{\Delta}\left(N, \left\lceil \frac{u}{\Delta} \right\rceil \Delta\right) = \max\{G-(j-1)p - Np, 0\} \text{ for } u \in (\bar{\xi}_{j}, \bar{\xi}_{j-1}]$$

exist. This implies that the pointwise limits $D_{+}(N, D, , u; w) := \lim_{\Delta\to 0} D^{\Delta}_{+}\left(N, D, \lceil \frac{u}{\Delta} \rceil \Delta; w\right)$, $b(N, D, u) = \lim_{\Delta\to 0} b^{\Delta}\left(N, D, \lceil \frac{u}{\Delta} \rceil \Delta\right)$, and $F(w; (N, D, u)) = \lim_{\Delta\to 0} F^{\Delta}\left(w; \left(N, D, \lceil \frac{u}{\Delta} \rceil \Delta\right)\right)$ exist. This concludes the proof of Proposition 2 ii).

Minimization of probability of success

Next, we show that the equilibrium just constructed minimizes the probability of success in the class of PBE. To this end, we consider an arbitrary PBE $(\tilde{b}^{\Delta}, \tilde{D}^{\Delta}_{+}, \tilde{F}^{\Delta})$. We show by backward induction in t that for any investor history $h^{B,\Delta}_{t} = \prod_{s\in\mathbb{T}^{\Delta}, s\leq t}(N_{s-\Delta}, D_{s-\Delta})$ an equilibrium investor history must satisfy

$$D_{t-\Delta} > \overline{D}^{\Delta}(N_{t-\Delta}, T-(t-\Delta)) \Rightarrow \tilde{b}^{\Delta}(h^{B,\Delta}_{t}) = 1. \tag{B10}$$

(a) Induction start ($t = T$): $\overline{D}^{\Delta}(N,\Delta) = G-(N-1)p$, so Equation B10 is satisfied for any PBE.
(b) Induction hypothesis ($s \geq t$): Assume that Equation B10 is satisfied for any history $h_s^{B,\Delta}$ with $s \geq t$.
(c) Induction step ($t \rightsquigarrow t-\Delta$): For an arbitrary history $h_{t-\Delta}^{B,\Delta}$, from an investor's perspective in period $t-\Delta$, the probability of success after a contribution is bounded from below by $\overline{\pi}(N_{t-2\Delta}, D_{t-2\Delta}, T-(t-\Delta))$ by the induction hypothesis. Thus, the investor must contribute if $\overline{\pi}(N_{t-2\Delta}, D_{t-2\Delta}, T-(t-\Delta)) \geq \frac{v_0}{v-p}$. Since for the constructed PT equilibrium,

$$D > \overline{D}^{\Delta}(N, T-2t) \Rightarrow \overline{\pi}(N, D, T-(t-\Delta)) \geq \frac{v_0}{v-p},$$

we have $D_{t-2\Delta} > \overline{D}^{\Delta}(N_{t-2\Delta}, T-(t-2\Delta)) \Rightarrow \tilde{b}^{\Delta}(h_{t-\Delta}^{B,\Delta}) = 1$.

Finally, if Equation B10 is satisfied, then the probability of success in the PBE must be at least as in the constructed PT equilibrium, since investors contribute whenever they contribute in the PT equilibrium and the donor contributes up to his wealth at the deadline in any PBE whenever necessary for success.

B4. Proof of Proposition 3 (Donor-Preferred Equilibrium)

PROOF OUTLINE:

Given any assessment, we use the same class of reduced histories and systems of probabilities $\kappa(\tilde{h}_t^B; w)$ and $\mathbb{P}(\tilde{h}_t^D, N_t; w)$ as in the proof of Proposition 1. Just as in the equilibrium that maximizes the probability of success, in a donor-preferred equilibrium, the investor always pledges when she is indifferent between pledging and not pledging, so we can assume that $b_s \in \{0,1\}$ for all histories. The induced probability measure $\mathbb{P}$ allows us to define $(\boldsymbol{\kappa}_{\Delta}(0; w))_w$, which determines the outcome of the game except for the donation amount.

The proof proceeds in four steps. **Step 1** establishes that donor-preferred equilibrium outcomes can be attained by PBE in a smaller class of assessments. In **Step 2**, we formulate a relaxed donor problem (analogously to Proposition 1). In **Step 3**, we solve the donor's problem and show that the success-maximizing solution also corresponds to a solution of the donor's problem. We also prove that all solutions that are PT equilibria converge to the same limit as $\Delta \to 0$. Finally, in **Step 4**, we verify that the donor strategy constructed in **Step 3** of the proof of Proposition 1 is consistent with the donor-preferred solution.

Proof:
Step 1: Limiting the class of assessments

To find a donor-preferred equilibrium, we first show (in Lemmata 8 and 9 below) that donor-preferred equilibrium outcomes can be attained by PBE in a smaller class of assessments. First, at histories at which investors are induced to pledge, all donor types that donate positive amounts make the same cumulative donation. Second, if a donor does not incentivize pledging, he donates nothing. Within the class of assessments satisfying these two properties, the mapping from reduced histories to donations becomes unique, a fact we use when we formulate the donor's maximization problem.

Lemma 8. *For any donor-preferred PBE* $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$*, there exists a donor-preferred PBE* $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$ *such that*

i) both assessments generate the same probability measures $(\boldsymbol{\kappa}_{\Delta}(0; w))_w$*,*

ii) for each $h_t^{D,\Delta}$*, there exists a* $D_*(h_t^{D,\Delta}) \in \mathbb{R}$ *such that*

(B11)

$$\begin{aligned} \hat{D}_{+}^{\Delta}(h_t^{D,\Delta}; w) &= \begin{cases} \tilde{D}_{+}(h_t^{D,\Delta}; w) & \text{if } \tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(h_t^{D,\Delta}; w)) = 0 \\ D_*(h_t^{D,\Delta}) & \text{if } \tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(h_t^{D,\Delta}; w)) = 1 \end{cases}, \text{ and} \\ \hat{b}^{\Delta}(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) &= \begin{cases} 1 & \text{if } D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta}) \\ 0 & \text{otherwise} \end{cases}. \end{aligned}$$

Proof of Lemma 8. Given a donor-preferred PBE $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$, define

$$D_*(h_t^{D,\Delta}) := \inf\left\{\tilde{D}_{+}^{\Delta}(h_t^{D,\Delta}; w) \,\middle|\, \tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(h_t^{D,\Delta}; w)) = 1\right\},$$

which is the smallest donation amount that incentivizes pledging at a history $h_t^{D,\Delta}$. Donating this amount is feasible for all donor types $w \geq D_*(h_t^{D,\Delta})$. Moreover, it is consistent with play on the equilibrium path. In particular, donating this amount is feasible for all types that incentivize pledging after $h_t^{D,\Delta}$ in $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$.

Then, define a new assessment $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$, where $\hat{b}^{\Delta}$ and $\hat{D}_{+}^{\Delta}$ are given by Equation B11. On the equilibrium path, $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ is derived by Bayes' rule. Off path, if $D_{t-\Delta} > D_*(h_{t-\Delta}^{D,\Delta})$, then let $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ be such that it is optimal for the investor not to pledge (e.g., $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \mathbb{1}(w = 0)$), and let $\hat{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \tilde{F}(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ otherwise.

Note that the strategies are such that $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$ and $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ result in the same probability measures $(\boldsymbol{\kappa}_{\Delta}(0; w))_w$, i.e., the same purchasing outcome after any realization of arrivals and donor type. The donation amount with $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ is by definition weakly lower after any arrival and donor type realization. Hence, if $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$ is a PBE, then it must be donor-preferred. It remains to be shown that $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$ is a PBE.

First, consider donor incentives. Given a PBE $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$, a donor type w with $\tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(h_t^{D,\Delta}; w)) = 0$ does not find it profitable to incentivize pledging after a history $h_t^{D,\Delta}$. Pledging can be incentivized by donations of at least $D_*(h_{t-\Delta}^{D,\Delta})$. Hence, also with assessment $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$, deviating to incentivize pledging cannot be profitable. For a donor type w with $\tilde{b}^{\Delta}(h_t^{D,\Delta}, \tilde{D}_{+}(h_t^{D,\Delta}; w)) = 1$, it is optimal to donate in the PBE $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$. Given the assessment $(\hat{b}^{\Delta}, \hat{D}_{+}^{\Delta}, \hat{F}^{\Delta})$, the donor can donate weakly less and still incentivize pledging, but the donor has a larger set of feasible donations in any future period. Thus, no donor type has an incentive to deviate given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$.

Next, consider investor incentives. Investors at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, where $D_{t-\Delta} < D_*(h_{t-\Delta}^{D,\Delta})$, have identical beliefs about donor types in both assessments, and the purchasing outcome is also identical, as argued above. Hence, the probability of success is the same across assessments and an investor with such a history must prefer not to pledge given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ because $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ is a PBE. Investors at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, where $D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta})$, believe that they face donor types that they would face if they played a PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$, and if they were at any of the histories $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ after which an investor pledges. Hence, investors must prefer to pledge at a history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$, where $D_{t-\Delta} = D_*(h_{t-\Delta}^{D,\Delta})$, given the assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$. A history $(h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ with $D_{t-\Delta} > D_*(h_{t-\Delta}^{D,\Delta})$ is now off the equilibrium path for assessment $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$, and we assumed that $\hat{F}$ is such that the investor does not wish to pledge in this case.

It follows that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ is a PBE.

Hence, to find a donor-preferred equilibrium, it suffices to restrict attention to assessments $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ such that for any $h_t^{D,\Delta}$, there exists a $D_*(h_t^{D,\Delta}) \in \mathbb{R}$ with

$$\tilde{D}_+^\Delta(h_t^{D,\Delta}; w) = D_*(h_t^{D,\Delta}), \text{ whenever } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+^\Delta(h_t^{D,\Delta}; w)) = 1, \tag{B12}$$

and $\tilde{b}^\Delta$ as is defined in Equation B11. Indeed, the success-maximizing equilibrium constructed in Proposition 1 is in this class.

Lemma 9. *For any donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ for which the donor strategy satisfies Equation B12 and the investor strategy satisfiesEquation B11, there exists a donor-preferred PBE $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ so that*

i) *both assessments generate the same probability measures $(\boldsymbol{\kappa}_\Delta(0; w))_w$,*

ii) *$\hat{b}^\Delta = \tilde{b}^\Delta$ and for each $h_t^{D,\Delta}$,*

$$\hat{D}_+^\Delta(h_t^{D,\Delta}; w) = \begin{cases} D_{t-\Delta} & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 0 \\ \tilde{D}_+(h_t^{D,\Delta}; w) & \text{if } \tilde{b}^\Delta(h_t^{D,\Delta}, \tilde{D}_+(h_t^{D,\Delta}; w)) = 1 \end{cases}. \tag{B13}$$

Proof of Lemma 9. Given the donor-preferred PBE $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$ satisfying Equation B12, let $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ be given by Equation B13, $\hat{b}^\Delta = \tilde{b}^\Delta$, and $\tilde{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ so that it is consistent with Bayes' rule on the equilibrium path, and $\tilde{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta}) = \hat{F}^\Delta(w; h_{t-\Delta}^{D,\Delta}, D_{t-\Delta})$ off the equilibrium path. Then, it follows immediately that the two assessments generate the same outcomes and, hence, the same probability measures $(\boldsymbol{\kappa}_\Delta(0; w))_w$. It remains to show that $(\hat{b}^\Delta, \hat{D}_+^\Delta, \hat{F}^\Delta)$ constitutes a PBE. The donor does not have a profitable deviation in histories after which the investor is incentivized to pledge, because the donor plays exactly the same strategy as in $(\tilde{b}^\Delta, \tilde{D}_+^\Delta, \tilde{F}^\Delta)$.

Whenever the donor does not incentivize pledging, the donor cannot have a profitable deviation, because incentivizing pledging is not profitable for $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$, and, moreover, $\hat{D}_{+}^{\Delta}(h_t^{D,\Delta}; w) = D_{t-\Delta} \le \tilde{D}_{+}^{\Delta}(h_t^{D,\Delta}; w)$ implies that every donor type w has a weakly larger set of feasible donations in the future under $\hat{D}_{+}^{\Delta}$ than under $\tilde{D}_{+}^{\Delta}$. Each investor is also best-responding, because she pledges after the same histories in both assessments, and whenever she does not pledge, her belief is a mixture of beliefs in histories after which she did not pledge in $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$.

Hence, in the following, we restrict attention to assessments $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ that satisfy Equation B13 and Equation B12. The donor strategy in such assessments depends only on the reduced history $\mathscr{R}_{\tilde{b}^{\Delta}}(h_t^{D,\Delta})$, so we can define $\mathscr{D}(\tilde{h}_t^{D,\Delta}) := D_*(h_t^{D,\Delta})$ for $\tilde{h}_t^{D,\Delta} = \mathscr{R}_{\tilde{b}^{\Delta}}(h_t^{D,\Delta})$. Indeed, the platform-optimal equilibrium from Proposition 1 satisfies Equation B13.

Step 2: Relaxed donor problem

Consider an arbitrary assessment $(\tilde{b}^{\Delta}, \tilde{D}_{+}^{\Delta}, \tilde{F}^{\Delta})$ that satisfies Equation B13. Recall that, analogously to Proposition 1, we can define reduced histories, systems of probabilities $\kappa(\tilde{h}_t^B; w)$, $\mathbb{P}(\tilde{h}_t^D, N_t; w)$, the mapping $\mathscr{R}_{\tilde{b}^{\Delta}}$ that maps general histories to the corresponding reduced history, and $\mathscr{D}(\tilde{h}_t^{D,\Delta})$, the corresponding donation threshold for reduced history $\tilde{h}_t^{D,\Delta}$. In order to formulate the donor's payoff, we write for $t' \le t$ that $\tilde{h}_{t'}^D \subseteq \tilde{h}_t^D$ if $\tilde{h}_{t'}^D$ is a sub-history that leads to $\tilde{h}_t^D$. Then, let

$$\bar{\mathscr{D}}(\tilde{h}_t^D) := \max_{\substack{\tilde{h}_{t'}^D \subset \tilde{h}_t^D, \\ t' \le t \, b_{t'} = 1}} \mathscr{D}(\tilde{h}_{t'}^D)$$

be the cumulative donations after period t if the donor follows a donation strategy as specified in Equation B13 so that he donates in all periods t' in which the reduced history $\tilde{h}_t^D$ dictates that $b_{t'} = 1$.

The donor's problem can be written as

$$\begin{aligned}\max_{\substack{(\boldsymbol{\kappa}_{\Delta}(0;w))_w, \\ (\mathscr{D}(\tilde{h}_t^D))_{\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D,\, t \in \mathbb{T}^{\Delta}}}} \quad & \sum_{\tilde{h}_{T-\Delta}^D \in \tilde{\mathscr{H}}_{T-\Delta}^D} \Delta\lambda \, \mathbb{E}^{F_0}\left[\kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}\left(G - (N_{T-\Delta}+1)p \le W\right)\left(W - \bar{\mathscr{D}}(\tilde{h}_{T-\Delta}^D)\right)\right] + \\ & (1-\Delta\lambda)\, \mathbb{E}^{F_0}\left[\kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}\left(G - N_{T-\Delta}p \le W\right)\left(W - \bar{\mathscr{D}}(\tilde{h}_{T-\Delta}^D)\right)\right] + \\ & \mathbb{E}^{F_0}\left[\left(\mathbb{P}(\tilde{h}_{T-\Delta}^D; w) - \kappa(\tilde{h}_{T-\Delta}^D, 1; w)\right) \mathbb{1}\left(G - N_{T-\Delta}p \le W\right)\left(W - \bar{\mathscr{D}}(\tilde{h}_{T-2\Delta}^D)\right)\right],\end{aligned}$$

subject to $\mathbb{P}(0; w) = 1$, Equation $\mathbb{P}$, Equation $\mathbb{P} - t$, and for all $\tilde{h}_t^D \in \tilde{\mathscr{H}}_t^D$, $t \in \mathbb{T}^{\Delta}$, $N_t \in \mathbb{N}$, $w \in [0, \infty)$ Equation Investor IC, and given

$$\begin{aligned}d_t(\tilde{h}_t^D; w) \quad := \quad & \sum_{\tilde{h}_{T-\Delta}^D \in \tilde{\mathscr{H}}_{T-\Delta}^D(\tilde{h}_t^D)} \Delta\lambda \, \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}\left(G - (N_{T-\Delta}+1)p \le w\right)\left(w - \bar{\mathscr{D}}(\tilde{h}_{T-\Delta}^D)\right) + \\ & (1-\Delta\lambda)\, \kappa(\tilde{h}_{T-\Delta}^D, 1; w) \mathbb{1}\left(G - N_{T-\Delta}p \le w\right)\left(w - \bar{\mathscr{D}}(\tilde{h}_{T-\Delta}^D)\right) + \\ & \left(\mathbb{P}(\tilde{h}_{T-\Delta}^D; w) - \kappa(\tilde{h}_{T-\Delta}^D, 1; w)\right) \mathbb{1}\left(G - N_{T-\Delta}p \le w\right)\left(w - \bar{\mathscr{D}}(\tilde{h}_{T-2\Delta}^D)\right),\end{aligned}$$

we can formulate a donor incentive compatibility constraint for all $\tilde{h}^D_{t-\Delta} \in \tilde{\mathscr{H}}^D_t$
(Donor IC)

$$\begin{aligned} & d_t(\tilde{h}^D_{t-\Delta}, 0, N_{t-\Delta}; w)) < \Delta\lambda d_t(\tilde{h}^D_{t-\Delta}, 1, N_{t-\Delta}+1; w)) + (1-\Delta\lambda) d_t(\tilde{h}^D_{t-\Delta}, 1, N_{t-\Delta}; w)) \\ \Rightarrow \quad & \kappa(\tilde{h}^D_{t-\Delta}, 1; w) = \mathbb{P}(\tilde{h}^D_{t-\Delta}; w). \end{aligned}$$

This donor IC constraint puts a lower bound on donations, because it imposes that the donor must donate whenever it is optimal to do so, but does not impose that the donor does not donate if it is optimal not to donate. Hence, this donor problem is a relaxed maximization problem.

We denote a solution to the above problem by $\boldsymbol{\kappa}^{**}_\Delta$ and $\left((\mathscr{D}_{**}(\tilde{h}^D_t, 1))_{\tilde{h}^D_t \in \tilde{\mathscr{H}}^D_t}\right)_{t\geq 0}$. Recall that the solution that we presented to the platform's relaxed problem was denoted $\boldsymbol{\kappa}^*_\Delta$.

Step 3: Solution to the relaxed problem

Next, we show the following two statements are true:

i) Any solution of this relaxed problem must satisfyEquation PT-κ for $\tilde{D}^*$ such that Equation $\tilde{D}^*$;

ii) $\boldsymbol{\kappa}_\Delta$ as in Equation PT-κ with $\tilde{D}^*((\mathbb{P}(\tilde{h}^D_t; w))_w, N_t) = \underline{W}((\mathbb{P}(\tilde{h}^D_t; w))_w)$ is a solution.

Given these two statements, it follows immediately that in the limit as $\Delta \to 0$, the outcome is unique by the proof of Proposition 1.

Analogously to the proof of Proposition 1, we show that the solution must satisfy Equation PT-κ with ($\tilde{D}^*$) by contradiction. Consider an arbitrary solution κ^{**}_Δ, corresponding to $\mathbb{P}^{**}$ and $\left((\mathscr{D}_{**}(\tilde{h}^D_t, 1))_{\tilde{h}^D_t \in \tilde{\mathscr{H}}^D_t}\right)_{t\geq 0}$, that does not satisfy Equation PT-κ with ($\tilde{D}^*$). Consider the latest period $\bar{t}$ in time after which Equation PT-κ with ($\tilde{D}^*$) is satisfied for all histories, and consider a period $\bar{t}-\Delta$ history $\tilde{h}^D_{\bar{t}-\Delta}$ such that $\kappa^{**}(\tilde{h}^D_{\bar{t}-\Delta}, 1; w)$ does not satisfy Equation PT-κ with ($\tilde{D}^*$). Then, the probability of success conditional on reaching history $\tilde{h}^B_{\bar{t}} = (\tilde{h}^D_{\bar{t}-\Delta}, 1)$ given by $\frac{q_t(\tilde{h}^D_{\bar{t}-\Delta}, 1, N_{t-\Delta}+1; w)}{\kappa(\tilde{h}^D_{\bar{t}-\Delta}, 1; w)}$ is increasing in w and is independent of the choice of $\kappa(\tilde{h}^D_{\bar{t}-\Delta}, 1; w)$. We can also again define $c(\tilde{h}^D_{\bar{t}-\Delta}) := \int \kappa^{**}(\tilde{h}^D_{\bar{t}-\Delta}, 1; W)\, dF_0(W)$. Note that by Equation Donor IC, it must be that for $t \geq \bar{t}$, $\tilde{D}^{**}(\tilde{h}^D_t) = \underline{W}((\mathbb{P}(\tilde{h}^D_t; w))_w)$. Further, by Equation Donor IC,

$$\begin{aligned} \tilde{D}^{**}(\tilde{h}^D_{\bar{t}}) \quad = \min\{\tilde{D}^*(\tilde{h}^D_{\bar{t}}) \mid \quad & d_{\bar{t}+\Delta}(\tilde{h}^D_{\bar{t}}, 0, N_{\bar{t}}; w)) \geq \\ & \Delta\lambda d_{\bar{t}+\Delta}(\tilde{h}^D_{\bar{t}}, 1, N_{\bar{t}}+1; w)) + (1-\Delta\lambda) d_t(\tilde{h}^D_{\bar{t}}, 1, N_{\bar{t}}; w)) \\ & \text{for all } w \text{ such that } \kappa(\tilde{h}^D_{\bar{t}}, 1; w) < \mathbb{P}(\tilde{h}^D_{\bar{t}}; w)\}. \end{aligned}$$

We now construct a $\boldsymbol{\kappa}'_\Delta$ such that the donor's objective function is higher than with $\boldsymbol{\kappa}^*_\Delta$, while keeping $\int \kappa'(\tilde{h}^D_{\bar{t}-\Delta}, 1; W)\, dF_0(W) \leq c(\tilde{h}^D_{\bar{t}-\Delta})$ in all histories. Analogously to

Proposition 1, we can uniquely define $\underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta})$ by

$$\int_{\underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta})}^{\infty} \mathbb{P}^{**}(\tilde{h}^D_{\bar{t}-\Delta}; W)\, dF_0(W) = c(\tilde{h}^D_{\bar{t}-\Delta}).$$

Since $\frac{q_t(\hat{h}^D_{\bar{t}-\Delta},1,N_{t-\Delta}+1;w)}{\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w)}$ is increasing in w, $\kappa(\tilde{h}^D_{\bar{t}-\Delta},1;w) = \mathbb{P}^{**}(\tilde{h}^D_{\bar{t}-\Delta};w)\mathbb{1}(w \geq \underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta}))$ satisfies Equation Investor IC. We set $\kappa'(\tilde{h}^D_t;w) := \kappa^{**}(\tilde{h}^D_t;w)$ for all histories $\hat{h}^D_t$ at $t < \bar{t}-\Delta$ and all histories $\hat{h}^D_t \notin \tilde{\mathscr{H}}^1_t(\tilde{h}^D_{\bar{t}-\Delta})$, $t \geq \bar{t}-\Delta$. Further, let

$$\kappa'(\tilde{h}^D_{\bar{t}-\Delta},1;w) := \begin{cases} \mathbb{P}^{**}(\hat{h}^D_{t-\Delta};w) & \text{for } w \geq \underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta}) \\ 0 & \text{otherwise,} \end{cases}$$

and for histories $\hat{h}_t \in \tilde{\mathscr{H}}^1_t(\tilde{h}^D_{\bar{t}-\Delta})$ where $t > \bar{t}-\Delta$, we set $\kappa'(\hat{h}^D_t;w) := \mathbb{P}'(\hat{h}^D_t;w)\frac{\kappa^*(\hat{h}^D_t;w)}{\mathbb{P}^*(\hat{h}^D_t;w)}$ so that all constraints remain satisfied and the transition probabilities remain unchanged. Further, the lowest donation amount by Equation Donor IC is then

$$\tilde{D}'(\tilde{h}^D_{\bar{t}}) \quad = \underline{W}_c(\tilde{h}^D_{\bar{t}-\Delta}).$$

In the objective function, this $\boldsymbol{\kappa}'$ achieves states with higher $N_{T-\Delta}$ more frequently and $\tilde{D}'(\tilde{h}^D_{\bar{t}}) < \tilde{D}^{**}(\tilde{h}^D_{\bar{t}})$, so $\boldsymbol{\kappa}'$ yields strictly higher donor payoffs than $\boldsymbol{\kappa}^{**}$ does. Thus, any solution $\boldsymbol{\kappa}^*_\Delta$ must satisfy Equation PT-κ with ($\tilde{D}^*$) almost surely.

Step 4: Implementation by equilibrium

We have already shown in Proposition 1 that $(\kappa^*_\Delta(0;w))_w$ is induced by the constructed assessment and established that the wealth threshold $\underline{D}^\Delta(N,u)$ corresponds to $\underline{W}(\tilde{h}^D_t)$ if there is a history $h^{D,\Delta}_t$ with $\mathscr{R}_{b^\Delta}(h^{D,\Delta}_t) = \tilde{h}^D_t$ and $u = T-t$, $N_t = N$. This concludes the proof.

B5. Proof of Proposition 4 (Investor-Preferred Equilibrium)

Finding an equilibrium that maximizes the sum of investor surplus is a complex problem since each investor's decision has externalities both on past investors who have pledged already and future investors. For a sufficiently small period length Δ, we separately construct a PT equilibrium yielding higher investor surplus than the success-maximizing equilibrium and one yielding higher surplus than the success-minimizing equilibrium.

We start with the construction of a PT equilibrium with higher investor surplus than the success-minimizing equilibrium for a general contribution game. First, note that if the realized donor valuation was known to be $w \in [G-2p, G-p)$, then the campaign would require exactly two investor pledges to succeed. Since the second investor can always lead the campaign to succeed, the first investor pledges if and only if $(v-p)\big(1-(1-\Delta\lambda)^{u/\Delta}\big) = v_0$. Conditional on such a W, investor surplus is maximized if the first investor pledges if

$$(v-p)\underbrace{\left(1-(1-\Delta\lambda)^{u/\Delta}\right)}_{\substack{\text{prob. of at least}\\ \text{one more arrival}}}-v_0+\underbrace{(v-p-v_0)\lambda u}_{\substack{\text{externality on}\\ \text{future investors}}}\geq 0 \quad\Leftrightarrow\quad \frac{(1-\Delta\lambda)^{u/\Delta}}{1+\lambda u}\leq 1-\frac{v_0}{v-p},$$

because the expected number of arrivals time u is $\frac{u}{\Delta}\Delta\lambda$. Denote $\bar{u}$ to be the smallest $u\in\mathbb{U}^{\Delta}$ such that the above inequality is satisfied, i.e., the inequality is equivalent to $u\geq\bar{u}$ (noting that $\frac{(1-\Delta\lambda)^{u/\Delta}}{1+\lambda u}$ is decreasing in u). Note that $\bar{\xi}_2^{\Delta}(G-2p)>\bar{u}$ because $\bar{\xi}_2^{\Delta}(G-2p)$ solves $(v-p)\left(1-(1-\Delta\lambda)^{u/\Delta}\right)=v_0$. We define a donation threshold $\overline{D}_{\epsilon}^{\Delta}$ as follows:

- $\overline{D}_{\epsilon}^{\Delta}(N,u):=\overline{D}^{\Delta}(N,u)$ for $N>0$, and for $N=0$ with $u\in[0,\bar{u})\cup[\bar{\xi}_2^{\Delta}(G-2p),\infty)$,
- $\overline{D}_{\epsilon}^{\Delta}(0,u):=\overline{D}^{\Delta}(0,u)-\epsilon=G-p-\epsilon$ for $u\in\left[\bar{u},\bar{\xi}_2^{\Delta}(G-2p)\right)$.

Consider a sufficiently small $\Delta>0$. Then, the PT assessment with donation threshold $\overline{D}_{\epsilon}^{\Delta}(N,u)$ for small $\epsilon>0$ still defines an equilibrium: All investors' incentives to pledge except the incentives of a first investor arriving at $u\in\left[\bar{u},\bar{\xi}_2^{\Delta}(G-2p)\right)$ do not change. If the first investor arrives at $u\in\left[\bar{u},\bar{\xi}_2^{\Delta}(G-2p)\right)$ and the donor has wealth $W\geq G-p-\epsilon$, then the donor can contribute $G-p-\epsilon=\overline{D}_{\epsilon}^{\Delta}(0,u)-\epsilon$ and incentivize the investor to pledge. Indeed, the probability of success is simply a truncation of F_0 at $G-p-\epsilon$, which is close to 1 for small ϵ, so

$$\left(1-(1-\Delta\lambda)^{u/\Delta}\right)+(1-\Delta\lambda)^{u/\Delta}\frac{1-F_0(G-p)}{1-F_0(G-p-\epsilon)}\geq v_0.$$

If the donor has valuation $W<G-p-\epsilon$, then the first investor does not want to contribute, as she knows that $W<G-p$, by definition of $\bar{\xi}_2^{\Delta}(G-p)>\bar{\xi}_2^{\Delta}(G-2p)$. Furthermore, by definition of $\bar{u}$, this PT equilibrium makes investors collectively better off.

Next, we construct a PT equilibrium with higher investor surplus than the success-maximizing equilibrium. We define a donation threshold $\underline{D}_{\epsilon,\delta}^{\Delta}$ for small $\epsilon>0$, $\delta>\Delta$ as follows:

- $\underline{D}_{\epsilon,\delta}^{\Delta}(N,u):=\underline{D}^{\Delta}(N,u)$ for $N>0$ and $(N,u)=(0,u)$ with $u\geq\delta$, and
- $\underline{D}_{\epsilon,\delta}^{\Delta}(0,u):=\underline{D}^{\Delta}(0,u)+\epsilon$ for $u<\delta$.

This defines a PT equilibrium because the incentive to pledge changes only if the first investor arrives in $[0,\delta)$ and if the donor valuation is in $W\in[\underline{D}^{\Delta}(0,u),\underline{D}^{\Delta}(0,u)+\epsilon)$. The probability of success in the success-maximizing equilibrium satisfies

$$\underline{\pi}^{\Delta}(0,\underline{D}^{\Delta}(0,u),u)=\frac{v_0}{v-p},$$

so if investors knew $W \in [\underline{D}^{\Delta}(0,u), \underline{D}^{\Delta}(0,u)+\epsilon)$, then the probability of success would be smaller than $\frac{v_0}{v-p}$ for sufficiently small ϵ, so it is optimal for the investor not to pledge. If $W \geq \underline{D}^{\Delta}(0,u)+\epsilon$, the donor can keep incentivizing investors to pledge in states $(0,u)$, $u<\delta$. Furthermore, the equilibrium outcome of this PT equilibrium yields higher investor surplus than the success-maximizing equilibrium, since if $W \in [\underline{D}^{\Delta}(0,u), \underline{D}^{\Delta}(0,u)+\epsilon)$, $N=0$ and $u<\delta$, then contributing creates collective investor surplus of less than

$$(v-p)\big(1-(1-\Delta\lambda)^{\delta/\Delta}\big)+(v-p)\lambda\delta \xrightarrow[\Delta\to 0]{} (v-p)(1-e^{-\lambda\delta}+\lambda\delta)$$

and not contributing a surplus of $v_0(1+\lambda\delta)$. Hence, for δ sufficiently small (and Δ sufficiently small), there is a PT equilibrium with higher investor surplus than the surplus-maximizing equilibrium.

B6. *Alternative Campaign Designs*

Recall the relaxed problem in the optimality proof of Proposition 1. The control variables are simply probabilities of reaching reduced histories given realized w that ignore donation amounts and donor incentives. The objective is to maximize the probability of success subject to investor participation. As a result, this relaxed problem can also be viewed as a constrained information or mechanism design problem that maximizes the probability of success.

Formally, let an allocation be a sequence $(a_t)_{t\in\mathbb{T}} \in [0,1]^{\mathbb{T}^{\Delta}}$ that determines whether a period-t investor (if she arrives) takes the outside option ($a_t=0$) or stays in the game ($a_t=1$), and a variable $\bar{a}$ that determines whether the project is successful. An allocation is feasible if, given the realized arrival process A_t, $\bar{a}=1 \Leftrightarrow \sum_t (A_t - A_{t-\Delta})a_t p + D_T \geq G$.

Let us first assume that the mechanism designer knows the donor's type. Given her beliefs, an investor in period t can decide whether to participate in the mechanism or not. Additionally, beliefs are formed based on the chosen probabilities of reaching reduced histories. Note that we cannot allow for transfers between investors. Then, it follows that the relaxed problem in the proof of Proposition 1 corresponds to an information design problem where the designer chooses an optimal dynamic signal structure representing the information released about W over time. This shows that, for example, revealing the donor's valuation prior to the campaign is not profit-maximizing.

Alternatively, we can consider a mechanism design problem, assuming that the donor's valuation is private information. Consider direct mechanisms where the donor sends a message $m \in [0,\infty)$ about his type. An investor in period t can decide whether to participate in the mechanism or not. Then, a direct, donor-incentivizing mechanism is given by a message strategy of the donor, a participation strategy of investors, an allocation mapping that maps messages and participation decisions to feasible allocations, and a donor transfer $D \in [0,\infty)$. Again, we cannot allow for transfers between investors. Then, it follows that the relaxed problem in the proof of Proposition 1 is a relaxed problem of the mechanism design problem that finds the success-maximizing, donor-incentivizing mechanism. This shows that, for example, allowing the donor to donate only before or after the crowdfunding stage is not profit-maximizing.

ADDITIONAL PROOFS AND EXAMPLES

C1. *Proof of Lemma 6*

(a) Induction start ($j \le 1 \Leftrightarrow D \ge G-(N+1)p$): For $j \le 1$ and $\mathbf{x} = (N, D, u)$ with $\underline{M}(D) - N \le 1$, it is immediate that the pointwise limits in (B8) exist and are given by

$$b(\mathbf{x}) := \lim_{\Delta\to 0} b^{\Delta}(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta) \equiv 1 \qquad D_+(\mathbf{x}; w) := \lim_{\Delta\to 0} D_+^{\Delta}(N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta; w) = D$$
$$\xi_j(w) := \lim_{\Delta\to 0} \xi_j^{\Delta}(w) \equiv 0 \qquad F(w; \mathbf{x}) := \lim_{\Delta\to 0} F^{\Delta}\Big(w; (N, D, \lceil \tfrac{u}{\Delta} \rceil \Delta)\Big) = \tfrac{F_0(w) - F_0(D)}{1 - F_0(D)} \mathbb{1}(w \ge D),$$

where $\lceil \frac{u}{\Delta} \rceil \Delta$ is the smallest multiple of Δ that is larger than u. Further, $\pi(\mathbf{x}) := \lim_{\Delta\to 0} \pi^{\Delta}(N, D, \lceil \frac{u}{\Delta} \rceil \Delta) = 1$ uniformly in $D \ge G-(N+1)p$ and u.

(b) Induction hypothesis ($j-1$): We assume that the pointwise limits (B8) exist for all $\boldsymbol{x} = (N, D, u)$ with $N \ge \underline{M}(D) - (j-1)$ and $j' \le j-1$, where for $w < G-p$,

$$\pi(\underline{M}(w) - j', w, \xi_{j'}(w)) = \frac{v_0}{v-p}.$$

Further, assume that the pointwise limit $\underline{D}(N, u) := \lim_{\Delta\to 0} \underline{D}^{\Delta}(N, \lceil \frac{u}{\Delta} \rceil \Delta)$ exists for $u \le \xi_{j-1}(G-(N+j-1)p)$. If $\pi(N, 0, u) < \frac{v_0}{v-p}$, then $\underline{D}$ is strictly decreasing in N and u, and

$$\pi(N, \underline{D}(N, u), u) = \frac{v_0}{v-p}.$$

Further, the uniform limit in $D \ge G-(N+j-1)p$ and u, $\pi(N, D, u) := \lim_{\Delta\to 0} \pi^{\Delta}(N, D, u)$, exists and is equal to

(C1)
$$\mathbb{E}^{F_0}\left[\int_0^{\max\{u - \xi_{\underline{M}(W)-(N+1)}(W)\}} \lambda e^{-\lambda s} \pi\Big(N+1, \max\{D, \underline{D}(N+1, u-s)\}, u-s\Big) ds \,\Big|\, W \ge D \right].$$

Finally, $\pi(N, D, u)$ is strictly increasing in N, D, u.

(c) Induction step ($j-1 \rightsquigarrow j$, $j \ge 2$): Consider a state (N, D, u) with $N \ge \underline{M}(D) - j$, i.e.,

$$G-(N+j)p \le D.$$

L.1) *Uniform convergence (in D and u) of $\tilde{\pi}^{\Delta}\big(N, D, \lceil \frac{u}{\Delta} \rceil \Delta\big)$ for $D \ge G-(N+j)p$:*

Recall that the auxiliary probability of success is given by

$$\lim_{\Delta\to 0}\tilde{\pi}^{\Delta}\big(N,D,\lceil \tfrac{u}{\Delta}\rceil\Delta\big)=$$
$$\lim_{\Delta\to 0}\mathbb{E}^{F_0}\Bigg[\sum_{i=1}^{\max\left\{\lceil \frac{u}{\Delta}\rceil\Delta-\xi^{\Delta}_{\underline{M}(W)-(N+1)}(W),0\right\}/\Delta}(1-\Delta\lambda)^{i-1}\Delta\lambda$$
$$\Big(\pi^{\Delta}\big(N+1,D,\Delta(\lceil \tfrac{u}{\Delta}\rceil-i)\big)\mathbb{1}\big(\Delta(\lceil \tfrac{u}{\Delta}\rceil-i)\geq\xi^{\Delta}_{j'-1}(D)\big)+\tfrac{v_0}{v-p}\mathbb{1}\big(\Delta(\lceil \tfrac{u}{\Delta}\rceil-i)<\xi^{\Delta}_{j'-1}(D)\big)\Big)$$
$$+(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W\geq G-(N+1)p)\Big|W\geq D\Bigg],$$

where $j':=\underline{M}(D)-N\leq j$. The uniform convergence of $\pi^{\Delta}(N+1,D,u')$ in D (by the induction hypothesis) and the Arzelà-Ascoli theorem imply that the family of functions $D\mapsto\pi^{\Delta}(N+1,D,u)$ is equicontinuous with respect to Δ. Hence, we may replace π^{Δ} by π. Finally, because $\lim_{\Delta\to 0}\xi^{\Delta}_{j'-1}(w)=\xi_{j'-1}(w)$, the dominated convergence theorem allows us to conclude that

$$\tilde{\pi}_j(N,D,u):=\lim_{\Delta\to 0}\tilde{\pi}^{\Delta}_j\big(N,D,\lceil \tfrac{u}{\Delta}\rceil\Delta\big)=\mathbb{E}^{F_0}\Bigg[\int_0^{\max\{u-\xi_{\underline{M}(W)-(N+1)}(W),0\}}\lambda e^{-\lambda s}\cdot$$
$$\Big(\pi\big(N+1,D,u-s\big)\mathbb{1}\big(u-s\geq\xi_{j'-1}(D)\big)+\tfrac{v_0}{v-p}\mathbb{1}\big(u-s<\xi_{j'-1}(D)\big)\Big)ds$$
$$+e^{-\lambda u}\mathbb{1}(W\geq G-(N+1)p)\Bigg|W\geq D\Bigg].$$

Note that $\tilde{\pi}^{\Delta}(N,D,\lceil \frac{u}{\Delta}\rceil\Delta)$ indeed converges uniformly in $D\geq G-(N+j)p$ for fixed u because the sum is bounded by one, F_0 is (uniformly) continuous on $[0,G]$, and $F_0(G)<1$. Then, since

$$\pi\big(\underbrace{\underline{M}(D)-(j'-1)}_{N+1},D,\xi_{j'-1}(D)\big)=\frac{v_0}{v-p},$$

for $u'<\xi_{j'-1}(D)$, $D<\underline{D}(N+1,u')$ $\pi(N+1,\underline{D}(N+1,u'),u')=\frac{v_0}{v-p}$ and for $u'\geq\xi_{j'-1}(D)$, $D\geq\underline{D}(N+1,u')$. Hence, we have

$$\tilde{\pi}(N,D,u):=\lim_{\Delta\to 0}\tilde{\pi}^{\Delta}\big(N,D,\lceil \tfrac{u}{\Delta}\rceil\Delta\big)=\mathbb{E}^{F_0}\Bigg[\int_0^{\max\{u-\xi_{\underline{M}(W)-(N+1)}(W),0\}}\lambda e^{-\lambda s}\cdot$$
$$\pi\big(N+1,\max\{D,\underline{D}(N+1,u-s)\},u-s\big)ds+e^{-\lambda u}\mathbb{1}(W\geq G-(N+1)p)\Bigg|W\geq D\Bigg]. \tag{C2}$$

L.2) *Continuity and strict monotonicity of $\tilde{\pi}$ in $D\geq G-(N+j)p$ and u:* First, $\tilde{\pi}(N,D,u)$ is continuous in D and u because $\tilde{\pi}_j(N+1,D,u)$ is continuous in D and u, $\underline{D}(N+1,u)$ is continuous in u by the induction hypothesis, and F_0 is continuous.

Furthermore, $\tilde{\pi}_j(N,D,u)$ is strictly increasing in $D\geq G-(N+j)p$ because $\tilde{\pi}(N+1,D,u)$ is weakly increasing in D by the induction hypothesis and $\frac{1}{1-F_0(D)}$

is strictly increasing.

Now the integrand is strictly positive as long as $u > \xi_{\underline{M}(w)-(N+1)}(w)$. Hence, $\tilde{\pi}(N,D,u)$ is strictly increasing in $u > \xi_{\underline{M}(w)-(N+1)}(w)$ because $\tilde{\pi}_j(N+1,D,u)$ is weakly increasing in u by the induction hypothesis and because $u-\xi_{\underline{M}(w)-(N+1)}(w)$ is strictly increasing in u.

L.3) *Pointwise convergence of* $\underline{D}^{\Delta}(N,\lceil \frac{u}{\Delta}\rceil\Delta)$ *and* $D^{\Delta}_{+,j}(N,D,\lceil \frac{u}{\Delta}\rceil\Delta;W)$*:* First, note that if $\tilde{\pi}^{\Delta}_j\big(N,0,\lceil \frac{u}{\Delta}\rceil\Delta\big) \geq \frac{v_0}{v-p}$, then $\tilde{\pi}_j(N,0,u) \geq \frac{v_0}{v-p}$ and, hence, $\underline{D}(N,u) := \lim_{\Delta\to 0}\underline{D}^{\Delta}(N,\lceil \frac{u}{\Delta}\rceil\Delta) = 0$. If $\tilde{\pi}^{\Delta}(N,0,u) < \frac{v_0}{v-p}$, then $\tilde{\pi}(N,0,u) \leq \frac{v_0}{v-p}$. Then, since $\tilde{\pi}(N,D,u)$ is continuous and strictly increasing in D, there is a unique solution $D'(N,u)$ to

$$\tilde{\pi}_j(N,D'(N,u),u) = \frac{v_0}{v-p}.$$

Since $\tilde{\pi}^{\Delta}(N,D,\lceil \frac{u}{\Delta}\rceil\Delta)$ converges uniformly, we have $\underline{D}(N,u) := \lim_{\Delta\to 0}\underline{D}^{\Delta}(N,\lceil \frac{u}{\Delta}\rceil\Delta) = D'(N,u)$. It follows immediately that for all $u > 0$,

$$\begin{aligned} D_+(N,D,u;w) &:= \lim_{\Delta\to 0} D^{\Delta}_{+,j}\big(N,D,\lceil \tfrac{u}{\Delta}\rceil\Delta;w\big) \\ &= \lim_{\Delta\to 0}\min\big\{\max\big\{D,\underline{D}^{\Delta}\big(N,\lceil \tfrac{u}{\Delta}\rceil\Delta\big)\big\},w\big\} \\ &= \min\{\max\{D,\underline{D}(N,u)\},w\}. \end{aligned}$$

L.4) *Pointwise convergence of* $b^{\Delta}_j\big(N,D,\lceil \frac{u}{\Delta}\rceil\Delta\big)$*:* Note that $b^{\Delta}_j\big(N,D,\lceil \frac{u}{\Delta}\rceil\Delta\big) = 1$ if $D \geq \underline{D}^{\Delta}\Big(N,\big(\lceil \frac{u}{\Delta}\rceil\Delta+1\big)\Delta\Big)$, and $b^{\Delta}_j\Big(N,D,\big(\lceil \frac{u}{\Delta}\rceil\Delta+1\big)\Delta\Big) = 0$ otherwise. Since $\lim_{\Delta\to 0}\underline{D}^{\Delta}\Big(N,\big(\lceil \frac{u}{\Delta}\rceil\Delta+1\big)\Delta\Big) = \underline{D}(N,u)$, $b^{\Delta}_j(N,D,u)$ converges pointwise to

$$\lim_{\Delta\to 0} b^{\Delta}_j(N,D,u) = \begin{cases} 1 & \text{if } D \geq \underline{D}(\underline{M}(D)-(j-1),u) \\ 0 & \text{if } D < \underline{D}(\underline{M}(D)-(j-1),u) \end{cases}.$$

L.5) *Pointwise convergence of* $\xi_j(w)$ *and* $\pi(\underline{M}(w)-j',w,\xi_{j'}(w)) = \frac{v_0}{v-p}$*:* If $\tilde{\pi}^{\Delta}(\underline{M}(w)-j,w,0) \geq \frac{v_0}{v-p}$, then it follows immediately that $\xi^{\Delta}_j(w) = 0$. If $\tilde{\pi}^{\Delta}(\underline{M}(w)-j,w,0) < \frac{v_0}{v-p}$, it follows that $\xi^{\Delta}_j(w) > 0$ and

$$\begin{cases} \tilde{\pi}^{\Delta}(\underline{M}(w)-j,W,\xi^{\Delta}_j(w)) \geq \frac{v_0}{v-p} \\ \hat{\pi}^{\Delta}(\underline{M}(w)-j,W,\xi^{\Delta}_j(w)-\Delta) < \frac{v_0}{v-p}. \end{cases}$$

Furthermore, since $\hat{\pi}(\underline{M}(w)-j,w,u)$ is continuous and strictly increasing in u for $u \geq \xi_{j-1}(\overline{W})$ and weakly increasing for $u < \xi_{j-1}(\overline{W})$, there is a unique solution $\xi'(w)$ to

$$\hat{\pi}(\underline{M}(w)-j,W,\xi'(w)) = \frac{v_0}{v-p}.$$

Hence, as $\Delta \to 0$, it must be that $\lim_{\Delta\to 0} \xi_j^{\Delta}(w) = \xi'(w)$.

L.6) *Pointwise convergence of* $F^{\Delta}\left(w;\left(\underline{M}(D)-j, D, \lceil \frac{u}{\Delta} \rceil \Delta\right)\right)$*:* It follows immediately from pointwise convergence of $\underline{D}^{\Delta}\left(\underline{M}(D)-j, \lceil \frac{u}{\Delta} \rceil \Delta\right)$ that

$$
\begin{aligned}
&F(w;(\underline{M}(D)-j, D, u)) := \lim_{\Delta\to 0} F^{\Delta}\left(w;\left(\underline{M}(D)-j, D, \lceil \tfrac{u}{\Delta} \rceil \Delta\right)\right) \\
&= \begin{cases} \frac{F_0(w)-F_0(D)}{1-F_0(D)} \mathbb{1}(w \geq D) & \text{if } D \geq \underline{D}(\underline{M}(D)-j, u) \\ \mathbb{1}(w \geq D) & \text{otherwise} \end{cases}.
\end{aligned}
$$

L.7) $\pi(N, D, u)$ *is strictly increasing in* N*,* D*, and* u*, as long as* $G-(N+1)p > D \geq \underline{D}(N, u)$*:* By Definition 2, $\underline{D}(N, u) \geq \underline{D}(N+1, u-\Delta) \geq \underline{D}(N+1, u)$ and $\underline{D}(N, u) \geq \underline{D}(N=1, u)$. An analogous argument to Lemma 2 iii) and iv) implies monotonicity in N, D, u.

L.8) $\underline{D}(N, u)$ *is strictly decreasing in* N *and* u*, as long as* $\pi(N, 0, u) < \frac{v_0}{v-p}$*:* Strict monotonicity of $\underline{D}(N, u)$ in N and u follows from the strict monotonicity properties in N, D, and u of $\tilde{\pi}(N, D, u)$ and because $\tilde{\pi}(N, \underline{D}(N, u), u) = \frac{v_0}{v-p}$ for $\pi(N, 0, u) < \frac{v_0}{v-p}$.

L.9) $\xi_j(w)$ *is strictly increasing in* j *as long as* $\xi_j(w) > 0$.

Since $\pi(N+1, w, \xi_{j-1} w)) = \frac{v_0}{v-p}$ and $\pi(N, D, u)$ is strictly increasing in N, $\xi_j(w) > \xi_{j-1}(w)$.

C2. *Application: Crowdfunding*

A widely mentioned benefit of crowdfunding is that it enables potential investors to learn about product quality from the behavior of other investors. In this section, we illustrate how social learning interacts with the signaling incentive of the donor, by presenting a two-period example. We highlight two insights. First, in the presence of social learning, the donor is less effective in solving the coordination problem. Second, our analysis is robust to some amount of social learning.

Let $q \in \{0, 1\}$ denote the unknown quality of the product. All players (i.e., the donor and investors) share the prior that $q = 1$ with probability $\mu_0 \in (0, 1)$. We view q as the inherent quality of the product or an unknown common value component of demand. In order to keep the example simple, we assume that the quality of the product affects investors' payoffs but not the donor's payoff. Investors value a product of quality q at $v(q) = v \cdot q$. So, if an investor pledges, she gets payoff $vq - p$ if the campaign is successful and zero otherwise. If she does not pledge, she receives the outside option v_0. As before, the donor values a successful campaign at $w \sim F_0$. He receives a payoff $w - D_T$ if the campaign succeeds, and zero otherwise. In the following, we set $v = 3$, $p = 1$, $v_0 = 1$, and $1 - F_0(0.5) = 0.3$. For simplicity, let us define $\phi := \frac{\mu_0}{1-\mu_0}$.

In every period $t = 1,2$, an investor arrives with probability $\Delta\lambda = 0.9$. On arrival, each investor privately observes a signal $s \in \{0,1\}$. For simplicity, we consider a “bad news" signal process: An investor who receives a bad signal $s = 0$ knows with certainty that quality is low ($q = 0$). Specifically, we set $\Pr(s = 1|q = 1) = 1$ and $\Pr(s = 1|q = 0) = 0.5$.

First, we consider $G = 1.5$ so that the campaign is successful if at least two investors pledge or if one investor pledges and the donor valuation w is greater than or equal to 0.5.[18] An investor in period $t = 2$ can socially learn only if the period-1 investor's strategy is to pledge if $s = 1$ and not to pledge if $s = 0$. In that case, the posterior belief of a period-2 investor if the period-1 investor has pledged is, by Bayes' rule,

$$\mu_2(1) = \frac{\mu_0}{\mu_0 + (1-\mu_0)\cdot 0.25} = \frac{4}{4+\phi^{-1}},$$

and if no pledge occurred in period 1,

$$\mu_2(0) = \frac{\mu_0 \cdot 0.1}{\mu_0 \cdot 0.1 + (1-\mu_0)(0.1 + 0.9\cdot 0.5)\cdot 0.5} = \frac{4}{4+11\phi^{-1}}.$$

Let us assume that ϕ is such that $3\mu_2(1) \geq p + v_0 = 2$ so that the second investor with a positive signal always pledges after a pledge in the first period. Let us further assume that $3\mu_2(0) < 2$ so that the second investor never pledges if no pledge occurred in the first period. Hence, $2 \geq \phi^{-1} > 2/11$.

In the first period, an investor with a positive signal pledges if she believes that given cumulative donations D, the donor valuations are distributed according to $w \sim F(\cdot|D)$ if

$$\frac{4}{4+2\phi^{-1}}(0.9 + 0.1(1 - F(0.5|D)))(3-1) - \frac{2\phi^{-1}}{4+2\phi^{-1}}(0.45 + 0.55(1 - F(0.5|D))) \geq 1.$$

The left-hand side is decreasing in $1 - F(0.5|D)$ if $\phi^{-1} > 8/11$. Furthermore, let

$$\phi^{-1} \leq \frac{2((0.9 + 0.1(1 - F_0(0.5))2 - 1)}{0.45 + 0.55(1 - F_0(0.5)) + 1} \approx 1.07 \tag{C3}$$

to make it worthwhile for the investor to pledge absent donations. Hence, for example, for $\phi^{-1} = 0.8 > 8/11$, in the success-maximizing equilibrium the donor optimally donates nothing until the deadline. The campaign succeeds if either $w > 1.5$ or if two investors with a high signal realization arrive.

If the scope of social learning is small, e.g., $\phi^{-1} = 0.5 < 8/11$, then a PT equilibrium in which the donor donates just enough to make the next investor buy exists. Hence, our analysis is robust to some amount of social learning.

The example highlights several new forces: First, increasing donations is less ef-

[18] The campaign can also succeed if no investor pledges and the donor valuation exceeds the goal amount, but this case is irrelevant for the strategic pledging incentives of investors.

fective in increasing the probability of success because it also increases the probability of buying the product when its quality is actually low. Second, the benefit from pledging might even be decreasing in cumulative donations.

C3. Application: Industrial Policy

We first consider the setting in which the government's payoff is given by $(W - D_T)\mathbb{1}(R_T \geq G)$. Because there is uncertainty about the goal, participants know if the goal is reached only once $D + Np + X \geq G$. We can construct a PT equilibrium analogously to the success-maximizing PT equilibrium in Proposition 1 such that the donation threshold makes investors just indifferent between pledging and not. All expressions are analogous to the case where the inductive definition of the probability of success contains an additional integral:

$$\begin{aligned}
&\tilde{\pi}^{\Delta}_{j-1}(N,D,u) = \\
&\mathbb{E}^{F_0}\Bigg[\int \sum_{i=1}^{\max\{(u-\xi^{\Delta}_{\underline{M}(W)-(N+1)}(W))/\Delta,0\}} (1-\Delta\lambda)^{i-1}\Delta\lambda\Big(\mathbb{1}(D^{\Delta}_{*,j-1}(N+1,u-\Delta(i-1))+Np \geq G+X)+ \\
&\pi^{\Delta}_{j-1}\big(N+1,\max\{D,D^{\Delta}_{*,j-1}(N+1,u-\Delta(i-1))\},u-\Delta i\big)\mathbb{1}(D^{\Delta}_{*,j-1}(N+1,u-\Delta(i-1))+Np < G+X)\Big) \\
&+(1-\Delta\lambda)^{u/\Delta}\mathbb{1}(W+X \geq G-(N+1)p)\,dH(X)\Big| W \geq D\Bigg].
\end{aligned}$$

Similarly, we can construct a PT equilibrium analogous to Proposition 2 using cutoff times ξ that correspond to a game without a donor but an uncertain goal. In this equilibrium, investors pledge even if they believed there were no additional donations. Hence, this equilibrium coincides with one in which investors know W. By construction, the donation thresholds for this equilibrium are higher for any state (N,u) than the threshold for the equilibrium that corresponds to Proposition 1. Thus, the probability of success in this equilibrium yields a higher probability of success than the equilibrium in which there was no signaling and W was made public. The same equilibria could be supported if the donor was maximizing the probability of success subject to a budget constraint.

Next, we consider alternative donor payoffs $(W-\gamma D_T)\mathbf{1}(R_T \geq G)-(1-\gamma)D_T$, where $\gamma \in [0,1]$ is the scrap value of investment if the goal is not reached. Solving a fully dynamic game is beyond the scope of this paper, but we can highlight that there is value in signaling, using a two-period example.

We assume $t = 1,2$ and that G is uniformly distributed on $\{2,3\}$. We also assume that the arrival rate of investors is $\lambda = 1$, and each investment requires a contribution of $p = 1$. We assume a cutoff probability of 0.5 (e.g., $v = 2$, $v_0 = 0.7$, so $\frac{v_0}{v-p} = 0.5$) and W to be uniformly distributed on $\{0,2\}$.

If W is announced ex-ante, then investors recognize that the donor will commit up to W at $t = 2$ to ensure success. Thus, investors effectively face a goal of $G - W$. If $W = 2$, one investment suffices for success, so investors always contribute. Conversely, if $W = 0$, the second-period investor never contributes, as the probability of success is at most $0.5 < 0.7$. Consequently, the first-period investor also refrains from contributing even if $G = 2$.

If W remains private information, we can construct an equilibrium such that the project succeeds if the realized donor valuation is $W = 0$ and the realized goal is $G = 2$. If an investment occurred in period 1, then the probability of success for an investor at $t = 2$ is $0.5 + 0.5 \cdot 0.5 = 0.75 \geq 0.7$, so she invests. A period-1 investor also contributes, with a calculated success probability of $0.5+0.5 \cdot 0.5 \cdot 0.9 = .725 \geq 0.7$. An optimal strategy for the donor is to only contribute at $t = 2$ and only if it facilitates success. Thus, the project succeeds if $G = 2$ and $W = 0$. Thus, the probability of success is greater when W is undisclosed.

C4. Proof of Proposition 5 (Donation Dynamics of PT Equilibria)

i) **Claim:** $\mathbb{P}(N_T p + D_{T-\Delta} < G|\mathscr{S}_T) > 1-\Delta\lambda$ and $\mathbb{P}(D_T = G - N_T p|\mathscr{S}_T) \geq 1-\Delta\lambda$.

If the campaign has not succeeded by the beginning of the last period, then $D_{T-\Delta} + N_{T-\Delta} p < G$. Then, it can only be that $N_T p + D_{T-\Delta} \geq G$ if a consumer arrives in the last period, which occurs with probability $\Delta\lambda$. Even if a consumer arrives, the campaign remains unsuccessful without a donation. If $N_T p + D_{T-\Delta} < G$, then the donor donates exactly such that $D_T = G - N_T p$ if his valuation w is large enough. If w is smaller, the campaign fails.

ii) **Claim:** $\mathbb{P}(D_{\tau-\Delta} < G - N_\tau p) = 1$ if $\tau < T$.

In any PT equilibrium with donation threshold D_*^Δ, the donor never donates more than $\max\{D, D_*^\Delta(N,u)\}$ at $u > \Delta$, where $D_*^\Delta(N,u) < G-(N+1)p$. Thus, if the campaign succeeds for $u > \Delta$, it must be due to a purchase.

iii) **Claim:** $D_*^\Delta(N, u+\Delta) \geq D_*^\Delta(N+1, u)$

This is simply Condition i) in Definition 2 of PT assessments.

iv) **Claim:** Given donor realizations $w > w'$, if a campaign is unsuccessful for both w and w', then the failure time ι is larger for w than for w'.

We can write the failure time of a campaign in a PT equilibrium as $\iota = \min_j \left\{\tau_j \geq 0 \,\middle|\, W < D_*^\Delta(j, T-\tau_j)\right\}$. Hence, it follows immediately that a donor with valuation w fails later than a donor with valuation w'.

v) **Claim:** In success-minimizing PT equilibria, all donations are at least p.

This follows immediately from the definition of the success-minimizing threshold $\overline{D}^\Delta(N,u)$ in Section II.D.

DATA APPENDIX

D1. Data Construction

We directly observe pledge counts for each reward level (buyer pledges) as well as total revenues. Total revenues are inclusive of donations and shipping costs—on Kickstarter, shipping costs are included in the progress towards the goal but are not included in the prices for rewards. This means that we observe both left-hand-side variables individually in the equation

$$\text{Total Revenue}_t - \text{Buyer Revenue}_t = \text{Donor Revenue}_t + \text{Shipping Costs}_t,$$

but we observe only the sum of the right-hand-side variables. In order to recover the amount of donations, we need an estimate of shipping costs.

We collect shipping costs for every campaign-reward-country combination and then assign a shipping cost to every observed pledge. Since donations are positive contributions to campaigns, we also incorporate the constraint $\text{Shipping Costs}_t \leq \text{Total Revenue}_t - \text{Investor Revenue}_t$.

In total, we collect more than 516,000 shipping quotes. The most frequently observed shipping options are free shipping, single-rate shipping, or worldwide shipping with region-specific or country-specific prices. We complete our analyses under three shipping-cost assignments: (i) least-expensive shipping, (ii) assuming all buyers are located in the United States, and (iii) most-expensive shipping. Specifications (i) and (iii) provide lower and upper bounds on the importance of donations. We use (ii) as our main specification because most campaigns originate in the U.S.

We define a buyer to be an individual who pledges for any reward; however, some rewards may be better classified as a donation. For example, if the lowest reward is a thank-you card but the main reward is a novel product, the lowest reward may be better treated as a donation. Another example may be the existence of an expensive option that includes the main reward but also allows the buyer to meet with the entrepreneur. We repeat all of our analyses treating the most-expensive, the least-expensive rewards, or both the least- and most-expensive rewards as donations.

For our empirical analysis, we use the following cleaning criteria:

i) Some entrepreneurs request that buyers pledge in excess of the posted price if they are interested in obtaining additional product features—called “add-ons" or "optional buys." Other campaigns have “stretch goals,” which means that the entrepreneur informally adjusts the goal and if met, adjusts the final product. Unfortunately, we do not have access to individual-level data to measure the prominence of buyers contributing in excess of the goal. We attempt to minimize the presence of these contributions by removing any campaigns whose HTML pages include words related to add-ons, optional buys, and stretch goals.

ii) We winsorize the sample by dropping the bottom 0.5% and the top 0.5% of campaigns in terms of the goal amount. This removes campaigns with low

$1 goals and campaigns with several-million-dollar goals (one in the billions). These extreme values impact some means, such as average goal, but medians are unchanged.

iii) We drop campaigns that were removed by the creator, campaigns under copyright dispute, and campaigns with optional add-ons.

D2. Additional Tables and Figures

Table D1—: Top Category Summary Statistics

	Design	Film & Video	Music	Technology
Project Length	33.8	33.0	33.1	35.9
	(10.9)	(12.5)	(12.3)	(12.2)
Goal ($)	19185.6	18003.5	9849.1	36482.4
	(32655.5)	(44140.1)	(27223.1)	(61063.7)
Number of Rewards	8.1	7.4	7.1	6.2
	(5.2)	(5.6)	(5.8)	(4.6)
Donor Revenue	31.1	26.2	17.8	17.9
(per period)	(525.8)	(286.2)	(160.6)	(238.7)
Buyer Revenue	320.9	55.9	50.1	213.2
(per period)	(2112.4)	(488.7)	(360.6)	(1881.7)
Percentage Donations	17.3	40.9	37.8	29.5
at Deadline	(25.1)	(32.3)	(31.5)	(36.2)
Percentage Donations	18.5	22.6	24.5	8.3
of Goal	(52.8)	(51.6)	(103.0)	(28.6)
Percentage Successful	49.1	44.0	54.0	24.6
Number of Campaigns	4819	5176	4328	4804

Note: Summary statistics for the top four Kickstarter categories, based on the number of campaigns within a category. Standard deviation reported in parentheses. Donor and buyer revenue are reported as 12-hour averages. Rows involving percentages use final campaign outcomes.

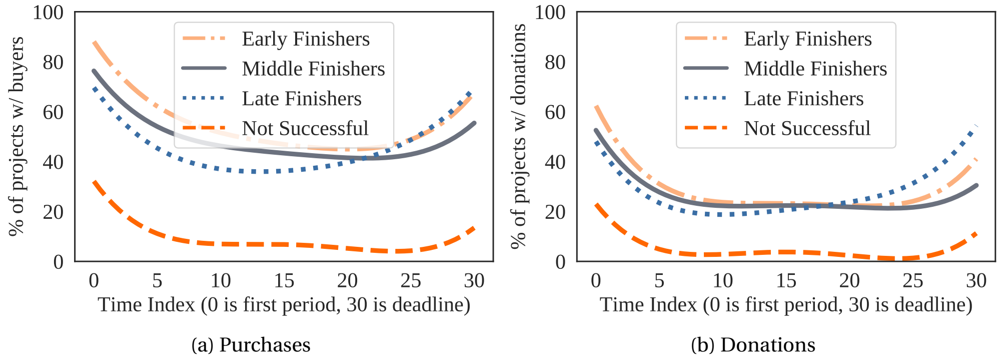


Figure D1. : Percentage of Campaigns that Receive Purchases or Donations over Time

Note: These figures show the percentage of campaigns that have purchases or donations over time, for 30-day campaigns; 30 denotes the campaign deadline. Four lines are shown: early finishers (campaigns that succeed within 3 days of launching), middle finishers (campaigns that succeed within 3–27 days of launching), late finishers (campaigns that succeed in the last 3 days before the deadline), and unsuccessful campaigns. The lines are fitted values of polynomial regressions.

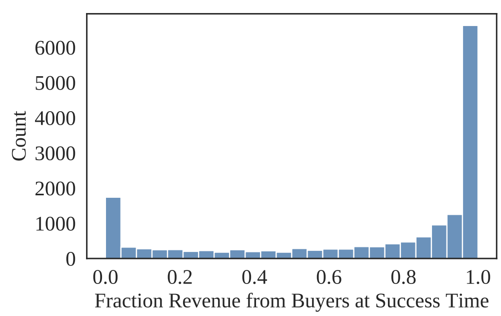
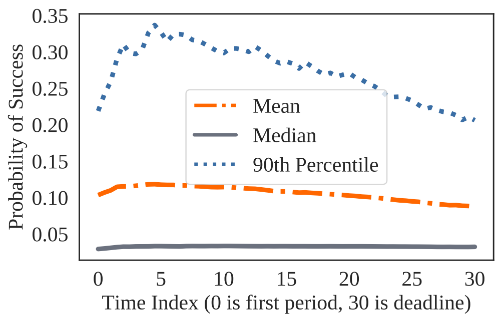
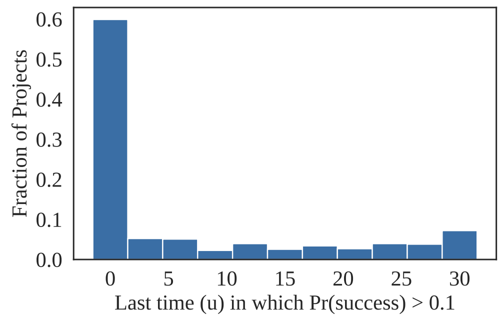


Figure D2. : Percentage of Revenue from Buyers at Success Time

Note: Histogram of the fraction of revenue from buyers in the period in which a campaign succeeds. Selected campaigns finish at least one day before the deadline.



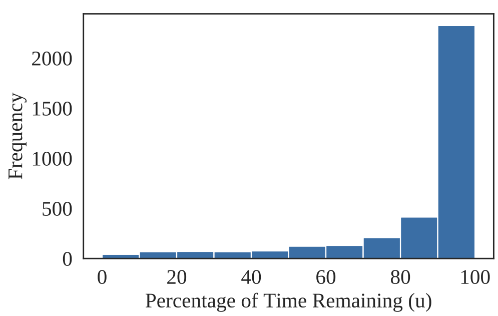
(a) Campaign Success Probability over Time



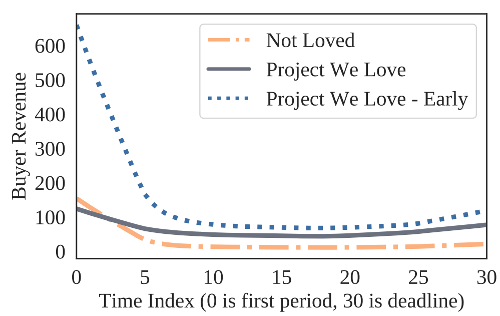
(b) Last Period Alive for each Failed Campaign

Figure D3. : Logistic Regression: Probability of Success for Campaigns that Eventually Fail

Note: (a) Fitted values of a logistic regression of campaign success over time for campaigns that eventually fail. Controls are fraction of total revenues over the goal amount (or 1 if greater) interacted with time. Plotted are the mean project, the median campaign, and the 90th percentile of projects. The results suggest that more than half of projects have a low probability of success at the start. (b) A histogram of the last time a campaign had a probability of success greater than 10%. Time rounded to three-day bins.



(a) Timing



(b) Investor Contributions

Figure D4. : "Projects We Love", Timing, and Investor Contributions

Note: "Projects We Love" is a designation assigned to campaigns by Kickstarter staff. These campaigns may be featured on the site homepage as well as advertised in emails. Panel (a) presents a histogram of when the designation is applied, as a function of time remaining in the campaign. Panel (b) presents average buyer revenue for three scenarios: (1) campaigns that never receive the designation, (2) campaigns that receive the designation after 10% of time has elapsed, and (3) campaigns that receive the designation within the first 10% of time.

Table D2—: Dynamic Donation Regressions

	(1)	(2)	(3)	(4)
Lag1 Above Median	10.327	11.437	-9.401	-8.200
	(2.415)	(2.415)	(2.452)	(2.452)
Lag2 Above Median	-2.147	-1.888	-6.387	-6.017
	(3.001)	(3.000)	(2.937)	(2.936)
Lag3 Above Median	-0.304	-0.084	-4.471	-4.136
	(2.875)	(2.874)	(2.814)	(2.812)
Lag4 Above Median	3.428	4.110	-9.886	-8.853
	(2.205)	(2.205)	(2.210)	(2.211)
Time Fixed Effects	–	✓	–	✓
Campaign Fixed Effects	–	–	✓	✓

Note: Results of panel data regressions where the dependent variable is $D_{j,t}$ and the independent variables are $\mathbf{1}\left[R_{j,t-k}/G_j > \text{median}\left[R_{\cdot,t-k}/G_{\cdot}\right]\right]$, for $k = 1,2,3,4$. That is, these variables mark if a campaign's cumulative revenue over the goal amount is above the median. We calculate the median at the time–category level for 30-day campaigns that eventually succeed. We run regressions for 30-day campaigns that have not yet reached success. Therefore, the interpretation of the model is donations as a function of whether or not a given campaign is above the median in terms of reaching the goal. The number of observations in each regression is 907,183. Checkmarks denote variables included in the regression. Dashes denote variables excluded from the regression.

D3. Bounding Donations

Donations on Kickstarter can come either from contributors entering an amount in the donation box or from contributors paying more than the reward price. However, some rewards may be interpreted as donations (e.g., low-priced rewards that approximate a thank-you, or an expensive reward that includes the product but also includes special recognition). The bias is in only one direction: we are possibly understating the magnitude of donations. This is not a problem per se, but we investigate how it affects our results.

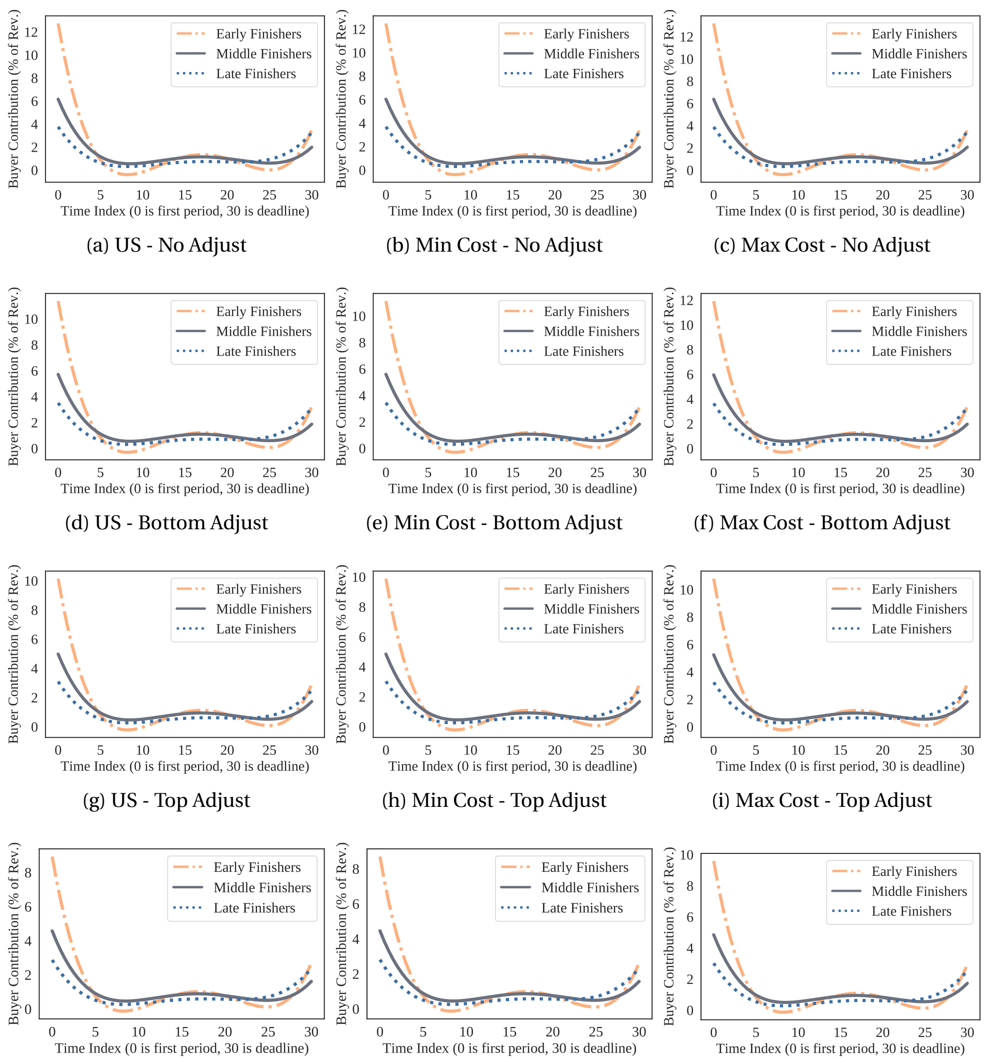
Given the number of projects and buckets per project, manually assigning a reward or part of a reward as a donation is infeasible. There are over 500,000 rewards in the data. Instead, we perform the following analyses. First, we assume the least expensive bucket represents a donation. Next, we assume the most expensive bucket represents a donation. Finally, we assume both the least and most expensive buckets constitute donations.

We also conduct robustness to our calculation of shipping costs. This is important because donations are determined after subtracting shipping costs. If we understate shipping costs, we overstate donations. We reprocess all the data assuming all purchases are made from the country with the lowest, and then the most expensive, shipping costs.

In Figure D5 and Figure D6, we plot average contributions divided by total campaign revenue over time for pledges and donations, respectively, under 12 scenarios. In Figure D7 and Figure D8, we plot the percentage of campaigns that see pledges and donations over time, respectively, under 12 scenarios. The labeling in the figures uses the legend in Table D3.

Table D3—: Robustness Analysis Figure Legend

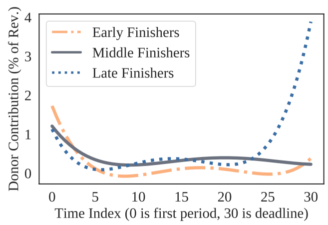
US	Assumes all rewards shipped under USA shipping costs
Min Cost	Assumes all rewards shipped to least expensive country
Max Cost	Assumes all rewards shipped to most expensive country
No Adjust	Treats all rewards as products
Bottom Adjust	Treats the least expensive reward as a donation
Top Adjust	Treats the most expensive reward as a donation
All Adjust	Treats both the least and most expensive reward as a donation



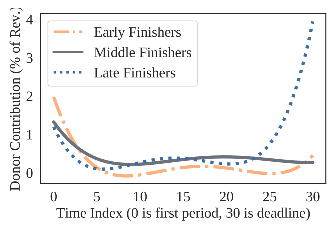
(a) US - No Adjust
(b) Min Cost - No Adjust
(c) Max Cost - No Adjust
(d) US - Bottom Adjust
(e) Min Cost - Bottom Adjust
(f) Max Cost - Bottom Adjust
(g) US - Top Adjust
(h) Min Cost - Top Adjust
(i) Max Cost - Top Adjust
(j) US - All Adjust
(k) Min Cost - All Adjust
(l) Max Cost - All Adjust

Figure D5. : Robustness: Buyer Contributions over Time

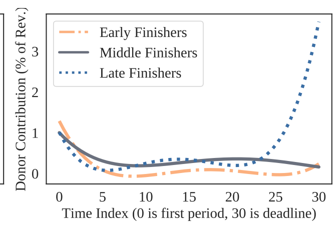
Note: These figures show average buyer contributions over time (over total campaign revenue) for different assumptions on shipping costs and what constitutes a donation. See Table D3 for label descriptions.



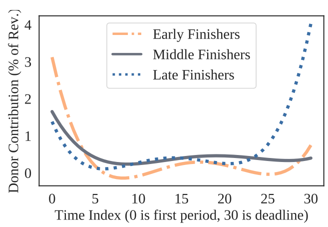
(a) US - No Adjust



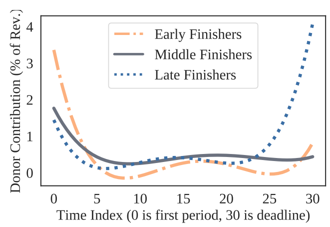
(b) Min Cost - No Adjust



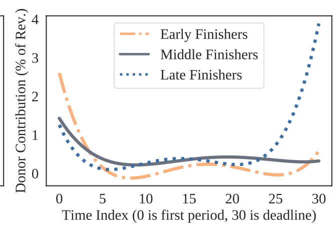
(c) Max Cost - No Adjust



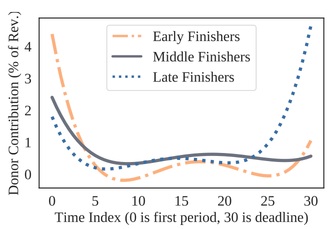
(d) US - Bottom Adjust



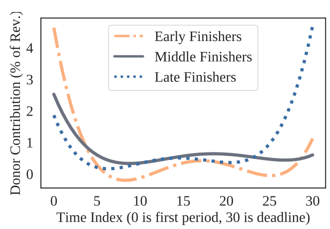
(e) Min Cost - Bottom Adjust



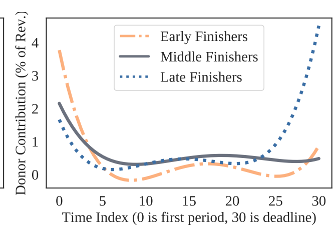
(f) Max Cost - Bottom Adjust



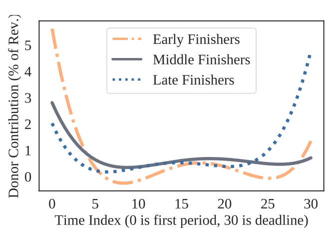
(g) US - Top Adjust



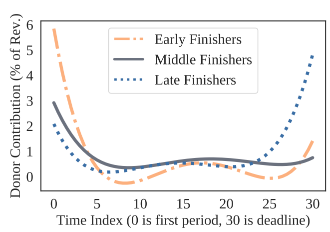
(h) Min Cost - Top Adjust



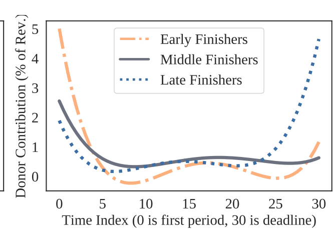
(i) Max Cost - Top Adjust



(j) US - All Adjust



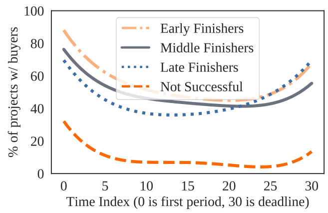
(k) Min Cost - All Adjust



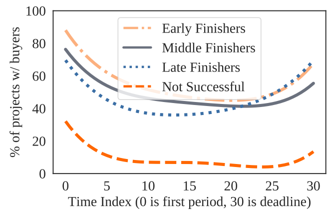
(l) Max Cost - All Adjust

Figure D6. : Robustness: Donor Contributions over Time

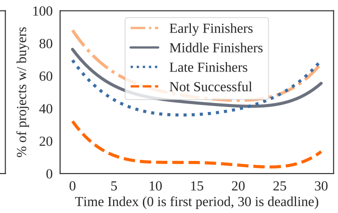
Note: These figures show average donor contributions over time (over total campaign revenue) for different assumptions on shipping costs and what constitutes a donation. See Table D3 for label descriptions.



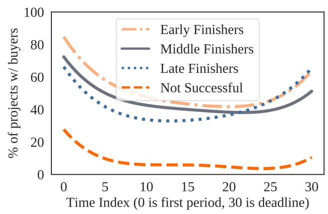
(a) US - No Adjust



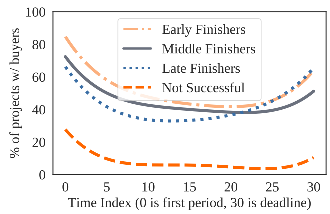
(b) Min Cost - No Adjust



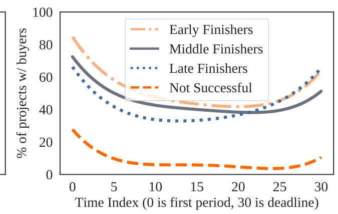
(c) Max Cost - No Adjust



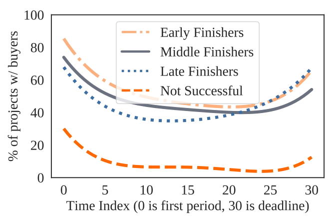
(d) US - Bottom Adjust



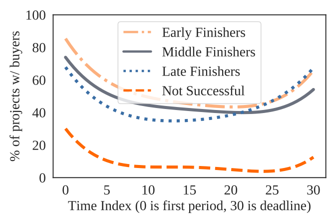
(e) Min Cost - Bottom Adjust



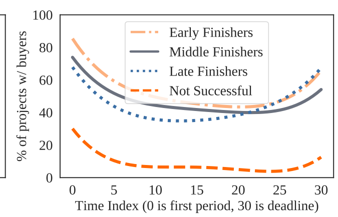
(f) Max Cost - Bottom Adjust



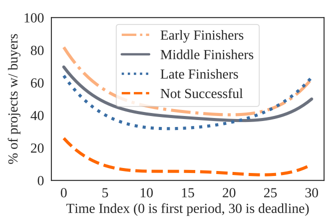
(g) US - Top Adjust



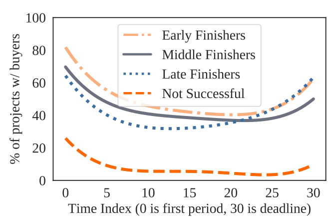
(h) Min Cost - Top Adjust



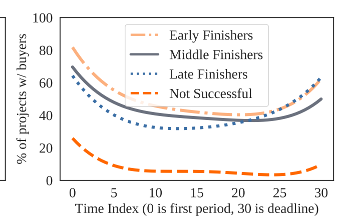
(i) Max Cost - Top Adjust



(j) US - All Adjust



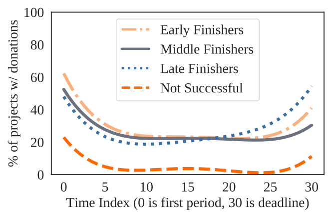
(k) Min Cost - All Adjust



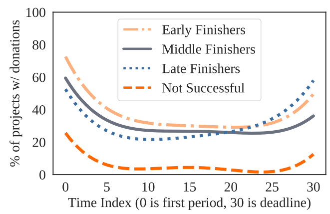
(l) Max Cost - All Adjust

Figure D7. : Robustness: Percentage of Projects that Receive Purchases over Time

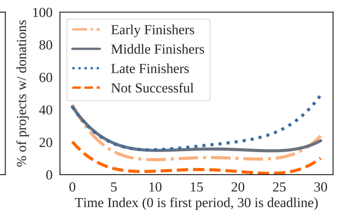
Note: These figures show the percentage of campaigns that receive pledges over time for different assumptions on shipping costs and what constitutes a donation. See Table D3 for label descriptions.



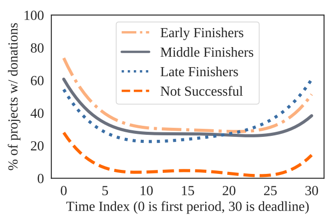
(a) US - No Adjust



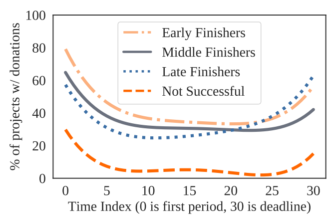
(b) Min Cost - No Adjust



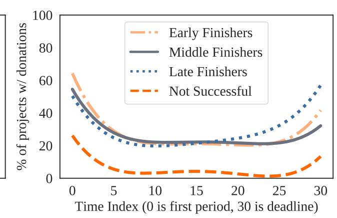
(c) Max Cost - No Adjust



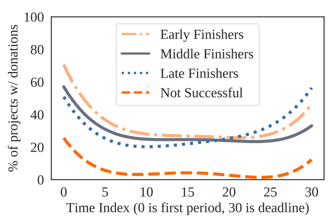
(d) US - Bottom Adjust



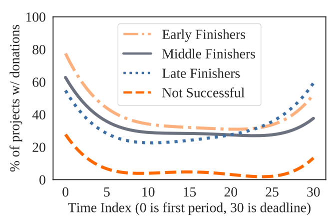
(e) Min Cost - Bottom Adjust



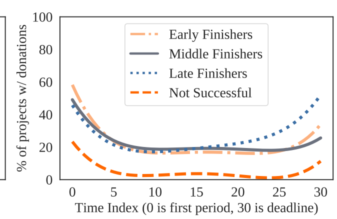
(f) Max Cost - Bottom Adjust



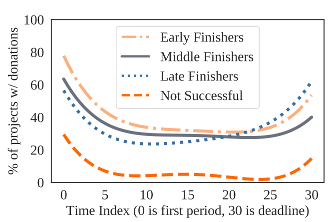
(g) US - Top Adjust



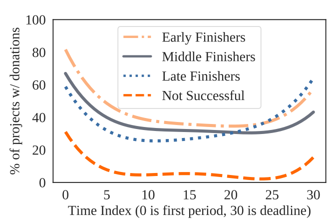
(h) Min Cost - Top Adjust



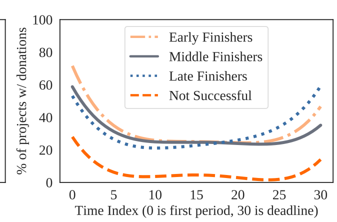
(i) Max Cost - Top Adjust



(j) US - All Adjust



(k) Min Cost - All Adjust



(l) Max Cost - All Adjust

Figure D8. : Robustness: Percentage of Projects that Receive Donations over Time

Note: These figures show the percentage of campaigns that receive donor contributions over time for different assumptions on shipping costs and what constitutes a donation. See Table D3 for label descriptions.

Table D4—: Robustness: Donation Revenue per Period

Shipping	Donation Adjust	Mean (All)	Mean (Uns.)	Mean (Suc.)	Median	5th %	95th %
US	No Adjust	19.1	3.3	43.7	0.0	0.0	44.0
Min Cost	No Adjust	20.7	3.5	47.4	0.0	0.0	50.0
Max Cost	No Adjust	14.9	2.9	33.5	0.0	0.0	15.0
US	Bottom Adjust	28.3	4.5	65.5	0.0	0.0	62.0
Min Cost	Bottom Adjust	29.8	4.7	69.0	0.0	0.0	72.0
Max Cost	Bottom Adjust	23.9	4.1	54.6	0.0	0.0	39.0
US	Top Adjust	33.6	5.6	77.1	0.0	0.0	75.0
Min Cost	Top Adjust	35.1	5.8	80.8	0.0	0.0	85.2
Max Cost	Top Adjust	28.1	5.2	63.7	0.0	0.0	46.0
US	All Adjust	42.1	6.7	97.2	0.0	0.0	100.0
Min Cost	All Adjust	43.5	6.9	100.5	0.0	0.0	100.0
Max Cost	All Adjust	36.9	6.3	84.5	0.0	0.0	70.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

Table D5—: Robustness: Pledge Revenue per Period

Shipping	Donation Adjust	Mean (All)	Mean (Uns.)	Mean (Suc.)	Median	5th %	95th %
US	No Adjust	119.5	14.0	283.8	0.0	0.0	391.0
Min Cost	No Adjust	117.7	13.8	279.8	0.0	0.0	384.0
Max Cost	No Adjust	123.9	14.4	294.6	0.0	0.0	408.5
US	Bottom Adjust	113.2	13.0	269.3	0.0	0.0	368.2
Min Cost	Bottom Adjust	111.5	12.8	265.4	0.0	0.0	360.0
Max Cost	Bottom Adjust	118.4	13.5	282.0	0.0	0.0	389.0
US	Top Adjust	107.7	11.9	257.0	0.0	0.0	347.1
Min Cost	Top Adjust	105.9	11.7	252.9	0.0	0.0	339.4
Max Cost	Top Adjust	113.6	12.4	271.4	0.0	0.0	369.5
US	All Adjust	101.8	11.0	243.4	0.0	0.0	325.0
Min Cost	All Adjust	100.1	10.8	239.5	0.0	0.0	316.6
Max Cost	All Adjust	108.0	11.5	258.4	0.0	0.0	348.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

Table D6—: Robustness: Percentage of Revenues from Donations

Shipping	Donation Adjust	Mean (All)	Mean (Uns.)	Mean (Suc.)	Median	5th %	95th %
US	No Adjust	29.2	33.1	24.6	15.8	0.0	100.0
Min Cost	No Adjust	30.5	34.3	26.1	17.9	0.0	100.0
Max Cost	No Adjust	26.9	31.5	21.5	11.4	0.0	100.0
US	Bottom Adjust	36.9	43.3	29.4	25.5	0.0	100.0
Min Cost	Bottom Adjust	38.2	44.4	30.9	27.1	0.3	100.0
Max Cost	Bottom Adjust	34.5	41.7	26.1	21.9	0.0	100.0
US	Top Adjust	41.1	43.8	38.0	33.7	0.0	100.0
Min Cost	Top Adjust	42.5	44.9	39.6	35.4	0.0	100.0
Max Cost	Top Adjust	38.4	42.0	34.2	29.8	0.0	100.0
US	All Adjust	47.8	52.3	42.4	43.5	0.0	100.0
Min Cost	All Adjust	49.0	53.3	43.8	44.8	1.0	100.0
Max Cost	All Adjust	45.1	50.6	38.6	40.3	0.0	100.0

Note: Summary statistics for the 42,462 campaigns in the sample. Means for all statistics are computed for all campaigns (All), unsuccessful campaigns (Uns.), and successful campaigns (Suc.). Also reported are the 50th, 5th, and 95th percentiles.

D4. Equilibrium Construction using Kickstarter Data

Estimation Sample: We incorporate five selection criteria when estimating the model:

i) Select projects that have a deadline of 30 days;

ii) Select projects in which the maximum number of pledges per period is below the 95th percentile;

iii) Select projects in which the maximum amount of donations per period is below the 95th percentile;

iv) Select projects in which the maximum ending revenue over the goal amount is below the 75th percentile;

v) After implementing (i)–(iv), select the first quartile of projects by goal amount for each category.

Estimation of Donor Valuations: We estimate the selection model (Heckman, 1979) in a single step.
Estimation of Arrival Process: We estimate the selection model (Terza, 1998) in a single step. We do not estimate the variance-covariance parameters directly. Instead, we estimate the variance as $\log(\sigma)$. We estimate the covariance term as $\tanh(\rho)$. We approximate the integrals in the log-likelihood using the Gauss–Hermite quadrature with 25 integration points.
Calibration of Investor Utility: We calibrate the model using the method of simulated moments (MSM). To calculate the success-maximizing equilibrium, we implement the dual induction argument of Proposition II.C with the following adjustments:

i) We discretize donations in increments of \$1. The donor's strategy is defined over the estimated log-normal distribution up to the 99.9th percentile.

ii) We adjust the length of a time interval so that arrival rates are less than 1, i.e., we define $\lambda^{\max} = \lceil \max_{t=0,\dots,T} \widehat{\lambda}_t \rceil$, and then define the length of a period to be $1/\lambda^{\max}$ for all t. Estimated arrival rates are multiplied by $1/\lambda^{\max}$.

iii) The goal is set to be $\lceil G \rceil$ for each category and goal quartile. Donation thresholds, time cutoffs, and beliefs are defined up to $\lceil G \rceil$.

We use derivative-free search over the parameter space, initially selecting candidate solutions between $v_0 + p$ and $v_0 + p + 1$. Our method searches over larger values of v if the objective is minimized beyond $v_0 + p + 1$. We also account for potential flat spots in the objective function. For example, the probability of success may be equal to zero over a range of potential solutions because beliefs are sufficiently low. We use a stopping criterion of 1e-6 in our procedure.

We do not report results for the Dance category because the valuation distribution and arrival process parameters are such that the objective is flat over all potential v.