Online Appendix

Central Bank Credibility and Fiscal Responsibility

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January 2, 2024

Proof of Lemma 1

Necessity follows from our discussion in the text. Sufficiency follows by using $\left\{ \{\tau_t, T_t, P_t, C_t, G_t, N_t\}_{t=0,1}, B, i \right\}$ given (8) and (9) to construct the values of $\{C_{j,t}, N_{j,t}, W_t, P_{j,t}\}_{t=0,1}$ that satisfy all optimality conditions and budget constraints.■

Proof of Lemma 2

Step 1. Let us consider how G_1 is determined. The relaxed problem is strictly concave which means that the first order condition defines the unique global optimum. Equation (18) implies that C_1 and G_1 are negatively related, which means that $N_1 = C_1 + G_1$ is strictly increasing in G_1. Therefore, the left hand side of (23) is decreasing in G_1 and the right hand side of (23) increasing in G_1. Since the right hand side of (23) is increasing in B, this implies that G_1 is decreasing in B.

Step 2. Analogous argument to step 1 imply that G_1 is decreasing in λ.

Step 3. Let us consider how P_1 is determined. Substitute (20) into (23) to achieve

$$\left[-(1-\mu)(C_1+G_1)^{\varphi} G_1 + \mu \frac{1}{G_1} \right] B = \frac{\lambda}{1-\lambda} H'(P_1) P_1^2. \tag{A.1}$$

From step 1, higher B is associated with lower G_1, which means that the left hand side of ($A.1$) is increasing in B. Therefore, since the right hand side of ($A.1$) is increasing in P_1, this means that P_1 is increasing in B.

Step 4. To consider how P_1 changes with respect to λ, we first establish that $P_1 > 1$. Suppose by contradiction that $P_1 \leq 1$. Consider a perturbation that increases P_1 in order to increase G_1 by some $\varepsilon > 0$ arbitrarily small. The change in welfare taking into account (18) is

$$-(1-\mu)\frac{1}{C_1} + \mu \frac{1}{G_1}.$$

We can establish that $G_1/C_1 < \mu/(1-\mu)$, implying that this term is positive and that the perturbation raises welfare. To see why, note that (18) implies that

$$G_1^{1+\varphi} \frac{C_1}{G_1} \left(\frac{C_1}{G_1} + 1 \right)^{\varphi} = 1. \tag{A.2}$$

Suppose by contradiction that $C_1/G_1 \leq (1-\mu)/\mu$. Taking into account that (9) and (20) implies that $G_1 < \mu (1-\mu)^{-\frac{1}{1+\varphi}}$, it follows that

$$G_1^{1+\varphi} \frac{C_1}{G_1} \left(\frac{C_1}{G_1} + 1 \right)^{\varphi} < \mu^{1+\varphi} (1-\mu)^{-1} \left(\frac{1-\mu}{\mu} \right) \left(\frac{1}{\mu} \right)^{\varphi} = 1,$$

which violates ($A.2$). Therefore, $G_1/C_1 < \mu/(1-\mu)$ and the perturbation strictly increases welfare. Therefore, $P_1 > 1$ for all $\lambda \in (0,1)$.

Consider a central bank with hawkishness λ' choosing $P_1(\lambda')$ and another central bank with hawkishness $\lambda'' > \lambda'$ choosing $P_1(\lambda'')$. For both central banks to be weakly prefering their policy choice, it is necessary that they weakly prefer to not mimic each other, which means that

$$\left(\frac{\lambda''}{1-\lambda''}-\frac{\lambda'}{1-\lambda'}\right)\left(H(P_1(\lambda')-H(P_1(\lambda''))\right)\geq 0$$

Since $\lambda''>\lambda'$ and $P_1(\lambda')$ and $P_1(\lambda'')$ both exceed 1, with $H(P)$ increasing for $P>1$, it follows that this condition can only hold if $P_1(\lambda')\geq P_1(\lambda'')$. Therefore, P_1 decreases in λ. ■

Proof of Lemma 3

Proof of part (i). If $B=0$ then $G_1=T$ and $G_0=T$, and the first best allocation conditional on $G_0=T$ can be implemented with $P_0=1$.

Proof of part (ii) Suppose that $\forall B/P_1\in(0,\upsilon)$ for some $\upsilon>0$ arbitrarily small. We establish this result in two steps.

Step 1. We first establish that $P_0\neq 1$. Consider a perturbation that increases P_0 by some $\varepsilon>0$ arbitrarily small. Using implicit differentiation taking into account (16), (17), and (24), we can derive the ensuing change in C_0, G_0, and N_0. The change in central bank welfare from the perturbation taking into account (17) is

$$(1-\lambda)\left(-(1-\mu)N_0^{\varphi}+\mu\frac{1}{G_0}\right)\frac{\partial G_0}{\partial P_0}-\lambda H'(P_0)\,. \tag{A.3}$$

Equations (16), (17), and (24) imply that $\frac{\partial G_0}{\partial P_0}>0$. Moreover, analogous reasoning to Step 4 in the proof of Lemma 2 taking into account that $P_0=1$ implies that $-(1-\mu)N_0^{\varphi}+\mu\frac{1}{G_0}>0$. Taking into account that $H'(1)=0$, it follows that the sign of ($A.3$) is strictly positive.

Step 2. We next establish that $P_0>1$. We first show that constraint (17) is equivalent to

$$C_0(C_0+G_0)^{\varphi}\leq 1+\frac{\alpha}{\sigma-1}(P_0-1)P_0\,. \tag{A.4}$$

Suppose that the solution to the relaxed problem admits ($A.4$) as a strict inequality. Then necessarily, the solution admits $P_0=1$. Consider a perturbation which increases G_0 by some ε arbitrarily small and which also increases C_0 so as to satisfy (24). The change in welfare is

$$(1-\lambda)\left(\left(\frac{1}{C_0}-(1-\mu)N_0^{\varphi}\right)\frac{\partial C_0}{\partial G_0}-(1-\mu)N_0^{\varphi}+\mu\frac{1}{G_0}\right)-\lambda H'(P_0)\,. \tag{A.5}$$

Given $P_0=1$, ($A.4$) which holds as a strict inequality, and the fact that $\frac{\partial C_0}{\partial G_0}>0$, it follows that ($A.5$) is strictly larger than

$$(1-\lambda)\left(-(1-\mu)\frac{1}{C_0}+\mu\frac{1}{G_0}\right). \tag{A.6}$$

Observe that as $B/P_1\to 0$, satisfaction of (24) requires $C_0\to C_1$ and $G_0\to T$. Using this observation, it follows that satisfaction of (24) requires $C_0/G_0>(1-\mu)/\mu\ \forall B/P_1\in(0,\upsilon)$ for some $\upsilon>0$ arbitrarily small. Thus, analogous reasoning to Step 4 in the proof of Lemma 2 implies that ($A.6$) is strictly positive. Therefore, the solution to the relaxed problem is equal to the solution to constrained problem.

Now suppose by contradiction that the solution admits $P_0<1$. Consider a perturbation that increases P_0 to 1, holding C_0 and G_0 constant. This perturbation satisfies all constraints of the relaxed problem and strictly increases welfare. Therefore, $P_0\geq 0$ and by Step 1, $P_0>0$.

Proof of part (iii). This follows from analogous reasoning to Step 4 in the proof of Lemma 2.■

Proof of Proposition 1

Proof of part (i). Take $\lambda \to 1$, where $\underline{G}_1(\lambda) \to 0$, $P_0 \to 1$, $P_1 \to 1$. Consider the program of the fiscal authority which can be rewritten as

$$\max_{C_0,G_0,N_0,C_1,G_1,N_1} \left\{ \begin{array}{c} (1-\mu)\left(\log C_0 - \frac{N_0^{1+\varphi}}{1+\varphi}\right) + \mu \log G_0 \\ (1-\mu)\left(\log C_1 - \frac{N_1^{1+\varphi}}{1+\varphi}\right) + \mu \log G_1 \end{array} \right\}$$

s.t.

$$C_t + G_t = N_t \text{ for } t = 0, 1,$$

$$C_t N_t^{\varphi} = 1 \text{ for } t = 0, 1, \text{ and} \tag{A.7}$$

$$\frac{T - G_0}{C_0} + \frac{T - G_1}{C_1} = 0. \tag{A.8}$$

Observe that (*A.*8) is equivalent to a weak inequality constraint

$$\frac{T - G_0}{C_0} + \frac{T - G_1}{C_1} \geq 0. \tag{A.9}$$

This is because the solution in the absence of this constraint admits

$$C_t N_t^{\varphi} = \frac{1-\mu}{\mu} G_t N_t^{\varphi} = 1,$$

which is the first best allocation, which violates (*A.*9). Therefore, the solution to the relaxed problem with (*A.*9) is equivalent to the solution to the constrained problem. Observe that (*A.*9) can be rewritten as

$$C_1 (T - G_0) + C_0 (T - G_1) \geq 0, \tag{A.10}$$

which is a globally convex constraint. Let ψ correspond to the Lagrange multiplier on (*A.*10), and consider the relaxed problem that ignores (*A.*7). First order conditions yield

$$\begin{aligned} \frac{1}{C_0} - (C_0 + G_0)^{\varphi} + \psi (T - G_1) &= 0 \\ \frac{1}{C_1} - (C_1 + G_1)^{\varphi} + \psi (T - G_0) &= 0 \\ \frac{\mu}{1-\mu}\frac{1}{G_0} - (C_0 + G_0)^{\varphi} - \psi C_1 &= 0 \\ \frac{\mu}{1-\mu}\frac{1}{G_1} - (C_1 + G_1)^{\varphi} - \psi C_0 &= 0 \end{aligned}$$

Since the program is concave and the constraint set convex, the solution is unique. Observe that $G_0 = G_1 = T$ satisfies the first order conditions so that it constitutes the solution. Moreover, condition (*A.*7) is satisfied, so that the solution to the relaxed problem is the solution to the constrained problem. Therefore, $B/P_1 = 0$. The statement of the proposition follows by continuity given that $B/P_1 \geq 0$.

Proof of part (ii). As $\lambda \to 0$, $\underline{G}_1(\lambda) \to T$, which means that $B/P_1 \to 0$. The statement of the proposition follows by continuity given that $B/P_1 \geq 0$.■

Proof of Proposition 2

The equilibrium value of B/P_1 is inversely proportional to the value of G_1. Therefore, we establish this result by focusing on the value of G_1. Define $G_1^*(\lambda)$ as the solution to the unconstrained problem of the fiscal authority. Observe that this value represents the solution to the below unconstrained problem:

$$\max_{G_1} \{\log G_0^*(G_1,\lambda) + \log G_1\}, \tag{A.11}$$

where $G_0^*(G_1,\lambda)$ denotes the best response of the date 0 monetary authority with hawkishness λ. First order conditions yield

$$\frac{1}{G_0}\frac{\partial G_0^*(G_1,\lambda)}{\partial G_1} + \frac{1}{G_1} = 0. \tag{A.12}$$

To determine $G_0^*(G_1,\lambda)$, note that the date 0 central banks' problem (25) can be represented as

$$\max_{G_0}\left\{\eta(\lambda)\log G_0 - \frac{\left(\frac{1}{T-G_1}(G_0-T)-1\right)^2}{2}\right\}$$

for

$$\eta(\lambda) = \frac{1}{\kappa}\frac{1-\lambda}{\lambda}\left(\frac{\alpha}{\sigma-1}\right)^2,$$

Observe that the function $\eta(\lambda)$ is a strictly decreasing function of λ. Define

$$\lambda^{**} = \left(1+\kappa\left(\frac{\sigma-1}{\alpha}\right)^2\right)^{-1}, \tag{A.13}$$

and observe that $\eta(\lambda^{**}) = 1$. The first order condition implies that

$$0 = G_0^2 - G_0(2T-G_1) - \eta(\lambda)(T-G_1)^2. \tag{A.14}$$

Implicit differentiation of ($A.14$) yields

$$\frac{\partial G_0^*(G_1,\lambda)}{\partial G_1} = -\frac{G_0+\eta(\lambda)2(T-G_1)}{G_1+2(G_0-T)} < 0. \tag{A.15}$$

After substitution, ($A.12$) can be rewritten as

$$\frac{1}{G_1}\left(-\frac{1+\eta(\lambda)2(T-G_1)G_0^{-1}}{1+2(G_0-T)G_1^{-1}}+1\right) = 0. \tag{A.16}$$

Observe that ($A.16$) is satisfied for $G_1 = T$. Thus, $G_1 = T$ is a local maximum or a local minimum in the date 0 fiscal authority's problem.

Using these observations, we prove the proposition in three steps. First, we establish that if $\lambda < \lambda^{**}$, then $G_1^*(\lambda) < T$ and is strictly increasing in λ. Second, we establish that if $\lambda \geq \lambda^{**}$, then $G_1^*(\lambda) = T$. Finally, we combine these results with the observation that $\underline{G}_1(\lambda)$ is strictly decreasing in λ to complete the proof.

Step 1. We establish that if $\lambda < \lambda^{**}$, then $G_1^*(\lambda) < T$ and is strictly increasing in λ.

Step 1a. We establish that $G_1^*(\lambda) < T$. Suppose by contradiction that $G_1^*(\lambda) = T$. Consider the necessary second order condition to the date 0 fiscal authority's problem by differentiating ($A.16$) with respect to G_1, taking into account that the term in parentheses in ($A.16$) evaluated at $G_1 = T$ is zero and that $\frac{\partial G_0^*(T,\lambda)}{\partial G_1} = -1$:

$$\frac{1}{G_1}\left(-\partial\frac{\left(\frac{1+\eta(\lambda)2(T-G_1)G_0^{-1}}{1+2(G_0-T)G_1^{-1}}\right)}{\partial G_1}+\partial\frac{\left(\frac{1+\eta(\lambda)2(T-G_1)G_0^{-1}}{1+2(G_0-T)G_1^{-1}}\right)}{\partial G_0}\right)<0. \tag{A.17}$$

Inequality ($A.17$) evaluated at $G_0 = G_1 = T$ yields

$$\frac{2}{T^2}\left(\eta\left(\lambda\right)-1\right)<0. \tag{A.18}$$

However, ($A.18$) cannot hold if $\lambda < \lambda^{**}$ since $\eta(\lambda) > 1$. Therefore, $G_1 = T$ is a local minimum if $\lambda < \lambda^{**}$, which means that $G_1^*(\lambda) < T$.

Step 1b. We establish that $G_1^*(\lambda) < T$ is uniquely determined. Note that ($A.16$) taking into account that $G_1 < T$ can be rewritten as

$$\eta\left(\lambda\right)=\frac{G_0^2-TG_0}{TG_1-G_1^2}. \tag{A.19}$$

Combining ($A.14$) and ($A.19$), we achieve:

$$G_0=\eta\left(\lambda\right)\left(2G_1-T\right), \tag{A.20}$$

which implies that since $G_0 > 0$, it follows that $G_1 > T/2$. Substitution of ($A.20$) into ($A.19$) yields an equation defining G_1:

$$\left(4\eta\left(\lambda\right)+1\right)G_1^2-\left(4\eta\left(\lambda\right)+3\right)TG_1+\left(\eta\left(\lambda\right)+1\right)T^2=0. \tag{A.21}$$

Observe that the left hand side of ($A.21$) is convex in G_1, exceeds 0 if $G_1 = 0$ and $G_1 = T$ (since $\lambda < \lambda^{**}$), and is below 0 for $G_1 = T/2$. It thus follows that there is a unique value of $G_1 > T/2$ that satisfies ($A.21$).

Step 1c. Equation ($A.21$) defines $G_1^*(\lambda)$. Given Step 1b, observe that from the convexity of the left hand side of ($A.21$), it follows that the the left hand side of ($A.21$) is strictly increasing in G_1 at $G_1 = G_1^*(\lambda)$, so that

$$\left(4\eta\left(\lambda\right)+1\right)2G_1-\left(4\eta\left(\lambda\right)+3\right)T>0. \tag{A.22}$$

Implicit differentiation of ($A.21$) with respect to λ yields

$$\frac{\partial G_1^*\left(\lambda\right)}{\partial\lambda}=-\eta'\left(\lambda\right)\frac{\left(2G_1-T\right)^2}{\left(4\eta\left(\lambda\right)+1\right)2G_1-\left(4\eta\left(\lambda\right)+3\right)T}>0, \tag{A.23}$$

where we have applied ($A.22$) and the fact that $G_1 > T/2$ to sign ($A.23$). This establishes $G_1^*(\lambda)$ is strictly increasing in λ for $\lambda < \lambda^{**}$.

Step 2. We now establish that if $\lambda \geq \lambda^{**}$, then $G_1^*(\lambda) = T$.

Step 2a. We first establish that if $\lambda = \lambda^{**}$, then $G_1^*(\lambda) = T$. Suppose that this were not the case and that the solution admits $G_1^*(\lambda) < T$. Equation ($A.21$) then defines $G_1^*(\lambda)$ and the same arguments as in Step 2b imply that $G_1^*(\lambda)$ is uniquely determined. Observe that if $\lambda = \lambda^{**}$, then $G_1 = T$ solves ($A.21$), contradicting the fact that the solution admits $G_1^*(\lambda) < T$. Therefore, $G_1^*(\lambda) = T$.

Step 2b. We now establish that $G_1^*(\lambda) = T$ for all $\lambda > \lambda^{**}$. Consider the contradiction assumption that $G_1^*(\lambda') = \hat{G}_1 < T$ for some $\lambda' > \lambda^{**}$. Weak optimality for the fiscal authority at date 0 conditional on $\lambda = \lambda'$ requires

$$\log\left(G_0^*\left(\hat{G}_1\right), \lambda'\right) + \log \hat{G}_1 \geq 2\log T. \tag{A.24}$$

Strict optimality for the fiscal authority at date 0 conditional on $\lambda = \lambda^{**}$ requires

$$2\log T > \log\left(G_0^*\left(\hat{G}_1\right), \lambda^{**}\right) + \log \hat{G}_1. \tag{A.25}$$

Combining ($A.24$) and ($A.25$) we achieve

$$\log\left(G_0^*\left(\hat{G}_1\right), \lambda'\right) > \log\left(G_0^*\left(\hat{G}_1\right), \lambda^{**}\right). \tag{A.26}$$

Implicit differentiation of ($A.14$) yields

$$\frac{\partial G_0^*\left(G_1, \lambda\right)}{\partial \lambda} = -\frac{1}{\lambda^2}\frac{\left(T - G_1\right)^2}{G_1 + 2\left(G_0 - T\right)} < 0,$$

which contradicts ($A.26$). Therefore, $G_1^*(\lambda) = T$ for all $\lambda > \lambda^{**}$.

Step 3. Observe that the constrained problem of the first authority at date 1 implies that the equilibrium value of G_1 must satisfy

$$G_1 = \max\left\{G_1^*(\lambda), \underline{G}_1(\lambda)\right\}.$$

Observe that $\lim_{\lambda\to 0} \underline{G}_1(\lambda) = T > \lim_{\lambda\to 0} G_1^*(\lambda)$ (from step 1a). Moreover, $\lim_{\lambda\to 1} \underline{G}_1(\lambda) < T < \lim_{\lambda\to 1} G_1^*(\lambda) = T$ (from step 2b). Therefore, $G_1^*(\lambda) = \underline{G}_1(\lambda)$ for some interior value of λ. Moreover, since $G_1^*(\lambda)$ and $\underline{G}_1(\lambda)$ are both monotonic, this interior point is unique, and can be labeled by λ^*. It follows that $G_1 = \underline{G}_1(\lambda)$ if $\lambda < \lambda^*$, with G_1 decreasing in λ if $\lambda < \lambda^*$. Moreover $G_1 = G_1^*(\lambda)$ if $\lambda > \lambda^*$, with G_1 strictly increasing in λ for $\lambda \in (\lambda^*, \lambda^{**})$ and $G_1 = T$ for $\lambda > \lambda^{**}$. ■