Online Appendix to "Misallocation Under Trade Liberalization"

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This appendix is organized as follows.

A Equilibrium of the baseline mode

Closed Economy Equilibrium. In a closed economy, taking as given the aggregates prices (P, w) and demand Q, the problem of a firm with (φ, τ) implies the optimal price

$$p(\varphi, \tau) = \frac{\sigma}{\sigma - 1} \frac{w\tau}{\varphi} \tag{A.1}$$

and optimal profit $\pi(\varphi,\tau) = [\sigma^{-\sigma}(\sigma-1)^{\sigma-1}P^{\sigma}Qw^{1-\sigma}]\varphi^{\sigma-1}\tau^{-\sigma} - wf$. The cutoff of production is given by $\varphi^*(\tau) = con_v \times P^{-1}(PQ)^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{\sigma-1}}$ with the normalization of $w = 1$ and the constant $con_v = \sigma^{\frac{\sigma}{\sigma-1}}(\sigma-1)^{-1}f^{\frac{1}{\sigma-1}}$.

Let $\mu(\varphi,\tau)$ be the distribution of operating firms $\mu(\varphi,\tau) = \frac{g(\varphi,\tau)}{\int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)d\varphi d\tau} = \frac{g(\varphi,\tau)}{\omega_e}$ if $\varphi \geq \varphi^*(\tau)$; and 0 otherwise. Define M_e and M as a measure of entrants and operative firms, respectively.

An equilibrium is characterized by an aggregate price index, a free entry condition, and a labor market clearing condition. The aggregate price index is the weighted average of the prices (A.1) of the operating firms:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \int\int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau)d\varphi d\tau. \quad \text{(A.2)}$$

The free entry condition requires that the present value of producing equals the entry cost, i.e.,

$$\omega_e E[\pi(\varphi,\tau)] = wf_e, \quad \text{(A.3)}$$

where ω_e is the probability of entry, $\omega_e = \int\int_{\varphi^*(\tau)}^{\infty} g(\varphi,\tau)\, d\varphi d\tau$, and the expected profit is given by $E[\pi(\varphi,\tau)] = \int\int_{\varphi^*(\tau)} \pi(\varphi,\tau)\mu(\varphi,\tau)d\varphi d\tau$.

The labor market clearing condition requires

$$L = ME\left[\frac{q}{\varphi} + f\right] + M_e f_e, \quad \text{(A.4)}$$

where the average labor demanded by firms is $E\left[\frac{q}{\varphi} + f\right] = \int\int_{\varphi^*(\tau)}^{\infty}\left[\frac{q}{\varphi} + f\right]\mu(\varphi,\tau)\, d\varphi d\tau$. In equilibrium, the number of producers equals the number of entrants multiplied by the probability of producing, such that

$$\omega_e M_e = M. \quad \text{(A.5)}$$

Noting that $\omega_e E(q/\phi) = (\sigma-1)(\omega_e f + f_e)$, which can be obtained through optimal profit function and the free entry condition, we arrive at

$$M_e = \frac{L}{\sigma\left(f_e + \omega_e f\right)}. \tag{A.6}$$

Open Economy Equilibrium. Optimal prices and cutoff functions are straightforward analogs of the closed economy case. An equilibrium of the open economy consists of seven aggregate conditions: two free entry conditions for Home and Foreign, two aggregate price indexes for Home and Foreign, two labor market conditions for Home and Foreign, and one balanced trade condition.

Home's free entry condition is given by

$$\begin{aligned}
&\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\int\int_{\varphi^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]g\left(\varphi,\tau\right)d\varphi d\tau - wf\int\int_{\varphi^*(\tau)}^{\infty}g\left(\varphi,\tau\right)d\varphi d\tau \\
&+\left[\frac{P_fQ_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\tau_x w\right)^{1-\sigma}\int\int_{\varphi_x^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]g\left(\varphi,\tau\right)d\varphi d\tau - wf_x\int\int_{\varphi_x^*(\tau)}^{\infty}g\left(\varphi,\tau\right)d\varphi d\tau\right] = w
\end{aligned}$$

Rewriting this equation

$$\begin{aligned}
&w^{1-\sigma}\left[P^{\sigma}Q\int\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau + P_f^{\sigma}Q_f\tau_x^{1-\sigma}\int\int_{\varphi_x^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}g(\varphi,\tau)d\varphi d\tau\right] \\
&= \sigma^{\sigma}(\sigma-1)^{1-\sigma}\left(wf_e + \omega_e wf + \omega_x\omega_e wf_x\right)
\end{aligned}$$

where $\omega_e = \int\int_{\varphi^*(\tau)}^{\infty} g\left(\varphi,\tau\right)d\varphi d\tau$ and $\omega_x = \int\int_{\varphi_x^*(\tau)}^{\infty}\mu\left(\varphi,\tau\right)d\varphi d\tau = \frac{\int\int_{\varphi_x^*(\tau)}^{\infty}g(\varphi,\tau)d\varphi d\tau}{\int\int_{\varphi^*(\tau)}^{\infty}g(\varphi,\tau)d\varphi d\tau}$ are the entry probability and the export probability conditional on entry, respectively. Similarly, we can write Foreign's free entry condition

$$\begin{aligned}
&\frac{P_fQ_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w_f^{1-\sigma}\int\int_{\varphi_f^*(\tau)}^{\infty}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]g_f\left(\varphi,\tau\right)d\varphi d\tau - w_ff\int\int_{\varphi_f^*(\tau)}^{\infty}g_f\left(\varphi,\tau\right)d\varphi d\tau \\
&+\left[\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\left(\tau_x w_f\right)^{1-\sigma}\int\int_{\varphi_{xf}^*(\tau)}^{\infty}\varphi^{\sigma-1}\tau^{-\sigma}g_f\left(\varphi,\tau\right)d\varphi d\tau - w_ff_x\int\int_{\varphi_{xf}^*(\tau)}^{\infty}g_f\left(\varphi,\tau\right)d\varphi d\tau\right] \\
&= w_ff_e.
\end{aligned} \tag{A.8}$$

Home and foreign aggregate prices are

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Big[M \int \int_{\varphi^*(\tau)}^{\infty} (\frac{w\tau}{\varphi})^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau \\ + M_f \int \int_{\varphi^*_{xf}(\tau)}^{\infty} (\frac{w_f \tau\tau_x}{\varphi})^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau \Big], \quad \text{(A.9)}$$

$$P_f^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \Big[M_f \int \int_{\varphi^*_f(\tau)}^{\infty} (\frac{w_f\tau}{\varphi})^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau \\ + M \int \int_{\varphi^*_{x}(\tau)}^{\infty} (\frac{w \tau\tau_x}{\varphi})^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau \Big]. \quad \text{(A.10)}$$

Using the free entry and labor market clearing, we have the home and foreign analogs:

$$M_e = \frac{L}{\sigma\left(f_e + \omega_e f + \omega_x \omega_e f_x\right)}. \quad \text{(A.11)}$$

Lastly, the balanced trade condition requires

$$P_f^{\sigma} Q_f M \int \int_{\varphi^*_x(\tau)}^{\infty} \left(\frac{w\tau_x\tau}{\varphi}\right)^{1-\sigma} \mu\left(\varphi,\tau\right) d\varphi d\tau = P^{\sigma} Q M_f \int \int_{\varphi^*_{xf}(\tau)}^{\infty} \left(\frac{w_f\tau_x\tau}{\varphi}\right)^{1-\sigma} \mu_f\left(\varphi,\tau\right) d\varphi d\tau. \quad \text{(A.12)}$$

B Proofs for the welfare analysis of the baseline model

B.1 Proof for Proposition 2

Proof. To derive the effect of trade cost shock in the economy, let λ be the share of the expenditure on domestic goods as in ACR, using balanced trade condition:

$$\lambda = \frac{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau}{\int \int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q} \tau_x^{1-\sigma} \int \int_{\varphi^*_x(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau}. \quad \text{(A.13)}$$

We also define S to be the share of variable labor used in producing domestic goods,

$$S = \frac{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q} \tau_x^{1-\sigma} \int\int_{\varphi_x^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}. \tag{A.14}$$

Note that without distortions, $\lambda = S$.

First, we make use of the following equations: the price index (A.9) and the balance trade condition (A.12), we get

$$P^{1-\sigma} = con_p M_e w^{1-\sigma} \left[\int\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi,\tau) d\varphi d\tau + \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q} \tau_x^{1-\sigma} \int\int_{\varphi_x^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} g(\varphi,\tau) d\varphi d\tau \right]. \tag{A.15}$$

Combine with the definition of λ,

$$P^{1-\sigma} = con_p M_e w^{1-\sigma} \frac{\int\int_{\varphi^*(\tau)} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} g(\varphi,\tau) d\varphi d\tau}{\lambda}.$$

Take the log and differentiation of the above equation:

$$(1-\sigma) d\ln P = d\ln M_e + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG(\varphi,\tau)\right] - d\ln\lambda \tag{A.16}$$

Second, use the free entry condition (A.7), the labor market condition, hence the number of firms (A.11) to get

$$w^{1-\sigma}\left[P^{\sigma} Q \int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau + P_f^{\sigma} Q_f \tau_x^{1-\sigma} \int\int_{\varphi_x^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau\right]$$
$$= \sigma^{\sigma}(\sigma-1)^{1-\sigma} \frac{wL}{\sigma M_e}$$

Combine with the definition of S,

$$w^{1-\sigma} P^{\sigma} Q \frac{\int\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}{S} = \sigma^{\sigma}(\sigma-1)^{1-\sigma} \frac{wL}{\sigma M_e}$$

Take log and differentiation of the above equation:

$$d\ln P^{\sigma}Q + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma}dG(\varphi,\tau)\right] - d\ln S = -d\ln M_e \tag{A.17}$$

In sum, we have two equations, and using the definition of γ:

$$(1-\sigma)d\ln P = d\ln M_e - d\ln\lambda - \gamma_\lambda(\hat{\varphi}^*)d\ln\hat{\varphi}^* \tag{A.18}$$

$$d\ln(PQ) = (1-\sigma)d\ln P - d\ln M_e + d\ln S + \gamma_s(\hat{\varphi}^*)d\ln\hat{\varphi}^*. \tag{A.19}$$

Hence

$$d\ln Q = -d\ln P + (-d\ln\lambda + d\ln S) + (\gamma_s(\hat{\varphi}^*) - \gamma_\lambda(\hat{\varphi}^*))d\ln\hat{\varphi}^*, \tag{A.20}$$

where from the cutoff equation, $\hat{\varphi}^* = con_v \times P^{-1}(PQ)^{\frac{1}{1-\sigma}}$, we have

$$d\ln\hat{\varphi}^* = -d\ln P - \frac{1}{\sigma-1}d\ln(PQ). \tag{A.21}$$

Solving equations (A.18)-(A.21) gives Proposition 2:

$$d\ln W = \underbrace{\frac{1}{\gamma_\lambda+\sigma-1}[-d\ln\lambda + d\ln M_e]}_{(ACR/MR)} + \underbrace{\left(\frac{\gamma_\lambda/(\sigma-1)}{\gamma_\lambda+\sigma-1}+1\right)d\ln PQ}_{(distortions)}, \tag{A.22}$$

where the last term captures the deviation from ACR and MR, and

$$d\ln PQ = \frac{\gamma_s-\gamma_\lambda}{\gamma_s+\sigma-1}[-d\ln\lambda + d\ln M_e] + \left(\frac{\gamma_\lambda+\sigma-1}{\gamma_s+\sigma-1}\right)(-d\ln\lambda + d\ln S).$$

□

B.2 Proof for Corollary 1

Proof. Under the special case, $\gamma_\lambda = \frac{\sigma-1}{\sigma}(\theta-\sigma+1)$ and $\gamma_s = \frac{\sigma-1}{\sigma}(\theta-\sigma)$, and the change in welfare becomes $d\ln W = \frac{\sigma}{\sigma-1}[d\ln S - d\ln\lambda]$.

1. Welfare change from a closed to an open economy:

Because domestic shares are

$$\lambda = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \left(\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P^\sigma Q}{P_f^\sigma Q_f} \right)^{\frac{\sigma-1-\theta}{\sigma}} + 1 \right]^{-1}$$

$$S = \left[\frac{P_f^\sigma Q_f}{P^\sigma Q} \tau_x^{1-\sigma} \left(\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P^\sigma Q}{P_f^\sigma Q_f} \right)^{\frac{\sigma-\theta}{\sigma}} + 1 \right]^{-1},$$

we know that $\lambda > S$ as long as there is a selection to export, i.e., $\frac{\tau_x^{\sigma-1} f_x}{f} \frac{P^\sigma Q}{P_f^\sigma Q_f} > 1$. In an open economy, the input share used to produce for exports exceeds the export share under the special case where reallocation is driven purely by distortions. Thus, $d \ln S$ is more negative than $d \ln \lambda$ when moving from a closed to open economy. Hence, the open economy has an unambiguously lower welfare.

2. The distortion term is always negative:

In the welfare expression of Prop 2, the distortion term becomes

$$d \ln PQ = \frac{\gamma_s - \gamma_\lambda}{\gamma_s + \sigma - 1} [-d \ln \lambda] + \left(\frac{\gamma_\lambda + \sigma - 1}{\gamma_s + \sigma - 1} \right) (-d \ln \lambda + d \ln S),$$

$$= -d \ln \lambda + \left(\frac{\gamma_\lambda + \sigma - 1}{\gamma_s + \sigma - 1} \right) d \ln S.$$

Because

$$d \ln \lambda = (1 - \lambda) \frac{\theta + 1}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f}$$

$$d \ln S = (1 - S) \frac{\theta}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f},$$

substitute for γ_s, γ_λ, $d \ln \lambda$ and $d \ln S$, the fiscal externality term is

$$-d \ln \lambda + \frac{\theta + 1}{\theta} d \ln S = \frac{(\theta + 1)(\lambda - S)}{\sigma} d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f}.$$

$\lambda > S$, hence as long as the trade cost reduction induces a larger fraction of exporters, i.e., $d \ln \frac{\tau_x^{\sigma-1} P^\sigma Q}{P_f^\sigma Q_f} < 0$, the distortion term is always negative. Q.E.D. □

B.3 Proof for Corollary 2

Proof. Recall the producing cutoff is given by $\varphi^*(\tau) = \hat{\varphi}^* \tau^{\frac{\sigma}{\sigma-1}}$ where $\hat{\varphi}^* = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} \left[\frac{wf}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}} w$. $I(\hat{\varphi})$ is the cumulative input/labor share in the domestic market, and $O(\hat{\varphi})$ is the cumulative sales share in the domestic market.

$$I(\hat{\varphi}) = \frac{\int \int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int \int_0^{\inf} \varphi^{\sigma-1}\tau^{-\sigma} g(\varphi,\tau) d\varphi d\tau}$$

$$O(\hat{\varphi}) = \frac{\int \int_0^{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{1-\sigma} g(\varphi,\tau) d\varphi d\tau}{\int \int_0^{\inf} \varphi^{\sigma-1}\tau^{1-\sigma} g(\varphi,\tau) d\varphi d\tau}.$$

Let $i(\hat{\varphi}) = I'(\hat{\varphi})$ and $o(\hat{\varphi}) = O'(\hat{\varphi})$. The hazard functions γ_s and γ_λ are

$$\gamma_s = -\frac{d\ln(1-I(\hat{\varphi}))}{d\ln\hat{\varphi}} = \frac{i(\hat{\varphi})}{1-I(\hat{\varphi})},$$

$$\gamma_\lambda = -\frac{d\ln(1-O(\hat{\varphi}))}{d\ln\hat{\varphi}} = \frac{o(\hat{\varphi})}{1-O(\hat{\varphi})},$$

When $\frac{i(\hat{\varphi})}{o(\hat{\varphi})}$ increases with $\hat{\varphi}$, i.e. I is likelihood ratio dominates O, then

$$\frac{1-I(\hat{\varphi})}{i(\hat{\varphi})} = \int_{\hat{\varphi}} \frac{i(\hat{\varphi}')}{i(\hat{\varphi})} d\hat{\varphi}' \geq \int_{\hat{\varphi}} \frac{o(\hat{\varphi}')}{o(\hat{\varphi})} d\hat{\varphi}' = \frac{1-O(\hat{\varphi})}{o(\hat{\varphi})},$$

that is, $\gamma_s \leq \gamma_\lambda$.

Let $x = \log\varphi, y = \log\tau$, then $x = \hat{\varphi} + \frac{\sigma}{\sigma-1}y$. Under joint-normal distribution of (x, y), define

$$V(\hat{\varphi}) \equiv \frac{i(\hat{\varphi})}{o(\hat{\varphi})} = \frac{\int \exp(\sigma x(\hat{\varphi}, y) - \sigma y) g(x(\hat{\varphi}, y), y) dy}{\int \exp(\sigma x(\hat{\varphi}, y) + (1-\sigma)y) g(x(\hat{\varphi}, y), y) dy}$$

where

$$g(x, y) = \exp\left[-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_\varphi^2} + \frac{y^2}{\sigma_\tau^2} - \frac{2\rho xy}{\sigma_\varphi\sigma_\tau}\right)\right].$$

When $\sigma_\tau \geq \frac{\sigma-1}{\sigma}\rho\sigma_\varphi$, $V'(\hat{\varphi}) \geq 0$. Then, the cumulative labor share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order, and the hazard functions satisfy $\gamma_s \leq \gamma_d$.

Furthermore,

$$\frac{d\ln\frac{1-I(\hat{\varphi})}{1-O(\hat{\varphi})}}{d\ln\hat{\varphi}} = \frac{d\ln(1-I(\hat{\varphi}))}{d\ln\hat{\varphi}} - \frac{d\ln(1-O(\hat{\varphi}))}{d\ln\hat{\varphi}} = -\gamma_s + \gamma_d \geq 0$$

then it follows

$$\frac{1-I(\hat{\varphi}_x^*)}{1-I(\hat{\varphi}^*)} \geq \frac{1-O(\hat{\varphi}_x^*)}{1-O(\hat{\varphi}^*)}$$

and $S \leq \lambda$.

$$d\ln PQ = \frac{\gamma_s - \gamma_\lambda}{\gamma_s + \sigma - 1}[-d\ln\lambda + d\ln M_e] + \left(\frac{\gamma_\lambda + \sigma - 1}{\gamma_s + \sigma - 1}\right)(-d\ln\lambda + d\ln S).$$

Moving from a closed economy to an open economy, as long as $-d\ln\lambda + d\ln M_e > 0$, the distortion term is always negative. Q.E.D. □

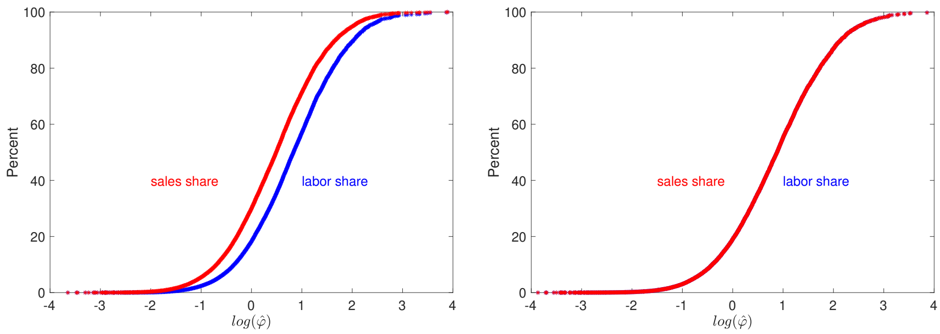
C Numerical example with symmetric countries

To unpack the theoretical results and to provide more intuition for the mechanisms that underpin these results, we next turn to a numerical example of the benchmark model with symmetric countries, i.e., both face domestic distortions. The assumption of symmetry abstracts from terms of trade effect and highlights the role of misallocation in generating loss from trade. Specifically, If Home suffers a loss from trade, it is not because Home is subsidizing firms' exports and Foreign gains due to terms of trade effect. This symmetric example emphasizes that loss from trade comes from negative selection and the deterioration of resource allocations.

The joint distribution between productivity and distortions is taken to be joint log-normal with standard deviations of $\sigma_\tau = \sigma_\varphi = 0.5$ and correlation of φ and τ of $\rho = 0.8$. The elasticity of substitution σ equals 3, the entry cost and fixed costs of domestic producing are 1, and the fixed cost for exporting f_x is 1.5.

Corollary 2 applies here as the distribution of (φ, τ), and the parameters satisfy its conditions. We plot the cumulative variable input and sales share under any $\log(\hat{\varphi}) \propto \log(\varphi^{\sigma-1}\tau^{-\sigma})$ in panel (a) of Figure A-1. According to proof for Corollary 2 B.3, the cu-

Figure A-1: Accumulated Labor Share vs Sales Share in a Market



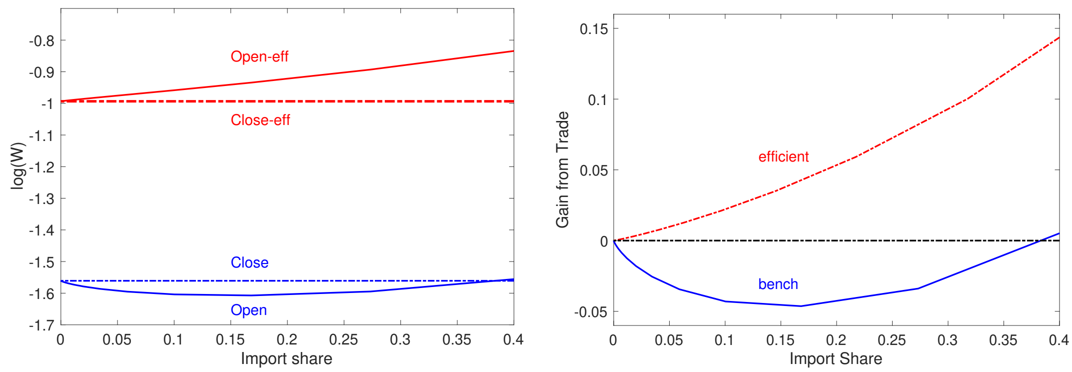
(a) With Distortion (b) Without Distortion

mulative variable input share distribution stochastically dominates the cumulative sales share distribution according to the likelihood ratio order, which implies first-order stochastic dominance. In contrast, without distortions with $\tau = 1$, these two distributions are identical, as shown in panel (b) of Figure A-1. When the economy opens to trade, firms that export are those with high profits and also use a large share of labor to produce. Overall, the share of labor used to produce exports would exceed the export share; exporting is more subsidized than domestic production.

The example helps illustrate a few points. First, welfare (Eq. 9) can fall when the economy opens up to trade. Figure A-2 (a) plots the level of welfare against import shares under the alternative scenarios: the efficient case without distortions, the case with distortions, and when the economy is closed or open. Three observations immediately follow: 1) that there is a welfare loss in the case with distortions compared to the case without; 2) opening up to trade leads to welfare gains in the efficient case; however, 3) opening up engenders a welfare *loss* in the presence of distortions. Taking the differences between the open and closed economy in either case, with or without distortion, we plot the welfare change after trade in Figure A-2 (b). It is clear that there is welfare loss with distortions.

Second, the numerical example also demonstrates that using import shares to infer welfare changes can give rise to markedly different results when there are distortions, as in Figure A-3 (a), which decomposes welfare into ACR and a distortion term, compared against the benchmark. Using ACR under distortions leads to a large departure: welfare *losses* become welfare gains in this case. Thus, using aggregate observables to infer welfare gains as in ACR can be very misleading in the presence of distortions, unlike in the efficient case

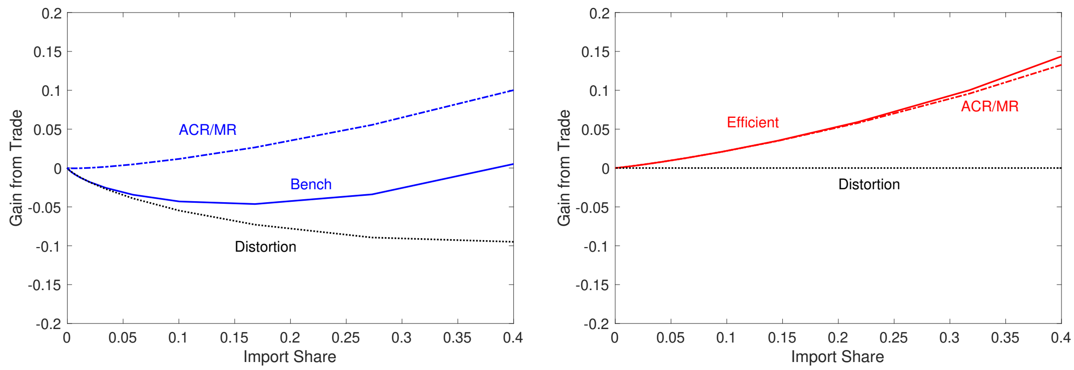
Figure A-2: Welfare and the Change from Trade



(a) Welfare (b) Gain from Trade

where ACR is a good approximation (Figure A-3 (b)).

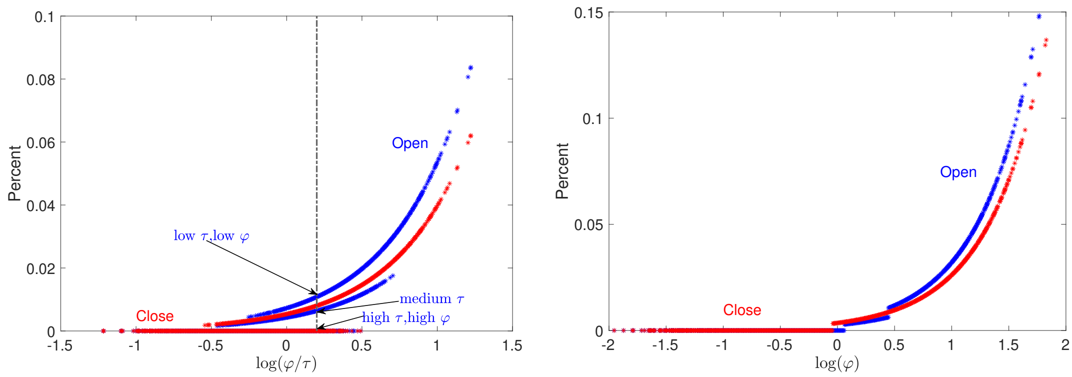
Figure A-3: Welfare Decomposition



(a) With distortion (b) Without distortion

In our model, the productivity cutoff for production and exports is no longer determined solely by productivity, but also by domestic distortion. One way to show the impact on selection is to examine firms' market share. The two panels in Figure A-4 plot firms' market share, both in the closed and open economy. The left panel is the case without distortions. Without distortion, the marginal cost is the inverse of the productivity φ. Firms with the same productivity level have the same marginal cost; their market share, above a cutoff productivity, rises with their productivity. Comparing the blue and red lines shows that above the export cutoff, more productive firms have higher market shares in the open economy than in the closed economy, demonstrating that these firms expand under trade liberalization. This happens at the cost of displacing other less productive firms' market share or driving them out of the market entirely. Here, the example clearly demonstrates that resources move from less productive to more productive firms as an economy opens up to trade.

Figure A-4: Selection Effects



(a) With distortion (b) Without distortion

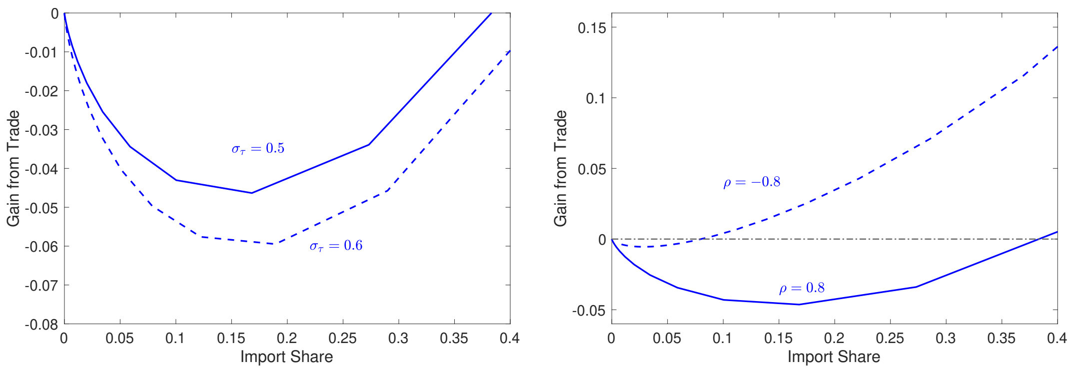
The right panel shows the firm's market share in the case with distortions. Firms may share the same marginal cost τ/φ and face the same potential revenues. However, their after-tax profits may differ, thus, their market share can also differ. Consider the point at which $\log(\varphi/\tau)$ is at 0.2. At this point, a firm with high, medium, and low levels of productivity faces the same marginal costs. However, the high-productivity firm is also subject to high taxes and thus low after-tax profit and does not make the cut for production. The medium-tax-medium-productivity firm has a positive market share but loses out to the low-tax-low-productivity firm when the economy opens up. Resources are reallocated from the more productive to the less productive firms. Also, there is no longer a neat lineup of market shares according to productivity: there is a wide range of productivities for which production is excluded.[1] Aggregate welfare effect depends on how trade alters the aggregate domestic labor share and sales share.

Distribution of Distortions. The distribution of distortions is an important determinant of the gains to trade. There are two key parameters: ρ, the correlation of τ and φ, and σ_τ, the dispersion of τ. Figure A-5 (a) compares the gains from trade under different σ_τ, while the other parameters remain the same as in the benchmark example. The welfare gain (loss) from trade is always larger (smaller) when σ_τ is smaller.

[1]This is also true if the distortions are input wedge on all the labor a firm uses. Firms that face a higher input wedge would have a lower profit in a market.

The correlation between distortion and productivity is important insofar as a higher correlation means that more productive firms are more likely to be excluded from the market. However, a reduction in welfare is possible even when the correlation is negative. The reason is that for any given productivity, it is always the more subsidized firms that can export and the highly taxed ones that exit— leading to a possible worsening of misallocation. In fact, as shown in Corollary 2, when the correlation is negative, more productive firms are highly subsidized. Exporters are those more productive and highly subsidized ones. Hence, their labor share is larger than the sales share, and the distortion term is always negative. Overall effects combine the price effect and the negative distortion effect. Figure A-5(a) illustrates this. It compares the gains from trade for $\rho = 0.8$, under our benchmark numerical example, and for $\rho = -0.8$, where productivity and distortion are highly negatively correlated. Under $\rho = -0.8$, the welfare gain (loss) from trade is always larger (smaller) than that in the case of $\rho = 0.8$. But when the import share is small, there are still losses from trade, even under a negative correlation.

Figure A-5: Gains/Loss from Trade



(a) Dispersion Matters (b) Correlation Matters

In sum, the size of welfare loss after opening up depends on the correlation of φ and τ and the dispersion of τ. The firm-level data helps us identify these parameters. Specifically, in the quantitative section, we will use the firm-level output, its dispersion, and its correlation with firm inputs to estimate the underlying distribution of productivity and distortions.

D Extended model with heterogeneous exporting wedges

In the open economy, an entrant firm draws from a quadruple of productivity φ, wedge of domestic sales τ, wedge of foreign sales τ_{ex}, and wedge of fixed cost in foreign sales τ_{fx}, i.e. $(\varphi, \tau, \tau_{ex}, \tau_{fx})$, from a distribution with pdf $g(\varphi, \tau, \tau_{ex}, \tau_{fx})$ and cdf $G(\varphi, \tau, \tau_{ex}, \tau_{fx})$. Foreign firms draw the quadruple from a pdf g_f and cdf G_f. The foreign country has total labor L_f and endogenous prices of P_f and w_f. Export is subject to an iceberg exporting cost τ_x and f_x, which are the same for all the firms.

A domestic exporting firm solves the following problem:

$$\max_{p_x, q_x} \frac{1}{\tau_{ex}} p_x q_x - \frac{w}{\varphi} \tau_x q_x - \tau_{fx} w f_x$$

subject to the foreign demand function $q_x = \frac{p_x^{-\sigma}}{P_f^{-\sigma}} Q_f$. The optimal exporting price is

$$p_x = \frac{\sigma}{\sigma - 1} \frac{w \tau_x \tau_{ex}}{\varphi},$$

and the optimal sales is

$$p_x q_x = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} w^{1-\sigma} \tau_x^{1-\sigma} (P_f^{\sigma} Q_f) \left(\frac{\varphi}{\tau_{ex}} \right)^{\sigma - 1}.$$

The optimal exporting profit is

$$\pi_x = \sigma^{-\sigma} (\sigma - 1)^{\sigma - 1} P_f^{\sigma} Q_f (w \tau_x)^{1-\sigma} \varphi^{\sigma - 1} \tau_{ex}^{-\sigma} - \tau_{fx} w f_x.$$

Cutoffs The two cutoff productivities in the home country entering the domestic market, $\varphi^*(\tau)$, and foreign markets, $\varphi_x^*(\tau_{ex}, \tau_{fx})$, are:

$$\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left[\frac{wf}{P^{\sigma} Q} \right]^{\frac{1}{\sigma-1}} w \tau^{\frac{\sigma}{\sigma-1}}, \qquad \varphi_x^*(\tau_{ex}, \tau_{fx}) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left[\frac{\tau_{fx} w f_x \tau_x^{\sigma - 1}}{P_f^{\sigma} Q_f} \right]^{\frac{1}{\sigma-1}} w \tau_{ex}^{\frac{\sigma}{\sigma-1}}. \quad \text{(A.23)}$$

Similarly, the two cutoffs for the foreign country are

$$\varphi_f^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{w_f f}{P_f^\sigma Q_f}\right]^{\frac{1}{\sigma-1}} w_f \tau^{\frac{\sigma}{\sigma-1}}, \qquad \varphi_{xf}^*(\tau_{ex}, \tau_{fx}) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}\left[\frac{\tau_{fx} w_f f_x \tau_x^{\sigma-1}}{P^\sigma Q}\right]^{\frac{1}{\sigma-1}} w_f \tau_{ex}^{\frac{\sigma}{\sigma-1}}. \tag{A.24}$$

Free Entry Conditions

$$\begin{aligned}
&\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG - wf\int_{\varphi^*(\tau)}^{\infty} dG \\
&+\Big[\frac{P_f Q_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG - wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG\Big] = wf_e,
\end{aligned} \tag{A.25}$$

and similarly for the foreign country:

$$\begin{aligned}
&\frac{P_f Q_f}{\sigma}\left(P_f\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w_f^{1-\sigma}\int_{\varphi_f^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG_f - w_f f\int_{\varphi_f^*(\tau)} dG_f \\
&+\Big[\frac{PQ}{\sigma}\left(P\frac{\sigma-1}{\sigma}\right)^{\sigma-1}(\tau_x w_f)^{1-\sigma}\int_{\varphi_{xf}^*(\tau_{ex},\tau_f)}\varphi^{\sigma-1}\tau_{ex}^{-\sigma}dG_f - w_f f_x\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG_f\Big] = w_f f_e
\end{aligned} \tag{A.26}$$

Measure M and M_f Define the fraction of firms operating for the domestic market and the fraction exporting, conditional on producing to be:

$$\omega_e = \int_{\varphi^*(\tau)} dG(\varphi, \tau, \tau_{ex}, \tau_{fx}), \qquad \omega_x = \frac{\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} dG\left(\varphi, \tau, \tau_{ex}, \tau_{fx}\right)}{\int_{\varphi^*(\tau)} dG\left(\varphi, \tau, \tau_{ex}, \tau_{fx}\right)},$$

$$\omega_{ef} = \int_{\varphi_f^*(\tau)} dG_f(\varphi, \tau, \tau_{ex}, \tau_{fx}), \qquad \omega_{xf} = \frac{\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})} dG_f\left(\varphi, \tau, \tau_{ex}, \tau_{fx}\right)}{\int_{\varphi_f^*(\tau)} dG_f\left(\varphi, \tau, \tau_{ex}, \tau_{fx}\right)}.$$

Home's free entry condition implies

$$\int_{\varphi^*(\tau)}\left(\frac{1}{\sigma-1}\frac{q}{\varphi} - f\right)dG + \int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{1}{\sigma-1}\tau_x\frac{q_x}{\varphi} - \tau_{fx}f_x\right)dG = f_e,$$

where we replaced the optimal profits π with $\frac{1}{\sigma-1}\frac{wq}{\varphi} - wf$ and π_x with $\frac{1}{\sigma-1}\frac{\tau_x w q_x}{\varphi} - w\tau_{fx} f_x$. Home's labor market clearing condition requires

$$L = M_e \left[\int_{\varphi^*(\tau)} \left(\frac{q}{\varphi} + f \right) dG + \int_{\varphi_x^*(\tau_{ex}, \tau_{fx})} \left(\tau_x \frac{q_x}{\varphi} + f_x \right) dG + f_e \right].$$

Using the free-entry condition and the labor market clearing condition, we have

$$M_e = \frac{L}{\sigma \left[f_e + \omega_e f + \omega_x \omega_e f_x + \frac{\sigma-1}{\sigma} f_x \int_{\varphi_x^*(\tau_{ex}, \tau_{fx})} (\tau_{fx} - 1) dG \right]}, \tag{A.27}$$

and similarly for foreign:

$$M_{ef} = \frac{L_f}{\sigma \left[f_e + \omega_{ef} f + \omega_{xf} \omega_{ef} f_x + \frac{\sigma-1}{\sigma} f_x \int_{\varphi_{xf}^*(\tau_{ex}, \tau_{fx})} (\tau_{fx} - 1) dG_f \right]}. \tag{A.28}$$

We can then get $M = \omega_e M_e$ and $M_f = \omega_{ef} M_{ef}$.

Aggregate price level We can write the aggregate prices of home and foreign as:

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left[M w^{1-\sigma} \frac{\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG}{\int_{\varphi^*(\tau)} dG} + M_f (\tau_x w_f)^{1-\sigma} \frac{\int_{\varphi_{xf}^*(\tau_{ex}, \tau_{fx})}^{\infty} (\frac{\varphi}{\tau_{ex}})^{\sigma-1} dG_f}{\int_{\varphi_f^*(\tau)}^{\infty} dG_f} \right] \tag{A.29}$$

$$P_f^{1-\sigma} = \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left[M_f w_f^{1-\sigma} \frac{\int_{\varphi_f^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG_f}{\int_{\varphi_f^*(\tau)} dG_f} + M (\tau_x w)^{1-\sigma} \frac{\int_{\varphi_x^*(\tau_{ex}, \tau_{fx})}^{\infty} (\frac{\varphi}{\tau_{ex}})^{\sigma-1} dG}{\int_{\varphi^*(\tau)}^{\infty} dG} \right]. \tag{A.30}$$

Summary of equilibrium conditions The equilibrium consists of $(P, P_f, M, M_f, Q, Q_f, w_f)$ with $w = 1$ as normalization. The equations consist of two free entry conditions (A.25) and (A.26), two labor clearing conditions (A.27) and (A.28), two price indices (A.29) and (A.30), and the balanced trade condition

$$P_f^{\sigma} Q_f M_e \int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \left(\frac{w\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma} dG \quad = \quad P^{\sigma} Q M_{ef} \int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})} \left(\frac{w_f\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma} dG_f. \quad \text{(A.31)}$$

Finally, the cutoff functions are given by (A.23) and (A.24).

E Proof of general welfare formula in the extended model

Proof. 1. Define input S and output λ shares

$$\lambda = \frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG}{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right] + \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q}\tau_x^{1-\sigma}\left[\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \varphi^{\sigma-1}\tau_{ex}^{1-\sigma} dG\right]}$$

$$S = \frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG}{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] + \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q}\tau_x^{1-\sigma}\left[\int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \varphi^{\sigma-1}\tau_{ex}^{-\sigma} dG\right]}$$

2. Define $\gamma_\lambda(\hat{\varphi})$ and $\gamma_s(\hat{\varphi})$

   $\gamma_\lambda(\hat{\varphi})$—the elasticity of the cumulative sales within the domestic market for firms above a cutoff, and $\gamma_s(\hat{\varphi})$—the elasticity of the cumulative domestic (variable) labor for firms above any cutoff $\hat{\varphi}$, both with respect to the cutoff.

$$\gamma_\lambda(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} (\frac{\varphi}{\tau})^{\sigma-1} dG\right]}{d\ln\hat{\varphi}}, \quad \gamma_s(\hat{\varphi}) = -\frac{d\ln\left[\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} \varphi^{\sigma-1}\tau^{-\sigma} dG\right]}{d\ln\hat{\varphi}}. \quad \text{(A.32)}$$

   Note $\int\int_{\hat{\varphi}\tau^{\frac{\sigma}{\sigma-1}}} (\frac{\varphi}{\tau})^{\sigma-1} dG$ is proportional to the cumulative market share (in any given market) of firms above any cutoff $\hat{\varphi}$. Therefore, $\gamma_\lambda(\hat{\varphi})$ represents the hazard function for the distribution of log firm sales within a market. Similarly, $\gamma_s(\hat{\varphi})$ represents the hazard function for the distribution of log firm variable labor within a market. $\gamma_\lambda(\hat{\varphi}^*)$ and $\gamma_s(\hat{\varphi}^*)$ are these elasticity evaluated at the domestic production cutoff.

3. Free entry condition

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\left[\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG+\frac{P_f^{\sigma}Q_f}{P^{\sigma}Q}(\tau_x)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG\right]$$
$$=wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\tau_{fx}dG+w\omega_e f+wf_e$$

We can rewrite the equilibrium condition (A.27) of M_e

$$M_e=\frac{L}{\sigma f_e+\omega_e\sigma f+\left[(\sigma-1)\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\tau_{fx}dG+\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}dG\right]f_x},$$

as the following one

$$\omega_e wf+wf_e=\frac{wL}{\sigma M_e}-\left[\frac{\sigma-1}{\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\tau_{fx}dG+\frac{1}{\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}dG\right]wf_x. \qquad \text{(A.33)}$$

Replacing $\omega_e wf+wf_e$ in the free-entry condition using (A.33), we have

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} w^{1-\sigma}\left[\int_{\varphi^*(\tau)}\left[\varphi^{\sigma-1}\tau^{-\sigma}\right]dG+\frac{P_f^{\sigma}Q_f}{P^{\sigma}Q}(\tau_x)^{1-\sigma}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left[\varphi^{\sigma-1}\tau_{ex}^{-\sigma}\right]dG\right]$$
$$=\frac{1}{\sigma}wf_x\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG+\frac{wL}{\sigma M_e}$$

Using the definition of S and normalizing $w=1$, we reach the following equation:

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1}\frac{\left[\int_{\varphi^*(\tau)}\varphi^{\sigma-1}\tau^{-\sigma}dG\right]}{S}=\frac{L}{\sigma M_e}\left[1+\frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG\right]$$

4. Price index:

$$P^{1-\sigma}=\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left[M_e w^{1-\sigma}\int_{\varphi^*(\tau)}\left(\frac{\varphi}{\tau}\right)^{\sigma-1}dG+M_{ef}(\tau_x w_f)^{1-\sigma}\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\frac{\varphi}{\tau_{ex}}\right)^{\sigma-1}dG_f\right]$$

Replacing the second term with the following balance trade condition

$$P_f^{\sigma}Q_f M_e\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}\left(\frac{w\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma}dG=P^{\sigma}QM_{ef}\int_{\varphi_{xf}^*(\tau_{ex},\tau_{fx})}\left(\frac{w_f\tau_x\tau_{ex}}{\varphi}\right)^{1-\sigma}dG_f$$

we have

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\int_{\varphi^*(\tau)} (\frac{\varphi}{\tau})^{\sigma-1} dG + (\tau_x)^{1-\sigma} \frac{P_f^{\sigma} Q_f}{P^{\sigma} Q} \int_{\varphi_x^*(\tau_{ex},\tau_{fx})} \left(\frac{\varphi}{\tau_{ex}}\right)^{\sigma-1} dG\right].$$

Using the definition of λ, the above equation becomes

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG}{\lambda}\right].$$

5. Summary of two equations: from free-entry and pricing index, we have

$$\frac{P^{\sigma}Q}{\sigma}\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \frac{\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right]}{S} = \frac{L}{\sigma M_e}\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx} - 1\right) dG\right]$$

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} M_e \left[\frac{\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG(\varphi,\tau)}{\lambda}\right]$$

Taking log and differentiation of the above two equations:

$$\begin{aligned} d\ln P^{\sigma}Q + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] - d\ln S & \\ = -d\ln M_e + d\ln & \left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty} \left(\tau_{fx} - 1\right) dG\right] \end{aligned}$$

$$(1-\sigma) d\ln P = d\ln M_e + d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right] - d\ln\lambda$$

The term $d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{-\sigma} dG\right] = -\gamma_s(\hat{\varphi}^*) d\ln\hat{\varphi}^*$ where the last equality uses the cutoff condition: $\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1}[wf]^{\frac{1}{\sigma-1}} w(P^{\sigma}Q)^{\frac{1}{1-\sigma}}\tau^{\frac{\sigma}{\sigma-1}}$. Similarly, in the second equation, $d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right]$ is such that

$$d\ln\left[\int_{\varphi^*(\tau)} \varphi^{\sigma-1}\tau^{1-\sigma} dG\right] = -\gamma_{\lambda}(\hat{\varphi}^*) d\ln\hat{\varphi}^* = \gamma_{\lambda}\frac{1}{\sigma-1}\left(\sigma d\ln P + d\ln Q\right).$$

6. Plugging γ_s and γ_λ back into the two equations we have

$$\sigma d\ln P + d\ln Q + \gamma_s \frac{1}{\sigma-1}\left(\sigma d\ln P + d\ln Q\right) - d\ln S$$
$$= -d\ln M_e + d\ln\left[1 + \frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG\right] \quad \text{(A.34)}$$

$$(1-\sigma)d\ln P = d\ln M_e + \gamma_\lambda \frac{1}{\sigma-1}\left(\sigma d\ln P + d\ln Q\right) - d\ln\lambda \quad \text{(A.35)}$$

7. Finally, solve the above two equations, we have $d\ln W = d\ln Q$ and

$$d\ln W = \underbrace{\frac{1}{\gamma_\lambda+\sigma-1}\left[-d\ln\lambda + d\ln M_e\right]}_{(ACR/MR)} + \underbrace{\left(\frac{\gamma_\lambda/(\sigma-1)}{\gamma_\lambda+\sigma-1}+1\right)d\ln PQ}_{(distortions)}, \quad \text{(A.36)}$$

where the last term captures the deviation from ACR and MR, and

$$d\ln PQ = \frac{\gamma_s-\gamma_\lambda}{\gamma_s+\sigma-1}\left[-d\ln\lambda + d\ln M_e\right]$$
$$+\left(\frac{\gamma_\lambda+\sigma-1}{\gamma_s+\sigma-1}\right)\left[-d\ln\lambda + d\ln S + d\ln(1+\frac{M_e f_x}{L}\int_{\varphi_x^*(\tau_{ex},\tau_{fx})}^{\infty}\left(\tau_{fx}-1\right)dG)\right]. \quad \text{(A.37)}$$

□

F Output and input distortions

We want to make three points here. First, for an individual firm, the problem with only output distortions is equivalent to the problem with input distortions. Second, when the measure of firms or entrants is fixed, and hence no f_e, the welfare expression in the input-distortion model is the same as that in the output-distortion model. Third, with endogenous entry, one needs to specify how the entry cost f_e is affected by the wedges. Given that f_e is paid before the realization of productivities and wedges, we assume f_e is in terms of inputs and not subject to any wedges in our benchmark.

Consider a model with Cobb-Douglas production function $y = \varphi\left(\frac{k}{\alpha}\right)^{\alpha}\left(\frac{\ell}{1-\alpha}\right)^{1-\alpha}$ and factor distortions of τ_k on rental capital and τ_ℓ on labor input. The fixed cost of production is assumed to use both capital and labor. Let $\hat{w}$ and $\hat{r}$ be the wage rate and rental return. The optimization of a firm is given by

$$\max_{p,q} pq - \frac{(\tau_k \hat{r})^\alpha (\tau_\ell \hat{w})^{1-\alpha}}{\varphi} q - (\tau_k \hat{r})^\alpha (\tau_\ell \hat{w})^{1-\alpha} f$$

subject to the demand function $q = (p/P)^{-\sigma} Q$.

Let $w = \hat{r}^\alpha \hat{w}^{1-\alpha}$ and $\tau = \tau_k^\alpha \tau_\ell^{1-\alpha}$. It is easy to see that the firm's problem under input wedges is isomorphic to equation (3) in our baseline model with only output wedges, given by

$$\max_{p,q} \frac{pq}{\tau} - \frac{w}{\varphi} q - f.$$

Importantly, the cutoff of production $\varphi^*(\tau)$ is the same as equation (4) in the paper, i.e.,

$$\varphi^*(\tau) = \frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma - 1} \left[\frac{wf}{P^\sigma Q} \right]^{\frac{1}{\sigma-1}} w \tau^{\frac{\sigma}{\sigma-1}}.$$

Furthermore, the firm's TFPR still maps to a composite wedge, and aggregate TFPR relates to the fiscal externality in the model.

The only complication is the entry cost $r^\alpha w^{1-\alpha} f_e$, which is in terms of inputs and is paid before firms draw their productivity and wedges in our model. Hence, we need to specify how the entry cost is affected by these input wedges.

One way to avoid this complication is to consider a case with a fixed measure of *entrants*. In this case, the measure of producers is still endogenous in that firms can choose not to produce if the fixed cost is too high. We have derived a welfare expression for this case and found that the fiscal externality can still be expressed by the gap between an input and output share $d \ln S - d \ln \lambda$ and the gap between domestic extensive elasticities of input and output. Here, the input share is defined as the variable inputs used in the domestic production over total inputs. If, instead, we consider a case with a fixed measure of *firms*, the measure of producers is fixed. The fiscal externality then only depends on $d \ln S - d \ln \lambda$. Therefore, when the measure of firms/entrants is fixed, the welfare expressions for models with output wedges and input wedges are the same. With endogenous entry, the two expressions may differ, and the expression with input wedges depends on the assumption of how wedges affect the entry cost. Nonetheless, in both cases, the key is $d \ln S - d \ln \lambda$. As demonstrated in Table 2 of quantitative analysis, the second term involving $d \ln S - d \ln \lambda$ accounts for about 94% of change in fiscal externality ($13.65/14.53 = 0.94$).

G Implied and non-targeted moments

This appendix examines some implied and non-targeted moments. We also compare the distribution of productivity and wedges for non-exporters in both the model and the data.

Table A-1 reports our benchmark model's implied and non-targeted moments. Some of the moments are the implied moments, in the sense that if we match very well the joint distributions of observed TFPQ, TFPR, and trade, we match well these moments, for example, the dispersion of value-added and its correlation with TFPR, TFPQ, and trade. The reason is that when constructing measures like TFPR and TFPQ, we use both value-added and inputs. Specifically, the logarithm of value added is proportional to the log difference between TFPR and TFPQ. By matching the joint distribution of TFPR and TFPQ, we are able to generate the observed standard deviation of value added.

Overall, the model tightly matches the standard deviation of value added among all the firms. It generates the observed correlations of value added with TFPR, TFPQ, export intensity, and export participation. On average, exporters have 6% lower TFPR and 17% higher TFPQ than non-exporters in the data. Our model generates the same magnitudes.

Some of the moments are non-targeted, including TFPR and TFPQ within each group, exporters and non-exporters, and among exporters. The export intensity negatively correlates with both TFPR and TFPQ in the data. Our model matches these non-targeted moments well.

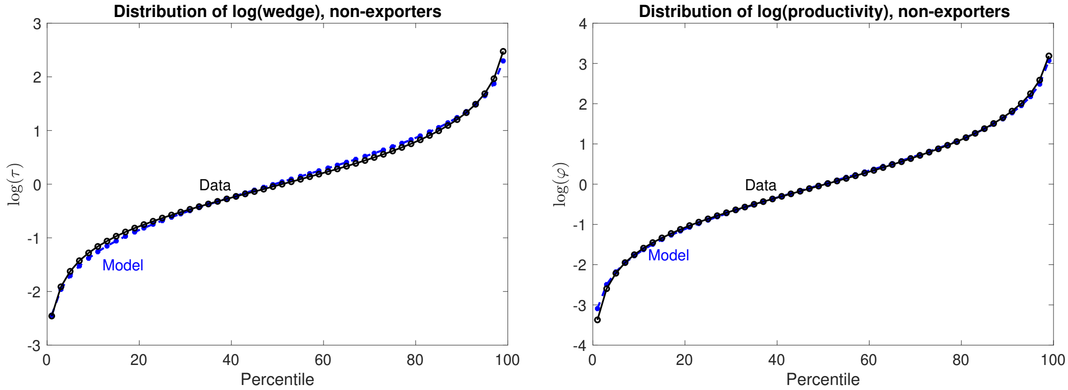
Figure A-6 presents the distribution of domestic wedge τ and productivity φ, which are backed out using a near non-parametric method, as described in the main text. In this method, we make no assumptions about the distribution of productivity and wedges in the data. Nonetheless, the comparison between the model and data distributions indicates a close match, as illustrated in Figure A-6. The standard deviation of $\log(\varphi)$ is 1.36 in the

Table A-1: Other Moments

	Data	Model
Implied moments		
TFPQ gap (ex−nonex)	0.17	0.18
TFPR gap (ex−nonex)	−0.06	-0.06
Export intensity	0.47	0.47
Std. value added	1.19	1.19
Corr (value added, TFPQ)	0.77	0.77
Corr (value added, TFPR)	0.45	0.44
Corr (value added, ex-int)	0.08	0.02
Corr (value added, ex-part)	0.17	0.18
Non-targeted moments		
Among Exporters		
Std. value added	1.20	1.36
Std. TFPQ	1.25	1.33
Corr (ex. intensity, TFPQ)	−0.13	-0.17
Corr (ex. intensity, TFPR)	−0.06	-0.03
Among Non-Exporters		
Std. value added.	1.16	1.08
Std. TFPQ.	1.34	1.31
Std. TFPR	0.96	0.98
Corr (TFPR, TFPQ)	0.93	0.93

Note: Data moments are for the 2005 Chinese National Bureau of Statistics. Value added, TFPR, and TFPQ are logged. Corr denotes correlation, Std for standard deviation, ex for export, ex.intensity for export intensity, ex-part for export participation. TFPR gap is the difference between the average TFPR of exporters and that of non-exporters. Similarly, for the TFPQ gap.

Figure A-6: Data point estimated τ and φ comparing with the Model



(a) $log(\tau)$ (b) $log(\varphi)$

data and 1.32 in the model, while the standard deviation of $\log(\tau)$ is 1.01 in the data and 1.02 in the model. Moreover, the correlations between productivity and wedge are also comparable, with a value of 0.92 in the data and 0.93 in the model.

Note that we estimate each non-exporter's (φ, τ) using the model estimated fixed cost f and observed value-added and input for non-exporters. This task is more challenging for exporters, who have four characteristics $(\phi, \tau, \tau_{ex}, \tau_{fx})$. First, conditional on exporting, the wedge on the fixed cost τ_{fx} cannot be uncovered with observables. Second, we don't have separated labor inputs to uncover (ϕ, τ, τ_{ex}); we have to use more model solutions like $\{P, Q, P_f, Q_f\}$ and the estimated trade cost τ_x. Furthermore, we cannot fully point-estimate in a non-parametric way due to export selection. Selection creates a need for extrapolation through functional form assumptions. In our benchmark model, we made distributional assumptions for the underlying wedges, and selection generates the observed differences in TFPR or TFPQ between exporters and non-exporters. With the model aggregates in equilibrium, the iceberg trade cost is estimated to match the import share. In short, in our benchmark, we extrapolate export wedges through distributions for non-exporters, so that 30% of firms export, and they differ from non-exporters as observed.

H Heterogeneous trade costs model

In this section, we re-estimate the *hetero-trade-costs model*, which replaces the firm-specific export taxes with firm-specific iceberg trade cost and firm-specific fixed cost of exporting.

There is tension in estimating this model, and the new estimations have a bit worse match to the data. Below, we explain the tension and go over the estimation results in Table A-2.

In our benchmark model, wedges in export, τ_{ex}, help the model match the distribution of exporters' TFPR, and wedges in fixed exporting cost, τ_{fx}, help the model explain the export participation pattern including their correlations with TFPR. And τ_{fx} does not enter the calculation of exporters' TFPR. In the hetero-trade-costs model, τ_{ex} and τ_{fx} are 'technology' factors. For exporters, we need to rely on their domestic wedges τ and high fixed costs f, f_x to generate the observed standard deviation of $TFPR$ and its correlations with export. As a result, the model generates either too low a dispersion of TFPR for exporters or too strong a correlation between TFPR and export participation.

To make it clear, we write down the solution for the hetero-trade-costs model here (τ_{ex} and τ_{fx} are 'technology' factors). An exporter i's optimal labor, sales, and export intensity are given by

$$\ell_i = \left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P^\sigma Q w^{-\sigma}\right] \varphi_i^{\sigma-1}\tau_i^{-\sigma} + \left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_f^\sigma Q_f w^{-\sigma}\tau_x^{1-\sigma}\right] \varphi_i^{\sigma-1}\tau_{ex,i}^{1-\sigma} + f + \tau_{fx,i} f_x$$

$$pq_i = \left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} P^\sigma Q w^{1-\sigma}\right] \varphi_i^{\sigma-1}\tau_i^{1-\sigma} + \left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} P_f^\sigma Q_f w^{1-\sigma}\tau_x^{1-\sigma}\right] \varphi_i^{\sigma-1}\tau_{ex,i}^{1-\sigma}$$

$$\text{export intensity}_i \equiv ex_i = \frac{pq_{ex,i}}{pq_i} = \frac{1}{1 + \frac{P^\sigma Q}{P_f^\sigma Q_f}\left(\tau_x \frac{\tau_{ex,i}}{\tau_i}\right)^{\sigma-1}}.$$

The exporter's TFPR is an arithmetic weighted average of its $TFPR_d$ for domestic production and $TFPR_x$ for foreign production,

$$TFPR_i = \frac{pq_{d,i} + pq_{ex,i}}{\ell_{d,i} + \ell_{ex,i}} = \frac{1}{\frac{\ell_{d,i}}{pq_i} + \frac{\ell_{ex,i}}{pq_i}} = \frac{1}{(1-ex_i)\frac{1}{TFPR_{d,i}} + ex_i\frac{1}{TFPR_{ex,i}}}. \tag{A.38}$$

Using our model, we can further write $TFPR_d$ and $TFPR_x$ as

$$TFPR_{d,i} = \frac{pq_{d,i}}{\ell_{d,i}} = \frac{\left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} P^\sigma Q w^{1-\sigma}\right] \varphi_i^{\sigma-1}\tau_i^{1-\sigma}}{\left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P^\sigma Q w^{-\sigma}\right] \varphi_i^{\sigma-1}\tau_i^{-\sigma} + f} = \left(\frac{\sigma w}{\sigma-1}\right)\left(\frac{\tau_i}{1+\zeta_d \varphi_i^{1-\sigma}\tau_i^\sigma f}\right)$$

$$TFPR_{ex,i} = \frac{pq_{ex,i}}{\ell_{ex,i}} = \frac{\left[\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} P_f^{\sigma} Q_f w^{1-\sigma}\tau_x^{1-\sigma}\right]\varphi_i^{\sigma-1}\tau_{ex,i}^{1-\sigma}}{\left[\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} P_f^{\sigma} Q_f w^{-\sigma}\tau_x^{1-\sigma}\right]\varphi_i^{\sigma-1}\tau_{ex,i}^{1-\sigma} + \tau_{Fx,i} f_x} = \left(\frac{\sigma w}{\sigma-1}\right)\left(\frac{1}{1+\zeta_x \tau_{fx,i}(\varphi_i/\tau_{ex,i})^{1-\sigma} f_x}\right)$$

where ζ_d and ζ_x depend on the aggregate variables of P, Q, P_f, Q_f. Note that unlike domestic distortion τ_i, the iceberg cost $\tau_{ex,i}$ does not show up in the numerator of $TFPR_{ex,i}$.

It is easy to see that without fixed exporting cost $f_x = 0$, $TFPR_{ex}$ is constant across exporters, and we need to use domestic distortion τ_i to generate both non-exporters and exporters $TFPR$. To be able to match both, we also need a high fixed exporting cost f_x and a large standard deviation of $\tau_{fx,i}$ and $\tau_{ex,i}$ so that we can generate disperse enough $TFPR_{ex,i}$ to help us match both exporters' and non-exporters $TFPR$ dispersion.

Large f_x and large standard deviation of $\tau_{fx,i}$ can fix the TFPR dispersion for exporters. However, there is another challenge in matching the correlation between export participation/intensity with $TFPR$. Consider the export participation rate. The export cutoff of firm i does not depend on its domestic wedge τ_i but exporting costs:

$$\varphi_{x,i}^{*} = \left[\frac{\sigma^{\frac{\sigma}{\sigma-1}}}{\sigma-1} w^{\frac{\sigma}{\sigma-1}} \tau_x f_x^{\frac{1}{\sigma-1}} (P_f^{\sigma} Q)^{\frac{1}{1-\sigma}}\right] \tau_{ex,i}\tau_{fx,i}^{\frac{1}{\sigma-1}} \equiv \zeta \tau_{ex,i}\tau_{fx,i}^{\frac{1}{\sigma-1}}.$$

Let Φ be the conditional distribution of φ, the export participation rate of firm i is $1 - \Phi\left[\zeta\tau_{ex,i}\tau_{fx,i}^{\frac{1}{\sigma-1}}\right]$, which decreases with the firm's iceberg $\tau_{ex,i}$ and fixed trade cost $\tau_{fx,i}$.

Hence, a lower $\tau_{ex,i}$ or $\tau_{fx,i}$ pushes up both export participation and $TFPR_{ex,i}$ of firm i. This generates a force to make $TFPR_{ex,i}$ and export participation positively correlated, which is counterfactual. In contrast, in our benchmark model, $\tau_{fx,i}$ is not part of labor and does not show up in $TFPR_{ex,i}$. Furthermore, $\tau_{ex,i}$ is a wedge and also shows up in the numerator of $TFPR_{ex,i}$. Hence, it is easier for our benchmark model to produce a negative correlation between $TFPR$ and export participation.

In addition, a larger variation in the domestic wedge τ and stronger positive correlations between τ_i and either $\tau_{ex,i}$ or $\tau_{fx,i}$ can lead to a higher dispersion of TFPR, as well as a negative correlation between TFPR (when τ_i is low) and export participation (when trade cost $\tau_{ex,i}$ is low). However, first, a very positive relationship between export intensity and TFPR ensues. Second, this suggests that heavily subsidized domestic firms are more technologically advanced when it comes to exporting, which we interpret as them also

Table A-2: Data, Benchmark, and Alternative Models

	Data	Bench	No τ_{fx} $\tau \neq \tau_{ex}$	No τ_{fx} $\tau = \tau_{ex}$	Heter-trade-cost
Parameters					
Fixed cost of producing f		0.07	0.06	0.09	0.12
Fixed cost of export f_x		0.09	0.05	0.42	0.20
Iceberg trade cost τ_x		2.85	3.07	1.85	3.45
Mean foreign prod $\mu_{f\varphi}$		2.47	2.37	3.92	5.32
Std. productivity σ_φ		1.36	1.39	1.33	1.40
Std. distortion on home sales σ_τ		1.13	1.15	1.01	0.90
Std. distortion or cost on export sales $\sigma_{\tau_{ex}}$		1.01	0.96	1.01	1.60
Corr(φ, τ) $\rho_{\varphi,\tau}$		0.90	0.90	0.90	0.90
Corr(φ, τ_{ex}) $\rho_{\varphi,\tau_{ex}}$		0.62	0.56	0.90	0.60
Corr(τ, τ_{ex}) $\rho_{\tau,\tau_{ex}}$		0.64	0.57	1.00	0.70
Std. distortion or cost on export fixed cost $\sigma_{\tau_{fx}}$		0.62	–	–	1.60
Corr(φ, τ_{fx}) $\rho_{\varphi,\tau_{fx}}$		0.30	–	–	0.40
Corr(τ, τ_{fx}) $\rho_{\tau,\tau_{fx}}$		−0.10	–	–	0.40
Corr(τ_{ex}, τ_{fx}) $\rho_{\tau_{ex},\tau_{fx}}$		0.01	–	–	0.17
Targeted Moments					
Fraction of firms producing	0.85	0.85	0.84	0.85	0.84
Fraction of firms exporting	0.30	0.30	0.30	0.30	0.30
Import share	0.23	0.23	0.23	0.23	0.23
Relative GDP of U.S. to China	1.79	1.79	1.79	1.79	1.79
Std. TFPQ	1.32	1.32	1.32	1.33	1.36
Std. TFPR	0.94	0.95	0.95	0.94	0.84
Std. TFPR, exporters	0.88	0.87	0.87	0.91	0.69
Corr (TFPR, TFPQ)	0.91	0.92	0.91	0.91	0.93
Corr (TFPR, TFPQ), exporters	0.90	0.89	0.89	0.97	0.87
Std. export intensity	0.38	0.33	0.33	0.00	0.28
Corr (ex. participation, TFPQ)	0.06	0.06	0.23	0.06	0.10
Corr (ex. participation, TFPR)	−0.03	−0.03	0.10	−0.31	−0.04
Corr (ex. intensity, TFPQ)	0.01	−0.01	0.09	0.06	0.02
Corr (ex. intensity, TFPR)	−0.04	−0.03	0.05	−0.31	−0.05
Non-Targeted Moments					
TFPQ gap (ex−nonex)	0.17	0.18	0.66	0.17	0.30
TFPR gap (ex−nonex)	−0.06	−0.06	0.21	−0.63	−0.08
Export intensity	0.47	0.47	0.40	0.33	0.41
Std. value added	1.19	1.19	1.20	1.21	1.33
Corr (value added, TFPQ)	0.77	0.77	0.76	0.77	0.88
Corr (value added, TFPR)	0.45	0.44	0.44	0.44	0.64
Corr (value added, ex-int)	0.08	0.02	0.12	0.60	0.11
Corr (value added, ex-part)	0.17	0.18	0.34	0.60	0.26
Among Exporters					
Std. value added	1.20	1.36	1.24	0.98	1.48
Std. TFPQ	1.25	1.33	1.26	1.33	1.25
Corr (ex. intensity, TFPQ)	−0.13	−0.17	−0.19	0.001	−0.11
Corr (ex. intensity, TFPR)	−0.06	−0.03	−0.06	0.00	−0.06
Among Non-Exporters					
Std. value added.	1.16	1.08	1.07	0.97	1.19
Std. TFPQ.	1.34	1.31	1.30	1.33	1.40
Std. TFPR	0.96	0.98	0.97	0.90	0.89
Corr (TFPR, TFPQ)	0.93	0.93	0.93	0.98	0.96
Distance with data		0.004	–	–	0.018

Note: TFPR and TFPQ are logged. Corr denotes correlation, Std for standard deviation, 'intensity' for export intensity, and 'part.' for export participation. For '(No τ_{fx}, $\tau \neq \tau_{ex}$)', we estimate the model with no τ_{fx} but allowing for differential τ_{ex} and τ. In this case, we do not target the four trade correlations. For '(No τ_{fx}, $\tau = \tau_{ex}$)', we estimate the model with no τ_{fx} and $\tau = \tau_{ex}$. In this case, we do not target within-group distributions of TFPR and TFPQ and the four trade correlations. For 'hetero-trade-costs,' we estimate a case without export wedges but with a heterogeneous iceberg and fixed exporting costs.

receiving subsidies for their exports in our benchmark.

In summary, it is hard for the estimation of the hetero-iceberg model to match the dispersion of exporters' TFPR and the observed correlations between TFPR and trade. We ran estimations using the simulated method of moments in Matlab, and we tried with various initial guesses under the global search method that allows for a broad range of parameter searches. The last column of Table A-2 presents the best estimation.

I Comparative static analysis over parameters calibrated to 1998 data

In this section, we investigate the factors behind the changes in welfare when the economy transitions from the 1998 scenario to our benchmark 2005 scenario. We achieve this by altering each parameter from its 1998 value to its corresponding value in 2005, one at a time, and measuring the resulting welfare gain or loss. Table A-3 displays the changes in the distribution parameters, namely, σ_{φ}, σ_{τ}, $\sigma_{\tau_{ex}}$, $\rho_{\varphi,\tau_{ex}}$, and $\sigma_{\tau_{fx}}$. For example, the column titled "lower σ_{φ}" represents the change in σ_{φ} from 1.59 in the 1998 calibration to 1.36 in the benchmark calibration. We do not provide a comparative analysis for the other distribution parameters since they remain almost the same in both the 1998 and 2005 scenarios. For each scenario, we report the key moments and the welfare relative to that in the calibration using 1998 data.

Figure A-7 plots the gain from trade and the decomposition. The left panel is when underlying firm distributions of productivity and wedges are fixed, as in 2005, the relatively large std. of exporting wedge compared to productivity generated a large negative fiscal externality. The right panel is the gain from trade when underlying firm distributions of productivity and wedges are fixed as in 1998, fiscal externality is small, and there are welfare gains with trade cost reduction.

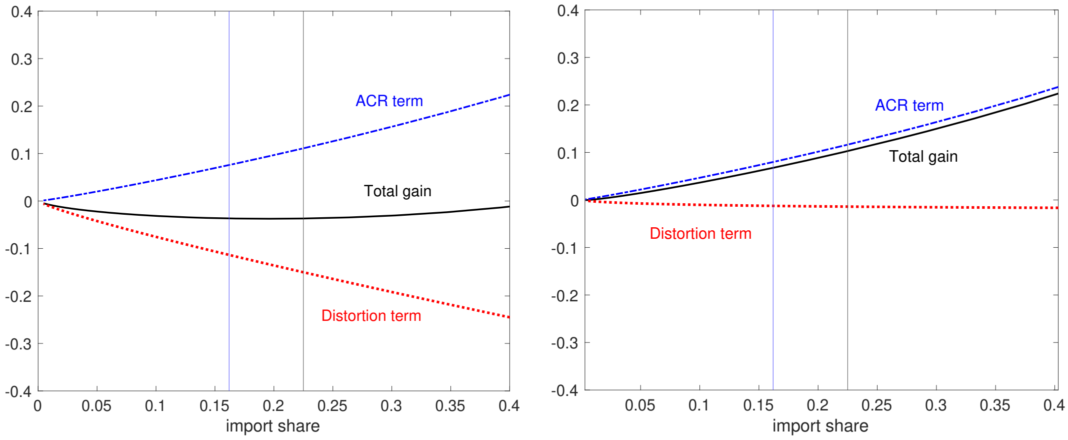
Note that it may look like there are significant differences in gains from trade with small changes in moments. This is due to the fact that welfare losses caused by distortions can be massive. In our benchmark calibration, the welfare level is approximately 200% lower than the efficient case. With small differences in data moments, the estimated welfare levels do

Table A-3: Comparative Static: 1998 to 2005

			98-model Comparative Static				
	05-model	98-model	lower σ_φ	lower σ_τ	lower $\sigma_{\tau_{ex}}$	lower $\rho_{\varphi,\tau_{ex}}$	increase $\sigma_{\tau_{fx}}$
98 value			1.59	1.34	1.11	0.68	0.56
05 value			1.36	1.13	1.01	0.62	0.62
Welfare wrt 98 (%)	57		-66	68	4.8	-3.1	-0.04
Key Moments							
Import share	0.23	0.16	0.18	0.12	0.16	0.19	0.16
Std. TFPQ	1.32	1.53	1.26	1.59	1.52	1.52	1.53
Std. TFPR	0.95	1.13	1.03	1.06	1.12	1.12	1.13
Corr (TFPR, TFPQ)	0.92	0.92	0.90	0.92	0.92	0.91	0.92
Std. export intensity	0.33	0.35	0.36	0.27	0.35	0.33	0.35
Among Exporters							
Std. TFPQ.	1.33	1.52	1.25	1.62	1.48	1.46	1.52
Std. TFPR.	0.87	1.02	0.92	1.04	0.97	1.00	1.02
Corr (TFPR, TFPQ)	0.89	0.92	0.89	0.93	0.91	0.90	0.92
Among Non-Exporters							
Std. TFPQ.	1.31	1.53	1.27	1.56	1.51	1.51	1.53
Std. TFPR	0.98	1.16	1.07	1.07	1.15	1.15	1.16
Corr. (TFPR, TFPQ)	0.93	0.92	0.91	0.93	0.92	0.93	0.92
Trade Correlations							
Corr (part., TFPQ)	0.06	0.09	-0.01	0.10	0.16	0.14	0.08
Corr (part., TFPR)	-0.03	0.01	-0.09	-0.005	0.09	0.02	0.01
Corr (intensity, TFPQ)	-0.01	0.01	-0.02	-0.04	0.06	0.06	0.01
Corr (intensity., TFPR)	-0.03	-0.003	-0.02	-0.09	0.06	0.02	-0.01

Note that the first column in the table, labeled as "05-model," presents the benchmark moments. The second column, labeled as "98-model," displays the moments obtained from the 1998 calibration. The remaining columns (3-7) present the moments obtained by changing only one parameter from the 98 calibrated value to the benchmark calibrated value while keeping all other distribution parameters the same as in 1998. For instance, the column labeled as "lower σ_φ" refers to the change in σ_φ from its value of 1.59 in the 98 calibration to 1.36 in the benchmark calibration. The table considers the comparative analyses in distribution parameters of σ_φ, σ_τ, $\sigma_{\tau_{ex}}$, $\rho_{\varphi,\tau_{ex}}$, and $\sigma_{\tau_{fx}}$. All other distribution parameters are similar in 1998 and in 2005. The statistics "welfare wrt. 98 (%)" is the difference between the welfare in each scenario and the welfare in 1998.

Figure A-7: Welfare Gain From Trade



(a) 2005 Calibration (Bench) (b) 1998 Calibration

not change much, in both the open and closed case. However, the difference between the two could have a couple of percentage changes, which are seemingly large.

J Other years' moments and calibrations

We chose 2005 because Chinese exports peaked in this year. As shown in Table A-4, both the fraction of firms importing and the import share rose until 2005, and then both fell in 2006 and 2007. We view 2005 as a period when China is more integrated with the world, while 1998 is a period when China is relatively close. In addition, the standard deviation of TFPQ and TFPR has been decreasing monotonically. The data moments before 2002 look similar to 1998, and in 2004 and 2006, they look very similar to 2005. Hence, we pick 2005 and 1998 as two example years. Nonetheless, we conduct a robustness check over other years.[2]

Table A-5 reports the gain from trade in 2006 after we calibrate the parameters to match the observed moments in 2006. The welfare loss in 2006 was similar to that of 2005. The changes are related to the change of the standard deviations of TFPQ and TFPR, as we discussed for the year 1998 vs 2005.

It is worth noting that our calibrations imply that the Chinese economy is very distorted.

[2] We didn't use 2004 due to the missing data in total sales, nor 2007 due to the poor quality of data in that year as it is the end of the sample.

Table A-4: Data Across Years

	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
Fraction of firms exporting	0.25	0.25	0.26	0.26	0.27	0.29	0.30	0.30	0.28	0.25
Import share	0.16	0.16	0.17	0.16	0.16	0.22	*n.a.*	0.23	0.22	0.20
Relative GDP of U.S. to China	2.61	2.58	2.53	2.38	2.17	2.06	1.93	1.79	1.64	1.50
Std. TFPQ	1.55	1.53	1.50	1.43	1.42	1.38	1.32	1.32	1.32	1.30
Std. TFPR	1.12	1.09	1.06	1.02	1.02	0.99	0.96	0.94	0.93	0.92
Std. TFPR, exporters	1.01	0.96	0.93	0.91	0.90	0.89	0.90	0.88	0.87	0.84
Corr (TFPR, TFPQ)	0.93	0.93	0.92	0.92	0.92	0.92	0.91	0.91	0.92	0.91
Corr (TFPR, TFPQ), exporters	0.92	0.91	0.90	0.90	0.90	0.89	0.90	0.90	0.90	0.88
Std. export intensity	0.38	0.38	0.38	0.38	0.38	0.37	*n.a.*	0.38	0.38	0.37
Corr (ex. participation, TFPQ)	0.08	0.09	0.08	0.07	0.07	0.06	0.04	0.06	0.06	0.03
Corr (ex. participation, TFPR)	−0.01	−0.01	−0.01	−0.02	−0.02	−0.02	−0.05	−0.03	−0.03	−0.07
Corr (ex. intensity, TFPQ)	0.04	0.04	0.04	0.03	0.03	0.02	*n.a.*	0.01	0.01	−0.01
Corr (ex. intensity, TFPR)	0.00	0.00	0.00	−0.02	−0.01	−0.02	*n.a.*	−0.04	−0.04	−0.06

Note: Data moments are constructed using the Chinese National Bureau of Statistics. Real GDP data is from PWT9.0. TFPR and TFPQ are logged. Corr denotes correlation, Std for standard deviation, ex. for export, ex.intensity for export intensity, ex.participation for export participation.

Table A-5: Parameters, Targeted Moments, and Welfare in 2006

Panel A: Parameters			Panel B: Targeted Moments				
	2005	2006		2005 (Bench)		2006	
				Data	Model	Data	Model
Fixed cost f	0.07	0.05	Fraction producing	0.85	0.85	0.85	0.86
Fixed export cost f_x	0.09	0.09	Fraction exporting	0.30	0.30	0.28	0.28
Iceberg cost τ_x	2.85	2.90	Import share	0.23	0.23	0.22	0.22
Foreign prod. $\mu_{f\varphi}$	2.47	2.38	U.S. GDP to China	1.79	1.79	1.64	1.67
Std prod. σ_φ	1.36	1.34	Std. TFPQ	1.32	1.32	1.32	1.30
Std home dist. σ_τ	1.13	1.12	Std. TFPR	0.94	0.95	0.93	0.94
Std export dist. $\sigma_{\tau_{ex}}$	1.01	0.98	Std. TFPR, exporter	0.88	0.87	0.87	0.87
$\rho_{\varphi,\tau}$	0.90	0.88	Corr (TFPR, TFPQ)	0.91	0.92	0.92	0.91
$\rho_{\varphi,\tau_{ex}}$	0.62	0.57	Corr (TFPR, TFPQ), ex	0.90	0.89	0.90	0.90
$\rho_{\tau,\tau_{ex}}$	0.64	0.52	Std. export intensity	0.38	0.33	0.38	0.35
Std. export cost $\sigma_{\tau_{fx}}$	0.62	0.66	Corr (ex-part., TFPQ)	0.06	0.06	0.06	0.09
$\rho_{\varphi,\tau_{fx}}$	0.30	0.04	Corr (ex-part., TFPR)	−0.03	−0.03	−0.03	0.00
$\rho_{\tau,\tau_{fx}}$	−0.10	−0.12	Corr (ex-int., TFPQ)	0.01	−0.01	0.01	−0.02
$\rho_{\tau_{ex},\tau_{fx}}$	0.01	−0.07	Corr (ex-int., TFPR)	−0.04	−0.03	−0.04	−0.06
Gains from Trade	−3.68	−2.69					
Distortion term	−15.01	−13.88					

Note: Data moments are constructed using the Chinese National Bureau of Statistics. TFPR and TFPQ are logged. Corr denotes correlation, Std for standard deviation, ex for export, ex-int for export intensity, ex-part for export participation.

Relative to the efficient-open case, the welfare is about 200% lower. On the other hand, in more integrated periods of 2005 and 2006, trade generated a welfare loss of about -3%.

K Discussions

K.1 Impact of Home distortions on foreign welfare

In the benchmark, foreign gain from trade is about 10% with or without Home distortions, though its gains are slightly higher when Home features distortions. Without distortions at Foreign, Foreign welfare still satisfies ACR/MR decomposition. But Home distortions have an impact on Foreign domestic sales share, entry, and cutoffs. To understand the impact, let's revisit Foreign welfare. From consumers' budget constraints and firms' free-entry conditions, we can write Foreign welfare as

$$W_f = C_f = \frac{w_f L_f}{P_f},$$

where w_f and P_f are Foreign wage and consumer price, respectively. We can further write Foreign aggregate price index as

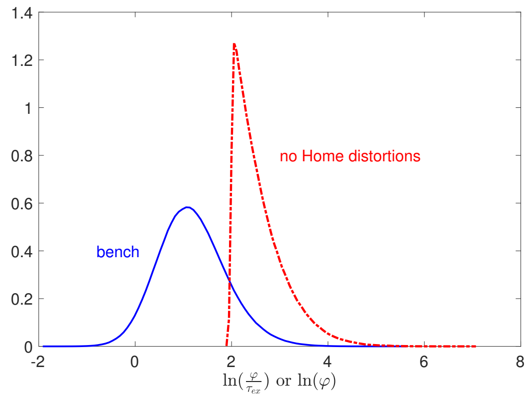
$$P_f = \left[M_{ef} \int_{\varphi_f^*} \left(\frac{\sigma}{\sigma - 1} \frac{w_f}{\varphi} \right)^{1-\sigma} dG_f + M_e \int_{\varphi_x^*(\tau_{ex}, \tau_{fx})} \left(\frac{\sigma}{\sigma - 1} \frac{w \tau_x \tau_{ex}}{\varphi} \right)^{1-\sigma} dG \right]^{\frac{1}{1-\sigma}}.$$

Plugging P_f back into the welfare equation and reorganizing it, we have

$$W_f = \frac{\sigma - 1}{\sigma} L_f \left[M_{ef} \int_{\varphi_f^*} \varphi^{\sigma-1} dG_f + M_e \tau_x^{1-\sigma} \int_{\varphi_x^*(\tau_{ex}, \tau_{fx})} \left(\frac{w_f}{w} \frac{\varphi}{\tau_{ex}} \right)^{\sigma-1} dG \right]^{\frac{1}{\sigma-1}}. \tag{A.39}$$

Hence, Home distortion affects foreign welfare through the import prices, the relative wage w_f/w, Foreign producing cutoff φ_f^*, and Home exporting cutoff φ_x^*. The import prices are proportional to firms' marginal cost of producing τ_{ex}/φ, or they are inversely related to firms' effective productivity φ/τ_{ex}. The higher the average effective productivity, the lower the import prices, and the higher the Foreign welfare. Also, the higher the relative wage, the higher the Foreign welfare.

Figure A-8: Distribution of Foreign Imported Goods



The prices Foreign faces are lower were Home firms to be taxed less (low τ_{ex}); on the other hand, some low-φ hence high-marginal-cost Home firms will be selected into exporting, making the Foreign's import prices higher. Figure A-8 depicts the distribution of the effective productivities (φ/τ_{ex}) of Foreign country's imported goods from Home country. The blue-solid line is for the benchmark, and the red-dashed line is for no Home distortions. The differences of the two lines reflect the different underlying distributions of φ/τ and φ, as well as the different cutoffs of Home exporting φ_x^* with and without distortions. The benchmark distribution is to the left of that when Home faces no distortions. These low effective productivity (or high marginal costs) tend to reduce Foreign welfare.

Meanwhile, Home distortions also have a general equilibrium effect on relative wages. When there are Home distortions, the relatively higher demand for foreign products induces a higher Foreign wage. Without Home distortions, its efficiency improves, and the Foreign wage would be lower.

In summary, Home distortions have two opposing effects on Foreign welfare. On the one hand, distortions push up the import prices (through low effective productivity or high marginal cost) of Foreign and lower Foreign welfare. On the other hand, distortions raise Foreign wages and welfare. These two effects cancel out in our estimation and lead to a similar welfare gain for Foreign country with or without Home distortions. One factor that affects the race of the two effects is the dispersion of τ_{ex}. More dispersed τ_{ex} pushes up the import prices of Foreign and leads to lower Foreign welfare under Home distortions.

K.2 Imbalanced trade

To see the quantitative impact of trade imbalances between China and the U.S., we follow Dekle, Eaton and Kortum (2007) and impose the observed imbalances in our equilibrium condition. Due to wealth transfer from trade imbalance, we would expect a decrease in Home import share and welfare and an increase in foreign wages. Quantitatively, under our benchmark parameters and trade surplus at Home (China), foreign wage increases by 1.6%, and Home welfare in the open economy decreases by 4.8% relative to our benchmark. This decline in welfare mainly comes from the wealth transfers from Home's trade surplus, as in Dekle et al. (2007) . Adding trade surplus to Home country slightly affects our model moments. We also reestimate the model parameters, and the quantitative results are similar to the case under benchmark parameters.

Note that our model is a static one. Under a dynamic model, a country that runs a trade surplus in the current period should run trade deficits in the future. Thus, the net present value of the trade imbalance should be close to zero. Let β be the countries' discount factor and r the world interest rate. Under a complete market model and $\beta(1+r) = 1$, the country's overall welfare gain or loss from trade would be roughly the same as our benchmark result.

K.3 Domestic distortion takes the form of iceberg cost

Here, we want to emphasize that the model with an iceberg-cost type of distortion does not produce any wedges. In this case, distortion works like a productivity shock. Hence, the welfare decomposition is equivalent to ACR or MR. There are always gains from trade. Most importantly, the welfare decomposition has no distortion term. In other words, using aggregates, as in the literature, can capture the gains from trade well.

To clearly make the point, we consider a closed economy, where distortions are modeled in the same way as the iceberg trade cost. Specifically, to produce q units, the firm has to use $\ell_v = \tau q/\varphi$ units of variable labor plus the fixed cost, where τ is the iceberg-type distortion and φ is the productivity. An intermediate-good firm (φ, τ) solves the following problem

$$\max_{p,q} \quad pq - \frac{w\tau}{\varphi}q - wf,$$

subject to the demand function $q = \frac{p^{-\sigma}}{P^{-\sigma}}Q$. We can characterize the optimal price p, variable labor ℓ_v, output q, and revenue pq as

$$p = \frac{\sigma}{\sigma - 1} w \left(\frac{\varphi}{\tau}\right)^{-1}, \tag{A.40}$$

$$\ell_v = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma - 1}, \tag{A.41}$$

$$q = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma}, \tag{A.42}$$

$$pq = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} (P^\sigma Q) w^{1-\sigma}\right] \left(\frac{\varphi}{\tau}\right)^{\sigma - 1}. \tag{A.43}$$

It is easy to see that all the endogenous variables here (p, ℓ_v, q, pq) only depend on the ratio of φ to τ, or the effective productivity $\tilde{\varphi} = \varphi/\tau$. Note that in our benchmark model with a 'tax' style of distortion, the optimal p, q, and pq take the same formula as above. However, optimal variable labor is given by,

$$\ell_v^{bench} = \left[\left(\frac{\sigma}{\sigma - 1}\right)^{-\sigma} (P^\sigma Q) w^{-\sigma}\right] \varphi^{\sigma - 1} \tau^{-\sigma}. \tag{A.44}$$

The distortion in our benchmark model is equivalent to a labor wedge. For one unit of labor, households receive w unit of payment, but firms pay for $w\tau$. With this one unit of labor, firms produce φ unit of goods. The marginal product of variable labor $pq/\ell_v^{bench} = \frac{\sigma}{\sigma-1} w\tau$ is firm-specific and is not equalized across firms. In contrast, the iceberg cost behaves like productivity. For one unit of labor, households receive w, and firms pay for w; there is no wedge between them. All firms have the same marginal product of variable labor, $pq/\ell_v = \frac{\sigma}{\sigma-1} w$. However, with one unit of labor, firms can only produce φ/τ unit of goods, which costs extra resources. However, there is no efficiency loss from misallocation (wedge) as in HK.

Hence, an open-economy model under iceberg-type distortion is equivalent to a Melitz model with productivity distribution on $\tilde{\varphi} = \varphi/\tau$. If $\tilde{\varphi}$ follows a Pareto distribution, we reach the ACR result, where the import share and trade elasticity can forecast the gain from trade. We do not need the underlying distribution of physical productivity φ and true distortion τ for measuring the gain from trade. If $\tilde{\varphi}$ follows a general distribution, the MR results hold. Still, there is no reallocation term as in our theory.

In summary, the iceberg type of distortion shows up like a technology shock. It lowers welfare because firms have to use more labor to produce the same unit of output. However, the iceberg cost does not generate misallocations across firms. Hence, the welfare decomposition does not consist of a term to reflect such misallocation. In contrast, our benchmark aims to examine the implication of HK type of distortion on gain from trade. This type of distortion generates misallocation, showing up as wedges across firms.

L TFPR and TFPQ in China and measurement error

We find large dispersions in measured TFPR in China, similar to the levels in HK for the years 1998 and 2007. TFPR can be written into two terms: revenue product of labor $ARPL_{ji} = p_{ji}q_{ji}/\ell_{ji}$ and revenue product of capital $ARPK_{ji} = p_{ji}q_{ji}/k_{ji}$, i.e. for any firm i in industry j,

$$\log(TFPR_{ji}) = \alpha_j \log(ARPL_{ji}) + (1-\alpha_j)\log(ARPK_{ji}).$$

where α_j is the industry-specific labor share. Both measured ARPL and ARPK have come down over time, between 1998 and 2007, as evident in Table A-6.[3] There is also greater dispersion in the average product of capital than there is in the average product of labor.

Table A-6: Dispersion of ARPK and ARPL

	1998	2001	2004	2007
std(ARPK)	1.348	1.306	1.241	1.185
std(ARPL)	1.184	1.039	0.940	0.923

We next turn to investigate further what factors are systematically related to measured TFPR. First, TFPR is highly correlated with TFPQ, as shown in Table A-4. Second, we

[3] Note that we trim the 1% tails of TFPR, output, and input in *each* year in the benchmark, and thus the standard deviations in the benchmark are slightly smaller than the numbers in Table A-6.

Table A-7: TFPR Regressions

VARIABLES	(1) $\ln(TFPR)$	(2) $\ln(TFPR)$	(3) $\ln(TFPR)$	(4) $\ln(TFPR)$	(5) $\ln(TFPR)$	(6) Data 2005 $\ln(TFPR)$	(7) Model (2005) $\ln(TFPR)$
$\ln(TFPQ)$	0.630***	0.635***	0.635***	0.635***	0.639***	0.656***	0.658
	(235.9)	(243.2)	(241.6)	(248.4)	(261.6)	(1121)	
Age			-0.00165***	-0.00163***	-0.00148***		
			(-9.736)	(-10.10)	(-10.05)		
SOE				-0.100***	-0.0930***		
				(-4.577)	(-4.481)		
Foreign owned				-0.230***	-0.156***		
				(-25.96)	(-24.60)		
Exporters					-0.213***	-0.167***	-0.175
					(-24.96)	(-99.33)	
Constant	-3.296***	-3.236***	-3.209***	-3.131***	-3.129***	0.0499***	
	(-106.2)	(-89.23)	(-87.12)	(-75.08)	(-77.04)	(54.29)	
Observations	1,587,629	1,479,528	1,478,648	1,478,648	1,478,648	233,225	
R-squared	0.812	0.822	0.823	0.831	0.837	0.844	
Time FE	Yes	Yes	Yes	Yes	Yes		
Industry FE	Yes	Yes	Yes	Yes	Yes		
Location FE	No	Yes	Yes	Yes	Yes		

Robust t-statistics in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

conduct the regression analyses of measured TFPR on TFPQ and a set of variables like age, ownership, exporter dummy with year, industry and location fixed effects. See Table A-7.

In all these regressions, the coefficient on firm TFPQ is large and significant; a 1 percent increase in TFPQ is associated with about a 60 percent increase in TFPR. Moreover, more than half of the variation in TFPR is explained by TFPQ alone. The positive relationship is consistent with the predictions of our model, as shown in the model regression (Column 6). The same is true for the results on exporters: given TFPQ, firms must have lower taxes on average and a lower TFPR in order to export. TFPR differences are also systematically related to firm characteristics: state-owned enterprises and Foreign-owned firms are subject to lower TFPR on average, given TFPQ.

Measurement error With the presence of fixed costs in producing and exporting in our model, the measured TFPR does not perfectly relate to the true wedges. In the data, there are other types of mismeasurements in output and input, which may also generate a dispersion in the average revenue products, and thereby affect the measured TFPR— as shown in Bils, Klenow and Ruane (2021) and Song and Wu (2015). Here, we use Bils et al. (2021)'s method to detect measurement errors. We find that even taking out the standard measurement errors, there are still large distortions remaining among Chinese firms.

The main approach involves using panel data to estimate the true marginal product dispersion among operating firms, rather than simply employing cross-sectional data. With this method, we find that the measurement errors are small in China, accounting for only 18% of the variation in the average product.[4] This 18% includes the mismeasurement of production inputs in the presence of fixed cost, which is accounted for in our benchmark.

We exploit three alternative methods to detect measurement error: average annual observations within firms, first differences over years within firms, and covariance between first differences and average products. All three approaches point to the same conclusion: 1) there is a large dispersion in marginal products in China; 2) measurement error only accounts for a small fraction of the dispersion in the measured marginal products (i.e., average products).

[4] Bils et al. (2021) finds measurement errors exaggerate misallocation in India by about 30% and about 70% of that in the U.S..

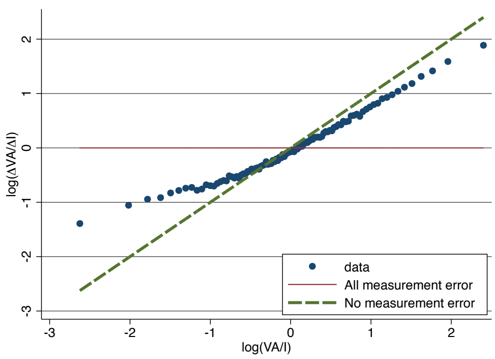
Table A-8: Detecting Measurement Errors

Average annual observation within firm				
std(ln(ARPK))	std(ln(ARPL))	$std(lnVA)$	$std(ln(VA/I))$	$corr(lnVA, ln(VA/I))$
1.19	0.96	1.19	0.94	0.4
	First level differences			
	2001	2004	2007	
$std(ln(\triangle VA/\triangle K))$	1.82	1.78	1.76	
$std(ln(\triangle VA/\triangle L))$	1.68	1.60	1.61	
	Regression			
	Ψ	$\Psi(1-\lambda)$		
	0.53^{***}	-0.0997^{***}		
	(34.58)	(-20.65)		

Note: This table reports three ways to detect measurement errors. The upper panel reports the average annual levels within firms. The middle panel reports the ratio of first differences as another measure of marginal product, where $\triangle VA$ denotes the first difference of value added. The lower panel reports regression coefficient as in equation (A.45). Robust t-statistics in parentheses.

First, if measurement errors were idiosyncratic across firms and over time, one can take the time average of annual observations within firms to wash out these errors, drastically reducing the dispersion of average products. The upper panel of Table A-8 reports the statistics when we take the average within firms. The average standard deviation is 1.19 for the average product of capital and 0.96 for the average product of labor. The standard deviations of value added and the average product of inputs are 1.19 and 0.94, where the correlation between the two variables is 0.4. These results mimic the moments in the year 2005. In particular, the dispersions of average products of inputs are still high. This implies that measurement errors of the iid type cannot explain the observed dispersions in the average products.

Figure A-9: Measured Marginal Product using First Differences vs TFPR



Second, as pointed out by Bils et al. (2021), the dispersion of first differences reflects the true distortion if marginal products are constant over time. Calculating the first differences of value-added $\triangle VA$, capital $\triangle K$, and labor $\triangle L$, and then taking the ratio $\triangle VA/\triangle K$ and $\triangle VA/\triangle L$ gives us an alternative measure of marginal products. The 1% tails of both ratios are trimmed, and the results are displayed in the middle panel of Table A-8 for the years 2001, 2004, and 2007. The dispersions are even higher than those in Table A-6 for the measured average product of inputs.

Table A-9: Measured Marginal Products using First Differences vs TFPR

VARIABLES	(1) $\log(\frac{\triangle VA}{\triangle I})$	(2) $\log(\frac{\triangle VA}{\triangle I})$	(3) $\log(\frac{\triangle VA}{\triangle I})$
$\log(TFPR)$	0.699***	0.715***	0.718***
	(132.2)	(158.6)	(135.3)
Constant	1.449***	0.331***	1.410***
	(81.68)	(17.49)	(78.31)
Observations	624,699	624,699	624,659
R-squared	0.168	0.269	0.173
Time FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes

Note: Specification (2) weights all the observations with the absolute value of composite input growth. Specification (3) weights all the observations with the share of aggregate value added. Robust t-statistics in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Moreover, the alternative measured marginal products are highly correlated with average products. Figure A-9 plots the $\ln(\triangle VA/\triangle I)$ against the benchmark average product of input $\ln(VA/I)$ where I is the composite of inputs, $I = K^{\alpha}L^{1-\alpha}$, where each dot corresponds to one of 100 percentiles of $\ln(VA/I)$. The regression coefficient at the firm level is 0.72, see Table A-9. Note that without measurement errors, the two measures are perfectly correlated. For the case with only measurement error, the two measures have no correlation. Hence, the high correlation between the alternative measure and the average products suggests small measurement errors and a large distortion-induced misallocation.

Lastly, we follow Bils et al. (2021) and run the following regression to further quantify the extent to which measured average products reflect marginal products:

$$\triangle\widehat{VA}_i = \Phi \cdot \log(TFPR_i) + \Psi \cdot \triangle\hat{I}_i - \Psi(1-\lambda) \cdot \log(TFPR_i) \cdot \triangle\hat{I}_i + D_s + \xi_i \tag{A.45}$$

Table A-10: Estimate Measurement Error

	(1)	(2)	(3)
VARIABLES	$\triangle\widehat{VA}$	$\triangle\widehat{VA}$	$\triangle\widehat{VA}$
$\log(TFPR)$	0.0376***	0.0144***	0.0616***
	(22.62)	(9.170)	(16.07)
$[\log(TFPR)]^2$			-0.0128***
			(-6.110)
$[\log(TFPR)]^3$			0.00152***
			(4.008)
$\triangle\widehat{input}$	0.530***	0.523***	0.524***
	(34.58)	(33.03)	(31.13)
$\log(TFPR) \times \triangle\widehat{input}$	-0.0997***	-0.0954***	-0.0893***
	(-20.65)	(-19.16)	(-6.420)
$[\log(TFPR)]^2 \times \triangle\widehat{input}$			-0.00611
			(-0.919)
$[\log(TFPR)]^3 \times \triangle\widehat{input}$			0.00108
			(1.040)
Constant	-0.0207***	0.0551***	-0.0241***
	(-3.125)	(8.231)	(-3.592)
Observations	1,106,982	1,106,914	1,106,982
R-squared	0.044	0.042	0.044
Time FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes

Robust t-statistics in parentheses. *** $p<0.01$, ** $p<0.05$, * $p<0.1$. Specification (2) weights all the observations with the share of aggregate value added.

where $\triangle \widehat{VA}_i$ and $\triangle \hat{I}_i$ are the growth rate of measured value-added and inputs, respectively, and $\log(TFPR_i)$ is the measured average products. The underlying assumption here is that the measurement errors are additive. The regression coefficient for $\triangle \hat{I}_i$ is 0.53, and for the interaction of $\log(TFPR_i)$ and $\triangle \hat{I}_i$ is -0.0997. Both are significant, and the robust t-statistics are reported in Table A-8. The implied λ is therefore 0.81. Hence, 81% of variation in $TFPR$ or average products is accounted for by distortions, and 19% is due to measurement errors.

The results are robust if we weigh the observations with their share of aggregate value added or if we control for higher orders of $\ln(TFPR)$ to allow for stationary shocks to firms' productivity and distortions. See Table A-10.

In summary, the three alternative ways of sifting out measurement errors using panel data all point to the result that the dispersion in the average product of inputs is mainly driven by distortions rather than measurement error typically conceived.

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