Online Appendix to
The Common-Probability Auction Puzzle

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1 Experimental Interfaces

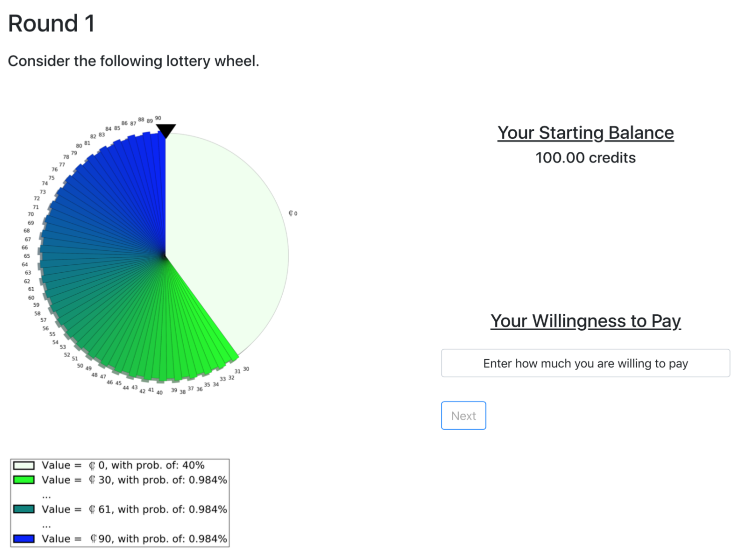


Figure A 1: Example for Reduced Lottery in Experiment II

Figure A 1 shows the display of a reduced CV lottery in Experiment II. The display of reduced CP lotteries was similar but with only two possible outcomes.

Figure A 2 shows the interface in the second part of Experiment IV. The two framed boxes correspond to the two pieces of information given to the subjects. The upper right box reminds the subject of their belief about the competitive bid for the exact same lottery. The lower box renders information about the lottery to be expected conditional on having the highest signal in the market.

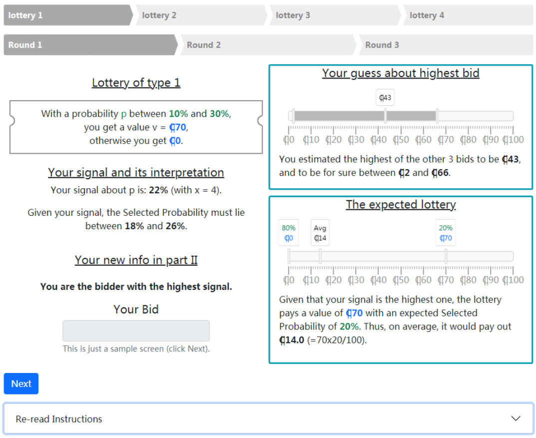


Figure A 2: Example of CP Interface in Part II of Experiment IV

2 Descriptive Statistics

2.1 Bid and Price Factors

Table A1: Bid Factors (BF) in Reduced Sample

		Naive bid (bid-bid^{Naive})		Break-Even bid (bid-bid^{BE})		Nash-Eq. bid (bid-bid^{RNNE})	
		mean	median	mean	median	mean	median
Exp. I	CV	10.53***	13.6***	12.26**	15.32***	13.33***	16.45***
		(1.872)		(1.871)		(1.870)	
	CP	-4.60***	-3.80**	-2.84**	-2.04	-1.77	-0.82
		(1.424)		(1.425)		(1.425)	
	Diff.	15.13***	17.4***	15.11***	17.36***	15.10***	17.26***
		(2.346)		(2.345)		(2.345)	
Exp. III	CV	16.14***	19***	17.80***	21***	17.53***	20.60***
		(2.570)		(2.559)		(2.791)	
	CP	-1.33	-0.40	-0.64	1	-0.31	0.80
		(1.225)		(1.178)		(1.621)	
	Diff.	17.47***	19.4***	17.86***	20***	17.84***	19.80***
		(2.819)		(2.789)		(3.199)	
Exp. IIIB	CV	12.95***	13.2***	14.55***	15***	15.9***	16.8***
		(1.550)		(1.558)		(1.689)	
	CP	-0.60	0	1.23	1**	0.84	1.80*
		(1.099)		(1.141)		(1.380)	
	Diff.	13.54***	13.2***	13.32***	14***	15.07***	15***
		(1.885)		(1.941)		(2.166)	
Exp. IV	CV	9.46***	8.4***	10.66***	9.36***	11.45***	10.00***
		(1.828)		(1.828)		(1.828)	
	CP	-1.61	-0.4	-0.41	0.76	0.39	1.60
		(1.750)		(1.750)		(1.750)	
	Diff	11.07***	8.80***	11.07***	8.60**	11.07***	8.40**
		(2.522)		(2.522)		(2.521)	

Note: Cluster robust standard errors (CRSE) clustered at subject level in parentheses. P-values: *: p-value<.1,**: p-value<.05, ***: p-value<.01. Clustering standard errors by sessions do not alter tests results and accounts for approximately 1% of the residual variance. In the remaining analyses standard errors are clustered at the subject level.

Tables A1 and A2 present the computation of bid factors relative to all three benchmarks: the naive, the break-even and the RNNE bid function. Positive (negative) bid factors imply that average bids are above (below) the benchmark.

Table A2: BID FACTORS (BF) – WINNING BIDS IN EXP. I

		CV	CP	Diff
Naive bid	mean	24.68***	9.50***	15.18***
(bid-bid^{Naive})		(0.731)	(1.076)	(1.299)
	median	23.40***	8.40***	5.00***
Break-Even bid	mean	26.41***	11.25**	15.17***
(bid-bid^{BE})		(0.727)	(1.099)	(1.316)
	median	25.16***	10.28***	14.88***
Nash-Eq. bid	mean	27.47***	12.33***	15.15***
(bid-bid^{RNNE})		(0.725)	(1.108)	(1.322)
	median	26.40***	11.52***	14.88***

Note: Cluster robust standard errors (CRSE) at subject level in parentheses. P-values: *: p-value<.1,**: p-value<.05, ***: p-value<.01.

Table A3: PRICE FACTORS IN EXP. II

		CV	CP	Diff
Part CL with signal				
Price Factor	mean	5.12***	-1.03	6.16***
(bid-$price - E[L\|s]$)		(1.676)	(1.001)	(1.945)
	median	2.4***	-1.8***	4.2***
Part CL without signal				
Price Factor	mean	0.75	-3.00**	3.75*
(bid-$price - E[L\|s]$)		(1.679)	(1.410)	(2.188)
	median	-4***	-4***	0
Part RL				
Price Factor	mean	-2.93*	-1.37	-1.56
(bid-$price - E[L\|s]$)		(1.684)	(1.128)	(2.022)
	median	-4**	-1	-3

Note: CRSE in parentheses. P-values: *: p-value<.1,**: p-value<.05, ***: p-value<.01.

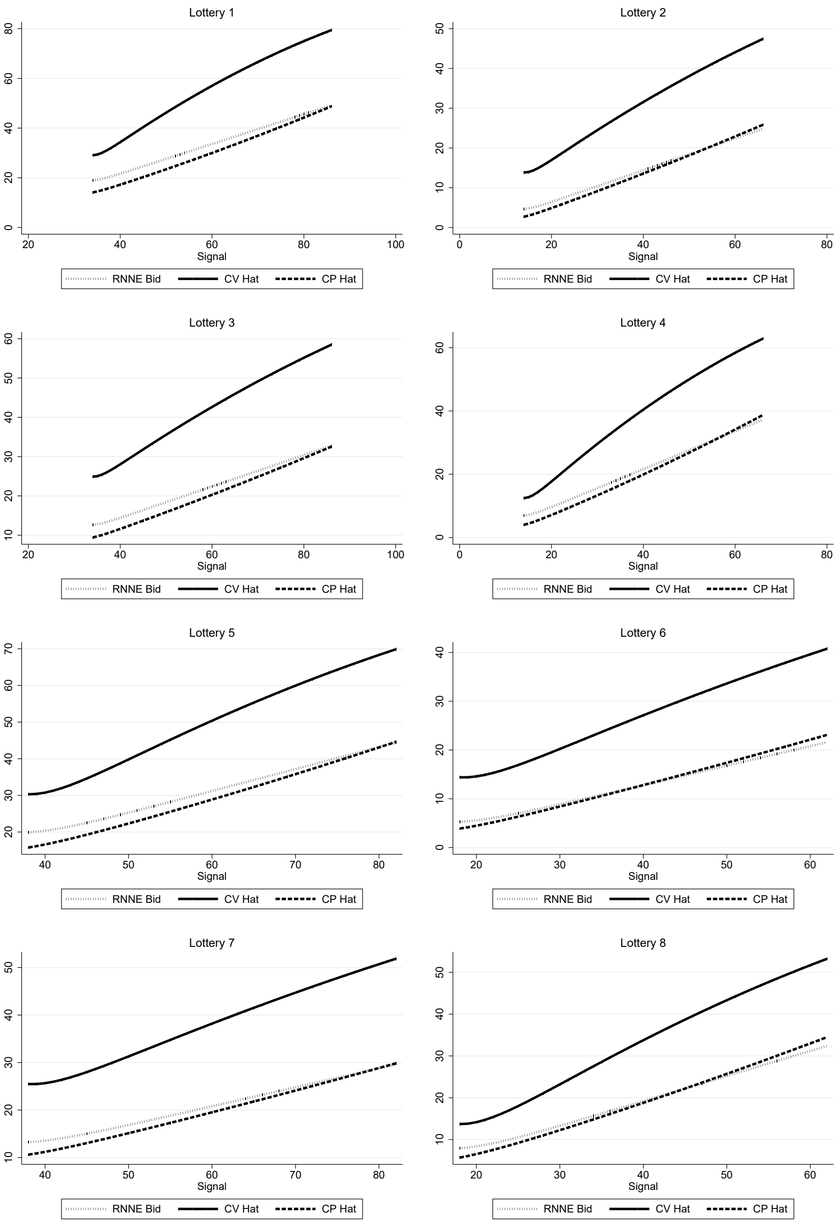


Figure A 3: Estimated Median Bids in CV and CP Auctions by Lottery Types (Experiment I)

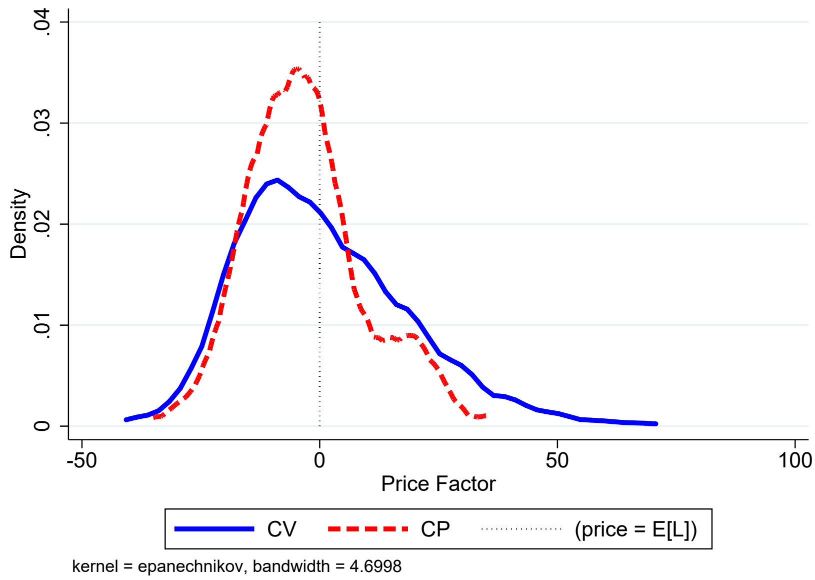


Figure A 4: Distribution of *Ex ante* Price Factors for Compound Lotteries ($=w_i - E[L]$) in Treatments CVL (solid) and CPL (dashed)

2.2 Adverse Selection

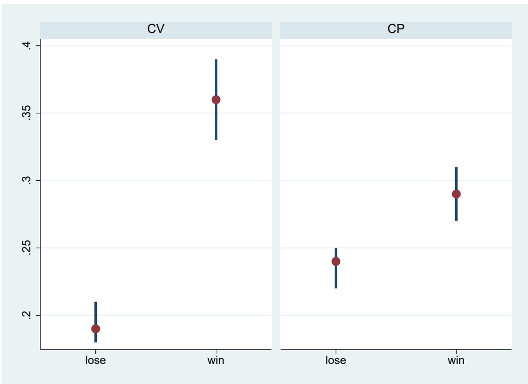


Figure A 5: Predicted probabilities of having the highest signal conditional on winning and losing

While the winner's curse is less severe in CP than in CV, it is not clear whether this is because subjects reason better through the adverse selection problem with probabilistic uncertainty or because winning reveals less information in the first place. To shed more light on the extent of the adverse selection problem, we juxtapose how informative two events are: the event of having the highest signal and the event of winning. The lottery's actual average payoff conditional on having the highest signal tells us how much, in each auction, subjects must actually shave their bids to break even. This is the empirical benchmark for updating in a Bayesian manner. To this end, we regress the lottery outcome on the signal and a dummy that takes the value one if the signal is the highest in the auction. The lottery's average payoff conditional on winning, on the other hand, tells us how much subjects can actually learn from winning the auction. In a similar manner, we regress the average lottery outcome on the signal and a dummy that takes the value one if the bidder with the same signal won the auction. Note that the two events of having the highest signal and of winning will convey the same information if the winner is always the bidder with the highest signal, thereby providing the need to account for an adverse selection effect in bidding. We find that in the data having the highest signal in CV (CP) requires adjusting expectations downward by an average of ₵-3.08 (₵-2.65, $p < 0.001$ in either case). In contrast, winning is not as informative, in particular in CP auctions. Winning an auction allows one to adjust expectations only by a fraction of what can actually be learned from having the highest signal: up to 35.71% in CV but only 10.07% in CP. Differences in the reduced sample are even more striking: 60.38% in CV versus 3.62% in CP. This is a direct result of a weaker correlation between signal rank and winning in the CP auction. Appendix Figure A 5 plots the predicted probabilities of having the highest signal conditional on winning. The risk of falling prey to the winner's curse is present in CV but marginal in CP. Winning in CV increases the likelihood of having the highest signal from 19% to 36%, which is three times more than in CP (24% to 28%).[1] We conclude that, in the aggregate, the winner's curse is mitigated in CP because, there, a winner is less likely to have the highest signal. This suggests that subjects in CP may not have been more sophisticated than those in CV, but it does not preclude the possibility that the ability to reason through the winner's curse differed.

[1]The probit estimation is done with the entire sample in Experiment I. The estimation with the reduced sample leads to more extreme results with a higher marginal effect of winning in CV, but a nonsignificant and weak effect in CP.

2.3 Decision weights

To assess the importance of signals relative to the known component of the lottery, we estimate the elasticity of the bid with respect to the known and the unknown (i.e., signal) component of the lottery. To this end, we use a simple Cobb-Douglas bidding function in the form of $b(s_i) = k^{\alpha} \cdot s^{\beta}$. A naive agent, for instance, would bid $E[L|s] = k^{\alpha} \cdot s^{\beta}$ with $\alpha = \beta = 1$.

Table A4: Median Regression Coefficients in Bidding

$ln(bid)$	(CV)	(CP)	(Diff)
$ln(k)$	$0.244^{*\dagger\dagger\dagger}$	$0.749^{***\dagger}$	-0.505^{**}
	(0.130)	(0.137)	(0.197)
$ln(s_i)$	$1.096^{***\dagger\dagger\dagger}$	1.254^{***}	-0.158
	(0.030)	(0.161)	(0.171)
$Cons$	-1.488^{***}	-4.818^{***}	3.331^{***}
	(0.570)	(0.687)	(0.901)
N	3253	2564	5817
$Subjects$	52	39	91
R^2	0.015	0.072	
$F-Test$	0.000	0.040	
MRS	$\approx 0.22\frac{s}{k}$	$\approx 0.60\frac{s}{k}$	

Note: Median regression with CRSE in parentheses. Significant difference from 0: *: p-value<.1,**: p-value<.05, ***: p-value<.01. Significant difference from 1: †: p-value<.1,††: p-value<.05, †††: p-value<.01. F-test refers to a test on equal weighting of known parameter and signal ($\alpha = \beta$).

We use the marginal rate of substitution (MRS) to compare the estimated bidding functions. The MRS represents here how much units of the signal subjects are willing to trade against a unit of the known parameter to maintain the same bid. For a naive bidder, the MRS equals $\frac{\alpha s}{\beta k} = \frac{s}{k}$. For our parameter variation, MRS under Nash equilibrium should be close to $\frac{s}{k}$. In both auction formats, the estimated MRS is smaller than $\frac{s}{k}$ ($\approx 0.23\frac{s}{k}$ in CV vs. $\approx 0.60\frac{s}{k}$ in CP in Appendix Table A4), indicating that subjects overweighted their private signal but underweighted the known component. Subjects in CV auctions put relatively more weight on the signal compared to those in CP auctions. Similar results are obtained with the pricing data,

Table A5: MEDIAN REGRESSION COEFFICIENTS IN PRICING

$ln(bid)$	CVL	CPL	Diff.
$ln(k)$	$0.546^{***\dagger\dagger\dagger}$	$0.810^{***\dagger\dagger\dagger}$	-0.264
	(0.157)	(0.056)	(0.067)
$ln(s_i)$	0.947^{***}	0.974^{***}	-0.027
	(0.069)	(0.044)	(0.066)
Cons	-2.500^{***}	-3.847^{***}	1.347
	(0.721)	(0.346)	
N	4256	4000	8256
$Subjects$	54	50	104
R^2	0.141	0.3919	
$F-Test$	0.0209	0.0017	
MRS	$\approx 0.57\frac{s}{k}$	$\approx 0.83\frac{s}{k}$	

Note: Median regression with cluster robust standard errors (CRSE) at subject-level in parentheses. Significant difference from 0: *: p-value<.1,**: p-value<.05, ***: p-value<.01. Significant difference from 1: †: p-value<.1,††: p-value<.05, †††: p-value<.01.

where MRS are closer to the naive benchmark $\frac{s}{k}$ (see Appendix Table A5). While subjects put more attention on signals in both CV and CP formats it is important to keep in mind that these signals are about different components of the lotteries. In CV treatments, subjects paid more attention to values in the lottery whereas in CP treatments they rather focused on the probabilities. In a nutshell, it appears that the uncertain component determines how subjects allocate their attention to different features of the auctioned item.

2.4 Information processing in Experiment II

We study the importance of information processing in the decision problem. The empirical value of a signal is obtained by comparing subjects' willingness to pay before and after receiving signal s_i. To this end, we regress subjects' willingness to pay w_i on objective measures like the prior expected value $E[L]$ and the information content of the signal given by the change in expectations $(E[L|s_i] - E[L])$. We also include a dummy D_{signal} that equals one when the willingness to pay was submitted after observing a signal.

Table A6: MEDIAN REGRESSION COEFFICIENTS

WTP	Rational	CV	CP	Diff
$E[L]$	1	$0.898^{***\dagger}$	$0.830^{***\dagger\dagger\dagger}$	0.068
		(0.053)	(0.039)	(0.069)
$E[L\|S]-E[L]$	1	1.051^{***}	$0.905^{***\dagger\dagger\dagger}$	0.146
		(0.084)	(0.030)	(0.103)
D_{signal}	0	5.292^{***}	2.074^{**}	3.218
		(2.027)	(0.947)	(2.419)
$Cons$	0	-0.335	0.074	-0.409
		(2.349)	(1.497)	(2.783)
N		4688	4400	9088
$Subjects$		54	50	104
R^2		0.234	0.390	
$F-Test$		0.0080	0.0001	

Note: Median regression with CRSE in parentheses. Significant difference from 0: *: p-value<.1,**: p-value<.05, ***: p-value<.01. Significant difference from NE coefficient: †: p-value<.1,††: p-value<.05, †††: p-value<.01.

As shown in Table A6, we do not find substantial differences in the way subjects processed these value and probability signals (consistent with our results in Table A5). Under risk-neutral expected utility, pricing occurs at the expected value. That is, an increase of ₵1 in prior and interim beliefs is reflected in an equivalent increase of ₵1 in prices, while uncertainty premia (captured by the constant and the dummy variable) should be zero (cf. first column of Table A6). In treatment CVL, subjects reacted reasonably to variations in both prior parameters and signals as the corresponding coefficients do not substantially differ from the RNEU benchmark. In treatment CPL, subjects slightly underreacted to variations in the parameters, but more importantly coefficients do not differ from the ones in CVL. Hence, subjects processed value and probability signals similarly.

A striking observation is that in both CVL and CPL, the mere fact of observing a signal significantly increased WTP by ₵5 and ₵2, respectively. In other words, even when objective prior and interim expectations coincided, subjects were willing to pay more after observing a signal. This could be rationalized to some extent with a reduced uncertainty premium in interim beliefs, as seen in the treatment CPL where after getting a signal subjects bid closer to expected value. Rather surprising is that in CVL subjects bid, on average, even above expected values after seeing a signal, implying that the mere fact of getting a signal led subjects to move from an average positive to a negative uncertainty premium.

2.5 Reduction of Compound Lotteries in Experiment II

In CVL, subjects made no such distinction when valuing reduced and compound CV lotteries. The median premium for compound risk in values is zero, suggesting that compound risk in values may not necessarily have been perceived as such. In CPL, they chose a small average compound risk premium of ₵ 2 for CP lotteries, pricing the reduced CP lotteries slightly higher than their compound analog. There are some order effects in the comparison of reduced and compound lotteries. Whether subjects first saw reduced or compound lotteries matters but only in the CV treatments. On average, subjects chose similar WTP with and without compound risk when they valued the compound lottery before its reduced form version (median compound risk premium of 0 in CV lotteries). Seeing the reduced lottery first, on the other hand, *increased* (rather than decreased) their WTP for the compound version of CV lotteries by ₵ 3.5. In other words, the median premium for compound risk defined over values is even negative, implying that the average subject was more averse to the reduced than to the compound version of the CV lottery.

2.6 Experiment III

Figures A 7a and A 7b show the robustness of our Experiment I results to our Experiment III design where subjects bid against previous Experiment I subjects. Like in Experiment I, in Experiment III subjects in CV bid significantly more than their peers in CP. While in both CV and CP auctions, subjects in Experiment III bid slightly higher than their peers in Experiment I, the difference in bids between CV and CP remains of similar magnitude: Subjects in CV bid, on average, ₵ 19.2 more than their peers

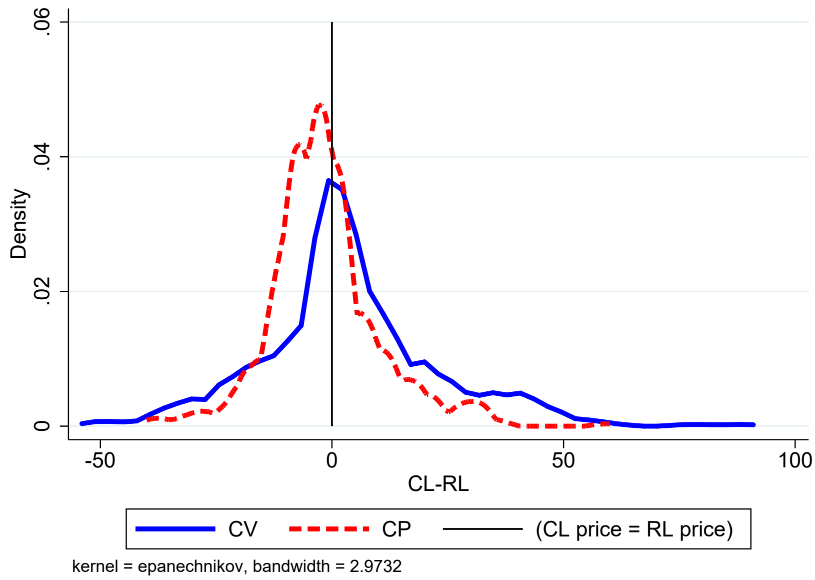
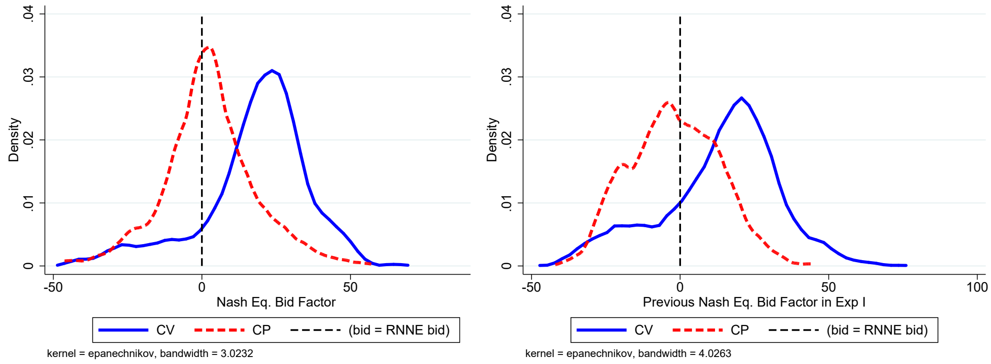


Figure A 6: Differences in Pricing of Compound and Reduced Lotteries in Treatments CVL (solid) and CPL (dashed)

in CP (compared to ₵ 17.60 for the same auctions in Experiment I). Hence, our results are robust to our design modification in Experiment III.



(a) Exp. III **(b)** Exp. I with identical signals

Figure A 7: Comparing bid factors in Exp. III & I

3 Individual Covariates

3.1 Attitudes toward risk, compound risk and ambiguity

In the last part of Experiments I & II, we elicited subjects attitudes toward risk, compound risk and ambiguity. Subjects started this part by first selecting the payoff relevant task. To this end, they threw a dice, knowing that the number on top of the dice would define the selected task. The correspondence between the dice numbers and the tasks were, however, revealed only at the end of the experiment (**?**). The exchange rate remained the same ($1 for 6 credits), but payoffs from the main part of the experiment were weighted more heavily than those in this last part (3:1).

This part consisted of only six decision problems. The six decision screens corresponded to three types of decision problems with two replicate measurements each.

3.2 Elicitation

We elicited risk attitudes with a multiple price list akin to Abdellaoui et al. (2011) and **?**. Subjects faced virtual bags with red and blue chips. First, subjects chose the color to bet on and then gave their certainty equivalent (henceforth CE) for their chosen bet. Risky bets were implemented with the following lottery (100:0.5;0) and (150:0.5;0) (i.e., a 50% chance of winning ₵100 / ₵150 or otherwise nothing).

To implement bets with compound risk, subjects were told that the computer would first randomly select one virtual bag from a set of virtual bags containing each a different mixture of red and blue balls (Figure A 8 shows an example of the screen for a bag with 20 chips), and would then randomly draw a chip from the selected bag. Subjects received ₵100 (₵150 in the replicate measurement) if the color of the drawn chip matched the color they bet on.

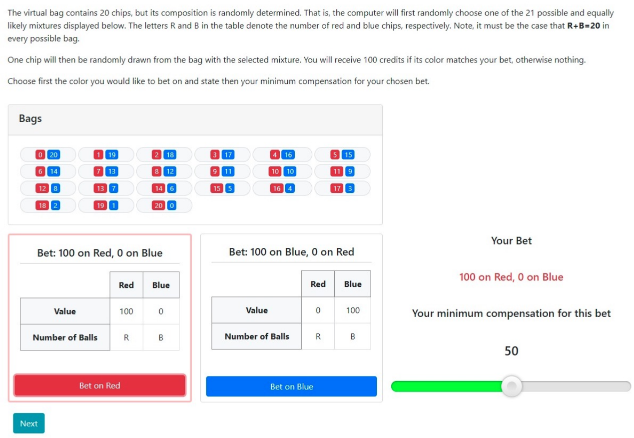


Figure A 8: Example for a decision screen to elicit attitudes toward compound risk (after selecting to bet on red and a certainty equivalent of 50 credits.)

The implementation of ambiguous bets was similar, except that the mixture of red and blue chips was determined ex ante by a research affiliate and was not known to subjects.

3.3 Descriptive statistics

Methods. We classify attitudes as averse toward a type of uncertainty if subjects' prices display a premium for the lottery. The premium is given by the difference between the lottery's expected value and the subject's CE. A positive (negative) premium reflects aversion (proclivity).

We mitigate possible measurement errors by taking the mean of the two replicate measurements: To this end, we first normalize the CE by the lottery's expected value and average the normalized CE across the two replicate measurements.[2,3] Note that all decisions under uncertainty should be affected by a risk premium, if a subject is not risk-neutral. In a crude attempt to control for risk attitudes in decisions with compound risk and ambiguity, we subtract the subject's average risk premium from the chosen premium for lotteries with compound risk and ambiguity (cf. **?**). This yields a conservative measure of the premia for compound risk and ambiguity since risk premia for binary lotteries should be highest when the success probability equals 50% (as in the risky lotteries). Thus, subjects who were less averse toward compound risk/ ambiguity than toward risk exhibit a negative premia for compound risk and ambiguity (applies to 59 (60) out of 195 subjects for the compound risk (ambiguity) premium).

Results. Figure A 9 shows the distribution of risk, compound risk and ambiguity premia, averaged across the two duplicate measures. In general, most subjects were averse toward uncertainty.

Distributions of premia are not significantly different from each other across treatments (the Kolmogorov-Smirnov statistics yields p-values of $p = 0.21, p = 0.45, p = 0.89$ for risk, compound risk and ambiguity premia, respectively). Most subjects chose a premium close to zero, and attitudes toward compound risk and ambiguity are positively correlated (consistent with **?**'s finding). The pairwise correlation coefficients are $\rho_{RC} = -0.24, \rho_{RA} = -0.10, \rho_{CA} = 0.54$.

3.4 Individual Characteristics

In general, individual characteristics do not significantly differ between the CV and CP treatments. The measures of the cognitive reflection tests (CRT) are higher in the treatments III-IV and have to be interpreted with caution

[2]For the ambiguous bets, we assume uniform beliefs over possible probabilities to compute the lotteries' expected value.

[3]Most subjects were also consistent in their attitudes, especially in their attitudes toward ambiguity. The redundant measures yield the same classification for 71.15%, 75.96% and 79.81% of the subjects regarding attitudes toward risk, compound risk and ambiguity, respectively (in the full sample).

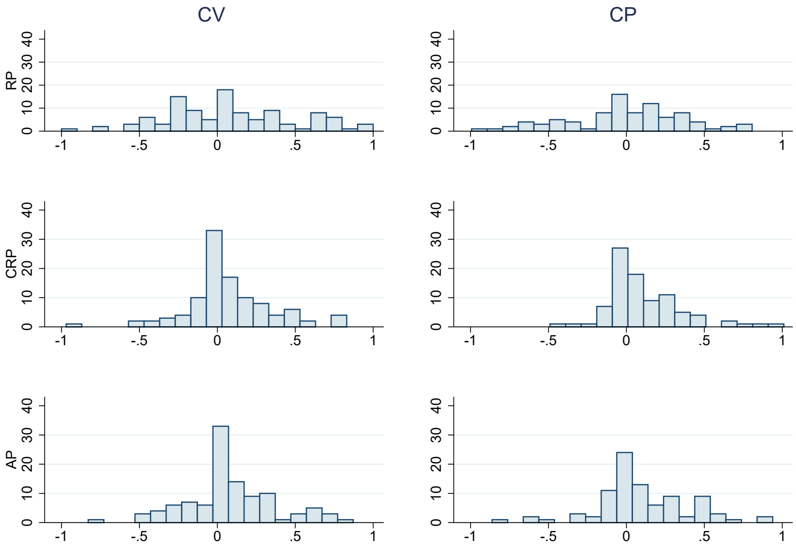


Figure A 9: Distribution of Premia in Exp. I & II – by Treatments CV (left) and CP (right).

because the experiment was conducted online.

Table A7: Means of Individual Characteristics by Treatment in Reduced Sample

		Male	Age	CRT	RP	CRP	AP
	CV	0.538***	21.962***	1.519***	0.006	0.095**	0.120***
		(0.070)	(0.355)	(0.149)	(0.053)	(0.041)	(0.036)
I	CP	0.615***	22.333***	1.308***	-0.076	0.153***	0.115**
		(0.079)	(0.460)	(0.160)	(0.065)	(0.033)	(0.050)
	Diff	-0.077	-0.372	0.212	0.082	-0.058	0.005
		(0.105)	(0.581)	(0.219)	(0.084)	(0.052)	(0.062)
	CV	0.434***	21.415***	1.389***	0.151**	0.057	0.043
		(0.069)	(0.386)	(0.164)	(0.059)	(0.035)	(0.043)
II	CP	0.480***	21.440***	1.5***	0.050	0.094**	0.090**
		(0.071)	(0.365)	(0.154)	(0.048)	(0.037)	(0.042)
	Diff	-0.046	-0.025	-0.111	0.101	-0.037	0.047
		(0.099)	(0.531)	(0.225)	(0.076)	(0.051)	(0.060)
	CV	0.450***	23.050***	2.300***	-0.028		
		(0.114)	(0.671)	(0.242)	(0.159)		
III	CP	0.389***	23.278***	2.500***	0.069		
		(0.118)	(0.645)	(0.217)	(0.124)		
	Diff	0.061	-0.228	-0.200	-0.097		
		(0.164)	(0.931)	(0.325)	(0.202)		
	CV	0.522***	21.913***	2.391***	0.068		
		(0.106)	(0.569)	(0.137)	(0.103)		
IIIb	CP	0.538***	21.346***	2.269***	0.223***		
		(0.100)	(0.474)	(0.172)	(0.724)		
	Diff	-0.017	0.567	-0.122	-0.155		
		(0.146)	(0.740)	(0.219)	(0.125)		
	CV	0.448***	23.279***	2.655***	-0.023		
		(0.094)	(0.660)	(0.114)	(0.177)		
IV	CP	0.333***	22.286***	2.048***	0.141		
		(0.105)	(0.492)	(0.243)	(0.169)		
	Diff	0.115	0.990	0.608**	-0.164		
		(0.141)	(0.825)	(0.268)	(0.219)		
	CV	0.480***	22.130***	1.865***	0.049	0.076***	0.081***
		(0.038)	(0.221)	(0.0084)	(0.043)	(0.027)	(0.028)
I-IV	CP	0.494***	21.981***	1.773***	0.062*	0.120***	0.101***
		(0.040)	(0.214)	(0.089)	(0.035)	(0.025)	(0.032)
	Diff	-0.013	0.150	0.092	-0.013	-0.040	-0.020
		(0.055)	(0.307)	(0.122)	(0.055)	(0.037)	(0.043)

Note: *: p-value<.1,**: p-value<.05, ***: p-value<.01. Robust standard errors clustered by subject in parentheses.