Ownership Concentration and Strategic Supply Reduction†

Appendix

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A Reservation values

In this appendix, we provide details on how we infer the reservation value of a TV station going into the reverse auction.

A.1 Cash flows

Specification. We parameterize $\alpha(X_{jt};\beta)$, $RT(X_{jt};\gamma)$, and $F(X_{jt};\delta)$ in the cash flow model in equation (8) as functions of station and market characteristics X_{jt} as

$$\begin{aligned}
\alpha(X_{jt};\beta) =& \sum_{a=1}^{9} \beta_0^a 1(\mathit{Affiliation}_{jt} = a) + \beta_1 \mathit{Fox}_{jt}(t-2002) + \beta_2 JSA/LMA_{jt} \\
&+ \sum_{s=2003}^{2012} \beta_3^s 1(t=s) + \beta_4 \mathit{CompIndex}_{jt} + \beta_5 \mathit{WealthIndex}_{jt}, \\
RT(X_{jt};\gamma) =& \exp\left(\sum_{h=1}^{3} \gamma_0^h 1(\mathit{Group}_{jt}^{RT} = h) + \gamma_1 \ln(\mathit{PopServed}_{jt}) \right. \\
&\left. + \gamma_2 \ln(\mathit{PopServed}_{jt})^2 + \gamma_3(t-2002) \right), \\
F(X_{jt};\delta) =& \exp\left(\sum_{h=1}^{3} \delta_0^h 1(\mathit{Group}_{jt}^{F} = h) + \delta_1 \ln(\mathit{PopServed}_{jt}) + \delta_2 \ln(\mathit{PopServed}_{jt})^2 \right),
\end{aligned}$$

or where $1(\cdot)$ is the indicator function and we use the shorthand

$$\begin{aligned}
\mathit{PopServed}_{jt} =& 1(\mathit{PowerOutput}_{jt} = \mathit{FullPower}) \cdot \mathit{DMAPop}_{jt} \\
&+ 1(\mathit{PowerOutput}_{jt} = \mathit{LowPowerClassA}) \cdot \mathit{InterferenceFreePop}_{jt}.
\end{aligned}$$

In specifying $\alpha(X_{jt};\beta)$, $\mathit{Affiliation}_{jt}$ refers to nine of the eleven affiliations in Table A20.[1] We normalize the parameter on the indicator for Spanish-language networks to zero. We include a full set of year fixed effects and a separate time trend for Fox affiliates as their profitability grew substantially over time. We include an indicator for the TV station being part of a joint sales or local marketing agreement.[2] We account for differences in the competitive environment and demographics across DMAs using the competitiveness and wealth indices $\mathit{CompIndex}_{jt}$ and $\mathit{WealthIndex}_{jt}$.[3] In specifying $RT(X_{jt};\gamma)$ and $F(X_{jt};\delta)$, we flexibly include the DMA population

[1] We exclude any TV station affiliated with other minor networks from the estimation in line with footnote 11. To predict the cash flow for such a TV station, we use its station and owner characteristics X_{jt} and the estimated parameter on the indicator for Independent.

[2] Under a local marketing agreement (LMA), a company operates the TV station owned by another company. Under a joint sales agreement (JSA), only certain functions are contracted, in particular advertising sales.

[3] To parsimoniously capture market characteristics, we conduct a principal component analysis of the log of the market-level variables prime-age (18-54) population, average per capita disposable personal income, retail expenditures, total market advertising revenues, number of primary TV stations, and number of major network affiliates. We define the time-varying number of primary TV stations and major network affiliates based on auction-eligible TV stations contained in the BIA data from 2003 to 2013 and for 2015 but rely on the BIA data for 2015 for the remaining market-level characteristics. The first principal component, denoted as $\mathit{CompIndex}_{jt}$, loads primarily on to

and interference free population for full-power stations and low-power class-A stations, respectively. Moreover, in specifying $RT(X_{jt};\gamma)$, we use $Group^{RT}_{jt}$ to group affiliations as (1) ABC, CBS, NBC, Fox, and Warner Bros; (2) CW, My Network TV, United Paramount, and Independents; (3) Spanish-language networks. We include a time trend because retransmission fees grew rapidly. In specifying $F(X_{jt};\delta)$, we use $Group^{F}_{jt}$ to group affiliations as (1) ABC, CBS, and NBC; (2) Fox, CW, and Warner Bros; (3) My Network TV, United Paramount, Spanish-language networks, and Independents.

Data and estimation. We combine the station-level data on advertising revenue, station characteristics, and market characteristics from BIA with the aggregated data from NAB. The NAB data yields 3,976 moments across aggregation categories and the ten years from 2003 to 2012. We drop the years 2013 and 2015 from the BIA data as 2012 is the latest year of availability for the NAB data. There are a total of 11,731 station-year observations from the BIA data that meet NAB's data collection and reporting procedure and therefore map into a table of a NAB report.

We use a simulated minimum distance estimator. We draw $N^s = 100$ vectors of cash flow error terms $\epsilon^s = \left(\epsilon^s_{jt}\right)$, where ϵ^s_{jt} is the cash flow error term of TV station j in year t in draw s. Denote by $\overline{CF}_{gt}$, CF^1_{gt}, CF^2_{gt}, and CF^3_{gt} the mean, first, second, and third quartiles of the cash flow distribution reported by NAB in year t for aggregation category $g = 1, \ldots, G_t$, where G_t is the number of aggregation categories in year t. Similarly, denote by $\widehat{\overline{CF}}_{gt}(\theta;\epsilon^s)$, $\widehat{CF}^1_{gt}(\theta;\epsilon^s)$, $\widehat{CF}^2_{gt}(\theta;\epsilon^s)$, and $\widehat{CF}^3_{gt}(\theta;\epsilon^s)$ the analogous moments of the predicted cash flow distribution for the TV stations that feature in aggregation category g in year t. Our notation emphasizes that the latter depend on the parameters $\theta = (\beta, \gamma, \delta, \sigma)$ and the vector of cash flow error terms ϵ^s in draw s. We use similar notation for the mean of the non-broadcast revenue and fixed cost distributions, replacing $\overline{CF}$ with $\overline{RT}$ and $\overline{F}$, respectively. To estimate θ, we match the moments of the predicted and actual distributions across aggregation categories and years and solve

$$\hat{\theta} = \arg\min_{\theta} \sum_{t=2003}^{2012} \sum_{g=1}^{G_t} \left(\overline{CF}_{gt} - \frac{1}{N^s}\sum_{s=1}^{N^s} \widehat{\overline{CF}}_{gt}(\theta;\epsilon^s)\right)^2 + \sum_{q=1}^{3}\left(CF^q_{gt} - \frac{1}{N^s}\sum_{s=1}^{N^s}\widehat{CF}^q_{gt}(\theta;\epsilon^s)\right)^2 + \left(\overline{RT}_{gt} - \widehat{\overline{RT}}_{gt}(\theta)\right)^2 + \left(\overline{F}_{gt} - \widehat{\overline{F}}_{gt}(\theta)\right)^2.$$

Our interior-point minimization algorithm terminates with a search step less than the specified tolerance of 10^{-12}. We use multiple starting values to guard against local minima.

Results. Table A1 reports the parameter estimates $\hat{\theta}$. We provide further details on predicted values and goodness of fit in Online Appendix A.3.

prime-age population, advertising revenues, number of primary TV stations, and number of major network affiliates. The second principal component, denoted as $WealthIndex_{jt}$, loads primarily on to average disposable income and retail expenditures.

Table A1: Cash flow parameters estimates

	Estimate
Retained share $\alpha(X_{jt};\beta)$	
ABC	-0.0417
CBS	-0.0521
NBC	-0.0500
Fox	-0.3545
CW	-0.0680
Warner Bros	-0.0255
MyNetwork TV	-0.2648
United Paramount	-0.3252
Spanish-language networks (normalized)	0
Independent	-0.0879
Fox × Trend	0.0113
JSA/LMA	0.2892
2003	0.5563
2004	0.5355
2005	0.5074
2006	0.4948
2007	0.4611
2008	0.4302
2009	0.3735
2010	0.4501
2011	0.4635
2012	0.4881
$CompIndex_{jt}$	0.0127
$WealthIndex_{jt}$	0.0028
Non-broadcast revenue $RT(X_{jt};\gamma)$	
Group 1	9.5292
Group 2	8.6304
Group 3	8.4692
$\ln(PopServed_{jt})$	0.4500
$\ln(PopServed_{jt})^2$	0.0116
Trend	0.1620
Fixed cost $F(X_{jt};\delta)$	
Group 1	1.4670
Group 2	0.6279
Group 3	0.2943
$\ln(PopServed_{jt})$	2.9244
$\ln(PopServed_{jt})^2$	-0.1413
Standard deviation σ	0.6896

A.2 Multiples

Data. BIA records 659 transactions in the eleven years from 2003 to 2013 with transaction prices, as opposed to station swaps, stock transfers, donations, etc. We exclude transactions for public stations, religious stations, and those with non-commercial owners.

In identifying transactions based on cash flow, we further exclude transactions for dark stations and for TV stations with negative predicted cash flows and transactions with a purchase price below \$1 million. In case of multi-station deals, we exclude transactions for TV stations with widely varying cash flows to facilitate allocating the purchase price in proportion to the population covered by the included TV stations. Lastly, we exclude four transactions with a cash flow multiple in excess of 250. This leaves us with a sample of 230 transactions between 2003 and 2012 based on cash flow.

In identifying transactions based on stick value, we include transactions for dark stations, for TV stations with negative predicted cash flows, and for TV stations that are not affiliated with a major network and have a purchase price of less than \$1 million. This leaves us with a sample of 168 transactions between 2003 and 2013 based on stick value.

For cash flow transactions, we infer the cash flow multiple from the transaction price and the cash flow $\widehat{CF}_{jt}$ predicted using equation (5), setting $\epsilon_{jt} = 0$. For stick value transactions, we infer the stick multiple from the transaction price, the population served, and the power output of the TV station using equations (6) and (7).

Specification and estimation. For cash flow transactions, we regress the log of the multiple on station, owner, and market characteristics using the specification:

$$\ln Multiple_{jt}^{CF} = \beta^{CF} X_{jt} + \epsilon_{jt}^{CF}. \tag{A1}$$

In X_{jt} we flexibly include the DMA population and interference free population for full-power stations and low-power class-A stations, respectively, interacted with network affiliation, where we group affiliations into major and minor networks according to Table A20 in Online Appendix C.1. We further include the wealth and competitiveness indices, the number of TV stations in the DMA, ownership category fixed effects (whether the owner owns between two and ten, or more than ten TV stations across DMAs), a low-power class-A fixed effect, a minor network fixed effect, a fixed effect for independent stations, and a full set of year fixed effects.

For stick value transactions, we use the specification:

$$\ln Multiple_{jt}^{Stick} = \beta^{Stick} X_{jt} + \epsilon_{jt}^{Stick}. \tag{A2}$$

In X_{jt} we flexibly include the DMA population and interference free population for full-power stations and low-power class-A stations, respectively. We further include the wealth and competitiveness indices, the number of TV stations in the DMA, ownership category fixed effects, the output power of the TV station and its interaction with an indicator for the period prior to the TV station's transition to digital transmission, a LPTV fixed effect, a full-power fixed effect, a fixed effect for satellite stations, and a full set of year fixed effects.

Table A2: Cash flow and stick value multiples parameter estimates

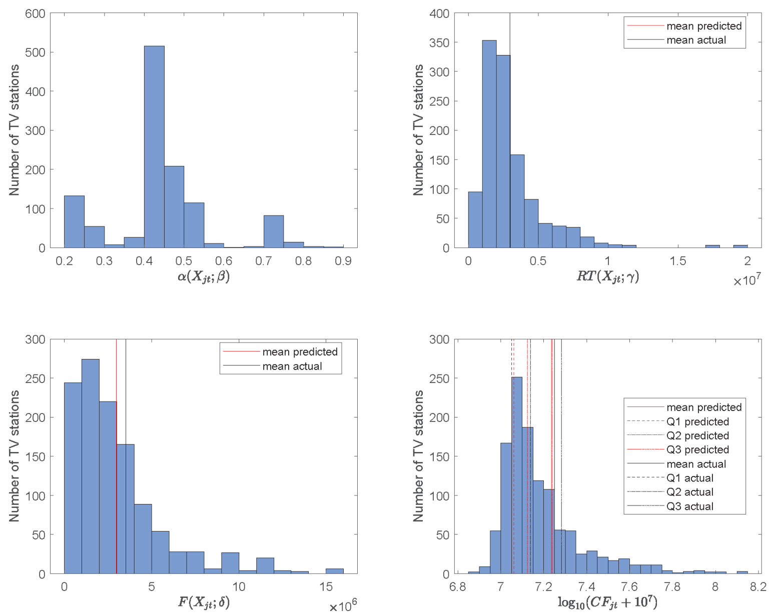
	Cash flow multiple		Stick multiple	
	Estimate	Std. Err.	Estimate	Std. Err.
$\ln(PopServed_{jt})$	0.3176**	(0.1350)	-0.6585***	(0.1982)
× Minor network	1.8581***	(0.5747)		
× Major network	0.3292	(0.3955)		
$\ln(PopServed_{jt})^2$	0.0106	(0.0152)	0.0241	(0.0198)
× Minor network	-0.1674***	(0.0438)		
× Major network	-0.0167	(0.0353)		
$WealthIndex_{jt}$	-0.0611	(0.0470)	0.0717	(0.0721)
$CompIndex_{jt}$	0.0518	(0.0896)	0.1588	(0.1928)
# Stations in DMA	0.0006	(0.0073)	-0.0076	(0.0162)
Owns 2-10 stations across DMAs	0.0021	(0.1527)	0.0617	(0.2736)
Owns >10 stations across DMAs	-0.2263	(0.1587)	0.0317	(0.3034)
$\ln(OutputPower_{jt})$			0.2452***	(0.0769)
$\ln(OutputPower_{jt})$ × Predigital			-0.1060	(0.0688)
Low-power class-A	-0.3335**	(0.1561)		
LPTV			-1.3881***	(0.2725)
Full-power			0.9531**	(0.3923)
Satellite			1.4541	(0.8805)
Independent	-4.3615**	(1.8785)		
Minor network	-1.4903	(1.1023)		
2004	-0.3205	(0.2877)	0.7308	(0.6316)
2005	0.2548	(0.2569)	1.1848**	(0.5373)
2006	-0.0359	(0.2815)	0.9274*	(0.5225)
2007	-0.1179	(0.2569)	1.3040**	(0.6037)
2008	-0.4977*	(0.2960)	0.0368	(0.5861)
2009	-0.435	(0.4586)	0.2331	(0.4798)
2010	-0.3297	(0.3282)	-1.1143**	(0.5508)
2011	-0.8047***	(0.2720)	-0.2103	(0.5562)
2012	-1.1719***	(0.2445)	0.1228	(0.5372)
2013	-0.8447***	(0.2306)	-0.7057	(0.4918)
Adjusted R^2	0.8192		0.8182	
N	402		253	

Results. Table A2 reports parameter estimates $\hat{\beta}^{CF}$and $\hat{\beta}^{Stick}$. The adjusted R^2 is 0.82 for the specifications in equations (A1) and (A2), suggesting that they fit the data well. We set $\epsilon_{jt}^{CF} = \epsilon_{jt}^{Stick} = 0$ to predict. We provide further details in Online Appendix A.3.

A.3 Goodness of fit

Here, we provide further details on predicted values and goodness of fit.

Figure A1: Predicted retained share $\alpha(X_{jt};\beta)$, non-broadcast revenue $RT(X_{jt};\delta)$, fixed cost $F(X_{jt};\delta)$, and cash flow CF_{jt} with moments in 2012



Notes: In the lower right panel, cash flow is reported as $\log_{10}\left(CF_{jt}+10^7\right)$ for visual clarity.

Cash flows. The parameter estimates $\hat{\theta}$ in Table A1 in Appendix A.1 indicate that Warner Bros and Spanish language networks affiliates retain the highest share of advertising revenues. Except for Fox affiliates, major network affiliates retain a higher share of advertising revenue than minor networks; however, the retained share of Fox affiliates rises over time. TV stations that are part of a joint sales or local marketing agreement retain a higher share of advertising revenue. The retained share falls over time, bottoming out in 2009 before bouncing back in recent years.

Figure A1 plots the distributions of the predicted retained share $\alpha(X_{jt};\beta)$ (upper left panel), non-broadcast revenue $RT(X_{jt};\gamma)$ (upper right panel), and fixed cost $F(X_{jt};\delta)$ (lower left panel) for the 1,172 commercial full-power stations surveyed by NAB in 2012. It also plots the distribution of predicted cash flow for a sample draw of the vector of cash flow error terms ϵ^s (lower right panel). We predict the retained share to be between 0.21 and 0.86 across TV stations, with an average of 0.44. We predict non-broadcast revenue to be between \$0.21 million and \$19.39 million, averaging \$2.98 million, and we predict fixed cost to be between \$0.00 million and \$15.78 million, averaging \$2.97 million. Finally, we predict cash flow to be between \$-2.58 million and \$129.77 million across TV stations, with an average of \$7.21 million.

The cash flow model fits the data well. In Figure A1, we overlay predicted moments as red lines and actual moments as reported in the NAB data (table "All Stations, All Markets") as black lines. NAB reports an average non-broadcast revenue of \$2.98 million in line with our prediction of \$2.98 million (upper right panel). We somewhat underestimate fixed cost, where NAB reports an average of \$3.53 million compared to our prediction of \$2.97 million (lower left panel). Turning to cash flow (lower right panel), NAB reports an average of \$7.80 million and first, second, and third quartiles of \$1.24 million, \$3.75 million, and \$9.18 million. This compares to our predictions of \$7.21 million, \$1.51 million, \$3.29 million, and \$7.38 million, respectively.

To further assess the fit of the cash flow model, Table A3 compares the cash flow, non-broadcast revenue, and fixed cost moments reported in the NAB data to the corresponding predicted moments, broken down by type of moment, affiliation, year, and market rank. It provides three different measures of fit: the correlation between predicted and data moments, the mean absolute deviation in levels in \$ million and as a percent of the data moments, and the mean deviation in levels and as a percent. Overall, our cash flow model predicts the 3,976 moments with a 0.99 correlation. The correlation between data and predicted moment ranges from 0.83 to 0.99 for the different types of moments. It is higher for the 2,394 moments pertaining to major network affiliates than for the 532 moments pertaining to minor network affiliates and independent stations. There are no systematic differences in the correlation between data and predicted moments across years. The correlation is higher for moments pertaining to larger markets. The remaining two measures of fit largely agree with the correlation.

Multiples. With the estimates for equations (A1) and (A2) in hand, we set $\epsilon_{jt}^{CF} = \epsilon_{jt}^{Stick} = 0$ and predict the cash flow and stick multiples for the 1,670 auction-eligible UHF stations that are located outside Puerto Rico and the Virgin Islands. Figure A2 illustrates the distributions of the predicted cash flow multiple (left panel) and stick multiple (right panel).

Figure A2: Distributions of predicted cash flow and stick multiples

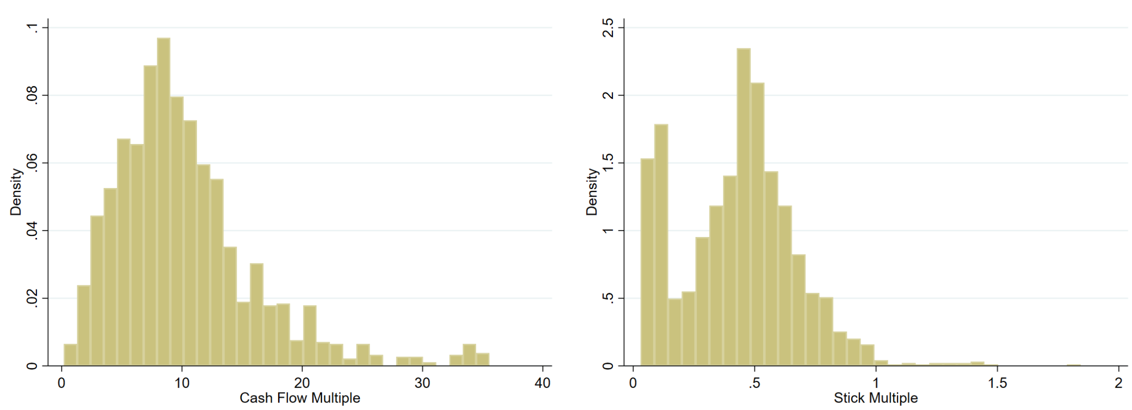


Table A3: Cash flow, non-broadcast revenue, and fixed cost moments and fit measures

	Number of moments	Correlation	Mean abs. deviation		Mean deviation	
			$ million	%	$ million	%
All moments	3976	0.984	0.746	0.157	-0.011	-0.002
Moments by type						
Cash flow, mean	663	0.989	0.815	0.121	-0.131	-0.019
Cash flow, first quartile	662	0.969	0.744	0.290	0.054	0.021
Cash flow, second quartile	663	0.980	0.881	0.174	0.036	0.007
Cash flow, third quartile	663	0.985	1.195	0.133	0.056	0.006
Non-broadcast revenue, mean	662	0.939	0.302	0.178	0.046	0.027
Fixed cost, mean	663	0.964	0.540	0.153	-0.125	-0.036
Moments by affiliation						
Major network	2394	0.986	0.833	0.142	0.037	0.006
Minor network	420	0.942	0.763	0.302	0.034	0.013
Independent	132	0.826	0.659	0.382	0.043	0.027
Moments by year						
2003	395	0.984	0.845	0.170	0.082	0.017
2004	390	0.989	0.713	0.133	-0.041	-0.008
2005	396	0.985	0.736	0.157	0.109	0.023
2006	372	0.990	0.681	0.124	-0.137	-0.025
2007	413	0.987	0.721	0.163	0.059	0.013
2008	420	0.980	0.735	0.178	-0.085	-0.021
2009	396	0.975	0.588	0.200	0.009	0.003
2010	396	0.982	0.746	0.153	-0.079	-0.016
2011	402	0.973	0.827	0.179	0.127	0.028
2012	396	0.985	0.867	0.139	-0.161	-0.026
Moments by market rank						
1-25	552	0.982	1.935	0.132	0.142	0.010
26-50	462	0.956	0.829	0.147	-0.115	-0.021
50-100	1116	0.937	0.518	0.167	-0.134	-0.043
101+	959	0.872	0.422	0.280	0.083	0.055

B Additional analysis of the model

In this appendix, we provide additional analysis of the model in Section 2.

B.1 Set of equilibria

Example in Section 2.1 with joint ownership. We derive the set of equilibria for the example in Section 2.1. The profit of firm 1 owning TV stations 1 and 3 is

$$\pi_1(b_1,b_2,b_3) = \begin{cases} 0 & \text{if} \quad \min\{b_1,b_2\} \geq 900 \\ & \quad \vee \min\{b_1,b_3\} \geq 900 \\ & \quad \vee \min\{b_2,b_3\} \geq 900, \\ \min\{b_1,900\}-300 & \text{if} \quad b_1 > \max\{b_2,b_3\}, \\ 2\min\{b_2,900\}-400 & \text{if} \quad b_2 > \max\{b_1,b_3\}, \\ \min\{b_3,900\}-100 & \text{if} \quad b_3 > \max\{b_1,b_2\}, \\ \frac{1}{2}(2b_2-400)+\frac{1}{2}(b_2-300) & \text{if} \quad b_1=b_2>b_3, \\ \frac{1}{2}(b_1-100)+\frac{1}{2}(b_1-300) & \text{if} \quad b_1=b_3>b_2, \\ \frac{1}{2}(2b_2-400)+\frac{1}{2}(b_2-100) & \text{if} \quad b_2=b_3>b_1, \\ \frac{1}{3}(2b_2-400)+\frac{1}{3}(b_2-100)+\frac{1}{3}(b_2-300) & \text{if} \quad b_1=b_2=b_3>0, \\ -400 & \text{if} \quad b_1=b_2=b_3=0 \end{cases} \tag{A3}$$

and the profit of firm 2 owning TV station 2 is

$$\pi_2(b_1,b_2,b_3) = \begin{cases} 0 & \text{if} \quad \min\{b_1,b_2\} \geq 900 \vee \min\{b_1,b_3\} \geq 900 \\ & \quad \vee \min\{b_2,b_3\} \geq 900, \\ 0 & \text{if} \quad b_2 > \max\{b_1,b_3\}, \\ \min\{\max\{b_1,b_3\},900\}-500 & \text{if} \quad b_2 < \max\{b_1,b_3\}, \\ \frac{1}{2}(\max\{b_1,b_3\}-500) & \text{if} \quad b_2=\max\{b_1,b_3\}>\min\{b_1,b_3\}, \\ \frac{2}{3}(b_1-500) & \text{if} \quad b_1=b_2=b_3>0, \\ -500 & \text{if} \quad b_1=b_2=b_3=0, \end{cases}$$

where we again assume that the relevant case is given by the first applicable if statement.

In Tables A4-A6, we again divide the strategy spaces of firms 1 and 2 as needed to either show that there is no profitable deviation for any firm or give an example of a profitable deviation. Combining the cells marked with ✓, the set of equilibria is as stated in equation (3).

Table A4: $b_2 \in [0, 600)$

$b_1 \setminus b_3$	$[0, b_2)$	b_2	$(b_2, 900)$	$[900, \infty)$
$[0, b_2)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
b_2	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
$(b_2, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
$[900, \infty)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$

Table A5: $b_2 = 600$

$b_1 \setminus b_3$	$[0, 500]$	$(500, 600)$	600	$(600, 900)$	$[900, \infty)$
$[0, 500]$	✓	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 900)$	✓
$(500, 600)$	$b_2 = 0$	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 900)$	✓
600	$b_2 = 0$	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 900)$	✓
$(600, 900)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 900)$	✓
$[900, \infty)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$

Table A6: $b_2 \in (600, \infty)$

$b_1 \setminus b_3$	$[0, 500]$	$(500, b_2)$	b_2	$(b_2, 900)$	$[900, \infty)$
$[0, 500]$	✓	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$
$(500, b_2)$	$b_2 = 0$	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$
b_2	$b_2 = 0$	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$
$(b_2, 900)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$
$[900, \infty)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$

Example in Section 2.1 imposing independently owned TV stations. We derive the set of equilibria for the example in Section 2.1 whilst imposing that all TV stations are independently owned. Assuming a random tie-breaking rule for bids above 0 and below 900 in line with footnote 26, the profit of TV station 1 is

$$\pi_1(b_1,b_2,b_3)=\begin{cases} 0 & \text{if} \quad \min\{b_1,b_2\}\geq 900 \vee \min\{b_1,b_3\}\geq 900 \\ & \qquad \vee \min\{b_2,b_3\}\geq 900, \\ 0 & \text{if} \quad b_1>\max\{b_2,b_3\}, \\ \min\{\max\{b_2,b_3\},900\}-100 & \text{if} \quad b_1<\max\{b_2,b_3\}, \\ \frac{1}{2}(\max\{b_2,b_3\}-100) & \text{if} \quad b_1=\max\{b_2,b_3\}>\min\{b_2,b_3\}, \\ \frac{2}{3}(b_2-100) & \text{if} \quad b_1=b_2=b_3>0, \\ -100 & \text{if} \quad b_1=b_2=b_3=0, \end{cases}$$

where we assume that the relevant case is given by the first applicable if statement. In particular, the first if statement covers the case where the reverse auction fails at the outset because at least two TV stations bid 900 or more. Consequently, in the subsequent if statements at most one TV station bids 900 or more. In the second if statement, TV station 1 is first to opt to remain on the air. In the third if statement, TV station 1 is frozen as either TV station 2 or 3 is first to opt to remain on the air. The remaining if statements cover ties. The profits of the remaining TV stations are analogous.

In Tables A7-A13, we divide the strategy space of TV station 2 into eight regions, namely $[0,100)$, 100, $(100,300)$, 300, $(300,500)$, 500, $(500,900)$, and $[900,\infty)$. We further divide the strategy spaces of TV stations 1 and 3 as needed to either show that there is no profitable deviation for any TV station (indicated by ✓ in the respective cell) or give an example of a profitable deviation.[4] Combining the cells marked with ✓, the set of equilibria is

$$\begin{aligned} &\left\{(b_1,b_2,b_3)\in[0,\infty)^3 | b_1\geq 500, b_2\leq 100, b_3\leq 100\right\} \\ &\cup\left\{(b_1,b_2,b_3)\in[0,\infty)^3 | b_1\leq 300, b_2\leq 300, b_3\geq 500\right\} \\ &\cup\left\{(b_1,b_2,b_3)\in[0,\infty)^3 | \max\{b_1,b_3\}< b_2, 300\leq b_2\leq 500\right\} \\ &\cup\left\{(b_1,b_2,b_3)\in[0,\infty)^3 | \max\{b_1,b_3\}\leq 500, b_2> 500\right\} \\ &\cup\left\{(b_1,b_2,b_3)\in[0,\infty)^3 | b_1\geq 900, b_2\geq 900, b_3\geq 900\right\}. \end{aligned} \tag{A4}$$

[4] The notation $\max\{b_1,b_3\}=0$ in Table A7 means that the TV station with the higher bid has a profitable deviation to zero, and similarly for the remaining tables.

Table A7: $b_2 \in [0, 100]$

$b_1 \setminus b_3$	$[0, b_2)$	$[b_2, 100]$	$(100, 500)$	$[500, \infty)$
$[0, b_2)$	$b_3 = 900$	$b_2 = 900$	$b_2 = 900$	✓
$[b_2, 300]$	$b_2 = 900$	$b_2 = 900$	$b_2 = 900$	✓
$(300, 500)$	$b_2 = 900$	$b_2 = 900$	$b_2 = 900$	$b_3 = 0$
$[500, \infty)$	✓	✓	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A8: $b_2 \in (100, 300)$

$b_1 \setminus b_3$	$[0, b_2)$	$[b_2, 500)$	$[500, \infty)$
$[0, b_2)$	$b_3 = 900$	$b_2 = 900$	✓
$[b_2, 300]$	$b_2 = 900$	$b_2 = 900$	✓
$(300, 500)$	$b_2 = 900$	$b_2 = 900$	$b_3 = 0$
$[500, \infty)$	$b_1 = 0$	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A9: $b_2 = 300$

$b_1 \setminus b_3$	$[0, 300)$	$[300, 500)$	$[500, \infty)$
$[0, 300)$	✓	$b_2 = 900$	✓
300	$b_2 = 900$	$b_2 = 900$	✓
$(300, 500)$	$b_2 = 900$	$b_2 = 900$	$b_3 = 0$
$[500, \infty)$	$b_1 = 0$	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A10: $b_2 \in (300, 500)$

$b_1 \setminus b_3$	$[0, b_2)$	$[b_2, 500)$	$[500, \infty)$
$[0, b_2)$	✓	$b_2 = 900$	$b_3 = 0$
$[b_2, 500)$	$b_2 = 900$	$b_2 = 900$	$b_3 = 0$
$[500, \infty)$	$b_1 = 0$	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A11: $b_2 = 500$

$b_1 \setminus b_3$	$[0, 500)$	$[500, \infty)$
$[0, 500)$	✓	$b_3 = 0$
$[500, \infty)$	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A12: $b_2 \in (500, 900)$

$b_1 \setminus b_3$	$[0, 500]$	$(500, b_2]$	(b_2, ∞)
$[0, 500]$	✓	$b_2 = 0$	$b_3 = 0$
$(500, b_2]$	$b_2 = 0$	$b_2 = 0$	$b_3 = 0$
(b_2, ∞)	$b_1 = 0$	$b_1 = 0$	$\max\{b_1, b_3\} = 0$

Table A13: $b_2 \in [900, \infty)$

$b_1 \setminus b_3$	$[0, 500]$	$(500, 900)$	$[900, \infty)$
$[0, 500]$	✓	$b_2 = 0$	$b_2 = 0$
$(500, 900)$	$b_2 = 0$	$b_2 = 0$	$b_2 = 0$
$[900, \infty)$	$b_2 = 0$	$b_2 = 0$	✓

Example in Section 2.1 with different reservation values. We derive the set of equilibria for the example in Section 2.1 whilst replacing the reservation value of TV station 2 by $v_2 = 700$. We came back to this variant of the example in Online Appendix B.3. The profit of firm 1 owning TV stations 1 and 3 is

$$\pi_1(b_1, b_2, b_3) = \begin{cases} 0 & \text{if} \quad \min\{b_1, b_2\} \geq 900 \\ & \quad\ \vee \min\{b_1, b_3\} \geq 900 \\ & \quad\ \vee \min\{b_2, b_3\} \geq 900, \\ \min\{b_1, 900\} - 300 & \text{if} \quad b_1 > \max\{b_2, b_3\}, \\ 2\min\{b_2, 900\} - 400 & \text{if} \quad b_2 > \max\{b_1, b_3\}, \\ \min\{b_3, 900\} - 100 & \text{if} \quad b_3 > \max\{b_1, b_2\}, \\ \frac{1}{2}(2b_2 - 400) + \frac{1}{2}(b_2 - 300) & \text{if} \quad b_1 = b_2 > b_3, \\ \frac{1}{2}(b_1 - 100) + \frac{1}{2}(b_1 - 300) & \text{if} \quad b_1 = b_3 > b_2, \\ \frac{1}{2}(2b_2 - 400) + \frac{1}{2}(b_2 - 100) & \text{if} \quad b_2 = b_3 > b_1, \\ \frac{1}{3}(2b_2 - 400) + \frac{1}{3}(b_2 - 100) + \frac{1}{3}(b_2 - 300) & \text{if} \quad b_1 = b_2 = b_3 > 0, \\ -400 & \text{if} \quad b_1 = b_2 = b_3 = 0 \end{cases} \tag{A5}$$

and the profit of firm 2 owning TV station 2 is

$$\pi_2(b_1, b_2, b_3) = \begin{cases} 0 & \text{if} \quad \min\{b_1, b_2\} \geq 900 \vee \min\{b_1, b_3\} \geq 900 \\ & \quad\ \vee \min\{b_2, b_3\} \geq 900, \\ 0 & \text{if} \quad b_2 > \max\{b_1, b_3\}, \\ \min\{\max\{b_1, b_3\}, 900\} - 700 & \text{if} \quad b_2 < \max\{b_1, b_3\}, \\ \frac{1}{2}(\max\{b_1, b_3\} - 700) & \text{if} \quad b_2 = \max\{b_1, b_3\} > \min\{b_1, b_3\}, \\ \frac{2}{3}(b_1 - 700) & \text{if} \quad b_1 = b_2 = b_3 > 0, \\ -700 & \text{if} \quad b_1 = b_2 = b_3 = 0, \end{cases}$$

where we again assume that the relevant case is given by the first applicable if statement.

In Tables A14-A17, we again divide the strategy spaces of firms 1 and 2 as needed to either show that there is no profitable deviation for any firm or give an example of a profitable deviation. Combining the cells marked with ✓, the set of equilibria is

$$\begin{gathered} \left\{(b_1, b_2, b_3) \in [0, \infty)^3 | b_1 < 900, b_2 \leq 600, b_3 \geq 900\right\} \\ \cup \left\{(b_1, b_2, b_3) \in [0, \infty)^3 | b_1 \leq 700, b_2 > 700, b_3 \leq 700\right\} \\ \cup \left\{(b_1, b_2, b_3) \in [0, \infty)^3 | \max\{b_1, b_3\} < b_2, 600 \leq b_2 \leq 700\right\}. \end{gathered}$$

Note that firm 1 never bids $b_3 = 900$ as long as firm 2 truthfully bids $b_2 = 700$.

Table A14: $b_2 \in [0, 600)$

$b_1 \setminus b_3$	$[0, b_2)$	b_2	$(b_2, 900)$	$[900, \infty)$
$[0, b_2)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
b_2	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
$(b_2, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	✓
$[900, \infty)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$	$(b_1, b_3) = (0, 900)$

Table A15: $b_2 = 600$

$b_1 \setminus b_3$	$[0, 600)$	$[600, 900)$	$[900, \infty)$
$[0, 600)$	✓	$(b_1, b_3) = (0, 0)$	✓
$[600, 900)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	✓
$[900, \infty)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$

Table A16: $b_2 \in (600, 700]$

$b_1 \setminus b_3$	$[0, b_2)$	$[b_2, \infty)$
$[0, b_2)$	✓	$(b_1, b_3) = (0, 0)$
$[b_2, \infty)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$

Table A17: $b_2 \in (700, \infty)$

$b_1 \setminus b_3$	$[0, 700]$	$(700, b_2)$	$[b_2, \infty)$
$[0, 700]$	✓	$b_2 = 0$	$(b_1, b_3) = (0, 0)$
$(700, b_2)$	$b_2 = 0$	$b_2 = 0$	$(b_1, b_3) = (0, 0)$
$[b_2, \infty)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$	$(b_1, b_3) = (0, 0)$

B.2 Overbidding and underbidding

We supplement the notation in Section 2 as follows: Let $Y_\tau \subseteq A_\tau$ be the set of active TV stations that withdraw from the reverse auction in round τ. In round $\tau + 1$, the set of inactive TV stations is thus $I_{\tau+1} = I_\tau \cup Y_\tau$; these are all the TV stations that have previously withdrawn and require channel assignments. Let $Z_\tau = \{j' \in A_\tau \setminus Y_\tau | S(I_{\tau+1} \cup \{j'\}, R) = 0\} \subseteq A_\tau$ be the set of active TV stations that are newly frozen in round τ because they cannot be repacked in addition to the TV stations that have previously withdrawn. In round $\tau + 1$, the set of frozen stations is thus $F_{\tau+1} = F_\tau \cup Z_\tau$ and the set of active stations is $A_{\tau+1} = A_\tau \setminus (Y_\tau \cup Z_\tau)$.

We partition the vector $b = (b_1, \ldots, b_N)$ as (b_j, b_{-j}), where b_j is the bid for TV station j and b_{-j} is the vector of bids of the other TV stations. In the interest of simplicity, we assume that different TV stations have different bids, i.e., $b_j \neq b_k$ for all $j \neq k$, except that we allow multiple TV stations to bid 0 or 900. Let $\pi_i(b)$ be firm i's profit from the reverse auction. Denoting as $J_i \subseteq \{1, \ldots, N\}$ the set of TV stations owned by firm i and as $F^* \subseteq \{1, ..., N\}$ the set of frozen TV stations at the conclusion of the reverse auction, we have

$$\pi_i(b) = \sum_{j \in J_i \cap F^*(b)} PO_j(b) - v_j,$$

where our notation emphasizes that the payout PO_j to TV station j as well as the set of frozen TV stations F^* depend on the vector of bids b.

We motivate the restriction to $b_j \in \{0, s_j, 900\}$ for a jointly owned TV station j with two propositions. Proposition 1 tackles the case of overbidding:

Proposition 1. *Suppose firm i owns multiple TV stations including TV station j, i.e., $|J_i| > 1$ and $j \in J_i$. Consider a vector of bids b with $s_j < b_j < 900$. If $S(Y_1(b) \cup \{j\}, R) = 1$ and $\pi_i(b_j, b_{-j}) > \pi_i(s_j, b_{-j})$, then $\pi_i(900, b_{-j}) \geq \pi_i(b_j, b_{-j})$.*

Proposition 1 assumes that it is feasible to repack TV station j in addition to any TV stations that withdraw in round 1 of the reverse auction. It states that if a firm owning multiple TV stations finds it more profitable to overbid $b_j > s_j$ than to truthfully bid $b_j = s_j$, then the firm may as well bid $b_j = 900$ and withhold TV station j from the reverse auction. In this sense, restricting the strategy space of the jointly owned TV station j from $b_j \in [s_j, 900]$ to $b_j \in \{s_j, 900\}$ does not make the firm worse off.

Proposition 1 is best thought of as characterizing the best reply of firm i and differs from the standard notion of weak dominance. While eliminating strictly (but not weakly) dominated strategies is innocuous and does not affect the set of equilibria, the restriction to $b_j \in \{0, s_j, 900\}$ for a jointly owned TV station j may well do so (see the example in Section 2.1). Alas, a stronger result than Proposition 1 has eluded us. We note that the notion of dominance in Milgrom and Segal (2020) is also weaker than strict dominance.

Proposition 2 tackles the case of underbidding and parallels Proposition 1:

Proposition 2. *Suppose firm i owns multiple TV stations including TV station j, i.e., $|J_i| > 1$ and $j \in J_i$. Consider a vector of bids b with $0 < b_j < s_j$. If $\pi_i(b_j, b_{-j}) > \pi_i(s_j, b_{-j})$, then $\pi_i(0, b_{-j}) \geq \pi_i(b_j, b_{-j})$.*

Turning to the proofs, we first state and prove two lemmas characterizing the impact of b_j on the payout to TV station j and on the profit of its owner, firm i. In a slight abuse of notation, we partition the vector $b = (b_1, \ldots, b_N)$ of bids as (b_i, b_{-i}), where b_i is the vector of bids of firm i and b_{-i} is the vector of bids of the other firms, and as (b_j, b_{-j}), where b_j is the bid of TV station j and b_{-j} is the vector of bids of the other TV stations. Let $\tau(j) \geq 1$ denote the round of the reverse auction where TV station j first opts to remain on the air (unless it has already been frozen), i.e., $P_{\tau(j)-1} > b_j \geq P_{\tau(j)}$ (and we set $P_0 = \infty$). Partition the set of frozen TV stations at the conclusion of the reverse auction as $F^*(b) = \bigcup_{j \in \{1,\ldots,N\}} F_j^*(b)$, where $F_j^*(b) \subseteq \{1, ..., N\}$ is the (possibly empty) set of TV stations that are frozen by TV station j given the vector of bids b.[5] Note that TV station j determines the payout $PO_k(b) = P_{\tau(j)}\varphi_k$ to all TV stations $k \in F_j^*(b)$. Finally, denote the set of inactive TV stations at the conclusion of the reverse auction as $I^*(b)$.

Lemma 3. *If $j \in J_i$ and $j \in F^*(b)$, then $\pi_i(b) = \pi_i(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j \leq b_j$.*

[5] If a TV station $k \in Z_1(b)$ is frozen at the outset of the reverse auction, then we assign it to a TV station $l \in Y_1(b)$ and say that $k \in F_l^*(b)$.

Proof. Because $j \in F^*(b)$, it must be that $j \in F_l^*(b)$ for some TV station l with $b_l > b_j$, i.e., TV station l freezes TV station j under the vector of bids b. Note that $j \in F_l^*(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j \le b_j$ and thus $F_j^*(b) = F_j^*(\tilde{b}_j, b_{-j}) = \emptyset$, i.e., TV station l continues to freeze TV station j under the vector of bids $(\tilde{b}_j, b_{-j})$ and TV station j does not freeze another TV station. Hence, we have to show that

$$\pi_i(b) = \sum_{k \neq j} \sum_{m \in J_i \cap F_k^*(b)} (P_{\tau(k)}\varphi_m - v_m) = \sum_{k \neq j} \sum_{m \in J_i \cap F_k^*(\tilde{b}_j, b_{-j})} (P_{\tau(k)}\varphi_m - v_m) = \pi_i(\tilde{b}_j, b_{-j})$$

for all $\tilde{b}_j \le b_j$. It suffices to show that $F_k^*(b) = F_k^*(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j \le b_j$ and $k \neq j$. First consider any TV station k with $b_k > b_j$. It is obvious that $F_k^*(b) = F_k^*(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j \le b_j$. Consider next any TV station k with $b_k < b_j$. Because $F_{\tau(l)+1}(b) = F_{\tau(l)+1}(\tilde{b}_j, b_{-j})$ and $A_{\tau(l)+1}(b) = A_{\tau(l)+1}(\tilde{b}_j, b_{-j})$, the reverse auction progresses the same from round $\tau(l)+1$ on under the vector of bids b as under the vector of bids $(\tilde{b}_j, b_{-j})$. Hence, $F_k^*(b) = F_k^*(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j \le b_j$. This completes the proof. □

Lemma 4. *If $j \in I^*(b)$ and $S(Y_1(b) \cup \{j\}, R) = 1$, then $F^*(b) = F^*(\tilde{b}_j, b_{-j})$ and $PO_k(b) \le PO_k(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j > b_j$ and $k \in \{1, ..., N\}$.*

Proof. The condition $S(Y_1(b) \cup \{j\}, R) = 1$ guarantees that the reverse auction does not fail at the outset for any vector of bids $(\tilde{b}_j, b_{-j})$. Consider first TV station j. Because $j \in I^*(b)$, it must be that $j \in I^*(\tilde{b}_j, b_{-j})$ and thus $PO_j(b) = 0 = PO_j(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j > b_j$. Next consider any TV station $k \neq j$. If $k \in I^*(b)$, then $k \in I^*(\tilde{b}_j, b_{-j})$ for all $\tilde{b}_j > b_j$ and thus $PO_k(b) = 0 = PO_k(\tilde{b}_j, b_{-j})$. Assuming $k \notin I^*(b)$ and therefore $b_k < 900$, we proceed in two cases, depending on whether or not there exists any inactive TV station with its bid between b_j and $\widetilde{b}_j$.

<u>Case 1:</u> There does not exist any inactive TV station with its bid between b_j and $\widetilde{b}_j$, i.e., $\{l | l \in I^*(b), b_j < b_l < \widetilde{b}_j\} = \emptyset$. Consider a TV station $k \neq j$. Figure A3 illustrates the possible subcases.

<u>Subcase 1a:</u> If $b_j < b_k$, then $k \in F_l^*(b)$ for some TV station l with $b_l \ge \widetilde{b}_j$. Thus $k \in F_l^*(\widetilde{b}_j, b_{-j}) \cup F_1(\widetilde{b}_j, b_{-j})$ and $PO_k(b) = P_{\tau(l)}\varphi_k = PO_k(\widetilde{b}_j, b_{-j})$.

<u>Subcase 1b:</u> If $b_k < b_j$ and $k \in F_j^*(b)$, then $k \in F_j^*(\widetilde{b}_j, b_{-j}) \cup F_1(\widetilde{b}_j, b_{-j})$ and $PO_k(b) = P_{\tau(j)}\varphi_k < PO_k(\widetilde{b}_j, b_{-j})$.

<u>Subcase 1c:</u> If $b_k < b_j$ and $k \in F_l^*(b)$ for some TV station $l \in I^*(b) \setminus \{j\}$, then $k \in F_l^*(\widetilde{b}_j, b_{-j}) \cup F_1(\widetilde{b}_j, b_{-j})$ and thus $PO_k(b) = P_{\tau(l)}\varphi_k = PO_k(\widetilde{b}_j, b_{-j})$.

<u>Case 2:</u> There exists at least one inactive TV station with its bid between b_j and $\widetilde{b}_j$, i.e., $M = \{m | m \in I^*(b), b_j < b_m < \widetilde{b}_j\} \neq \emptyset$. Let $M = \{m^1, ..., m^n\}$ and enumerate its members such that $b_j < b_{m^1} < b_{m^2} < ... < b_{m^n} < \widetilde{b}_j$. It suffices to show that $F^*(b) = F^*(b_{m^1} + \epsilon, b_{-j})$ and $PO_k(b) \le PO_k(b_{m^1} + \epsilon, b_{-j})$ for all $k \neq j$ and any sufficiently small $\epsilon > 0$; it then follows that $F^*(b) = F^*(b_{m^1} + \epsilon, b_{-j}) = ... = F^*(b_{m^n} + \epsilon, b_{-j}) = F^*(\widetilde{b}_j, b_{-j})$, where the last equality follows

from Case 1, and $PO_k(b) \leq PO_k(b_{m^1} + \epsilon, b_{-j}) \leq ... \leq PO_k(b_{m^n} + \epsilon, b_{-j}) \leq PO_k(\widetilde{b}_j, b_{-j})$ for all $k \neq j$ for the same reason.

Consider a TV station $k \neq j$. Figure A4 illustrates the possible subcases.

Subcase 2a: If $k \in F_l^*(b)$ for some TV station l with $b_{m^1} < b_l$, then $k \in F_l^*(b_{m^1} + \epsilon, b_{-j}) \cup F_1(\widetilde{b}_j, b_{-j})$ and $PO_k(b) = P_{\tau(l)}\varphi_k = PO_k(b_{m^1} + \epsilon, b_{-j})$.

Subcase 2b: If $k \in F_l^*(b)$ for some TV station l with $b_l < b_j$, then $k \in F_l^*(b_{m^1} + \epsilon, b_{-j})$ and $PO_k(b) = P_{\tau(l)}\varphi_k = PO_k(b_{m^1} + \epsilon, b_{-j})$.

Subcase 2c: If $k \in F_j^*(b) \cup F_{m^1}^*(b)$, then $k \in F_j^*(b_{m^1} + \epsilon, b_{-j}) \cup F_{m^1}^*(b_{m^1} + \epsilon, b_{-j}) \cup F_1(\widetilde{b}_j, b_{-j})$ and $PO_k(b) \leq P_{\tau(m^1)}\varphi_k = PO_k(b_{m^1} + \epsilon, b_{-j})$.

This completes the proof. □

Figure A3: Case 1 and subcases in proof of Lemma 4

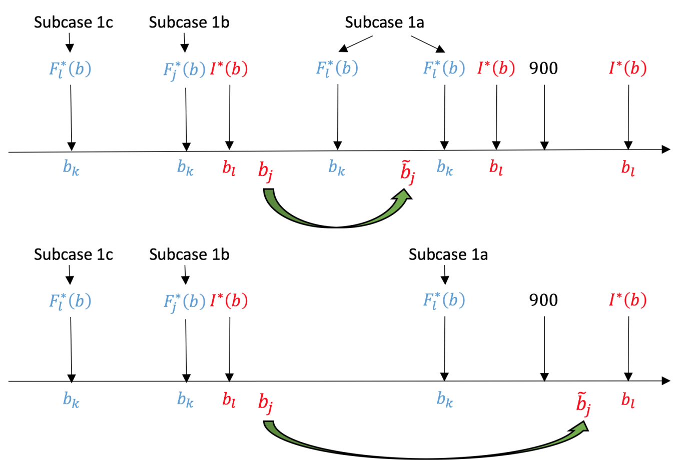
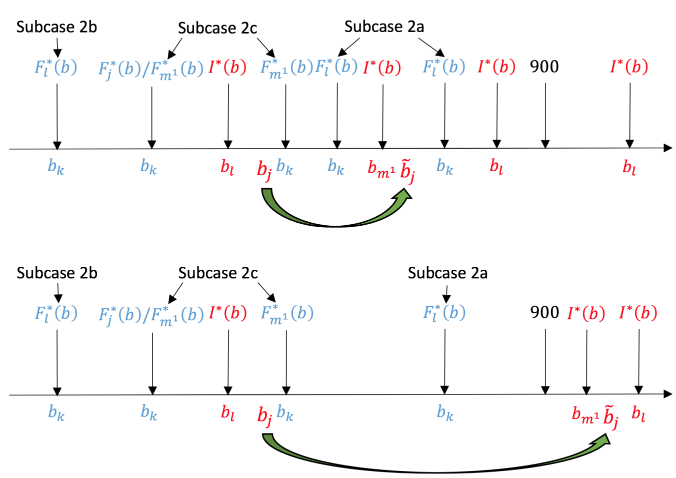


Figure A4: Case 2 and subcases in proof of Lemma 4



We are now ready to prove Proposition 1:

Proof of Proposition 1. We first show that $j \in I^*(b)$. Suppose to the contrary that $j \notin I^*(b)$. Then $j \in F^*(b)$ and Lemma 3 implies $\pi_i(b) = \pi_i(s_j, b_{-j})$, contradicting $\pi_i(b) > \pi_i(s_j, b_{-j})$. Hence, $j \in I^*(b)$ and it follows from Lemma 4 that

$$\begin{aligned}
\pi_i(b) &= \sum_{l \in J_i \cap F^*(b)} (PO_l(b) - v_l) \\
&\leq \sum_{l \in J_i \cap F^*(900, b_{-j})} (PO_l(900, b_{-j}) - v_l) \\
&= \pi_i(900, b_{-j}).
\end{aligned}$$

□

The proof of Proposition 2 largely parallels that of Proposition 1:

Proof of Proposition 2. Suppose to the contrary that $\pi_i(0, b_{-j}) < \pi_i(b)$. Then it must be that $j \in I^*(b)$; otherwise, $j \in F^*(b)$ and it follows from Lemma 3 that $\pi_i(0, b_{-j}) = \pi_i(b)$. Hence, $j \in I^*(b)$ and it follows from Lemma 4 that

$$\begin{aligned}
\pi_i(b) &= \sum_{l \in J_i \cap F^*(b)} (PO_l(b) - v_l) \\
&\leq \sum_{l \in J_i \cap F^*(s_j, b_{-j})} (PO_l(s_j, b_{-j}) - v_l) \\
&= \pi_i(s_j, b_{-j}),
\end{aligned}$$

contradicting $\pi_i(b) > \pi_i(s_j, b_{-j})$. □

B.3 Incomplete information

It is well known that analyzing auctions involving multiple objects under the assumption of incomplete information is difficult (see Chapters 5 and 6 of Milgrom (2004) and Part II, especially Chapter 18, of Krishna (2010)). To make some headway, we recast the example in Section 2.1 as a game of incomplete information. We assume that the reservation value v_j of TV station j is privately known to its owner and specify another firm's belief about the reservation value of TV station j to be $\tilde{v}_j \sim N(v_j, \sigma^2)$, independent across TV stations.

The game of incomplete information gives rise to bidding functions, rather than bids, that depend on beliefs. As beliefs depend on σ, note that as σ goes to zero, beliefs collapse to the true reservation values. In this way, we are able to ascertain the relationship between bidding functions under the game of incomplete information and bids under the game of complete information. In the game of incomplete information, let $b_1(v_1, v_3, \sigma) \geq 0$ and $b_3(v_1, v_3, \sigma) \geq 0$ be the bidding functions of TV stations 1 and 3 that are owned by firm 1 and $b_2(v_2, \sigma) \geq 0$ the bidding function of TV

Table A18: Possible bid configurations

	TV station 1		TV station 2		TV station 3	
Bid configuration	$\Pr(1 \in F^*(b))$	$PO_1(b)$	$\Pr(2 \in F^*(b))$	$PO_2(b)$	$\Pr(3 \in F^*(b))$	$PO_3(b)$
$\min\{b_1, b_2\} = 900$ $\vee \min\{b_1, b_3\} = 900$ $\vee \min\{b_2, b_3\} = 900$	0	0	0	0	0	0
$b_1 > \max\{b_2, b_3\}$	0	0	1	b_1	1	b_1
$b_2 > \max\{b_1, b_3\}$	1	b_2	0	0	1	b_2
$b_3 > \max\{b_1, b_2\}$	1	b_3	1	b_3	0	0
$900 > b_1 = b_2 > b_3$	$\frac{1}{2}$	$b_1 \vee 0$	$\frac{1}{2}$	$b_1 \vee 0$	1	b_1
$900 > b_1 = b_3 > b_2$	$\frac{1}{2}$	$b_1 \vee 0$	1	b_1	$\frac{1}{2}$	$b_1 \vee 0$
$900 > b_2 = b_3 > b_1$	1	b_2	$\frac{1}{2}$	$b_2 \vee 0$	$\frac{1}{2}$	$b_2 \vee 0$
$900 > b_1 = b_2 = b_3 > 0$	$\frac{2}{3}$	$b_1 \vee 0$	$\frac{2}{3}$	$b_1 \vee 0$	$\frac{2}{3}$	$b_1 \vee 0$
$b_1 = b_2 = b_3 = 0$	1	0	1	0	1	0

station 2 that is owned by firm 2. In what follows, we characterize the bidding functions as $\sigma \to 0^+$. We show that firm 1 always bids $b_1 < b_3$. Its expected profit depends solely on b_3 and, as $\sigma \to 0^+$, closely resembles its profit under complete information. Moreover, for a wide range of values of σ, $b_3(100, 300, \sigma)$ is arbitrarily close to (but different from) $b_3 = 900$. Close to extreme overbidding thus arises in the game of incomplete information. In a variant of the example, we also show that close to extreme overbidding arises in the game of incomplete information when σ is large. In contrast, extreme overbidding does not arise in the game of complete information. Taken together, these results suggest that our notion of strategic supply reduction in settings with jointly owned TV stations extends beyond complete information.

To recast the example in Section 2.1 as a game of incomplete information, note that expected profit of firm 1 if it bids $b_1 \geq 0$ and $b_3 \geq 0$ is

$$\begin{aligned} E\pi_1(b_1, b_3; v_1, v_3, \sigma) &= \int_{\tilde{v}_2} (PO_1(b_1, b_2(\tilde{v}_2, \sigma), b_3) - v_1)\, 1\,(1 \in F^*(b_1, b_2(\tilde{v}_2, \sigma), b_3)) \\ &\quad + (PO_3(b_1, b_2(\tilde{v}_2, \sigma), b_3) - v_3)\, 1\,(3 \in F^*(b_1, b_2(\tilde{v}_2, \sigma), b_3))\, d\Phi_2(\tilde{v}_2), \end{aligned}$$

where $1(\cdot)$ is the indicator function and $\tilde{v}_2$ is distributed according to the cumulative distribution function $\Phi_2(\tilde{v}_2) = \Phi\left(\frac{\tilde{v}_2 - v_2}{\sigma}\right)$ with $\Phi(\cdot)$ being the standard normal cumulative distribution function. As firm 1 bids optimally, the bidding functions are given by $(b_1(v_1, v_3, \sigma), b_3(v_1, v_3, \sigma)) = \arg\max_{b_1, b_3 \geq 0} E\pi_1(b_1, b_3; v_1, v_3, \sigma)$. The expected profit of firm 2 if it bids $b_2 \geq 0$ is

$$\begin{aligned} E\pi_2(b_2; v_2, \sigma) &= \int_{\tilde{v}_1} \int_{\tilde{v}_3} (PO_2(b_1(\tilde{v}_1, \tilde{v}_3, \sigma), b_2, b_3(\tilde{v}_1, \tilde{v}_3, \sigma)) - v_2) \\ &\quad \cdot 1\,(2 \in F^*(b_1(\tilde{v}_1, \tilde{v}_3, \sigma), b_2, b_3(\tilde{v}_1, \tilde{v}_3, \sigma)))\, d\Phi_3(\tilde{v}_3) d\Phi_1(\tilde{v}_1). \end{aligned}$$

As firm 2 bids optimally, the bidding function is given by $b_2(v_2, \sigma) = \arg\max_{b_2 \geq 0} E\pi_2(b_2; v_2, \sigma)$.

In the interest of simplicity, we restrict $b_j \leq 900$ and consider the nine possible bid configurations in Table A18.[6] We determine $F^*(b)$ and $PO_j(b)$ from the bid configuration along with the specification of $S(X,R)$ in equation (2), assuming a random tie-breaking rule for bids above 0 and below 900 in line with footnote 26. The expected profit of firm 1 if it bids $b_1 \in [0,900]$ and $b_3 \in [0,900]$ is

$$\begin{aligned} E\pi_1(b_1,b_3;v_1,v_3,\sigma) \quad = \quad & \int_{\tilde{v}_2} (b_1 - v_3)\, 1\left(b_1 > \max\left\{b_2(\tilde{v}_2,\sigma), b_3\right\}\right) \\ & + (2b_2(\tilde{v}_2,\sigma) - v_1 - v_3)\, 1\left(b_2(\tilde{v}_2,\sigma) > \max\left\{b_1, b_3\right\}\right) \\ & + (b_3 - v_1)\, 1\left(b_3 > \max\left\{b_1, b_2(\tilde{v}_2,\sigma)\right\}\right) \\ & + \left(\frac{1}{2}(b_3 - v_1) + \frac{1}{2}(b_1 - v_3)\right) 1\left(900 > b_1 = b_3 > b_2(\tilde{v}_2,\sigma)\right) \\ & - (v_1 + v_3)\, 1\left(b_1 = b_2(\tilde{v}_2,\sigma) = b_3 = 0\right) d\Phi_2(\tilde{v}_2), \end{aligned}$$

where we anticipate that in equilibrium firm 2's bid does not have mass points above 0 and below 900 and therefore, from firm 1's perspective, cannot tie with firm 1's bids in this range.

The expected profit of firm 2 if it bids $b_2 \in [0,900]$ is

$$\begin{aligned} E\pi_2(b_2;v_2,\sigma) \quad = \quad & \int_{\tilde{v}_1}\int_{\tilde{v}_3} (b_1(\tilde{v}_1,\tilde{v}_3,\sigma) - v_2)\, 1\left(b_1(\tilde{v}_1,\tilde{v}_3,\sigma) > \max\left\{b_2, b_3(\tilde{v}_1,\tilde{v}_3,\sigma)\right\}\right) \\ & + (b_3(\tilde{v}_1,\tilde{v}_3,\sigma) - v_2)\, 1\left(b_3(\tilde{v}_1,\tilde{v}_3,\sigma) > \max\left\{b_1(\tilde{v}_1,\tilde{v}_3,\sigma), b_2\right\}\right) \\ & + \frac{1}{2}(b_1(\tilde{v}_1,\tilde{v}_3,\sigma) - v_2)\, 1\left(900 > b_1(\tilde{v}_1,\tilde{v}_3,\sigma) = b_2 > b_3(\tilde{v}_1,\tilde{v}_3,\sigma)\right) \\ & + (b_1(\tilde{v}_1,\tilde{v}_3,\sigma) - v_2)\, 1\left(900 > b_1(\tilde{v}_1,\tilde{v}_3,\sigma) = b_3(\tilde{v}_1,\tilde{v}_3,\sigma) > b_2\right) \\ & + \frac{1}{2}(b_3(\tilde{v}_1,\tilde{v},\sigma_3) - v_2)\, 1\left(900 > b_2 = b_3(\tilde{v}_1,\tilde{v}_3,\sigma) > b_1(\tilde{v}_1,\tilde{v}_3,\sigma)\right) \\ & + \frac{2}{3}(b_1(\tilde{v}_1,\tilde{v}_3,\sigma) - v_2)\, 1\left(900 > b_1(\tilde{v}_1,\tilde{v}_3,\sigma) = b_2 = b_3(\tilde{v}_1,\tilde{v}_3,\sigma) > 0\right) \\ & - v_2 1\left(b_1(\tilde{v}_1,\tilde{v}_3,\sigma) = b_2 = b_3(\tilde{v}_1,\tilde{v}_3,\sigma) = 0\right) d\Phi_3(\tilde{v}_3) d\Phi_1(\tilde{v}_1). \end{aligned}$$

Inspection of the expected profit of firm 2 almost immediately yields

Proposition 5. *Truthful bidding* $b_2(v_2,\sigma) = \max\left\{\min\left\{v_2, 900\right\}, 0\right\}$ *is a dominant strategy for firm 2.*

Proof. We show that for any given values of $b_1(\tilde{v}_1,\tilde{v}_3,\sigma)$ and $b_3(\tilde{v}_1,\tilde{v}_3,\sigma)$, firm 2 cannot do better than bid $b_2(v_2,\sigma) = \max\left\{\min\left\{v_2, 900\right\}, 0\right\}$. We proceed by enumerating the different possible cases for $b_1(\tilde{v}_1,\tilde{v}_3,\sigma)$, $b_3(\tilde{v}_1,\tilde{v}_3,\sigma)$, and v_2. We restrict attention to cases where $b_1(\tilde{v}_1,\tilde{v}_3,\sigma) \geq b_3(\tilde{v}_1,\tilde{v}_3,\sigma)$ because cases where $b_1(\tilde{v}_1,\tilde{v}_3,\sigma) \leq b_3(\tilde{v}_1,\tilde{v}_3,\sigma)$ are analogous. For each case, Table A19 lists the best response of firm 2. A blank cell indicates that the case cannot arise. As can be seen from Table A19, the best response contains $\max\left\{\min\left\{v_2, 900\right\}, 0\right\}$ for each case, thereby establishing the proposition.

In column (1) of Table A19, firm 2 prefers not to sell TV station 2 at the opening price of 900.

[6] While restricting $b_j \leq 900$ restricts the set of equilibria, it does not restrict the payouts to TV stations associated with these equilibria.

Table A19: Best response of firm 2

	(1) $v_2 > 900$	(2) $v_2 = 900$	(3) $900 > v_2 > b_1$	(4) $900 > v_2 = b_1 > 0$	(5) $v_2 = b_1 = 0$	(6) $v_2 < b_1$
$900 = b_1 > b_3 > 0$	900	$[0, 900]$				$[0, b_1)$
$900 > b_1 > b_3 > 0$	$(b_1, 900]$	$(b_1, 900]$	$(b_1, 900]$	$[0, 900]$		$[0, b_1)$
$900 = b_1 > b_3 = 0$	900	$[0, 900]$				$[0, b_1)$
$900 > b_1 > b_3 = 0$	$(b_1, 900]$	$(b_1, 900]$	$(b_1, 900]$	$[0, 900]$		$[0, b_1)$
$900 = b_1 = b_3$	900	$[0, 900]$				$[0, b_1)$
$900 > b_1 = b_3 > 0$	$(b_1, 900]$	$(b_1, 900]$	$(b_1, 900]$	$[0, 900]$		$[0, b_1)$
$b_1 = b_3 = 0$	$(0, 900]$	$(0, 900]$	$(0, 900]$		$[0, 900]$	

Firm 2 therefore either causes the reverse auction to fail at the outset if $b_1 = 900$ or withdraws first if $b_1 < 900$. In column (2), firm 2 is indifferent between selling TV station 2 at the opening price of 900 and not selling it. Firm 2 therefore bids anything if $b_1 = 900$ or withdraws first if $b_1 < 900$. In column (3), firm 2 prefers not to sell TV station 2 at a price of b_1. Firm 2 therefore withdraws first. In column (4) and (5), firm 2 is indifferent between selling TV station 2 at a price of b_1 and not selling it. Firm 2 therefore bids anything. In column (6), firm 2 prefers to sell TV station 2 at a price of b_1. Firm 2 therefore does not withdraw first. □

Using Proposition 5, the expected profit of firm 1 if it bids $b_1 \in [0, 900]$ and $b_3 \in [0, 900]$ can be written as

$$\begin{aligned} E\pi_1(b_1, b_3; v_1, v_3, \sigma) \quad = \quad & \int_{900}^{\infty} (2 \cdot 900 - v_1 - v_3)\, 1\,(900 > \max\{b_1, b_3\})\, d\Phi_2(\tilde{v}_2) \\ & + \int_{0}^{900} (b_1 - v_3)\, 1\,(b_1 > \max\{\tilde{v}_2, b_3\}) \\ & + (2\tilde{v}_2 - v_1 - v_3)\, 1\,(\tilde{v}_2 > \max\{b_1, b_3\}) \\ & + (b_3 - v_1)\, 1\,(b_3 > \max\{b_1, \tilde{v}_2\}) \\ & + \left(\frac{1}{2}(b_3 - v_1) + \frac{1}{2}(b_1 - v_3)\right) 1\,(900 > b_1 = b_3 > \tilde{v}_2)\, d\Phi_2(\tilde{v}_2) \\ & + \int_{-\infty}^{0} (b_1 - v_3)\, 1\,(b_1 > b_3) \\ & + (b_3 - v_1)\, 1\,(b_3 > b_1) \\ & + \left(\frac{1}{2}(b_3 - v_1) + \frac{1}{2}(b_1 - v_3)\right) 1\,(900 > b_1 = b_3 > 0) \\ & - (v_1 + v_3)\, 1\,(b_1 = b_3 = 0)\, d\Phi_2(\tilde{v}_2). \end{aligned} \tag{A6}$$

We assume $v_1 = 100$ and $v_3 = 300$ as in Section 2.1. Towards determining $b_1(100, 300, \sigma)$ and $b_3(100, 300, \sigma)$, the following propositions show that firm 1 always bids $b_1 < b_3$.

Proposition 6. *$E\pi_1(0, 0; 100, 300, \sigma) < E\pi_1(0, \epsilon; 100, 300, \sigma)$ and $E\pi_1(b, b; 100, 300, \sigma) < E\pi_1(b - \epsilon, b; 100, 300, \sigma)$ for all $b \in (0, 900]$ for any sufficiently small $\epsilon > 0$.*

Hence, firm 1 never bids $b_1 = b_3$.

Proof. First, consider $b = 0$. Then plugging into equation (A6) yields

$$\begin{aligned} E\pi_1(0,0;100,300,\sigma) &= \int_{900}^{\infty} (2\cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{0}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad - \int_{-\infty}^{0} (100 + 300)\, d\Phi_2(\tilde{v}_2) \\ &< \int_{900}^{\infty} (2\cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{\epsilon}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{-\infty}^{\epsilon} (\epsilon - 100)\, d\Phi_2(\tilde{v}_2) \\ &= E\pi_1(0,\epsilon;100,300,\sigma) \end{aligned}$$

for any sufficiently small $\epsilon > 0$. Consider next $b \in (0, 900)$. Then plugging into equation (A6) yields

$$\begin{aligned} E\pi_1(b,b;100,300,\sigma) &= \int_{900}^{\infty} (2\cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{b}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{-\infty}^{b} \left(b - \frac{1}{2}100 - \frac{1}{2}300\right) d\Phi_2(\tilde{v}_2) \\ &< \int_{900}^{\infty} (2\cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{b}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{-\infty}^{b} (b - 100)\, d\Phi_2(\tilde{v}_2) \\ &= E\pi_1(b-\epsilon,b;100,300,\sigma). \end{aligned}$$

Finally, consider $b = 900$. Then plugging into equation (A6) yields

$$\begin{aligned} E\pi_1(900,900;100,300,\sigma) &= 0 \\ &< \int_{-\infty}^{900} (900 - 100) d\Phi_2(\tilde{v}_2) \\ &= E\pi_1(900-\epsilon,900;100,300,\sigma). \end{aligned}$$

□

Proposition 7. $b_1 > b_3$ implies $E\pi_1(b_1, b_3; 100, 300, \sigma) > E\pi_1(b_3, b_1; 100, 300, \sigma)$.

Hence, firm 1 never bids $b_1 > b_3$. Taken together, Propositions 6 and 7 imply that firm 1 always bids $b_1 < b_3$.

Proof. Consider first $900 > b_1 > b_3 \geq 0$. Then plugging into equation (A6) yields

$$\begin{aligned}
E\pi_1(b_1, b_3; 100, 300, \sigma) &= \int_{900}^{\infty} (2 \cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\
&\quad + \int_{b_1}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\
&\quad + \int_{-\infty}^{b_1} (b_1 - 300)\, d\Phi_2(\tilde{v}_2) \\
&< \int_{900}^{\infty} (2 \cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\
&\quad + \int_{b_1}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\
&\quad + \int_{-\infty}^{b_1} (b_1 - 100)\, d\Phi_2(\tilde{v}_2) \\
&= E\pi_1(b_3, b_1; 100, 300, \sigma).
\end{aligned}$$

Next consider $900 = b_1 > b_3 \geq 0$. Then plugging into equation (A6) yields

$$\begin{aligned}
E\pi_1(900, b_3; 100, 300, \sigma) &= \int_{-\infty}^{900} (900 - 300)\, d\Phi_2(\tilde{v}_2) \\
&< \int_{-\infty}^{900} (900 - 100)\, d\Phi_2(\tilde{v}_2) \\
&= E\pi_1(b_3, 900; 100, 300, \sigma).
\end{aligned}$$

□

Using Propositions 6 and 7, the expected profit of 1 firm if $b_3 < 900$ becomes

$$\begin{aligned} E\pi_1(b_1, b_3; 100, 300, \sigma) &= \int_{900}^{\infty} (2 \cdot 900 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{b_3}^{900} (2\tilde{v}_2 - 100 - 300)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{0}^{b_3} (b_3 - 100)\, d\Phi_2(\tilde{v}_2) \\ &\quad + \int_{-\infty}^{0} (b_3 - 100)\, d\Phi_2(\tilde{v}_2) \\ &= 1400 \left(1 - \Phi\left(\frac{900 - v_2}{\sigma}\right)\right) \\ &\quad + (2v_2 - 400)\left(\Phi\left(\frac{900 - v_2}{\sigma}\right) - \Phi\left(\frac{b_3 - v_2}{\sigma}\right)\right) \\ &\quad + 2\sigma\left(\phi\left(\frac{b_3 - v_2}{\sigma}\right) - \phi\left(\frac{900 - v_2}{\sigma}\right)\right) \\ &\quad + (b_3 - 100)\,\Phi\left(\frac{b_3 - v_2}{\sigma}\right) \qquad \text{(A7)} \end{aligned}$$

and

$$\begin{aligned} E\pi_1(b_1, 900; 100, 300, \sigma) &= \int_{-\infty}^{900} (900 - 100)\, d\Phi_2(\tilde{v}_2) \\ &= 800\Phi\left(\frac{900 - v_2}{\sigma}\right) \end{aligned}$$

if $b_3 = 900$. Note that the expected profit of firm 1 depends solely on b_3; hence, $b_1 \in [0, b_3)$ is indeterminate. Note also that $\lim_{b_3 \to 900-} E\pi_1(b_1, b_3; 100, 300, \sigma) > E\pi_1(b_1, 900; 100, 300, \sigma)$; hence, firm 1 never bids $b_3 = 900$.

To explore the relationship between the game of incomplete information as $\sigma \to 0^+$ so that beliefs collapse at the true reservation values and the game of complete information, we first assume $v_2 = 500$ as in Section 2.1. The expected profit of firm 1 in the game of incomplete information becomes

$$\begin{aligned} & E\pi_1(b_1, b_3; 100, 300, \sigma) \\ &= \begin{cases} 1400 - 800\Phi\left(\frac{400}{\sigma}\right) + (b_3 - 700)\,\Phi\left(\frac{b_3 - 500}{\sigma}\right) + 2\sigma\left(\phi\left(\frac{b_3 - 500}{\sigma}\right) - \phi\left(\frac{400}{\sigma}\right)\right) & \text{if} \quad b_3 < 900, \\ 800\Phi\left(\frac{400}{\sigma}\right) & \text{if} \quad b_3 = 900. \end{cases} \end{aligned} \qquad \text{(A8)}$$

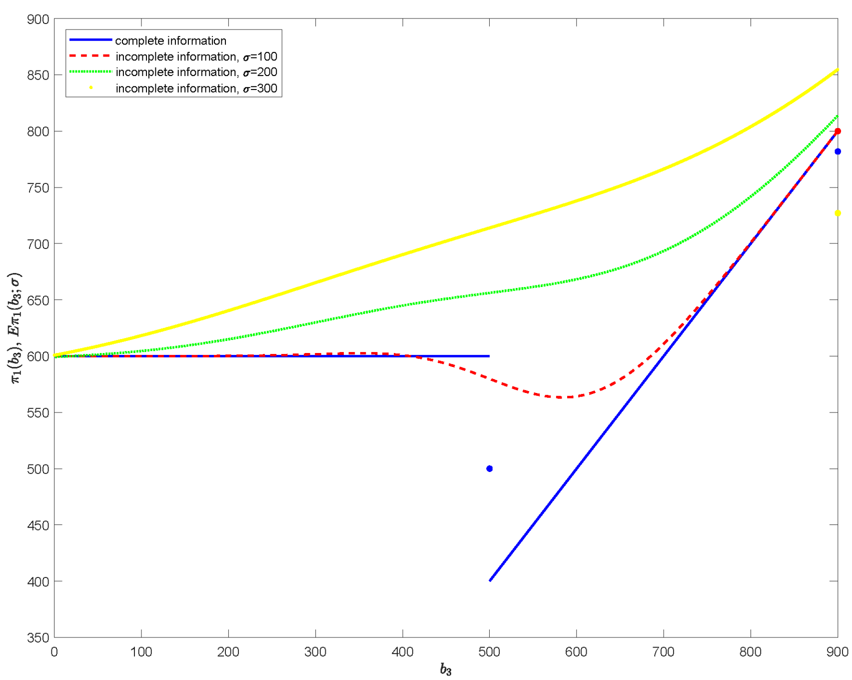
For comparison, in the game of complete information the profit of firm 1 in equation (A3) becomes

$$\pi_1(b_1, 500, b_3) = \begin{cases} 600 & \text{if} \quad b_3 < 500, \\ b_3 - 100 & \text{if} \quad b_3 > 500, \\ 500 & \text{if} \quad b_3 = 500, \end{cases} \tag{A9}$$

where we assume that firm 2 truthfully bids $b_2 = 500$ and firm 1 bids $b_1 < b_3$ as in the game of incomplete information. Note that in the game of complete information the profit of firm 1 again depends solely on b_3 and that firm 1 always bids such that $b_3 = 900$.

Figure A5 plots the expected profit of firm 1 in equation (A8) for various values of σ and the profit of firm 1 in equation (A9). As $\sigma \to 0^+$, the expected profit of firm 1 under incomplete information closely resembles the profit of firm 1 under complete information. Moreover, for a wide range of values of σ, $b_3(100, 300, \sigma)$ in the game of incomplete information is arbitrarily close to (but different from) $b_3 = 900$ in the game of complete information. Close to extreme overbidding thus arises in the game of incomplete information.

Figure A5: Expected profit and profit of firm 1 in Eqns (A8) and (A9) with $v_2 = 500$



To further explore the relationship between the games of complete and incomplete information, in Online Appendix B.1, we consider a variant the example in Section 2.1 in which we replace the reservation value of TV station 2 by $v_2 = 700$. The expected profit of firm 1 in the game of incomplete information becomes

$$E\pi_1(b_1, b_3; 100, 300, \sigma)$$
$$= \begin{cases} 1400 - 400\Phi\left(\frac{200}{\sigma}\right) + (b_3 - 1100)\,\Phi\left(\frac{b_3-700}{\sigma}\right) + 2\sigma\left(\phi\left(\frac{b_3-700}{\sigma}\right) - \phi\left(\frac{200}{\sigma}\right)\right) & \text{if} \quad b_3 < 900, \\ 800\Phi\left(\frac{200}{\sigma}\right) & \text{if} \quad b_3 = 900. \end{cases} \tag{A10}$$

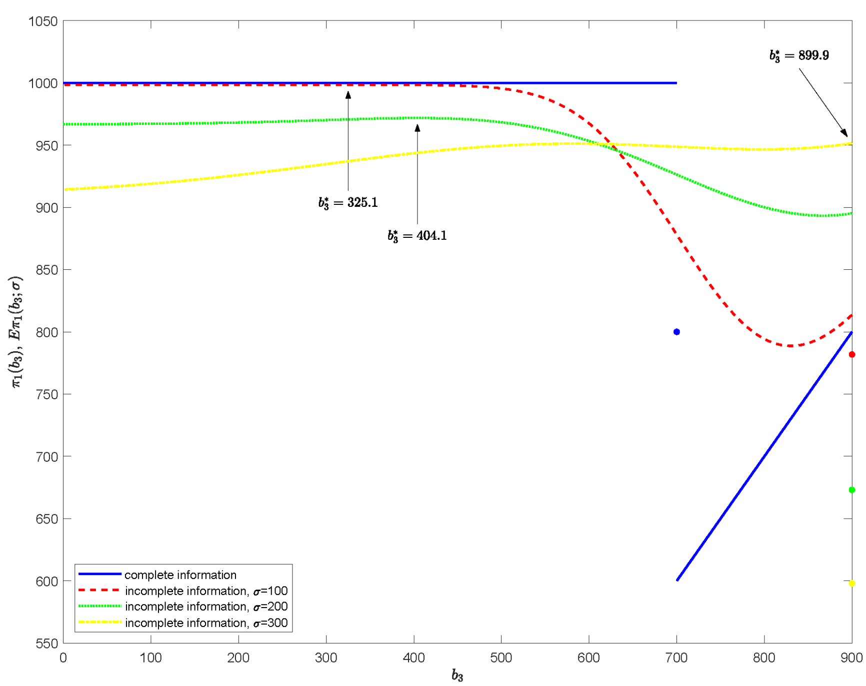
For comparison, in the game of complete information the profit of firm 1 in equation (A5) becomes

$$\pi_1(b_1, 700, b_3) = \begin{cases} 1000 & \text{if} \quad b_3 < 700, \\ b_3 - 100 & \text{if} \quad b_3 > 700, \\ 800 & \text{if} \quad b_3 = 700, \end{cases} \tag{A11}$$

where we assume that firm 2 truthfully bids $b_2 = 700$ and firm 1 bids $b_1 < b_3$ as in the game of incomplete information. Note that in the game of complete information the profit of firm 1 again depends solely on b_3 and that firm 1 always bids $b_3 \in [0, 700)$.

Figure A6 is analogous to Figure A5. As $\sigma \to 0^+$, the expected profit of firm 1 under incomplete information again closely resembles the profit of firm 1 under complete information. Figure A6 further shows that $b_3(100, 300, \sigma)$ in the game of incomplete information gets close to the reservation value $v_3 = 300$ of TV station 3 as $\sigma \to 0+$. In this example, a small amount of incomplete information thus appears to single out truthful bidding. Finally, Figure A6 shows that $b_3(100, 300, \sigma)$ gets close to 900 as $\sigma \to \infty$. A large amount of incomplete information thus appears to support close to extreme overbidding even though firm 1 never bids $b_3 = 900$ in the game of complete information as we show in Online Appendix B.1.

Figure A6: Expected profit and profit of firm 1 in Eqns (A10) and (A11) with $v_2 = 700$



C Primary data

In this appendix, we discuss several details of the data sources we rely on and describe how we construct our sample and primary variables.

C.1 BIA data

After restricting to full-power stations (primary and satellite stations) and low-power class-A and LPTV stations, the BIA data provides us with 66,078 station-year observations from 2003 to 2013 and for 2015. Commercial stations make up 56,856 observations and non-commercial stations, including dark stations, 9,222 observations.

The BIA data provides station, owner and market characteristics, as well as transaction histories covering the eight most recent changes in the ownership of a TV station. Advertising revenue and DMA rank are provided for each year from 2003 to 2013 and for 2015. DMA population is provided for 2007, 2008, 2012, 2013, and 2015. We use the data for 2007 and 2008 to extrapolate DMA population linearly to earlier years and the data for 2008 and 2013 to interpolate linearly to the years in-between. With few exceptions, other characteristics are provided only for 2012 and for 2015.[7] Transaction histories are provided from 2003 to 2013.

For commercial full-power and low-power class-A stations, advertising revenue is missing for 4,892, or 24.9%, station-year observations. Table A20 shows the share of station-year observations with missing advertising revenue for commercial stations. As the top panel shows, advertising revenue is missing for almost all satellite stations because BIA subsumes their advertising revenues into those of their parent primary stations.[8] Missing values are further concentrated among low-power class-A stations. Given this prevalence, we supplement the sample with data on 1,331 LPTV stations with non-missing revenue data. LPTV stations are not auction-eligible, but are more comparable to low-power class-A stations than full-power stations. Focusing only on full-power and low-power class-A stations, the bottom panel of Table A20 summarizes the prevalence of missing revenue data by affiliation. Revenue data is more frequently unavailable for Spanish-language networks (Azteca America, Independent Spanish, Telemundo, Unimas, and Univision), other minor networks, and independent stations. There are no discernible patterns in missing values along other dimensions of the data such as market size.

We impute missing advertising revenue for commercial full-power and low-power class-A stations as follows. For primary stations, we regress the log of advertising revenue (in $ thousand) $\ln AD_{jt}$ on station, owner, and market characteristics X_{jt}. We run this regression separately for each year from 2003 to 2013 and for 2015. We include in X_{jt} as station characteristics the log of the interference free population coverage (in thousand) of the TV station, an indicator for whether the TV station

[7] An "on air date" is provided and we drop observations for a TV station before it went on the air. A previous affiliation and the date of the affiliation change are provided. We manually fill in historical affiliations, including the merger of United Paramount and Warner Bros in 2006 to form CW and the creation of MyNetwork TV in 2006.

[8] We enforce this convention for the 84 station-year observations where a satellite station has non-missing advertising revenue. We manually link the 116 satellite stations to 78 primary stations because BIA does not provide this information.

Table A20: Missing advertising revenue for commercial stations

	Station-year obs.	Missing advertising revenue	
		Station-year obs.	%
Full-power			
Primary	14,698	967	6.58
Satellite	1,411	1,327	94.05
Low-power class-A	4,967	3,925	79.02
LPTV	37,191	35,860	96.42
Major networks			
ABC	2,690	433	16.10
CBS	2,640	339	12.84
Fox	2,471	344	13.92
NBC	2,664	403	15.13
Minor networks			
CW	950	112	11.79
MyNetwork TV	833	146	17.53
United Paramount	269	37	13.75
Warner Bros	269	26	9.67
Spanish-language networks	1,911	608	31.82
Other	3,225	1,631	50.57
Independent	3,133	2,140	68.31

has multicast sub-channels, an indicator for LPTV stations, an indicator for full-power stations, fixed effects for the eleven network affiliations in Table A20, fixed effects for the interaction of affiliation groups ((1) ABC, CBS, NBC, and Fox; (2) CW, My Network TV, United Paramount, Warner Bros, and Spanish-language networks; (3) Independents and other minor networks) with U.S. states, as owner characteristics an indicator for whether the owner owns more than one TV station in the same DMA, ownership category fixed effects (whether the owner owns between two and ten, or more than ten TV stations across DMAs), and as DMA characteristics the number of TV stations in the DMA, the number of major network affiliates in the DMA, the wealth and competitiveness indices for the DMA (see Appendix A.1), and the log of DMA population (in thousand). We report the parameter estimates in Table A21. The adjusted R^2 is 0.99 in all years, suggesting that we capture most of the variation in advertising revenue across TV stations and years.

With the parameter estimates in hand, we impute advertising revenue AD_{jt} for primary stations, where missing, as $\widehat{AD}_{jt} = e^{\ln \widehat{AD}_{jt} + \frac{\hat{\sigma}^2}{2}}$ to account for the non-zero mean of the log-normally distributed error term with estimated variance $\hat{\sigma}^2$. Where applicable, we then allocate revenue between the primary station and any affiliated satellite stations in proportion to their interference free population.

Table A21: Advertising revenue imputation by year

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2015
$\ln(InterferenceFreePop_{jt})$	0.481***	0.470***	0.406***	0.365***	0.333***	0.294***	0.304***	0.302***	0.307***	0.264***	0.268***	0.225***
	(0.048)	(0.047)	(0.044)	(0.043)	(0.042)	(0.040)	(0.040)	(0.038)	(0.038)	(0.035)	(0.035)	(0.038)
Multicast	0.214***	0.195***	0.203***	0.134**	0.150***	0.146***	0.150***	0.090*	0.112**	0.081	0.075	0.143**
	(0.052)	(0.049)	(0.051)	(0.053)	(0.053)	(0.052)	(0.052)	(0.051)	(0.052)	(0.050)	(0.050)	(0.061)
LPTV	-0.020	-0.277**	-0.241*	-0.304**	-0.310**	-0.316***	-0.268**	-0.120	-0.182	-0.132	-0.157	-0.164
	(0.146)	(0.138)	(0.134)	(0.130)	(0.121)	(0.116)	(0.114)	(0.108)	(0.110)	(0.105)	(0.104)	(0.105)
Full-power	0.735***	0.620***	0.753***	0.846***	0.864***	0.860***	0.888***	0.969***	0.909***	0.969***	0.952***	1.022***
	(0.115)	(0.114)	(0.110)	(0.111)	(0.104)	(0.100)	(0.101)	(0.095)	(0.096)	(0.093)	(0.092)	(0.094)
Owns >1 station in DMA	0.029	0.019	0.056	0.070	0.092*	0.106**	0.109**	0.081	0.107**	0.102**	0.116**	0.124**
	(0.056)	(0.055)	(0.055)	(0.054)	(0.054)	(0.052)	(0.052)	(0.051)	(0.052)	(0.050)	(0.050)	(0.052)
Owns 2-10 stations across DMAs	0.108	0.093	0.087	0.047	0.082	-0.005	0.111	0.090	0.180**	0.173**	0.165**	0.195**
	(0.080)	(0.079)	(0.082)	(0.084)	(0.084)	(0.081)	(0.085)	(0.083)	(0.083)	(0.083)	(0.084)	(0.093)
Owns >10 stations across DMAs	0.342***	0.304***	0.251***	0.247***	0.208**	0.181**	0.225***	0.240***	0.353***	0.277***	0.275***	0.342***
	(0.079)	(0.078)	(0.081)	(0.085)	(0.085)	(0.081)	(0.084)	(0.082)	(0.082)	(0.082)	(0.081)	(0.087)
# Stations in DMA	0.002	0.005	0.006	0.005	0.003	0.002	0.007	0.011**	0.012**	0.019***	0.010	0.005
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)	(0.006)	(0.006)	(0.006)	(0.007)	(0.005)
# Major network affiliates in DMA	-0.046	-0.032	-0.018	0.003	-0.003	-0.003	0.030	0.064	0.076*	0.146***	0.074	-0.004
	(0.032)	(0.032)	(0.032)	(0.033)	(0.033)	(0.034)	(0.038)	(0.040)	(0.041)	(0.045)	(0.047)	(0.034)
$WealthIndex_{jt}$	0.125***	0.120***	0.133***	0.123***	0.123***	0.133***	0.133***	0.148***	0.150***	0.157***	0.147***	0.149***
	(0.024)	(0.023)	(0.024)	(0.024)	(0.024)	(0.023)	(0.024)	(0.023)	(0.023)	(0.022)	(0.023)	(0.025)
$CompIndex_{jt}$	0.026	-0.017	-0.053	-0.092	-0.060	-0.048	-0.152	-0.252**	-0.275**	-0.462***	-0.245*	-0.075
	(0.081)	(0.084)	(0.088)	(0.091)	(0.090)	(0.093)	(0.105)	(0.115)	(0.121)	(0.130)	(0.134)	(0.086)
$\ln(DMAPop_{jt})$	0.361***	0.385***	0.409***	0.469***	0.502***	0.507***	0.500***	0.509***	0.493***	0.529***	0.528***	0.507***
	(0.053)	(0.052)	(0.051)	(0.051)	(0.049)	(0.048)	(0.047)	(0.045)	(0.045)	(0.043)	(0.043)	(0.046)
Network affiliation fixed effects	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Affiliation groups × U.S. states	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Adjusted R^2	0.994	0.994	0.993	0.993	0.992	0.993	0.992	0.993	0.992	0.992	0.992	0.991
N	1191	1215	1247	1307	1343	1364	1371	1379	1397	1415	1421	1454

C.2 NAB data

NAB collects financial information on cash flow, revenue, and expenses broken down into detailed source categories for commercial full-power stations. We define advertising revenue as the sum of local, regional, national, and political advertising revenue, commissions, and network compensation. We further define non-broadcast revenue as the sum of total trade-outs and barter, multicast revenue, and other broadcast related revenue. Finally, we define fixed cost as the sum of engineering expenses and general and administrative expenses.

NAB reports the data at various levels of aggregation. Table A22 shows the resulting 66 tables in 2012.[9] The number of tables fluctuates slightly year-by-year because NAB imposes a minimum of ten TV stations per aggregation category to ensure confidentiality.[10,11] Note that a TV station may feature in more than one table. For example, WABC-TV, the New York ABC affiliate, is used in calculating statistics for (1) markets of rank 1 to 10; (2) major network affiliates; (3) all ABC affiliates; and (4) ABC affiliates in markets with rank 1 to 25.

For each aggregation category, NAB reports the mean as well as the first, second, and third quartile for cash flow and the detailed source categories for revenue and expenses. Because we do not observe correlations between the categories, we can construct the mean of advertising revenue, non-broadcast revenue, and fixed cost but not the quartiles. We present a sample of the NAB data for select aggregation categories in Table A23.

To validate the data, first we compare the mean of advertising revenue from the NAB data to suitably averaged advertising revenue from the BIA data. The resulting 662 pairs of means from the two data sources exhibit a correlation of 0.980. Next, to investigate the consequences of imputing advertising revenue, where missing, in the BIA data, we equally split the sample into two groups based on the amount of imputation. For each of the 662 NAB tables, we calculate the share of stations in the BIA data that qualify for the table and have imputed advertising revenue. The 331 pairs of means with below-median amounts of imputation exhibit a correlation of 0.980 and the 331 pairs of means with above-median amounts of imputation exhibit a correlation of 0.975. This suggests that imputing advertising revenue does not significantly diminish the validity of the BIA data.

[9] We exclude 15 aggregation categories that are defined by total revenue because the BIA data is restricted to advertising revenue.

[10] In 2012, NAB received 785 responses to 1,288 questionnaires, a response rate of 60.9%.

[11] Some years, in particular, break out United Paramount and Spanish-language networks but not other minor networks. We conclude that the response rate of other minor networks is very low and thus exclude other minor networks from the cash flow estimation in Appendix A.1.

Table A22: NAB tables in 2012

Table	Description	Table	Description
1	All Stations, All Markets	34	ABC, CBS, FOX, NBC, Markets 176+
2	All Stations, Markets 1-10	35	ABC, All Markets
3	All Stations, Markets 11-20	36	ABC, Markets 1-25
4	All Stations, Markets 21-30	37	ABC, Markets 26-50
5	All Stations, Markets 31-40	38	ABC, Markets 51-75
6	All Stations, Markets 41-50	39	ABC, Markets 76-100
7	All Stations, Markets 51-60	40	ABC, Markets 101+
8	All Stations, Markets 61-70	41	CBS, All Markets
9	All Stations, Markets 71-80	42	CBS, Markets 1-25
10	All Stations, Markets 81-90	43	CBS, Markets 26-50
11	All Stations, Markets 91-100	44	CBS, Markets 51-75
12	All Stations, Markets 101-110	45	CBS, Markets 76-100
13	All Stations, Markets 111-120	46	CBS, Markets 101+
14	All Stations, Markets 121-130	47	FOX, All Markets
15	All Stations, Markets 131-150	48	FOX, Markets 1-50
16	All Stations, Markets 151-175	49	FOX, Markets 51-75
17	All Stations, Markets 176+	50	FOX, Markets 76-100
18	ABC, CBS, FOX, NBC, All Markets	51	FOX, Markets 101+
19	ABC, CBS, FOX, NBC, Markets 1-10	52	NBC, All Markets
20	ABC, CBS, FOX, NBC, Markets 11-20	53	NBC, Markets 1-25
21	ABC, CBS, FOX, NBC, Markets 21-30	54	NBC, Markets 26-50
22	ABC, CBS, FOX, NBC, Markets 31-40	55	NBC, Markets 51-75
23	ABC, CBS, FOX, NBC, Markets 41-50	56	NBC, Markets 76-100
24	ABC, CBS, FOX, NBC, Markets 51-60	57	NBC, Markets 101+
25	ABC, CBS, FOX, NBC, Markets 61-70	58	CW, All Markets
26	ABC, CBS, FOX, NBC, Markets 71-80	59	CW, Markets 1-25
27	ABC, CBS, FOX, NBC, Markets 81-90	60	CW, Markets 26-50
28	ABC, CBS, FOX, NBC, Markets 91-100	61	CW, Markets 51-75
29	ABC, CBS, FOX, NBC, Markets 101-110	62	MNTV, All Markets
30	ABC, CBS, FOX, NBC, Markets 111-120	63	MNTV, Markets 1-50
31	ABC, CBS, FOX, NBC, Markets 121-130	64	MNTV, Markets 51+
32	ABC, CBS, FOX, NBC, Markets 131-150	65	Independent, All markets
33	ABC, CBS, FOX, NBC, Markets 151-175	66	Independent, Markets 1-25

Table A23: Sample NAB data for select aggregation categories in 2012

	Advertising revenue ($ million)	Cash flow ($ million)				Non-broadcast revenue ($ million)	Fixed cost ($ million)
			Quartile				
	Mean	Mean	First	Second	Third	Mean	Mean
All Stations, All Markets	16.96	7.80	1.24	3.75	9.18	2.98	3.53
All Stations, Markets 101-110	8.27	4.12	1.70	3.62	6.44	2.10	2.46
ABC, CBS, FOX, NBC, All Markets	19.05	9.24	1.94	4.93	10.90	3.33	3.99
ABC, Markets 1-25	67.78	32.40	15.09	27.15	42.46	7.60	9.76
NBC, Markets 101+	7.57	3.65	1.29	3.28	5.90	1.88	2.19
CW, All Markets	13.35	3.93	0.35	1.80	3.22	2.88	2.60
MNTV, Markets 1-50	9.49	3.12	1.27	1.80	3.21	2.51	2.02
Independent, All Markets	13.43	2.79	-0.02	1.29	4.33	2.20	3.27

D Private equity firms

According to FCC filings, the Blackstone Group LP owns 99% of LocusPoint. NRJ is a media holding company funded through loans from Fortress Investment Group LLC according to a recent U.S. Securities and Exchange Commission filing. Lastly, OTA is a division of MSD Capital LP, which was formed to manage the wealth of Dell Computer founder Michael Dell.

D.1 Timeline of acquisitions and sales

Figures A7-A9 document the timeline of acquisitions (black) and sales (red) of TV stations by LocusPoint, NRJ, and OTA. As stated in the main text, from 2010 to 2015 these private equity firms acquired 48 UHF stations. In addition, LocusPoint acquired W33BY-D, WMJF-CD, and WBNF-CD for $4.8 million and sold them to HME Equity Fund II LLC for $23.75 million before the reverse auction;[12] we exclude these UHF stations from Figure A7. NRJ acquired KFWD for $9.9 million;[13] we include this VHF station in Figure A8. Finally, LocusPoint acquired WPHA-CD from D.T.V. LLC in a deal that apparently has not been consummated due to a law suit between the two parties; we exclude this UHF station from Figure A7.[14]

We obtain the holdings of LocusPoint, NRJ, and OTA as of 2015 from BIA. We rely on news coverage to confirm these holdings and identify any changes to them.[15] We have been unable to ascertain the purchase price for W24BB-D and thus set it to zero. If multiple TV stations were acquired in a single transaction, then we allocate the total purchase price to each acquired TV station in proportion to its interference free population.

The FCC released the identity of the TV stations that relinquished their licenses in the reverse auction along with their payouts. OTA voluntarily surrendered the license of WJPW-CD to the FCC.[16] We exclude from Table 2 and Figures A7-A9 any sales of non-spectrum assets such as programming contracts, or equipment.[17] We set the sales price of non-spectrum assets to zero if we cannot ascertain it separately in a transaction involving multiple TV stations.

[12]See http://www.tvnewscheck.com/article/92491/hme-equity-closes-on-purchase-of-3-lptvs, accessed on March 17, 2018.

[13]See http://www.tvnewscheck.com/article/89486/nrj-tv-buys-dallas-vhf-for-99-million, accessed on April 30, 2018.

[14]See https://publicfiles.fcc.gov/api/service/tv/application/1709537.html and Paragraph 81 of https://transition.fcc.gov/eb/Orders/2016/FCC-16-41A1.html, accessed on April 1, 2018.

[15]We primarily track TV station trading news through http://www.tvnewscheck.com/ and https://www.rbr.com/.

[16]See https://enterpriseefiling.fcc.gov/dataentry/public/tv/draftCopy.html?displayType=html&appKey=25076ff35f490dae015f4fa9968c0e0d&id=25076ff35f490dae015f4fa9968c0e0d&goBack=N, accessed on April 30, 2018.

[17]NRJ sold the non-spectrum assets of WGCB-TV, WMFP, and WTVE after relinquishing their licenses in the reverse auction and OTA sold the non-spectrum assets of KTLN-TV, WEBR-CD, WYCN-CD, and WLWC, see http://www.tvnewscheck.com/article/108526/station-trading-roundup-5-deals-259m, accessed on April 1, 2018, https://tvnewscheck.com/article/242153/station-trading-roundup-1-deal-81-2m/, accessed on July 14, 2020, https://tvnewscheck.com/article/108888/station-trading-roundup-1-deal-12500/, accessed on July 14, 2020, https://tvnewscheck.com/article/108526/station-trading-roundup-5-deals-25-9m/, accessed on July 14, 2020, and https://tvnewscheck.com/article/106271/nexstar-buys-zombie-station-wlwc-for-4-1m/, accessed on July 14, 2020.

Figure A7: Timeline of LocusPoint's acquisitions (black) and sales (red) of TV stations

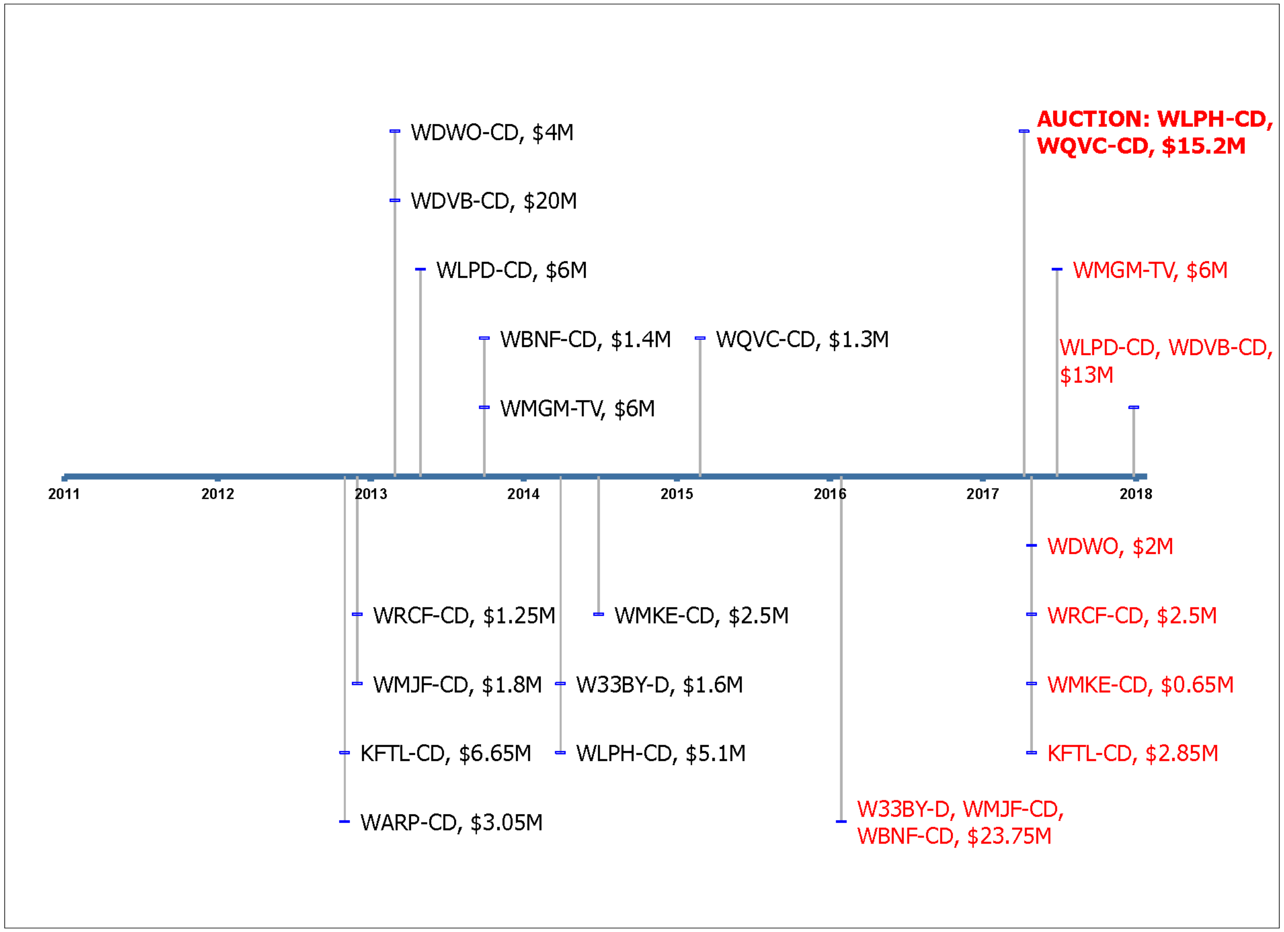


Figure A8: Timeline of NRJ's acquisitions (black) and sales (red) of TV stations

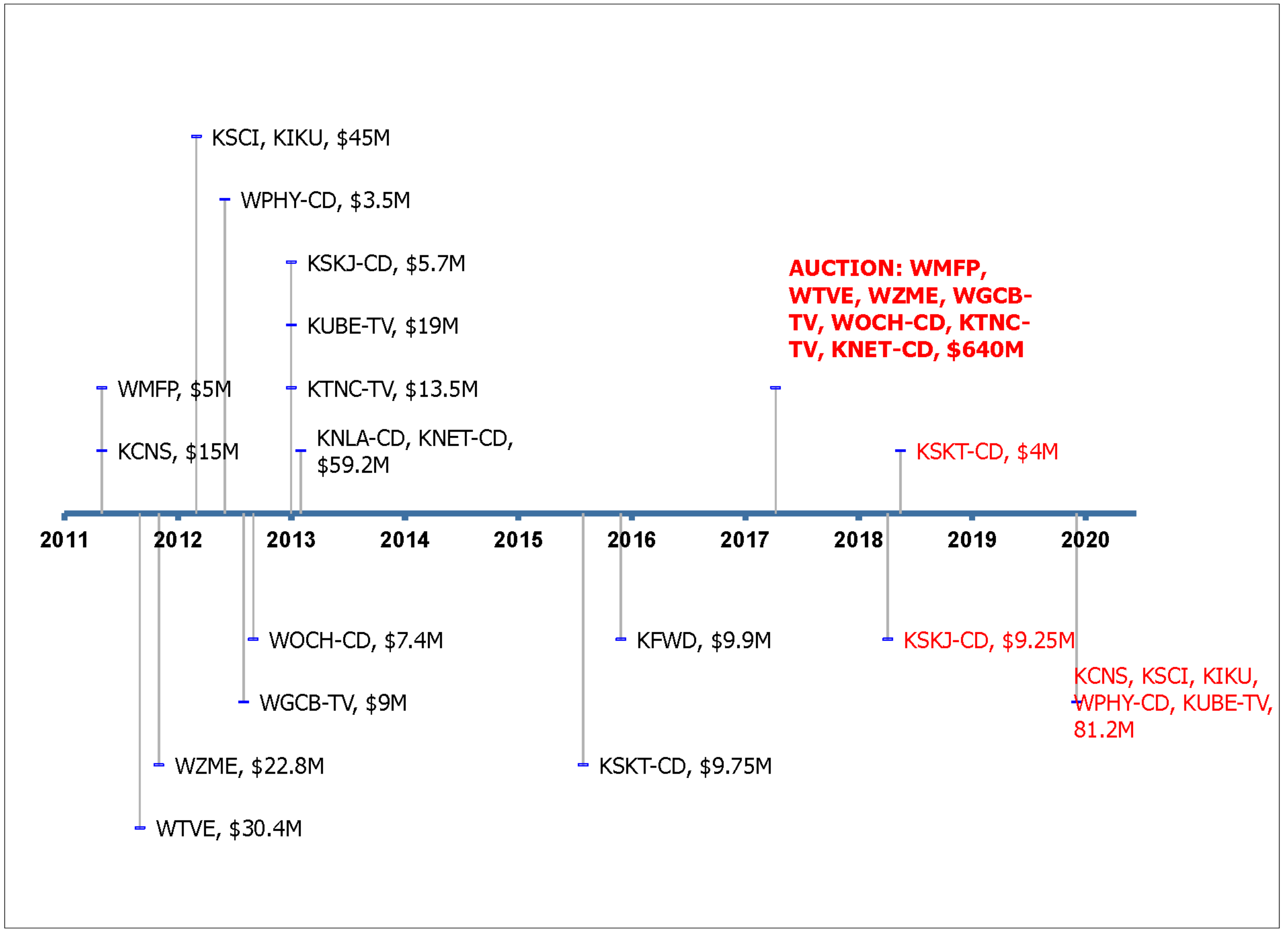
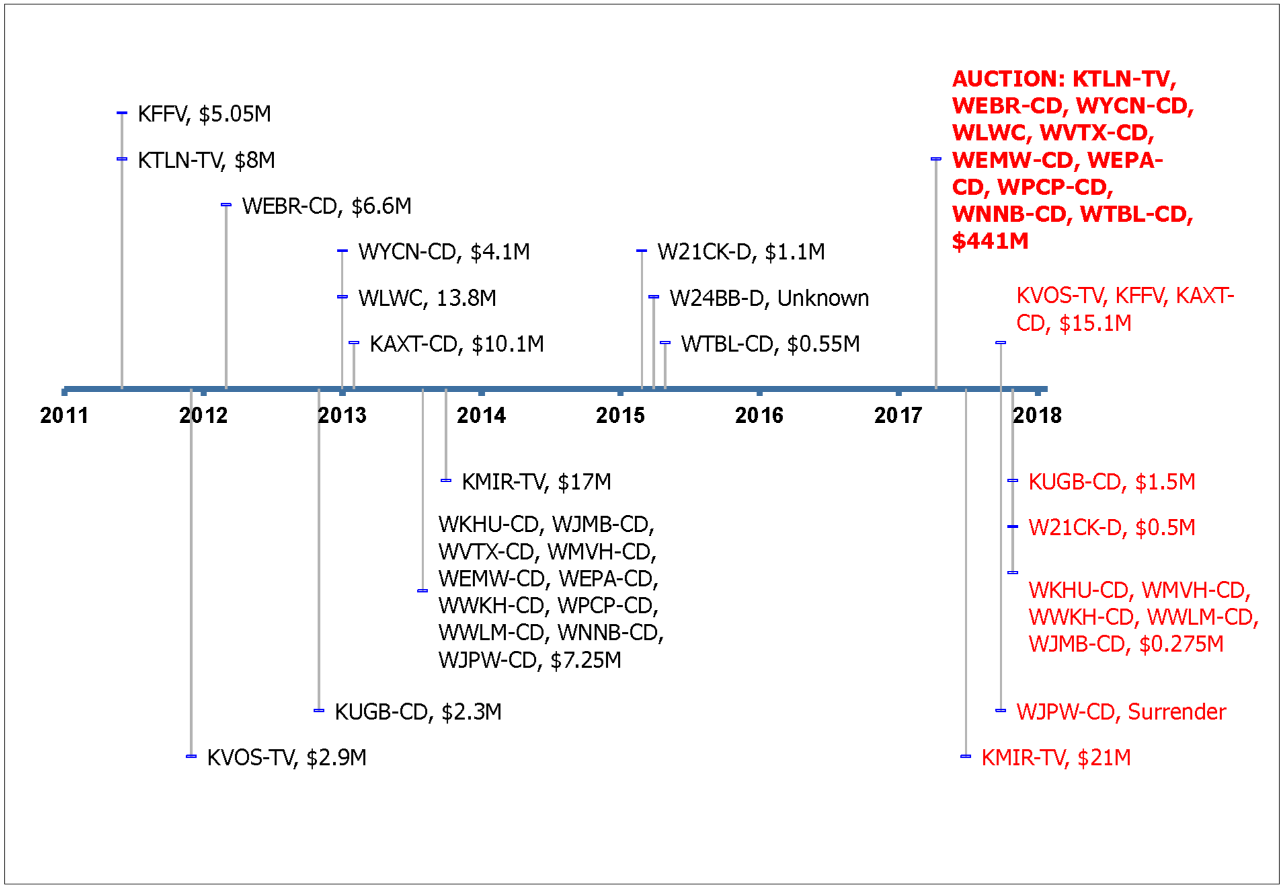


Figure A9: Timeline of OTA's acquisitions (black) and sales (red) of TV stations



D.2 Comparison of TV stations acquired by private equity firms and other transactions

Table A24 summarizes attributes of the 48 TV stations acquired by the three private equity firms and contrasts them with the 286 TV stations that were part of other transactions in the four years from 2010 to 2013. While there is considerable overlap in the distributions of transaction price and the other attributes between the two groups, the private equity firms acquired relatively cheaper TV stations. Moreover, the 48 TV stations acquired by the three private equity firms have higher broadcast volume, due to both higher interference free population and higher interference count.

Table A24: Comparison of TV stations acquired by private equity firms and other transactions from 2010 to 2013

	Private equity firms			Other transactions		
	Mean	Std. Dev.	Median	Mean	Std. Dev.	Median
Transaction price ($ million)	7.91	9.74	4.55	25.20	48.90	7.73
UHF	1	0	1	0.80	0.40	1
Commercial	0.98	0.14	1	0.98	0.13	1
Full-power	0.31	0.47	0	0.84	0.37	1
Major network	0.04	0.20	0	0.60	0.49	1
Broadcast volume (million)	0.28	0.16	0.28	0.17	0.13	0.14
Inference free population (million)	3.61	3.47	2.53	1.69	2.04	1.01
Interference count	104.10	35.41	101.50	79.44	47.35	72.50
Number of licenses		48			286	

E Regions

Table A25 summarizes the size of repacking regions for all 202 DMAs.

Table A25: Repacking regions for all 202 DMAs

			Quartile			
	Mean	Min	First	Second	Third	Max
Number of DMAs per region	11.6	1	6	12	17	26
Ratio between region and focal DMA						
Number of TV stations	18.8	1	6.9	13.6	21.6	160.0

F Pseudo code for algorithm

There are N TV stations in the focal DMA and its neighbors. Throughout we fix the vector $b = (b_1, \ldots, b_N)$ of their bids. Using the notation in Section 2, PO_j is the payout of TV station j from the reverse auction and π_j its profit. The base clock price is P, the set of active TV stations is A, the set of inactive TV stations is I, and the set of frozen TV stations is F, where we omit the dependence of these objects on the round τ of the reverse auction.

Full repacking. Algorithm 1 describes the algorithm that we use under full repacking as well as under naive bidding with $b = (s_1, \ldots, s_N)$. On line 1, $|Y| \leq 1$ by assumption, except possibly if $\tau = 1$, so that at most one active TV station opts to remain on the air.

Algorithm 1 Full repacking

Initialization: Set $\tau = 1$, $P = 900$, $A = \{1, \ldots, N\}$, $I = \emptyset$, and $F = \emptyset$.
Repeat

1. Let $Y = \{k \in A | b_k \geq P\}$ be the set of active TV stations that opt to remain on the air at a base clock price of P. Set $A \leftarrow A \setminus Y$, $I \leftarrow I \cup Y$, and $PO_j = \pi_j = 0$ for all $j \in Y$.
2. If $\tau = 1$ and $S(Y, R) \neq 1$, then these TV stations cannot be repacked and the reverse auction has failed at the outset (see footnote 26). Set a flag, $PO_j = \pi_j = 0$ for all $j \in A$, and terminate.
3. For all $k \in A$ do
   (a) If $S(I \cup \{k\}, R) \neq 1$, then active TV station k cannot additionally be repacked. In this case, set $A \leftarrow A \setminus \{k\}$, $F \leftarrow F \cup \{k\}$, $PO_k = \varphi_k P$, and $\pi_k = \varphi_k P - v_k$.
4. End
5. If $A \neq \emptyset$, then set $P = \max_{j \in A} b_j$, $\tau \leftarrow \tau + 1$, and continue with the decreased based clock price.
6. If $P = 0$, then the reverse auction concludes with a base clock price of 0 (see footnote 25). Set a flag, $F \leftarrow F \cup A$, $PO_j = 0$ and $\pi_j = -v_j$ for all $j \in A$, and $A = \emptyset$ (in this order).

Until $A = \emptyset$.

Limited repacking. Algorithm 2 describes the algorithm that we use under limited repacking. It takes the output of the algorithm under full repacking and naive bidding as an input.

We relabel TV stations such that TV stations $\{1, \ldots, K\}$ are in the focal DMA and TV stations $\{K + 1, \ldots, N\}$ are in the neighboring DMAs. We denote by $F^{*,full,naive}$ the (appropriately relabeled) set of frozen TV stations at the conclusion of the reverse auction from the algorithm under full repacking and naive bidding. In the initialization, $F^{*,full,naive} \cap \{K+1, \ldots, N\}$ is the set of TV stations in neighboring DMAs that have been frozen under full repacking and naive bidding; these TV stations cannot freeze another TV stations under limited repacking. On line 3, $A \cap \{1, \ldots, K\}$ is the set of active TV stations in the focal DMA; these are the only TV stations that can be frozen under limited repacking.

Algorithm 2 Limited repacking

Initialization: Set $\tau = 1$, $P = 900$, $A = \{1, \ldots, N\} \setminus \left(F^{*,full,naive} \cap \{K+1, \ldots, N\}\right)$, $I = \emptyset$, and $F = F^{*,full,naive} \cap \{K+1, \ldots, N\}$.

Repeat

1. Let $Y = \{k \in A | b_k \geq P\}$ be the set of active TV stations that opt to remain on the air at a base clock price of P. Set $A \leftarrow A \setminus Y$, $I \leftarrow I \cup Y$, and $PO_j = \pi_j = 0$ for all $j \in Y$.

2. If $\tau = 1$ and $S(Y, R) \neq 1$, then these TV stations cannot be repacked and the reverse auction has failed at the outset (see footnote 26). Set a flag, $PO_j = \pi_j = 0$ for all $j \in A$, and terminate.

3. For all $k \in A \cap \{1, \ldots, K\}$ do

   (a) If $S(I \cup \{k\}, R) \neq 1$, then active TV station k cannot additionally be repacked. In this case, set $A \leftarrow A \setminus \{k\}$, $F \leftarrow F \cup \{k\}$, $PO_k = \varphi_k P$, and $\pi_k = \varphi_k P - v_k$.

4. End

5. If $A \neq \emptyset$, then set $P = \max_{j \in A} b_j$, $\tau \leftarrow \tau + 1$, and continue with the decreased base clock price.

6. If $P = 0$, then the reverse auction concludes with a base clock price of 0 (see footnote 25). Set a flag, $F \leftarrow F \cup A$, $PO_j = 0$ and $\pi_j = -v_j$ for all $j \in A$, and $A = \emptyset$ (in this order).

Until $A = \emptyset$.

G Robustness

In this appendix, we explore the impact of limited repacking and underbidding on our results.

G.1 Limited repacking

We assess the effect of the limited repacking in two ways. First, we compare limited to full repacking for all 202 DMAs under naive bidding and both the 84 MHz and the 126 MHz clearing target. Table A26 shows that moving to full repacking reduces nationwide payouts by 0.2% under the 126 MHz clearing target and by 1.5% under the 84 MHz clearing target. This payout reduction is driven by the smaller number of TV stations that are acquired in the reverse auction under the more flexible full repacking, as Table A26 shows. A lowering of the clearing target, and the smaller number of TV stations that have to be acquired to meet it, amplifies this effect. Closer inspection shows that the differences in payouts under full and limited repacking are minor: the largest discrepancy across simulation draws is in the San Diego, CA, DMA ($341 thousand) at the 126 MHz clearing target and in the New York, NY, DMA ($41 thousand) at the 84 MHz clearing target. At the same time, the correlation between payouts under full and limited repacking is 1.0000 for the 126 MHz clearing target across DMAs and simulation draws and 0.9998 for the 84 MHz clearing target, suggesting that limited repacking captures the distribution of payouts well.

Table A26: Nationwide payouts to TV stations and number of TV stations acquired under naive bidding and full repacking

	Naive bidding	
	Payouts ($ billion)	Number of TV stations acquired
Panel A: 126 MHz clearing target		
Limited repacking	15.767	452.022
	(2.639)	(11.052)
Full repacking	15.734	441.600
	(2.637)	(9.153)
Panel B: 84 MHz clearing target		
Limited repacking	2.478	182.609
	(0.360)	(8.942)
Full repacking	2.441	160.580
	(0.356)	(4.985)

Second, we compare limited to full repacking for the New York, NY, DMA under strategic bidding, as doing so for all 202 DMAs is not computationally feasible. As Table A27 shows, limited repacking has a modest impact on payouts in the New York, NY, DMA and on the gains from strategic bidding for both the 126 MHz and the 84 MHz clearing target.

Table A27: Payouts to TV stations in New York, NY, DMA under strategic bidding and full repacking

Payouts ($ billion)	Naive bidding	Strategic bidding				Payout increase at mean (%)
		Mean	Min	Median	Max	
Panel A: 126 MHz clearing target						
Limited repacking	3.072	5.100	4.369	5.053	5.889	66.0
	(1.169)	(2.119)	(2.125)	(2.204)	(2.628)	
Full repacking	3.072	5.039	4.323	5.023	5.788	64.0
	(1.169)	(2.082)	(2.076)	(2.141)	(2.592)	
Panel B: 84 MHz clearing target						
Limited repacking	0.373	0.415	0.403	0.415	0.428	11.3
	(0.117)	(0.127)	(0.124)	(0.128)	(0.135)	
Full repacking	0.371	0.409	0.394	0.408	0.422	10.0
	(0.116)	(0.127)	(0.121)	(0.131)	(0.132)	

Notes: Payout increase at mean calculated as percent difference between mean payouts under strategic and naive bidding.

G.2 Underbidding

We investigate the impact of underbidding on payouts for the New York, NY, DMA under the 84 MHz clearing target and assume that the strategy space of TV station j is $b_j \in \{0, s_j, 900\}$ instead of $b_j \in \{s_j, 900\}$ if it is jointly owned. This increases the number strategy profiles from 189 to 8,575. To lighten the computational burden, we reduce to number of simulation draws from $N^S = 100$ to $N^S = 50$.

Table A28: Payouts to TV stations in New York, NY, DMA with underbidding

Payouts ($ billion)	Naive bidding	Strategic bidding: Mean	Min	Median	Max	Payout increase at mean (%)
Panel A: 84 MHz clearing target						
Base case	0.375	0.410	0.398	0.409	0.423	9.5
	(0.103)	(0.109)	(0.109)	(0.108)	(0.112)	
With underbidding	0.375	0.411	0.395	0.411	0.425	9.6
	(0.103)	(0.112)	(0.113)	(0.112)	(0.112)	

Notes: Payout increase at mean calculated as percent difference between mean payouts under strategic and naive bidding. Using $N^S = 50$ simulation draws.

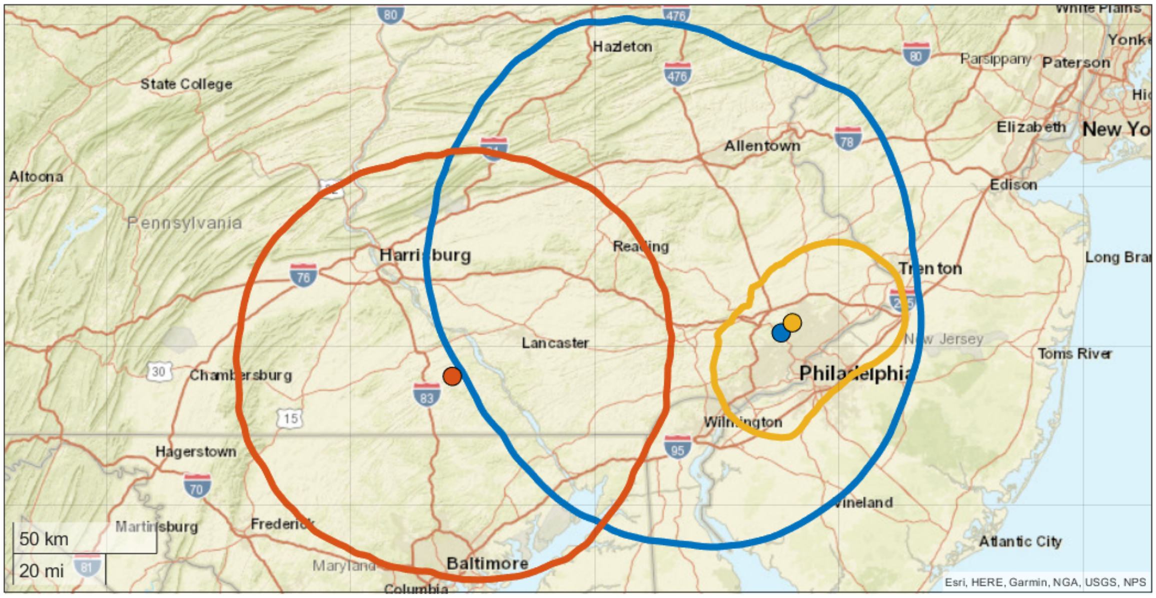
As Table A28 shows, allowing for underbidding has a small impact on payouts. Although allowing for underbidding enlarges the set of payout-unique equilibria, the overlap with the set of payout-unique equilibria in the base case that rules out underbidding is large. In the base case, we find 2,592 equilibria across simulation draws that map into 138 payout-unique equilibria. With underbidding, across the same draws, we find 13,234 equilibria that map into 200 payout-unique equilibria. Yet, 120 payout-unique equilibria appear in both the base case and with underbidding.

H Multi-market strategies

We continue with the Philadelphia, PA, DMA as a case study to illustrate how multi-market strategies may work. The 24 TV stations in the Philadelphia, PA, DMA are held by 18 owners. Twelve of these owners hold at least one additional license in the repacking region but outside the Philadelphia, PA, DMA. Abandoning the restriction from Section 6.2 that any TV station outside the focal DMA bids truthfully increases the number of strategy profiles from 729 to 8.80 trillion. As this is computationally infeasible, we focus on one of the twelve owners that hold at least one additional license in the repacking region, namely NRJ. This increases the number of strategy profiles from 729 to 1701.

In late 2012, NRJ purchased WGCB-TV in the Harrisburg, PA, DMA for $9 million. WGCB-TV is located in Red Lion, PA, towards both the Philadelphia, PA, and Baltimore, MD, DMAs. While NRJ owns no other TV station in the Harrisburg, PA, DMA, it had previously purchased WTVE and WPHY-CD in the Philadelphia, PA, DMA in late 2011 and early 2012 for $30.4 million and

Figure A10: Service contours of WGCB-TV, WTVE, and WPHY-CD



Notes: Dots denote facility locations. The red dot denotes WGCB-TV in the Harrisburg, PA, DMA. The blue dot denotes WTVE and the yellow dot denotes WPHY-CD in the Philadelphia, PA, DMA.

\$3.5 million, respectively. Figure A10 shows the overlap between the service contours of WGCB-TV (in red), WTVE (in blue), and WPHY-CD (in yellow).[18] WGCB-TV has a very high interference count and may interfere with 161 TV stations in the repacking process. Hence, if NRJ withholds WGCB-TV from the reverse auction, this may affect prices in the Philadelphia, PA, DMA and potentially other DMAs as well; alternatively, withholding a TV station in the Philadelphia, PA, DMA may increase the payout to WGCB-TV.

To investigate, we allow NRJ to bid strategically on WGCB-TV in concert with its TV stations in the Philadelphia, PA, DMA. Table A29 compares payouts to TV stations in the Philadelphia, PA, DMA under the multi-market strategy to payouts in our base case. On average across payout-unique equilibria and simulation draws, payouts increase by 4.8% under the 126 MHz clearing target and by 6.3% under the 84 MHz clearing target. The gains from strategic bidding increase as well under the multi-market strategy. The fact that accounting for a single case of cross-market multi-license ownership has a discernible impact suggests that accounting for all such cases—if it were computationally feasible—potentially has a dramatic impact on payouts in the reverse auction.

[18]We obtain service contours from the FCC's TV Query Broadcast Station Search at https://www.fcc.gov/media/television/tv-query, accessed on March 15, 2018.

Table A29: Payouts to TV stations in Philadelphia, PA, DMA under multi-market strategy

Payouts ($ billion)	Naive bidding	Strategic bidding: Mean	Min	Median	Max	Payout increase at mean (%)
Panel A: 126 MHz clearing target						
Base case	1.826	3.273	2.783	3.222	3.818	79.2
	(0.702)	(1.461)	(1.558)	(1.531)	(1.768)	
Multi-market strategy	1.826	3.431	2.829	3.449	4.039	87.9
	(0.702)	(1.482)	(1.533)	(1.567)	(1.811)	
Panel B: 84 MHz clearing target						
Base case	0.285	0.336	0.317	0.333	0.358	17.9
	(0.085)	(0.116)	(0.109)	(0.120)	(0.137)	
Multi-market strategy	0.285	0.357	0.335	0.352	0.384	25.3
	(0.085)	(0.120)	(0.117)	(0.118)	(0.146)	

Notes: Payouts under multi-market strategy exclude WGCB-TV for comparability to base case. Payout increase at mean calculated as percent difference between mean payouts under strategic and naive bidding.

I Efficiency

We say that an outcome is efficient if it meets the clearing target and minimizes the total reservation value of acquired TV stations or, equivalently, if it meets the clearing target and maximizes the total reservation value of TV stations that remain on the air. To obtain the efficient outcome, we follow Newman et al. (2017) and solve the binary programming problem detailed below. We compare the efficient outcome to the outcome of the reverse auction under naive bidding in terms of TV stations that go off the air and compute the value loss ratio, defined as the total reservation value of acquired TV stations in the reverse auction relative to the efficient outcome. We take the regional approach described in Section 5.2 by restricting the binary programming problem to a repacking region. Similar to Newman et al. (2017) in their analysis of New York, NY, we compute the value loss ratio considering all TV stations in the repacking region.[19]

Binary programming problem. There are N TV stations in the focal DMA and its neighbors with reservation values $(v_1, \ldots, v_N)$ in a given simulation draw. The clearing target defines the set of channels R that are available for repacking TV stations that remain on the air. Define the indicator $x_{j,c}$ to equal one if TV station j is assigned to channel c and zero otherwise. Consequently, TV station j remains on the air if $\sum_c x_{j,c} > 0$. Define $I(x) = \{j | \sum_c x_{j,c} > 0\}$ to be the set of all TV stations that remain on the air, where x is the vector of assignments of TV stations to channels. We solve the binary programming problem

$$\max_x \sum_j \sum_c x_{j,c} v_j \tag{A12}$$

subject to $S(I(x), R) = 1$ and $\sum_c x_{j,c} \leq 1$ for all j. The first constraint ensures that the assignment of TV stations to channels is feasible and the second constraint that a TV station is either assigned a single channel or goes off the air.

In practice, instead of calling the feasibility checker $SATFC$, we follow Newman et al. (2017) and add the underlying constraints from the domain and pairwise interference files described in Section 3.2 to the binary programming problem. For a given clearing target, define R_j to be the set of channels that are available for repacking TV station j per the domain file and Q to be the set of all pairs of TV stations and channel assignments that are not feasible per the pairwise interference file. We solve the binary programming problem in equation (3.2) subject to

[19] Restricting the computation of the value loss ratio to the TV stations in the focal DMA causes excess volatility and skewness for two reasons. First, as the binary programming problem considers all TV stations in the repacking region, the value loss ratio is no longer bounded below by one. Second, the value loss ratio becomes infinite if the efficient outcome does not entail acquiring any TV station in the focal DMA. As a result, the value loss ratio restricted to the TV stations in the focal DMA can be larger than what we report below.

$$x_{j,c} + x_{j',c'} \leq 1 \text{ for all } (j, c, j', c') \in Q,$$
$$\sum_c x_{j,c} \leq 1 \text{ for all } j,$$
$$x_{j,c} = 0 \text{ for all } c \notin R_j \text{ and all } j.$$

The first constraint enforces that TV stations j and j' cannot be assigned channels c and c', respectively, if this is not feasible per the pairwise interference file. In case of a same-channel constraint between TV stations j and j', we have $c = c'$, and in case of an adjacent-channel constraint, we have $c = c' \pm 1$. As both the objective function and the constraints are linear, we use CPLEX to solve the binary programming problem.

Results. Table A30 shows the value loss ratio, averaged across simulation draws, for select DMAs for the 84 MHz and 126 MHz clearing targets. We conduct the analysis for the top ten DMAs in terms of payouts in the actual reverse auction.[20] This set includes seven out of the ten largest DMAs, as well as Milwaukee, WI, Hartford-New Haven, CT, and Providence, RI-New Bedford, MA. The value loss ratios are between 1.05 and 1.15 for the 84 MHz clearing target and between 1.04 and 1.11 for the 126 MHz clearing target. By comparison, Newman et al. (2017) restrict attention to 218 TV stations in a neighborhood of New York, NY, and the 126 MHz clearing target and report a value loss ratio of 1.05. Overall, the potential efficiency gains from re-designing the reverse auction appear to be limited.

Table A30: Value loss ratio for top ten DMAs

		Clearing target	
	Payout rank	84 MHz	126 MHz
New York, NY	1	1.11	1.05
Los Angeles, CA	2	1.05	1.07
Philadelphia, PA	3	1.08	1.04
San Francisco, CA	4	1.06	1.05
Boston, MA	5	1.15	1.11
Washington, DC	6	1.09	1.04
Chicago, IL	7	1.11	1.07
Milwaukee, WI	8	1.11	1.06
Hartford, CT	9	1.08	1.04
Providence, RI	10	1.08	1.04

Notes: Using $N^S = 98$ simulation draws for the New York, NY, DMA and 84 MHz clearing target, as CPLEX did not solve the binary programming problem for the remaining draws within one month with 32 CPUs.

[20] To give a sense of the computational burden, the analysis took a total of roughly 13,000 CPU-days.

J Data references

Availability Statements for Proprietary Data Sources

BIA The BIA data are from the MEDIA Access Pro Database from BIA Kelsey. More information can be found at `https://www.bia.com/`.

NAB The NAB data are from the Television Financial Report by the National Association of Broadcasters. More information can be found at `https://my.nab.org/store/s/nab-publications`.

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