Supplemental Appendix to "Capital–Labor Substitution and Firms' Labor Shares"

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A Proofs and derivations for the model with CES demand

A.1 Policy functions and steady state

This sub-section proves Proposition 1 and characterizes the steady state of the economy. For these results, we work with a CES demand function

$$y_t = \left(\int_f y_{tf}^{\frac{\sigma-1}{\sigma}} \cdot df\right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma > 1$ denotes the elasticity of substitution across varieties.

Proof of Proposition 1. We first show that $\tilde{\alpha}_t(\alpha, z)$ weakly increases in z. We have

$$\tilde{\alpha}_t(\alpha, z) = \underset{\alpha' \in [\alpha, 1]}{\arg\max} -c_a \cdot y_t \cdot (\alpha' - \alpha) + \frac{1}{1+r}\mathbb{E}[V_{t+1}(\alpha', z')|z].$$

It is therefore sufficient to show that $\mathbb{E}[V_{t+1}(\alpha, z')|z]$ has increasing differences in (α, z). Let $\Omega_{t+1}(\alpha', z) = \partial_\alpha \mathbb{E}[V_{t+1}(\alpha', z')|z]$. The envelope theorem implies

$$\text{(A1)} \qquad \Omega_t(\alpha', z) = \mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right] + \mathbb{E}\left[P_t(z') \cdot \min\left\{c_a \cdot y_t, \frac{1}{1+r}\Omega_{t+1}(\alpha', z')\right\}\Big|z\right],$$

where $P_t(z')$ denotes the probability of survival given z', and the minimum operator accounts for the fact that the restriction $\alpha' \geq \alpha$ binds in some states.

Let's define a sequence of functions $\Omega_t^{(n)}$ of (α', z) as:

$$\begin{aligned}\Omega_t^{(1)}(\alpha', z) =& \mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right]\\ \Omega_t^{(n+1)}(\alpha', z) =& \mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right] + \mathbb{E}\left[P_t(z') \cdot \min\left\{c_a \cdot y_t, \frac{1}{1+r}\Omega_{t+1}^{(n)}(\alpha', z')\right\}\Big|z\right].\end{aligned}$$

We prove by induction in n that $\Omega_t^{(n)}(\alpha', z)$ weakly increases in z. The base case for $n = 1$ follows from the fact that $\mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right]$ increases in z. Because the process is assumed to be increasing (in a stochastic sense), this is equivalent to showing that $\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}$ increases in z'.

Let's write the marginal cost of firms as

$$c_t(\alpha, z) = \frac{1}{z}c_t(\alpha), \quad \text{with} \quad c_t(\alpha) = \min_{a \leq \alpha}\left(\Gamma_t^k(a) + \Gamma^\ell(a) \cdot w_t^{1-\eta}\right)^{\frac{1}{1-\eta}}.$$

This formulation is more general than the one from (1) in the main text. It allows for the possibility that firms might have paid the fixed cost to automate all tasks up to α, and yet, due to changing factor prices, choose to allocate some of these tasks to labor.

A second application of the envelope theorem (but now with respect to the optimal pricing decision of firms) implies

$$\frac{\partial \pi_t(\alpha', z')}{\partial \alpha} = -\frac{y_t(\alpha', z')}{z'} \cdot \frac{\partial c_t(\alpha')}{\partial \alpha},$$

where $y_t(\alpha', z')$ is the quantity sold by a firm with technology $\alpha_{tf} = \alpha'$ and $z_{tf} = z'$ at time t. Here, $\frac{\partial c_t(\alpha')}{\partial \alpha}$ is weakly negative (firms always get the option value of automating tasks if factor prices justify it). This means that $\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}$ is increasing in z' if $\frac{y_t(\alpha', z')}{z'}$ increases in z', which holds in the CES demand systems when $\sigma > 1$.

For the inductive step, suppose that $\Omega_t^{(n)}(\alpha', z)$ is weakly increasing in z for all (t, α) with $n \leq N$. We have

$$\Omega_t^{(N+1)}(\alpha', z) = \mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right] + \mathbb{E}\left[P_t(z') \cdot \min\left\{c_a \cdot y_t, \frac{1}{1+r}\Omega_{t+1}^{(N)}(\alpha', z')\right\}\Big|z\right].$$

As before, $\mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right]$ weakly increases in z. Moreover, $P_t(z') \cdot \min\{c_a \cdot y_t, (1/(1+r)) \cdot \Omega_{t+1}^{(N)}(\alpha', z')\}$ (weakly) increases in z' (due to the inductive hypothesis), and so the term $\mathbb{E}\left[P_t(z') \cdot \min\{c_a \cdot y_t, (1/(1+r)) \cdot \Omega_{t+1}^{(N)}(\alpha', z')\}|z\right]$ also (weakly) increases in z, which completes the inductive step.

Because the set of weakly increasing functions is closed, $\Omega_t(\alpha', z) = \lim_{n\to\infty} \Omega_t^{(n)}(\alpha', z)$ is also weakly increasing in z. It follows that $\mathbb{E}[V_{t+1}(\alpha', z')|z]$ has increasing differences in (α', z) as wanted.

We now turn to the limiting behavior of $\tilde{\alpha}_t(\alpha, z)$ as z grows to infinity. Automation decisions are guided by $\Omega_{t+1}(\alpha', z)$, which gives the marginal benefit to the firm of automating tasks up to α'. Suppose that $\alpha < \alpha_{t+1}^*$, and take any $\alpha' \in [\alpha, \alpha_{t+1}^*]$. With a CES demand system, $\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}$ is an increasing and unbounded function of z', unless $\alpha' = \alpha_{t+1}^*$, in which case this is zero. As required in footnote 11, this implies that $\mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right]$ and the right-hand side of equation (A1) converge to infinity as $z \to \infty$, unless $\alpha' = \alpha_{t+1}^*$. Optimal policy thus sets $\alpha' = \alpha_{t+1}^*$.

We conclude by exploring the limiting behavior of $\tilde{\alpha}_t(\alpha, z)$ as z goes to zero. The conditions in footnote 11 imply that $\mathbb{E}\left[\frac{\partial \pi_t(\alpha', z')}{\partial \alpha}|z\right]$ and the right-hand side of equation (A1) converge to zero as $z \to 0$. This implies $\Omega_{t+1}(\alpha', z) = 0$ for all α' and optimal policy keeps $\alpha' = \alpha$. ■

Remark: the above proof shows that automation decisions and productivity levels are complementary if $\frac{y_t(\alpha,z)}{z}$ increases in z. A demand system satisfies this property if the product of the demand elasticity and the passthrough of marginal costs into prices exceeds 1 for all firms. This holds with a CES demand (demand elasticity σ and passthrough 1), but does not hold generically with a log-concave demand system. In any case, the proof above also shows that this complementarity will only break down for extremely large firms for which σ approaches 1. These firms reduce their use of labor and capital as their productivity increases and have no incentives to automate further.

The next Proposition shows that the model admits a stationary equilibrium where all firms allocate all tasks below a common α^* to capital and the behavior of aggregates is identical to that from a model where firms face no fixed costs of automation.

Consider a version of our model where firms face no fixed costs of automation (and so they set $\alpha_{tf} = \alpha^*_t$) and capital prices are fixed, so that $q_t(x) = q(x)$. This version of our model is equivalent to a standard Hopenhayn model where firms marginal costs only depend on their productivity z and are given by

$$c_t(z) = \frac{1}{z} \cdot \min_{\alpha} \left(\Gamma^k(\alpha) + \Gamma^\ell(\alpha) \cdot w_t^{1-\eta}\right)^{\frac{1}{1-\eta}}. \tag{A2}$$

As shown in Hopenhayn (1992), this model has a unique stationary equilibrium. Let w^* denote the wage in this stationary equilibrium an α^* the common level of automation that minimizes marginal costs for this wage level.

Proposition A1 *Suppose $q_t(x) = q(x)$. The economy admits a unique stationary equilibrium with wage w^*. In this stationary equilibrium, $\alpha_{tf} \geq \alpha^*$ almost surely (i.e. for all firms except a set of measure zero) and produce tasks below α^* with capital and tasks above α^* with labor.*

Proof. Suppose the wage converges to w and let α denote the level of automation that minimizes marginal costs for this wage level.

Consider the path for $\bar{\alpha}_t$. Define

$$\alpha_{inf} = \lim_{T\to\infty} \inf_{t\geq T}\{\bar{\alpha}_t\}.$$

We first show that we cannot have $\alpha_{inf} < \alpha$ by way of contradiction.

Suppose $\alpha_{inf} < \alpha$. For large t, firms alive at t started with a level of automation of at least α_{inf} almost surely. This follows from the restrictions in footnote 11, which imply that all firms exit with positive probability, and so the probability that firms alive at time t where born at T or later converges to 1 as $t \to \infty$.

For these firms, $\alpha_{tf} \geq \alpha_{inf}$. Moreover, a positive mass m of these firms received positive productivity shocks and increased α_{tf} above $\alpha_{inf} + \delta$, for a positive but small δ such that $\alpha_{inf} + \delta < \alpha$. This follows from Proposition 1 and the restrictions in footnote 11, which imply that firms reach the requisite size to justify investing in increasing α_{tf} up to $\alpha_{inf} + \delta$ with positive probability. However, This would imply $\bar{\alpha}_t \geq \alpha_{inf} + m \cdot \delta$, contradicting the definition of α_{inf}.

This means we must have $\alpha_{min} \geq \alpha$. For large t, firms alive at t started with a level of automation of at least α almost surely. This means that $\alpha_{tf} \geq \alpha$ almost surely and the economy converges to a standard firm-dynamics model where firms costs are given by (A2), as wanted.

As shown in Hopenhayn (1992), this model features a unique stationary equilibrium, with wage w^*. In this stationary equilibrium, $\alpha_{tf} \geq \alpha^*$ and firms produce all tasks below α^* with capital and all tasks above α^* with labor almost surely. Note that there might be a measure-zero set of firms with $\alpha_{tf} < \alpha^*$ that do not exit, and that is why the proposition claims $\alpha_{tf} \geq \alpha^*$ almost surely. ■

A.2 Effects of q shocks

This subsection proves Propositions 2 and 3. We first provide a technical lemma that helps characterize the impact of q shocks on real wage levels. This lemma is not central to this paper, but is proven here for completeness.

Lemma A1 (Effects of q shocks on the stationary equilibrium) *Let c denote the (real) marginal cost for a firm with unit productivity in the stationary equilibrium. The stationary distribution of firm productivities and c remain unchanged following a q shock.*

Proof. The proof is by construction and involves showing that this outcome satisfies the equilibrium conditions E1–E6.

Let $f(z)$ denote the mass of firms with productivity z in the initial stationary distribution and c the (real) marginal cost for a firm with unit productivity in this equilibrium. By construction, this equilibrium satisfies E1–E5.

Consider an arbitrary q shock. We guess and verify that the wage adjusts in the new equilibrium so as to keep $f(z)$ and c unchanged. In turn, output adjusts as to ensure labor-market clearing.

We verify this conjecture in steps:

- First, note that the ideal-price index depends on c and the distribution of z, both of which we conjectured remain unchanged. This shows that our conjecture satisfies E1.

- Second, note that firm entry and exit decisions conditional on z remain unchanged. This is because the demand faced by firm and their operating costs scale with output y, while their marginal cost remains constant and equal to $(1/z) \cdot c$. This means that firms value functions will scale with y. Notice that in the initial and final equilibrium, firms set $\alpha_{t+1,f} = \alpha_{tf}$ (a consequence of Proposition A1). This means that there are no costs incurred for automating additional tasks in a stationary equilibrium. This shows that our conjecture satisfies E3 and E4.

- Third, because entry and exit decisions conditional on z remain unchanged, the stationary distribution of productivity $f(z)$ also remains unchanged. This shows that our conjecture satisfies E5.

- Finally, the change in output is pinned down by labor market clearing, which shows that our conjecture satisfies E2.

The same proof applies to any homothetic demand system. ■

Proof of Proposition 2. Write $q(x) = q \cdot q_0(x)$ for tasks below α^* and consider a permanent increase in q by $d \ln q_{int}$ for these tasks. The equilibrium impact of this shock is to change wages by $d \ln w$ and automation decisions by a common amount $d \ln \alpha^*$ (a consequence of Proposition A1).

We first characterize the effect of this q shock on wages. Lemma A1 shows that wages adjust so as to keep unit costs unchanged. An application of Shephard's lemma implies that

$$d \ln c = \varepsilon^\ell \cdot d \ln w - \varepsilon^k \cdot d \ln q_{int},$$

where, in addition, the envelope theorem ensures that the effect of changes in α^* on c are second order and can be ignored. Because $d \ln c = 0$, we can solve for the change in wages as

$$d \ln w = \frac{\varepsilon^k}{\varepsilon^\ell} \cdot d \ln q_{int}.$$

We now turn to the behavior of cost shares (or equivalently, output elasticities). In steady state, all firms have the same labor cost share (a consequence of Proposition A1), given by

$$\varepsilon^\ell = \frac{\Gamma^\ell(\alpha^*) \cdot w^{1-\eta}}{\Gamma_0^k(\alpha^*) \cdot q^{\eta-1} + \Gamma^\ell(\alpha^*) \cdot w^{1-\eta}}.$$

This common cost share for labor varies with wages and α^*. Equation (2) implies that the change in the optimal threshold α^* satisfies

$$d\ln\alpha^* = \frac{1}{\partial\ln\psi_\ell(\alpha^*)/q(\alpha^*)/\partial\ln\alpha}\cdot d\ln w.$$

Using this expression for $d\ln\alpha^*$ and the definition of η^*, we obtain

$$\begin{aligned}
d\ln\varepsilon^\ell &= \varepsilon^k\cdot d\ln\frac{\varepsilon^\ell}{\varepsilon^k}\\
&= \varepsilon^k\cdot(1-\eta)\cdot(d\ln q_{int}+d\ln w)+\varepsilon^k\cdot\frac{\partial\ln\Gamma^\ell(\alpha^*)/\Gamma_0^k(\alpha^*)}{\partial\ln\alpha}\cdot d\ln\alpha^*\\
&= \varepsilon^k\cdot(1-\eta)\cdot(d\ln q_{int}+d\ln w)+\varepsilon^k\cdot(\eta-\eta^*)\cdot d\ln w\\
&= \varepsilon^k\cdot(1-\eta^*)\cdot d\ln w+\varepsilon^k\cdot(1-\eta)\cdot d\ln q_{int}.
\end{aligned}$$

Along the transition, firms will differ in the extent to which they will automate their tasks. Let $d\ln\alpha_{tf}$ denote the additional tasks automated by firm f at time t. We have that

$$d\ln\varepsilon^\ell_{tf} = \varepsilon^k\cdot(1-\eta)\cdot(d\ln q_{int}+d\ln w_t)+\varepsilon^k\cdot\frac{\partial\ln\Gamma^\ell(\alpha^*)/\Gamma_0^k(\alpha^*)}{\partial\ln\alpha}\cdot d\ln\alpha_{tf}.$$

The expression for incumbents that do not automate follows from taking $d\ln\alpha_{tf}=0$. ∎

Proof of Proposition 3. Write $q(x)=q\cdot q_0(x)$ for tasks above α^* and consider a permanent increase in q by $d\ln q_{ext}$ for these tasks. The equilibrium impact of this shock is to change wages by $d\ln w$ and automation decisions by a common amount $d\ln\alpha^*$ (a consequence of Proposition A1).

We first characterize the effect of this q shock on wages. Lemma A1 shows that wages adjust so as to keep unit costs unchanged. The envelope theorem ensures that the effect of changes in α^* on c are negative but second order, while the effects of changes in α^* on c are positive and first order. Because $d\ln c=0$, wages must increase by a positive but second order amount, $d\ln w=\mathcal{O}(d\ln q_{ext}^2)>0$.

We now turn to the behavior of cost shares (or equivalently, output elasticities). In steady state, all firms have the same labor cost share (a consequence of Proposition A1), given by

$$\varepsilon^\ell = \frac{\Gamma^\ell(\alpha^*)\cdot w^{1-\eta}}{\Gamma_0^k(\alpha^*)+\Gamma^\ell(\alpha^*)\cdot w^{1-\eta}}.$$

This common cost share for labor will vary with prices and α^*. Equation (2) implies that the change in the optimal threshold α^* satisfies

$$d\ln\alpha^* = \frac{1}{\partial\ln\psi_\ell(\alpha^*)/q(\alpha^*)/\partial\ln\alpha}(d\ln w+d\ln q_{ext}).$$

Using this expression for $d\ln\alpha^*$ and the definition of η^*, we obtain

$$\begin{aligned}
d\ln\varepsilon^\ell &= \varepsilon^k \cdot d\ln\frac{\varepsilon^\ell}{\varepsilon^k} \\
&= \varepsilon^k \cdot (1-\eta)\cdot d\ln w + \varepsilon^k \cdot \frac{\partial \ln \Gamma^\ell(\alpha^*)/\Gamma_0^k(\alpha^*)}{\partial \ln\alpha}\cdot d\ln\alpha^* \\
&= \varepsilon^k \cdot (1-\eta)\cdot d\ln w - \varepsilon^k \cdot (\eta^*-\eta)\cdot(d\ln w + d\ln q_{ext}) \\
&= \varepsilon^k \cdot (1-\eta^*)\cdot d\ln w - \varepsilon^k \cdot (\eta^*-\eta)\cdot d\ln q_{ext}.
\end{aligned}$$

Along the transition, firms will differ in the extent to which they automate their tasks. Let $d\ln\alpha_{tf}$ denote the additional tasks automated by firm f at time t. We have that

$$d\ln\varepsilon^\ell_{tf} = \varepsilon^k \cdot (1-\eta)\cdot d\ln w_t + \varepsilon^k \cdot \frac{\partial \ln \Gamma^\ell(\alpha^*)/\Gamma_0^k(\alpha^*)}{\partial \ln\alpha}\cdot d\ln\alpha_{tf}.$$

The expression for incumbents that do not automate follows from taking $d\ln\alpha_{tf} = 0$. ■

A.3 The induced elasticity of substitution η^*

The text explains that η^* is the elasticity of substitution that one would estimate from permanent variation in wages. This subsection formalizes this connection.

The elasticity of substitution is defined as

$$\text{elasticity of substitution} = \frac{d\ln K/L}{d\ln w} = 1 - \frac{d\ln(\varepsilon^\ell/\varepsilon^k)}{d\ln w}.$$

Using the expression for cost shares in the text, the definition of η_t^* and the fact that

$$d\ln\alpha^* = \frac{1}{\partial\ln\psi_\ell(\alpha^*)/q(\alpha^*)/\partial\ln\alpha}\cdot d\ln w,$$

we get

$$\begin{aligned}
\text{elasticity of substitution} =& 1-(1-\eta) - \frac{\partial \ln \Gamma^\ell(\alpha^*)/\Gamma_t^k(\alpha^*)}{\partial \ln\alpha}\cdot\frac{d\ln\alpha^*}{d\ln w} \\
=& 1-(1-\eta)-(\eta-\eta_t^*) \\
=& \eta_t^*,
\end{aligned}$$

as wanted.

B Proofs and derivations for the model with log-concave demand

This section provides the details of the model with a log-concave demand system.

B.1 Micro-foundation for λ

We show that the demand system implied by (4) is isomorphic to one where firms compete against a growing mass of firms for a given consumer.

Let's first consider the demand system in the main text. Firm demand is derived from the following cost minimization problem:

$$\min_{y_{tf}} \int_f p_{tf} \cdot y_{tf} \cdot df \quad \text{s.t:} \quad \int_f \lambda \cdot H\left(\frac{y_{tf}}{\lambda_t \cdot y_t}\right) \cdot df = 1.$$

Let $\rho_t \cdot y_t$ denote the Lagrange multiplier on the constraint. The first-order condition for the choice of y_{tf} is then

$$y_{tf} = y_t \cdot \lambda \cdot D\left(\frac{p_{tf}}{\rho_t}\right), \tag{A3}$$

where D is decreasing and given by the inverse function of $H'(x)$. Plugging the demand for each variety in the constraint, we obtain

$$\int_f \lambda \cdot H\left(D\left(\frac{p_{tf}}{\rho_t}\right)\right) \cdot df = 1. \tag{A4}$$

Moreover, because the price of the final good is normalized to 1, we must have

$$1 = \int_f \lambda \cdot p_{tf} \cdot D\left(\frac{p_{tf}}{\rho_t}\right) \cdot df. \tag{A5}$$

In sum, the equilibrium for the Kimball demand system is summarized by equations (A3), (A4), and (A5).

Let's now show this system is equivalent to one with multiple consumers where firms can access and compete over a fixed mass of them.

There is a mass 1 of customers with equal incomes indexed by j whose flow utility u_{tj} from consuming a set of varieties $\mathcal{F}_{tj}$ is defined implicitly by

$$\int_{f \in \mathcal{F}_{tj}} H\left(\frac{y_{tfj}}{u_{tj}}\right) \cdot df = 1.$$

Consumers maximize their utility u_{tj} subject to their budget constraint. As above, consumer

j demand from firm $f \in \mathcal{F}_{tj}$ is

$$y_{tfj} = y_t \cdot D\left(\frac{p_{tf}}{\rho_{tj}}\right),$$

where y_t is income per consumer and ρ_{tj} satisfies

$$\int_{f \in \mathcal{F}_{tj}} H\left(D\left(\frac{p_{tf}}{\rho_{tj}}\right)\right) \cdot df = 1.$$

Firms and customers are randomly matched to each other, with each customer matched to a mass λ of firms. Random matching implies that all consumers face the same distribution of prices, and so they share a common $\rho_{tj} = \rho_t$.

As a result, total demand for firm f is

$$y_{tf} = y_t \cdot \lambda \cdot D\left(\frac{p_{tf}}{\rho_t}\right),$$

which coincides with equation (A3). The equation for ρ_{tj} can then be written as

$$\int_f \lambda \cdot H\left(D\left(\frac{p_{tf}}{\rho_t}\right)\right) \cdot df = 1,$$

which coincides with (A4). Finally, adding revenue across firms we get $y_t = \int_f y_{tf} \cdot p_{tf} \cdot df$, which implies

$$1 = \int_f \lambda \cdot p_{tf} \cdot D\left(\frac{p_{tf}}{\rho_t}\right) \cdot df. \tag{A6}$$

This equation coincides with (A5), establishing the aggregation result.

B.2 Implications for prices, sales, and markups

This subsection shows that Marshall's second laws imply properties P1–P3 in the text.

The formal assumptions behind Marshall's second laws are:

- **Weak second law:** demand elasticity $-x \cdot D'(x)/D(x)$ exceeds 1 and increases in x.
- **Strong second law:** marginal revenue $x + D(x)/D'(x)$ is positive and log-concave.

PROPOSITION A2 *Consider a firm with a constant marginal cost c and denote its optimal price by $p^*(c)$, markups by $\mu^*(c)$, and firm sales by $\omega^*(c)$. Under Marshall's weak second law, $p^*(c)$ is increasing and $\mu^*(c)$ is decreasing. Moreover, under Marshall's strong second law,* *markups and prices, $\mu^*(c)$ and $p^*(c)$, are log-convex functions of costs, which implies lower passthroughs for more productive firms. Finally, sales $\omega^*(c)$ are log-concave and decreasing functions of costs.*

Proof. Prices are given by

$$p^*(c) = \arg\max_p y \cdot \lambda \cdot D\left(\frac{p}{\rho}\right) \cdot (p - c).$$

This problem has increasing differences in p and c, which implies that $p^*(c)$ increases in c.

Moreover, the first order condition for this problem is

$$-\frac{1}{\rho} D'\left(\frac{p}{\rho}\right) \cdot (p - c) = D\left(\frac{p}{\rho}\right) \quad \Rightarrow \quad \frac{\mu^*(c)}{\mu^*(c) - 1} = -\frac{p^*(c)}{\rho} \frac{D'\left(\frac{p^*(c)}{\rho}\right)}{D\left(\frac{p^*(c)}{\rho}\right)}.$$

Marshall's weak second law combined with the fact that $p^*(c)$ increases in c implies that the right-hand side of the above equation increases in c. The left-hand side is a decreasing function of $\mu^*(c)$, which therefore implies that $\mu^*(c)$ is decreasing in c as wanted.

We can rewrite the first-order condition for prices as

$$\frac{p^*(c)}{\rho} + \frac{D(p^*(c)/\rho)}{D'(p^*(c)/\rho)} = \frac{c}{\rho}.$$

Differentiating this expression yields

$$\frac{\partial \ln p^*(c)}{\partial \ln c} = \frac{1}{d\left(\frac{p^*(c)}{\rho}\right)},$$

where

$$d(x) = \frac{\partial \ln\left(x + D(x)/D'(x)\right)}{\partial \ln x}$$

is a decreasing function according to Marshall's strong second law. It follows that $\ln p^*(c)$ is a convex function in $\ln c$. Moreover, $\ln \mu^*(c) = \ln p^*(c) - \ln c$ inherits this convexity.

Turning to sales shares, we have that $\omega^*(c)$ can be written as

$$\omega^*(c) = h(p^*(c))/y,$$

where $h(x) = xD(x)$ is a log-concave and decreasing function of x (from Marshall's weak second law). Thus, $\omega^*(c)$ is the composition of a log-concave and decreasing function $(h(x))$

with a log-convex and increasing function $p(c)$, which results in a log-concave and decreasing function. ■

B.3 Effects of rising market access

We now derive the effects of an increase in market access—Properties P4–P5 in the text.

We assume the economy starts from a stationary equilibrium where all firms have the same degree of automation. We then characterize the immediate effect of a λ shock. By design, this exercise does not account for any effect of λ shocks through subsequent entry, exit, or automation decisions. These effects are harder to study analytically, but are explored in our numerical exercises.

Let μ_z and p_z denote the markup and price charged by a firm of productivity z, and ω_z its sales share.

PROPOSITION A3 *An increase in λ has the following immediate effects:*

- *μ_z decreases for all z;*
- *for $z > z'$, $\mu_z/\mu_{z'}$ decreases;*
- *for $z > z'$, $\omega_z/\omega_{z'}$ increases.*

PROOF. Firms' marginal cost is $(1/z) \cdot c(w)$ for some common $c(w)$, which is a function of the equilibrium wage.

Let $c_{norm} = c/\rho$, where ρ is the value of ρ_t at the initial equilibrium. c_{norm} is an endogenous object pinned down by market access and the distribution of productivities, as we show later. It also summarizes the degree of competition in the economy.

We can rewrite firms' pricing problem as

$$\max_{p_{norm}} D(p_{norm}) \cdot \left(p_{norm} - \frac{1}{z} \cdot c_{norm}\right),$$

where $p_{norm} = p/\rho$ is a normalized firm price. Optimal firm prices are $p_z = \rho \cdot p^*(c_{norm}/z)$, markups are $\mu_z = \mu^*(c_{norm}/z)$, and sale shares are $\omega_z = \omega^*(c_{norm}/z)$.

We now show that λ increases c_{norm}. The implicit definition of ρ can be rewritten as

$$\int_z \lambda \cdot H\left(D\left(p^*\left(c_{norm}/z\right)\right)\right) \cdot m_z \cdot dz = 1.$$

From this equation we see that λ increases the equilibrium value of c_{norm} (keep in mind that, here, m_z is maintained constant, as we are characterizing only the immediate impact of a λ shock).

We now characterize the effects of the increase in c_{norm}.

First, for a given z, $\mu_z = \mu^*(c_{norm}/z)$ decreases in c_{norm}, as wanted.

Second, because $\mu^*(c)$ is log-convex, we have that, for $z > z'$,

$$\ln \mu_z - \ln \mu_{z'} = \ln \mu^* \left(c_{norm}/z\right) - \ln \mu^* \left(c_{norm}/z'\right)$$

is decreasing in c_{norm}, as wanted.

Third, because the function $\omega^*(c)$ is log-concave,

$$\ln \omega_z - \ln \omega_{z'} = \ln \omega^* \left(c_{norm}/z\right) - \ln \omega^* \left(c_{norm}/z'\right)$$

is increasing in c_{norm} for $z > z'$, as wanted. ∎

PROPOSITION A4 *Let $f(z)$ denote the mass of firms of productivity z. The aggregate labor share is $s^\ell = \varepsilon^\ell/\mu$, where the aggregate markup μ is a sales weighted harmonic mean of firm-level markups:*

$$\frac{1}{\mu} = \int_z \frac{1}{\mu_z} \cdot \omega_z \cdot f(z) \cdot dz.$$

The immediate effect of λ is to increase the aggregate markup if the distribution of productivity is log-convex (i.e., more convex than Pareto), lower it if the distribution of productivity is log-concave (i.e., less convex than Pareto), and leave it unchanged if the distribution of productivity is log-linear (i.e., Pareto).

Proof of Proposition A4. As before, we investigate the implications of an increase in c_{norm} holding the distribution of productivities constant at $f(z)$ and without accounting for subsequent automation decisions.

We can write the aggregate markup as

$$\frac{1}{\mu} = \int_z \frac{1}{\mu^* \left(c_{norm}/z\right)} \cdot \omega^* \left(c_{norm}/z\right) \cdot f(z) \cdot dz.$$

With the change of variable $x = c_{norm}/z$, we can rewrite this as

$$\frac{1}{\mu} = \int_x \frac{1}{\mu^* \left(x\right)} \cdot g(x, c_{norm}) \cdot dx,$$

where $g(x, c_{norm})$ is a density function given by

$$g(x, c_{norm}) = \omega^* \left(x\right) \cdot f(c_{norm}/x) \cdot \frac{c_{norm}}{x^2} \cdot dx.$$

First, suppose that $f(z)$ is log-concave. This implies that

$$\ln g(x, c_{norm}) = \ln \omega^* (x) + \ln f(c_{norm}/x) + \ln c_{norm} - 2 \ln x$$

has increasing differences in x and $\bar{c}$. This is equivalent to the following monotone likelihood ratio property (MLRP):

$$\frac{g(x, c_{norm})}{g(x', c_{norm})} \text{ increasing in } c_{norm} \text{ for } x > x'.$$

The MLRP property implies that an increase in c_{norm} generates a shift up (in the first-order stochastic dominance sense) in $g(x, c_{norm})$. Because the function $\frac{1}{\mu^*(x)}$ is increasing in x, the aggregate markup μ decreases in c_{norm} as wanted.

Second, suppose that $f(z)$ is log-convex. This implies that

$$\ln g(x, c_{norm}) = \ln \omega^* (x) + \ln f(c_{norm}/x) + \ln c_{norm} - 2 \ln x$$

has decreasing differences in x and c_{norm}. This is equivalent to the following monotone likelihood ratio property (MLRP):

$$\frac{g(x, c_{norm})}{g(x', c_{norm})} \text{ decreasing in } c_{norm} \text{ for } x > x'.$$

The MLRP property implies that an increase in c_{norm} generates a shift down (in the first-order stochastic dominance sense) in $g(x, c_{norm})$. Because the function $\frac{1}{\mu^*(x)}$ is increasing in x, the aggregate markup μ increases in c_{norm} as wanted.

Finally, suppose that $f(z)$ is log-linear. This implies that

$$\ln g(x, c_{norm}) = \ln \omega^* (x) + \ln f(c_{norm}/x) + \ln \bar{c} - 2 \ln x$$

is a linear function in $\ln c_{norm}$. Equivalently,

$$\frac{g(x, c_{norm})}{g(x', c_{norm})} \text{ is independent of } c_{norm}.$$

Thus, the integral defining μ is independent of c_{norm}. ■

B.4 Properties of the Klenow–Willis aggregator

As a functional form for the Kimball (1995) aggregator H we use the specification from Klenow and Willis (2016), defined as

$$H(q) \equiv 1 + (\sigma - 1) \cdot \exp\left(\frac{1}{\nu}\right) \cdot \nu^{\frac{\sigma}{\nu}-1} \cdot \left[\Gamma\left(\frac{\sigma}{\nu}, \frac{1}{\nu}\right) - \Gamma\left(\frac{\sigma}{\nu}, \frac{q^{\frac{\nu}{\sigma}}}{\nu}\right)\right],$$

where $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function,

$$\Gamma(s, x) \equiv \int_x^\infty t^{s-1} \cdot \exp(-t) dt.$$

This gives rise to the following (relative) demand function $D^{-1} = H'$:

$$D(x) = \left(1 - \nu \cdot \ln\left(x \cdot \frac{\sigma}{\sigma - 1}\right)\right)^{\frac{\sigma}{\nu}}.$$

The price elasticity of demand is

$$-\frac{x \cdot D'(x)}{D(x)} = \frac{\sigma}{1 - \nu \cdot \ln\left(x \cdot \frac{\sigma}{\sigma-1}\right)} = \sigma \cdot D(x)^{-\frac{\nu}{\sigma}}, \tag{A7}$$

which reduces to the constant σ if $\nu = 0$ (the benchmark case of a CES aggregator). In general, equation (A7) shows that under this parametrization, the super-elasticity of demand is equal to the constant $-\frac{\nu}{\sigma}$, and that larger firms will face more inelastic demand curves.

To conclude, we show that the Klenow-Willis aggregator satisfies Marshall's second laws. Equation (5) shows that the demand elasticity is increasing in the relative price and greater than 1 (Marshall's weak second law), imposing the restriction that $\sigma > 1$ and $\nu > 0$. To see that the strong law holds as well, write the logarithm of marginal revenue as

$$\begin{aligned} \ln\left(x + \frac{D(x)}{D'(x)}\right) &= \ln x + \ln\left(1 + \frac{D(x)}{x \cdot D'(x)}\right) \\ &= \ln x + \ln\left(\frac{\sigma + \nu \cdot \ln x + \nu \cdot \ln\left(\frac{\sigma}{\sigma-1}\right) - 1}{\sigma}\right), \end{aligned}$$

which is a concave function of $\ln x$ as desired.

B.5 Incorporating demand shocks

In our baseline model, firm dynamics are driven by productivity shocks z_{tf}. This subsection discusses the implications of allowing for firm-specific demand shocks z^d_{tf}. It shows that demand shocks produce identical responses to a productivity shock on revenue, markups, automation decisions, labor shares, and profits. The two shocks only differ in their implications for firm prices.

To introduce demand shocks, we modify the Kimball aggregator in (4) to

$$\int_f \lambda \cdot H\left(\frac{z^d_{tf} \cdot y_{tf}}{\lambda \cdot y_t}\right) \cdot df = 1,$$

where z^d_{tf} is a taste shifter for firm f's variety.

The demand curve faced by firms is now given by

$$y_{tf} = \lambda \cdot y_t \cdot \frac{1}{z^d_{tf}} \cdot D\left(\frac{p_{tf}}{\rho_t \cdot z^d_{tf}}\right),$$

and its profit maximization problem is modified to

$$\pi_t(\alpha_{tf}, z_{tf}, z^d_{tf}) = \max_{p_{tf}} \lambda \cdot y_t \cdot \frac{1}{z^d_{tf}} \cdot D\left(\frac{p_{tf}}{\rho_t \cdot z^d_{tf}}\right) \cdot \left(p_{tf} - \frac{1}{z_{tf}} c_t(\alpha_{tf})\right),$$

where $c_t(\alpha_{tf})$ is the unit cost of a firm with unitary productivity and $\pi_t(\alpha_{tf}, z_{tf}, z^d_{tf})$ the profit function.

Let $p^d_{tf} = p_{tf}/z^d_{tf}$ be a taste-adjusted price. We can rewrite profit maximization as

$$\pi_t(\alpha_{tf}, z_{tf}, z^d_{tf}) = \max_{p^d_{tf}} \lambda \cdot y_t \cdot D\left(\frac{p^d_{tf}}{\rho_t}\right) \cdot \left(p^d_{tf} - \frac{1}{z_{tf} \cdot z^d_{tf}} c_t(\alpha_{tf})\right).$$

This shows that firms' profits and optimal choices are functions of the composite $z_{tf} \cdot z^d_{tf}$. This implies that a productivity and a demand shock with the same persistence will generate the exact same responses in terms of firm profits, exit, and automation decisions, all of which depend entirely on the profit function.

Moreover, using the notation introduced in the proof of Proposition A2, we can write optimal taste-adjusted prices as

$$p^d_{tf} = p^*\left(\frac{1}{z_{tf} \cdot z^d_{tf}} c_t(\alpha_{tf})\right),$$

prices as

$$p_{tf} = z^d_{tf} \cdot p^*\left(\frac{1}{z_{tf} \cdot z^d_{tf}} c_t(\alpha_{tf})\right),$$

and markups as

$$\mu_{tf} = \frac{p_{tf}}{\frac{1}{z_{tf}} c_t(\alpha_{tf})} = \frac{p^d_{tf}}{\frac{1}{z_{tf} \cdot z^d_{tf}} c_t(\alpha_{tf})} = \mu^* \left(\frac{1}{z_{tf} \cdot z^d_{tf}} c_t(\alpha_{tf}) \right).$$

This shows that demand and productivity shocks have identical implications for markups but different implications for prices. Markups increase with both demand and productivity shocks. Instead, prices increase with demand shocks (since $p^*(c)$ is increasing in c but with a passthrough below one) and decrease with productivity shocks.

C Additional numerical exercises and details for manufacturing

C.1 Details of the q-shock:

A more general expression for the q shocks used in our quantitative exploration is

$$q_{I,t}(x) = \begin{cases} q_{int,t} & \text{if } x \leq \alpha_0^* \\ \min\{q_{int,t} \cdot \frac{\psi_\ell(x)/\psi_\ell(\alpha_0^*)}{\psi_k(x)/\psi_k(\alpha_0^*)}, q_{ext,t}\} & \text{if } x > \alpha_0^* \end{cases}$$

for some increasing $\{q_{int,t}, q_{ext,t}\}$ converging to $\{q_{int}, q_{ext}\}$.

The main text normalized baseline wages to 1 and assume that $q_0^I(x) = 1$. This implies $\psi_\ell(\alpha_0^*)/\psi_k(\alpha_0^*) = 1$, which simplifies the formulation of the shock.

Figure A1 represents the q shock graphically. Tasks are arranged in [0,1] in the horizontal axis. At time 0, we have $q_{I,t}(x) = 1$ for all tasks. Over time, the productivity with which the economy can produce the capital needed for task x rises. In the figure, we depict a case where capital advances are more pronounced at the extensive margin, as in our calibration.

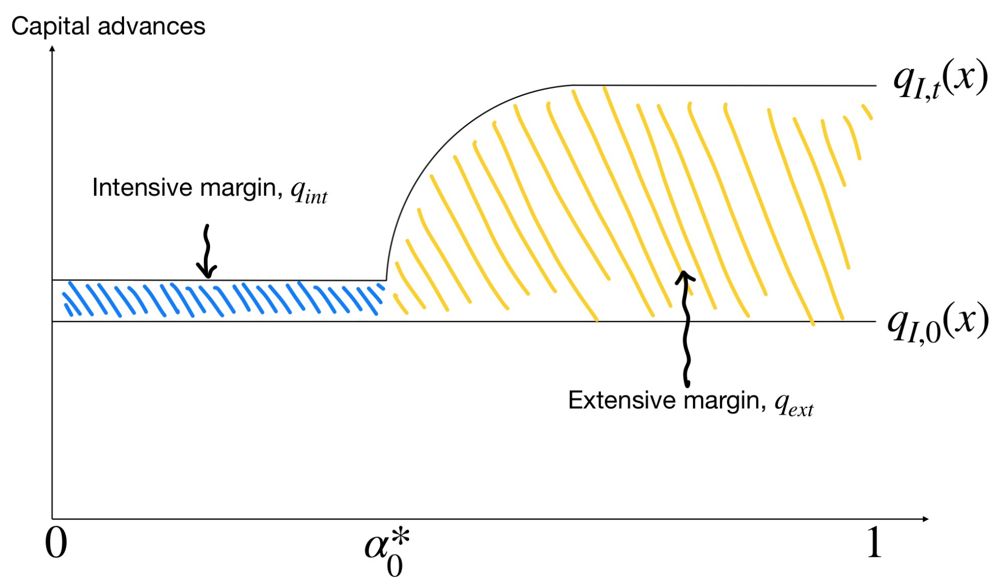

Figure A1: Representation of q shocks. Tasks are arranged in [0,1] in the horizontal axis. At time 0, we have $q_{I,0}(x) = 1$ for all tasks. Over time, the productivity with which the economy can produce the capital needed for task x rises to $q_{I,t}(x)$.

C.2 Shutting down diffusion

Columns (3) and (4) in Table A1 report two counterfactual exercises that assess the significance of the diffusion of automation assumption. In our baseline experiment (reproduced in column 2), technology diffuses via entry as entrants' initial automation level equals the unweighted average automation level in use in the economy, $\alpha_{entrant,t} = \bar{\alpha}_t$. In column (3) we maintain the diffusion assumption for the initial steady state, such that firms have identical labor shares in 1982, but then remove it over the transition. I.e., entering firms in any year $t > 1982$ are assigned the automation level of the initial steady state $\alpha^\star$, which is lower than $\bar{\alpha}_t$. We do not re-calibrate the inferred aggregate shocks and parameters. The results are very similar. In fact, the main difference is that in the counterfactual economy, without diffusion to entrants, the relative adoption gradient naturally increases.

Column (4) explores the implications of shutting down diffusion for the initial steady state as well. For this experiment, we assume that entrants start with a fraction of the optimal automation level $\alpha_{tf} = m \cdot \alpha^\star$, with $m < 1$. We calibrate $m = 0.5$ to match the ratio of the unweighted mean firm to the aggregate labor share of 1.11 in 1982. This modified version of the model generates an initial steady state with the same amount of labor share dispersion by size as in the data. The resulting dynamics in response to q shocks remain very similar to our baseline findings.

C.3 Calibration with $\eta^* = 1.45$

Columns (5) and (6) in Table A1 report results from an alternative parametrization with an induced elasticity of $\eta^* = 1.45$, as estimated by Karabarbounis and Neiman (2013) and Hubmer (2023).

For this parametrization, we normalize initial capital prices to 1 and set

$$\psi_k(x) = 1, \qquad\qquad \psi_\ell(x) = A \cdot \left(x^{\frac{1-\eta^*}{\eta^*-\eta}} - 1 \right)^{\frac{1}{1-\eta^*}} .$$

Note that this requires $\eta^* > 1$ so that $\psi_\ell(x)/\psi_k(x)$ is increasing in x.

With this specification, the cost function of firms in the initial steady state becomes

$$c_0(\alpha, z) = \frac{1}{z} \cdot \left(\alpha + \left(1 - \alpha^{\frac{\eta^*-1}{\eta^*-\eta}} \right)^{\frac{\eta^*-\eta}{\eta^*-1}} \cdot \left(\frac{w_0}{A} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}} .$$

Minimizing with respect to α, we obtain

$$\alpha_0^* = \left(\frac{(w_0/A)^{\eta^*-1}}{1 + (w_0/A)^{\eta^*-1}} \right)^{\frac{\eta^*-\eta}{\eta^*-1}}$$

TABLE A1: Robustness checks manufacturing: no diffusion and induced elasticity $\eta^* = 1.45$

	Data	Baseline	No diffusion over transition	No diffusion in initial st st	Both q shocks	Uniform q shock
		Induced elasticity $\eta^* = 1$ (baseline)			$\eta^* = 1.45$	
	(1)	(2)	(3)	(4)	(5)	(6)
I. Parameters and inferred aggregate shocks						
$d\ln q_{int}$		0.67	0.67	0.67	0.86	1.68
$d\ln q_{ext}$		5.48	5.48	5.48	1.63	1.68
c_a		0.19	0.19	0.19	0.32	0.35
II. Targeted moments, 1982–2012						
Δ aggregate labor share	-0.20	-0.20	-0.20	-0.21	-0.20	-0.20
Δ log average capital price	-1.08	-1.08	-1.08	-1.09	-1.09	-1.68
Relative adoption (P99+ vs. P50-75 firms)	1.71	1.71	4.27	1.67	1.70	1.71
III. Concentration 1982–2012 (from Autor et al., 2020; Decker et al., 2020)						
Δ log 4 firms' sales share	0.140	0.105	0.125	0.071	0.055	0.071
Δ log 20 firms' sales share	0.072	0.104	0.124	0.072	0.054	0.070
Δ log productivity dispersion	0.050	0.061	0.073	0.061	0.041	0.059
IV. Typical firm labor share from Kehrig and Vincent (2021), 1982–2012						
Δ median labor share	0.030	-0.005	0.084	-0.005	-0.042	-0.002
Δ unweighted mean	-0.017	-0.039	0.004	-0.077	-0.055	-0.023
V. Melitz–Polanec decomposition from Autor et al. (2020), 1982–2012						
Δ aggregate labor share	-0.185	-0.198	-0.199	-0.197	-0.201	-0.202
Δ unweighted incumbent mean	-0.002	-0.015	-0.023	-0.016	-0.029	0.006
Exit	-0.055	-0.006	-0.006	-0.005	-0.004	-0.004
Entry	0.059	0.006	0.007	0.006	0.006	0.006
Covariance term	-0.187	-0.183	-0.177	-0.183	-0.174	-0.210
VI. Covariance decomposition from Kehrig and Vincent (2021), 1982–2012						
Market share dynamics	0.047	0	0	0	0	0
Labor share by size dynamics	-0.043	-0.078	-0.077	-0.078	-0.059	-0.069
Cross-cross dynamics	-0.232	-0.066	-0.066	-0.064	-0.080	-0.099

Notes: Column (2) reproduces the findings from our baseline CES model. Column (3) features the same initial steady state but then removes diffusion of technology via entry over the transition. Column (4) in addition removes the diffusion assumption in the initial steady state. Columns (5-6) feature a higher induced elasticity of $\eta^* = 1.45$. (5) infers the combination of intensive and extensive margin capital price declines as in the baseline. (6) instead imposes that capital prices decline uniformly.

and

$$c_0(\alpha_0^*, z) = \frac{1}{z} \cdot \left(1 + \left(\frac{w_0}{A}\right)^{1-\eta^*}\right)^{\frac{1}{1-\eta^*}}.$$

This shows that the induced elasticity of substitution is η^*, as wanted.

As in our baseline, we set $\eta = 0.5$, which means that tasks are complements. We also calibrate A so that the labor share in the initial steady state is 67%.

In column (5) we follow the same inference procedure as in the baseline model. To match the manufacturing labor share decline and average capital price decline, we back out a combination of q shocks that loads relatively less on the extensive margin. This is expected since with $\eta^* > 1$, even a uniform capital price decline generates an aggregate labor share decline.

Column (6) demonstrates this point by fitting a uniform q shock to the manufacturing labor share decline. This column shows that one could also generate the observed labor share decline as a result of a uniform decline in capital prices of 168 log points. The fact that this exceeds the decline in capital prices seen in the data highlights the importance of allowing for differences in capital advances at the intensive and extensive margin, since a uniform shock would require more technological progress than inferred from capital price data.

C.4 Changing the super-elasticity of demand in the log-concave demand model

In the main text, we calibrated a demand super-elasticity of $\frac{\nu}{\sigma} = 0.22$ by matching the ratio of the (unweighted) mean firm labor share to the aggregate sectoral labor share. This appendix reports results for manufacturing using a lower super-elasticity of 0.16 as estimated by Edmond, Midrigan and Xu (2022).

For this robustness check, we focus on the exercise in section 3.3. This shows how a different super-elasticity changes our inference and the effects of q and λ shocks.

First, we re-calibrate the parameters in the initial steady state. The main difference is that a lower value of the super-elasticity requires less convexity in the productivity distribution, since the mapping from productivity to firm sales is less log-concave. For manufacturing, we infer $n = 0.91$ (instead of $n = 0.74$ as in Table 3). Thus, the inferred z-distributions are closer to the log-linear case (Pareto).

Table A2 reports the main results over the transition (1982–2012) for both sectors. Relative to the results in the main text, the inferred rising competition shock is somewhat larger, with $d \ln \lambda = 0.09$ (instead of $d \ln \lambda = 0.06$ as in Table 4). However, the lower log-convexity of the z-distribution implies that the λ shock generates a smaller increase in the aggregate markup, and correspondingly a smaller decrease in the aggregate labor share. The contribution of falling capital prices to the labor share decline is similar across parametrizations. As expected, the labor share dynamics across firms are in between those obtained in the main text with a higher super-elasticity $\frac{\nu}{\sigma} = 0.22$ and the CES demand case.

Table A2: Robustness checks manufacturing: lower super-elasticity of $\frac{\nu}{\sigma} = 0.16$

		Results from log-concave demand model		
	Data	Both shocks	Only effects of $d \ln q$	Only effects of $d \ln \lambda$
	(1)	(2)	(3)	(4)
I. Parameters and inferred aggregate shocks				
$d \ln q_{int}$		0.63	0.63	0
$d \ln q_{ext}$		5.29	5.29	0
$d \ln \lambda$		0.09	0	0.09
c_a		0.17	0.17	0.17
II. Targeted moments, 1982–2012				
Δ aggregate labor share	-0.199	-0.198	-0.206	0.005
Δ log average capital price	-1.081	-1.084	-1.031	0
Δ log 4 firms' sales share	0.140	0.143	0.094	0.056
Relative adoption (P99+ vs. P50-75 firms)	1.71	1.71	1.66	14.23
III. Typical firm labor share and other moments				
Δ median labor share	0.030	0.037	0.022	0.011
Δ unweighted mean	-0.017	-0.003	-0.017	0.010
Δ log 20 firms' sales share	0.072	0.139	0.111	0.035
Δ log productivity dispersion	0.050	0.074	0.065	0.000
IV. Melitz–Polanec decomposition from Autor et al. (2020)				
Δ aggregate labor share	-0.185	-0.195	-0.203	0.005
Δ unweighted incumbent mean	-0.002	0.023	0.009	0.012
Exit	-0.055	-0.011	-0.013	-0.013
Entry	0.059	0.009	0.011	0.012
Covariance term	-0.187	-0.216	-0.210	-0.006
V. Covariance decomposition from Kehrig and Vincent (2021)				
Market share dynamics	0.047	0.057	0.056	0.055
Labor share by size dynamics	-0.043	-0.042	-0.046	0.054
Cross-cross dynamics	-0.232	-0.179	-0.175	-0.115
VI. Change in markups, 1982–2012				
Δ log aggregate markup	-0.010	0.010	0.011	0.000
Within-firm change in markup	-0.075	-0.025	-0.021	-0.010
Reallocation to high-markup firms	0.065	0.035	0.032	0.010

Notes: The table reports the equivalent of Table 4 in the main text but imposes a lower super-elasticity of $\frac{\nu}{\sigma} = 0.16$ (instead of $\frac{\nu}{\sigma} = 0.22$ as in the main text). The parameters of the respective economies are re-calibrated, both in the steady state to match all other targeted moments, as well as in regards to the inferred shocks $d \ln q_{int}$, $d \ln q_{ext}$, $d \ln \lambda$ and the automation fixed cost c_a over the transition.

D Calibration and results for other sectors

This appendix summarizes the results of our decomposition for non-manufacturing sectors.

D.1 Retail

We follow the same calibration approach as in manufacturing, with the calibrated parameters and targets listed in Table A3. As explained in the text, we lack detailed data for some of the moments used in retail and so we keep some of the parameters or moments from manufacturing.

Table A3: Calibration of the log-concave demand model for retail

	Parameter		Moment	Data	Model
	I. Parameters related to production function				
η	Task substitution elasticity	0.5	From Humlum (2019)	0.5	0.5
γ_ℓ	Comparative advantage	0.22	Retail labor share (BLS/BEA)	0.72	0.72
	II. Parameters governing firm dynamics and productivities in 1982 steady state				
ν/σ	Demand super-elasticity	0.22	Imputed from manufacturing		
σ	Demand elasticity	8.95	Aggregate markup	1.15	1.15
ζ	Weibull scale	0.0128	Top 20 firms' sales share	29.9%	29.9%
n	Weibull shape	0.47	Top 4 firms' sales share	15.1%	15.1%
$\underline{c}_o$	Scale operating cost	$6.9 \cdot 10^{-6}$	Entry (=exit) rate	0.062	0.062
ξ_o	Tail index operating cost	0.320	Size of exiters	0.490	0.494
μ_e	Entrant productivity	0.855	Size of entrants	0.600	0.600
ρ_z	Productivity persistence	0.86	Revenue TFP persistence among retail firms		

Notes: The table reports the calibrated parameters and targets for our model with log-concave demand in retail. The data on top firms' sales share is from Autor et al. (2020)'s estimates for the US retail sector. The annual entry rate, as well as relative sizes of entrants and exiters, are from Lee and Mukoyama (2015) and imputed from manufacturing.

Table A4 summarizes the results for retail. Column 1 reports the available data. Column 2 reports the effects of the q and λ shocks backed out to match trends in retail's labor share and concentration. Columns 3 and 4 report the effects of the increase in q and λ shocks separately. The decline in capital prices at the extensive margin continues to be the dominant force in driving the labor share decline. However, the model attributes only a small fraction of the increase in sales concentration to q shocks.

Table A4: Effects of lower capital prices and rising competition: Retail 1982–2012

		Results from log-concave demand model		
	Data	Both shocks	Only effects of $d\ln q$	Only effects of $d\ln\lambda$
	(1)	(2)	(3)	(4)
I. Parameters and inferred aggregate shocks				
$d\ln q_{int}$		0.48	0.48	0
$d\ln q_{ext}$		3.25	3.25	0
$d\ln\lambda$		0.30	0	0.30
c_a		0.06	0.06	0.06
II. Targeted moments, 1982–2012				
Δ aggregate labor share	-0.127	-0.127	-0.122	-0.022
Δ log average capital price	-0.865	-0.864	-0.689	0.000
Δ log sales concentration	0.546	0.546	0.063	0.480
Relative adoption (P99+ vs. P50-75 firms)	1.71	1.71	1.12	2.54
III. Typical firm labor share and other moments				
Δ median labor share		0.048	-0.036	0.037
Δ unweighted mean		0.028	-0.046	0.035
Δ log productivity dispersion		0.033	0.016	0.001
IV. Change in markups				
Δ log aggregate markup	0.045	0.051	0.013	0.038
Within-firm change in markup	-0.018	-0.015	-0.010	-0.016
Reallocation to high-markup firms	0.063	0.066	0.023	0.054

Notes: Column (2) reports the findings from our benchmark model. Column (3) shows results when shutting down the market access shock, and column (4) when shutting down instead the price of capital shock. The data for markups comes from Compustat estimates and is described in Section 3.3.

D.2 Wholesale and Utilities & transportation

Table A5 summarizes the steady state calibration of the model with log-concave demand for wholesale as well as the utilities & transportation sector. The calibration strategy is identical to manufacturing and retail. Here too, we lack detailed data for some of the moments used and so we keep some of the parameters or moments from manufacturing.

For these two sectors, the log-convexity of the z-distribution is rather mild in these two sectors (n slightly below 1), more in line with manufacturing than retail.

Table A6 shows the decomposition exercise. In wholesale and in utilities & transportation, the labor share decline is mild, while the observed increase in sales concentration is moderate. Consequently, the inferred capital price declines at the extensive margin are smaller than in manufacturing, while the inferred increases in competition ($d\ln\lambda$) are weaker than in retail but stronger than in manufacturing. The inferred automation fixed costs (c_a) are comparable.

Figure A2 summarizes our findings across sectors.

Table A5: Calibration of the log-concave demand model for Wholesale and Utilities & Transportation

	Parameter		Moment	Data	Model
	I. Wholesale: steady state parameters and moments (1982)				
γ_ℓ	Comparative advantage	0.63	Wholesale labor share	0.53	0.53
ν/σ	Demand super-elasticity	0.22	Imputed from manufacturing		
σ	Demand elasticity	9.4	Aggregate markup	1.15	1.15
ζ	Weibull scale	0.071	Top 20 firms' sales share	42.9%	42.9%
n	Weibull shape	0.75	Top 4 firms' sales share	22.3%	22.3%
$\underline{c}_o$	Scale operating cost	$3.2 \cdot 10^{-7}$	Entry (=exit) rate	0.062	0.062
ξ_o	Tail index operating cost	0.235	Size of exiters	0.490	0.493
μ_e	Entrant productivity	0.889	Size of entrants	0.600	0.601
ρ_z	Productivity persistence	0.86	Revenue TFP persistence among wholesale firms		
	II. Utilities & Transportation: steady state parameters and moments (1992)				
γ_ℓ	Comparative advantage	0.72	Util.&transp. labor share	0.51	0.51
ν/σ	Demand super-elasticity	0.22	Imputed from manufacturing		
σ	Demand elasticity	10.7	Aggregate markup	1.15	1.15
ζ	Weibull scale	0.066	Top 20 firms' sales share	59.1%	58.0%
n	Weibull shape	0.74	Top 4 firms' sales share	30.4%	31.3%
$\underline{c}_o$	Scale operating cost	$9.0 \cdot 10^{-8}$	Entry (=exit) rate	0.062	0.063
ξ_o	Tail index operating cost	0.212	Size of exiters	0.490	0.489
μ_e	Entrant productivity	0.891	Size of entrants	0.600	0.600
ρ_z	Productivity persistence	0.86	Revenue TFP persistence among ut. & transp. firms		

Notes: The table reports the calibrated parameters and targets for our model with log-concave demand in wholesale (panel I) and utilities and transportation (panel II). The data on top firms' sales share comes from Autor et al. (2020)'s estimates for each sector. The annual entry rate, as well as relative sizes of entrants and exiters, are from Lee and Mukoyama (2015) and imputed from manufacturing.

Table A6: Effects of lower capital prices and rising competition: Wholesale, Utilities & Transportation 1982–2012

	(1) Data	Results from log-concave demand model: (2) Benchmark	(3) Only $d \ln q$	(4) Only $d \ln \lambda$
A. Wholesale (1982–2012)	*I. Parameters and inferred aggregate shocks*			
$d \ln q_{int}$		1.30	1.30	0
$d \ln q_{ext}$		2.36	2.36	0
$d \ln \lambda$		0.19	0	0.19
c_a		0.18	0.18	0.18
	II. Targeted moments, 1982–2012			
Δ aggregate labor share	-0.045	-0.045	-0.048	0.005
Δ log average capital price	-1.596	-1.593	-1.563	0
Δ log sales concentration	0.202	0.209	0.048	0.163
Relative adoption (P99+ vs. P50-75 firms)	1.71	1.71	1.42	3.52
	III. Typical firm labor share and other moments			
Δ median labor share		0.187	0.156	0.033
Δ unweighted mean		0.150	0.122	0.031
Δ log productivity dispersion		0.093	0.076	0.000
	IV. Change in markups			
Δ log aggregate markup	0.045	0.009	0.006	0.003
Within-firm change in markup	-0.018	-0.043	-0.032	-0.015
Reallocation to high-markup firms	0.063	0.052	0.039	0.018
B. Utilities & Transportation (1992–2012)	*I. Parameters and inferred aggregate shocks*			
$d \ln q_{int}$		0.62	0.62	0
$d \ln q_{ext}$		1.06	1.06	0
$d \ln \lambda$		0.12	0	0.12
c_a		0.10	0.10	0.10
	II. Targeted moments, 1982–2012			
Δ aggregate labor share	-0.028	-0.028	-0.029	0.002
Δ log average capital price	-0.684	-0.683	-0.676	0
Δ log sales concentration	0.108	0.104	0.025	0.079
Relative adoption (P99+ vs. P50-75 firms)	1.71	1.71	1.47	4.48
	III. Typical firm labor share and other moments			
Δ median labor share		0.101	0.088	0.017
Δ unweighted mean		0.083	0.070	0.016
Δ log productivity dispersion		0.029	0.023	0.000
	IV. Change in markups			
Δ log aggregate markup	0.045	0.004	0.002	0.001
Within-firm change in markup	-0.018	-0.015	-0.010	-0.008
Reallocation to high-markup firms	0.063	0.018	0.012	0.010

Notes: Column (2) contains the benchmark model. Due to data availability, the transition is over 1982–2012 for wholesale, resp. 1992–2012 for utilities & transportation. Column (3) shuts down the market access shock, and column (4) shuts down the price of capital shock.

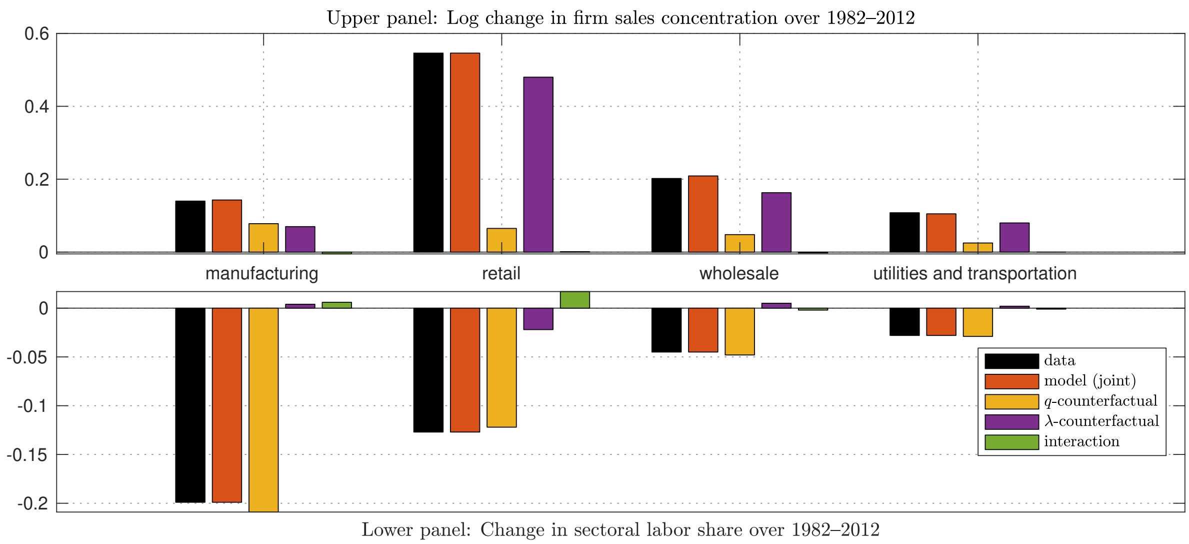

FIGURE A2: MODEL DECOMPOSITION OF LABOR SHARE AND SALES CONCENTRATION CHANGES.

For each sector, the upper panel displays the log change in firm sales concentration (i) in the data Autor et al. (from 2020), (ii) in the benchmark model with q and λ shocks jointly calibrated, (iii) in a model counterfactual that keeps only the q shock active, (iv) in a model counterfactual that keeps only the estimated λ shock active; (v) displays the interaction term, defined as (ii - iii - iv). The lower panel shows sectoral labor share changes in data (BEA-BLS) and model.

E Estimating Output Elasticities

E.1 Data description, sample, and definitions

We use data from Compustat for 1960–2016. We use the following variable definitions and conventions:

- Revenue y^R_{tf}: we measure revenue using firm sales—*SALES* in Compustat.

- Expenditures in variable inputs v_{tf}: we measure these expenditures using the cost of goods sold— *COGS* in Compustat.

- Stock of capital k_{tf}: we follow De Loecker, Eeckhout and Unger (2020) and measure the capital stock using the gross value of property, plants, and equipment—*PPEGT* in Compustat. We obtained similar results using an alternative measure of capital computed using the perpetual inventory method. For this measure, we use the gross value of property, plants, and equipment as our initial stock. We then measure net investment as the difference in the net capital stock—*PPENT* in Compustat—over consecutive periods and deflate this over time using the investment price deflator to compute the capital stock over time.

- Investment rate x_{tf}: we measure the investment rate as the percent change in capital; that is, $\ln x_{tf} = \ln k_{t+1,f} - \ln k_{tf}$

- Industry and firm groupings $c(f)$: we conduct our estimation separately for 23 NAICS industries, roughly defined at the 2-digit level. When grouping firms into size quintiles, we do so for each year and within each 3-digit NAICS industry. We also experimented with the classification of industries based on SIC codes used in Baqaee and Farhi (2020) and obtained very similar results.

- Sample definition and trimming: following De Loecker, Eeckhout and Unger (2020), we trim the sample by removing firms in the bottom 5th and top 5th percentiles of the *COGS*-to-*SALES* distribution. In addition, following Baqaee and Farhi (2020), we exclude firms in farm and agriculture, construction, real estate, finance, and utilities from our capital elasticity and markup calculations in Figures 3 and 6.

- Winsorizing: we winsorize the obtained revenue elasticities at zero, and take 5-year moving averages to smooth them. Moreover, following Baqaee and Farhi (2020), we winsorize our markup estimates at the 5th and 95th percentile of their distribution.

E.2 Estimation approach and details

Consider a firm that produces output by combining capital, k, and variable inputs, v, such as labor and materials. This section describes our approach for estimating the output-to-capital elasticity ε^k_{tf} and the output-to-variable-input elasticity ε^v_{tf} from firm-level data on revenue (y), expenditures in variable inputs (v), and capital (k). Following Olley and Pakes (1996) and Ackerberg, Caves and Frazer (2015), we make the following assumptions:[1]

A1 Differences across firms in the price of variable inputs reflect quality, which implies that we can treat expenditures in variable inputs as a measure of their quality-adjusted quantity.

A2 Revenue y^R_{tf} is given by a revenue production function of the form

$$\ln y^R_{tf} = z^R_{tf} + \varepsilon^{Rv}_{tc(f)} \cdot \ln v_{tf} + \varepsilon^{Rk}_{tc(f)} \cdot \ln k_{tf} + \epsilon_{tf},$$

where $c(f)$ denotes groups of firms with the same degree of automation and facing a common process for their revenue productivity, which only differ in their revenue productivity, z^R_{tf}, and an ex-post shock ϵ_{tf} that is orthogonal to k_{tf} and v_{tf}.

A3 Unobserved productivity z^R_{tf} evolves according to a Markov process of the form

$$z^R_{tf} = g(z^R_{ft-1}) + \zeta_{tf},$$

where ζ_{tf} is orthogonal to k_{tf} and v_{ft-1}, and the function g is common to all firms in the same group $c(f)$.

A4 True revenue, $\ln y^{R*}_{tf} = \ln y_{tf} - \epsilon_{tf}$ can be expressed as

$$\ln y^{R*}_{tf} = h(\ln x_{ft}, \ln k_{tf}, \ln v_{tf}),$$

where $\ln x_{tf} = \ln k_{t+1,f} - \ln k_{tf}$ denotes the investment rate of a firm and the function h is common to all firms in the same group $c(f)$.

[1] An alternative approach to estimating markups assumes constant returns to scale (as we do) and directly measures the user cost of capital as $R = r + \delta - \pi_k$, where r is a required rate of return inclusive of an industry-specific risk premium, δ is the depreciation rate, and π_k is the expected change over time in capital prices. One can then compute markups as revenue divided by total cost ($= V + RK$). The user-cost formula, which goes back to Hall and Jorgenson (1967) requires common and frictionless capital markets and assumes no adjustment costs for capital. This strikes us as restrictive when thinking about firms undergoing a costly automation process. Instead, the approach described below makes no assumptions about the marginal product of capital across firms, or the importance of adjustment costs.

A5 The gross output production function exhibits constant returns to scale in capital and variable inputs, which implies that output elasticities are given by

$$\text{(A8)} \qquad \varepsilon^{v}_{tf} = \varepsilon^{Rv}_{tc(f)} \Big/ \left(\varepsilon^{Rv}_{tc(f)} + \varepsilon^{Rk}_{tc(f)}\right) \qquad\qquad \varepsilon^{k}_{tf} = \varepsilon^{Rk}_{tc(f)} \Big/ \left(\varepsilon^{Rk}_{tc(f)} + \varepsilon^{Rk}_{tc(f)}\right).$$

Assumptions A1–A4 are standard in the literature. Assumption A4 justifies the use of the investment rate as a proxy variable. Economically, this assumption requires that all firms in a given group share the same investment policy function $k_{t+1,f} = \pi(k_{tf}, z^{R}_{tf})$, and that this common policy function is invertible. Under these assumptions, and given a grouping of firms $c(f)$, we can estimate revenue elasticities following the usual approach from Ackerberg, Caves and Frazer (2015), which uses the investment rate as a proxy variable to obtain true revenue and then estimates revenue elasticities by exploiting the orthogonality of ζ_{tf} to k_{tf} and $v_{t-1,f}$.

Assumption A5 is added to deal with the fact that we do not observe prices, such that the usual estimation procedure yields revenue elasticities, not the quantity elasticities that are relevant for computing markups (see Bond et al., 2021). Under Assumption A5 we can recover output elasticities from revenue elasticities using (A8). Suppose that revenue is given by $y^R = p(q) \cdot q$, where $p(q)$ is the inverse demand curve. Quantity elasticities and revenue elasticities are then linked according to $\varepsilon^{Rv} = (p'(q) \cdot q/p(q) + 1) \cdot \varepsilon^{v}$ and $\varepsilon^{Rk} = (p'(q) \cdot q/p(q) + 1) \cdot \varepsilon^{k}$, where $1/\mu = (p'(q) \cdot q/p(q) + 1)$. Assuming constant returns to scale implies that $\varepsilon^{v} = \varepsilon^{Rv}/(\varepsilon^{Rv} + \varepsilon^{Rk})$, as wanted.

Given a grouping of firms $c(f)$, we can estimate revenue elasticities following the usual approach from Ackerberg, Caves and Frazer (2015), which uses investment as a proxy variable for unobserved productivity. This requires a first-stage regression where we first compute "true" output as

$$\ln y^{R*}_{tf} = \mathbb{E}[\ln y^{R}_{tf} | \ln x_{tf}, \ln k_{tf}, \ln v_{tf}, t, c(f)] = h(\ln x_{tf}, \ln k_{tf}, \ln v_{tf}; \theta^{h}_{tc(f)}).$$

Here $\theta^{h}_{tc(f)}$ is a parametrization for a flexible function h that might vary over time and between groups of firms. For any pair of revenue elasticities $\varepsilon^{Rv}_{tc(f)}$ and $\varepsilon^{Rk}_{tc(f)}$, one can then compute revenue productivity as

$$z^{R}_{tf} = \ln y^{R*}_{tf} - \varepsilon^{Rv}_{tc(f)} \cdot \ln v_{tf} - \varepsilon^{Rk}_{tc(f)} \cdot \ln k_{tf},$$

estimate the flexible model

$$z^{R}_{tf} = g(z^{R}_{t-1,f}; \theta^{g}_{tc(f)}) + \zeta_{tf},$$

where $\theta^{g}_{tc(f)}$ is a parametrization for a flexible function g, and form the following moment conditions that identify the revenue elasticities:

$$\mathbb{E}\left[\zeta_{tf} \otimes (\ln k_{tf}, \ln v_{t-1,f})\right] = 0.$$

In our baseline approach, we parametrize the functions h and g using quadratic polynomials and conduct our estimation over 10-year rolling windows. More importantly, and in line with the emphasis in our model that large firms operate different technologies and face a different demand curve, we group firms by quintiles of sales in each industry. Thus, our estimation provides output elasticities that vary over time, by industry, and by quintiles of firm size in each industry. This represents a significant deviation from previous papers which assume that all firms in a given industry share the same output elasticities.

A byproduct of this estimation procedure are series for revenue TFP, z^{R}_{tf}. The estimated persistence of revenue TFP is 0.95 for manufacturing and 0.86 for retail, wholesale, utilities and transportation. These justifies the values of ρ_z used in our calibration approach.

Besides our main estimation approach, we also explored the following variations:

Estimates parametrizing g and h using cubic polynomials We estimate elasticities under the same assumptions outlined in the main text, but parametrize g and h using cubic polynomials. Figure A3 plots the behavior of the resulting output elasticities over time by firm size quintile. Figure A4 reports the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share.

Estimates assuming there are no ex-post shocks ϵ In the absence of ex-post shocks, we can treat observed revenue as true revenue and there is no need to use a proxy variable to recover productivity. Instead, we can compute revenue productivity directly as

$$z^{R}_{tf} = \ln y^{R}_{tf} - \varepsilon^{Rv}_{tc(f)} \cdot \ln v_{tf} - \varepsilon^{Rk}_{tc(f)} \cdot \ln k_{tf},$$

and proceed with the rest of the estimation in the same way as before.

Figure A5 plots the behavior of the resulting output elasticities over time by firm size quintile. Figure A6 reports the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share.

Estimates assuming a linear Markov process for productivity Suppose that productivity follows a linear Markov process

$$z_{tf}^{R} = \beta z_{t-1,f}^{R} + \zeta_{tf}.$$

Define $\upsilon_{tf} = z_{tf}^{R} + \epsilon_{tf}$. Because ex-post shocks are i.i.d, we have that υ_{tf} also follows a linear Markov process

$$\upsilon_{tf} = \beta \upsilon_{t-1,f} + \underbrace{\zeta_{tf} + \epsilon_{tf} - \beta \epsilon_{t-1,f}}_{=\iota_{tf}}.$$

Estimation proceeds as follows. First, we can compute υ_{tf} directly as

$$\upsilon_{tf} = \ln y_{tf}^{R} - \varepsilon_{tc(f)}^{Rv} \cdot \ln v_{tf} - \varepsilon_{tc(f)}^{Rk} \cdot \ln k_{tf}.$$

Then we estimate the linear model

$$\upsilon_{tf} = \beta \upsilon_{t-1,f} + \iota_{tf},$$

and base estimation on the moment conditions

$$\mathbb{E}\left[\iota_{tf} \otimes \left(\ln k_{tf}, \ln v_{t-1,v}\right)\right] = 0.$$

Figure A7 plots the behavior of the resulting output elasticities over time by firm size quintile. Figure A8 reports the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share.

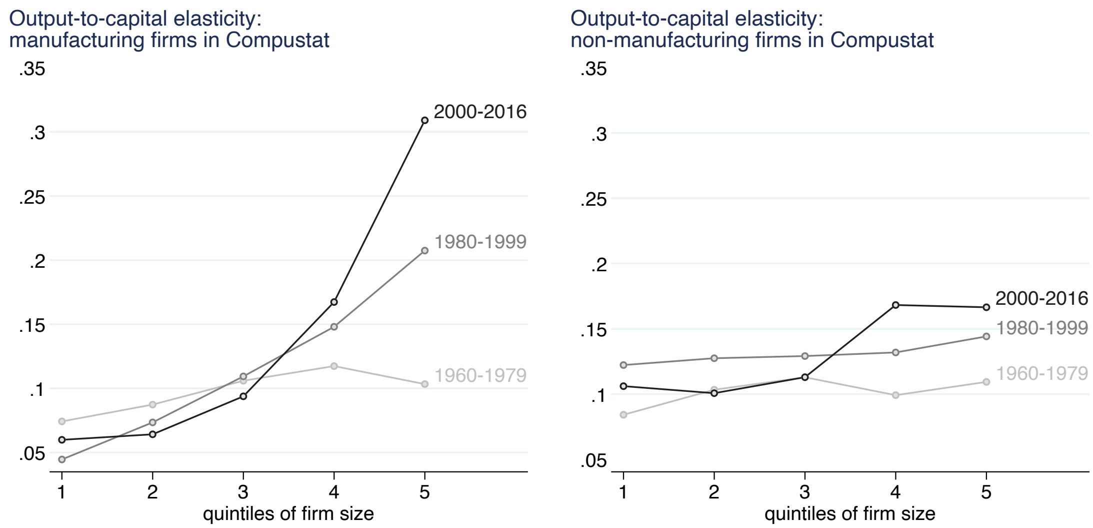

Figure A3: Output-to-capital elasticities for Compustat firms estimated using a cubic parametrization of g and h. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms. Firm-level elasticities are estimated using a cubic parametrization for g and h, as explained in Appendix E.

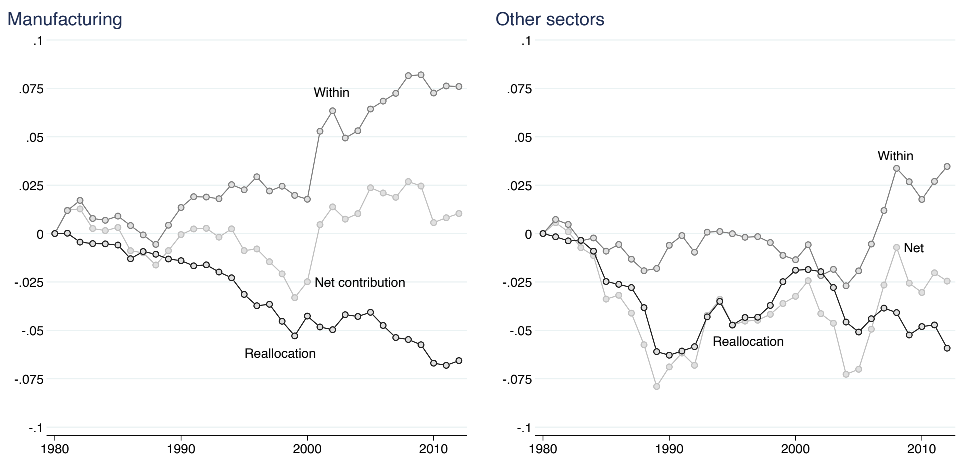

Figure A4: Decomposition of the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share. See the main text for details on this decomposition. Firm-level markups are estimated using a cubic parametrization for g and h, as explained in Appendix E. The left panel provides the decomposition for manufacturing firms in Compustat. The right panel provides the decomposition for Compustat firms in other economic sectors.

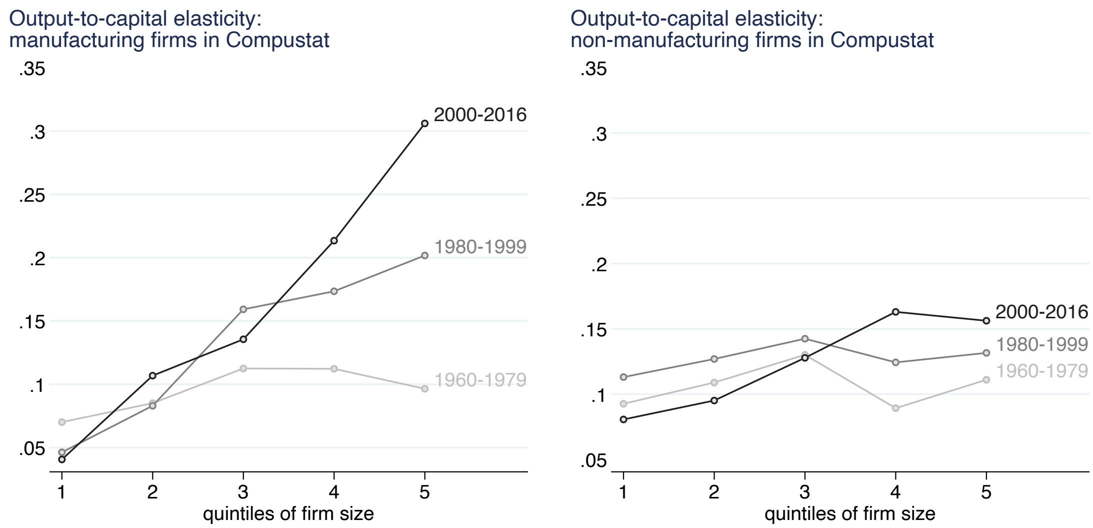

Figure A5: Output-to-capital elasticities for Compustat firms estimated under the assumption that there are no ex-post shocks. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms. Firm-level elasticities are estimated under the assumption of no ex-post shocks, as explained in Appendix E.

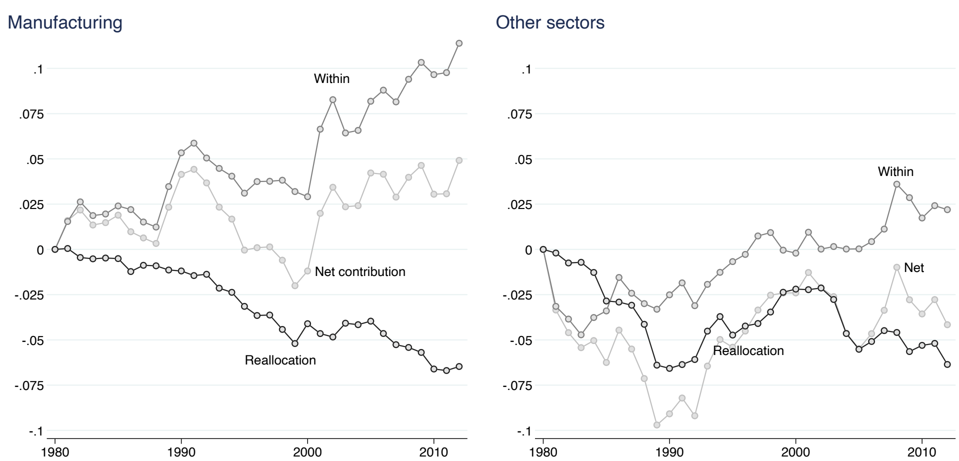

Figure A6: Decomposition of the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share. See the main text for details on this decomposition. Firm-level markups are estimated under the assumption of no ex-post shocks, as explained in Appendix E. The left panel provides the decomposition for manufacturing firms in Compustat. The right panel provides the decomposition for Compustat firms in other economic sectors.

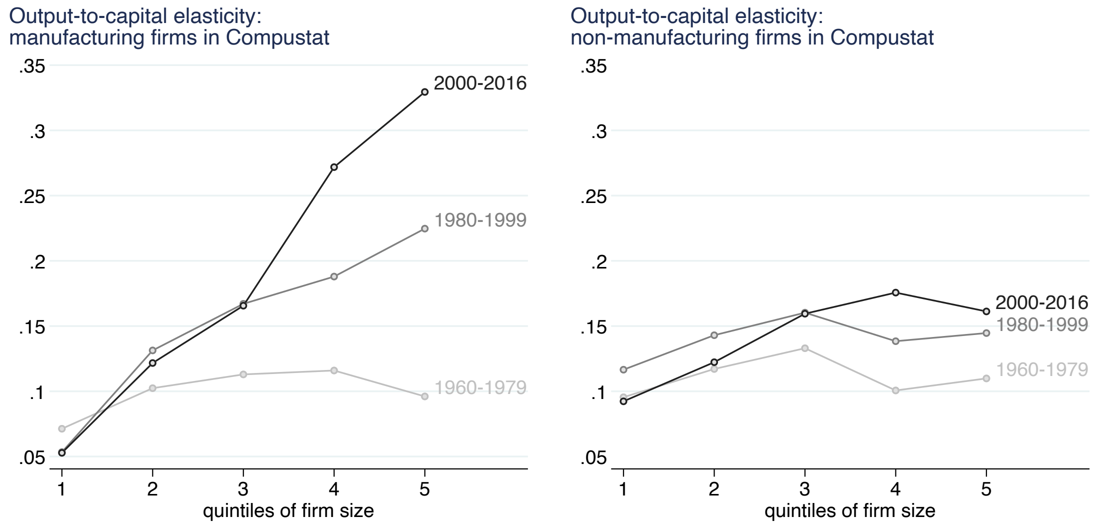

Figure A7: Output-to-capital elasticities for Compustat firms estimated under the assumption that productivity follows a linear Markov process. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms. Firm-level elasticities are estimated under the assumption that productivity follows a linear Markov process, as explained in Appendix E.

Figure A8: Decomposition of the contribution of within-firm changes in markups and between-firm reallocation to (percent) changes in the labor share. See the main text for details on this decomposition. Firm-level markups are estimated under the assumption that productivity follows a linear Markov process, as explained in Appendix E. The left panel provides the decomposition for manufacturing firms in Compustat. The right panel provides the decomposition for Compustat firms in other economic sectors.

E.3 Implications for the average markup in the economy as a whole

Figure A9 plots the implied time series for the economy-wide aggregate markup, computed as a sales-weighted *harmonic* mean of firm-level markups. Our estimates for markups suggest that they have been stable over time at around 1.2. This is in line with our quantitative exercise, which points to a modest increase in markups.

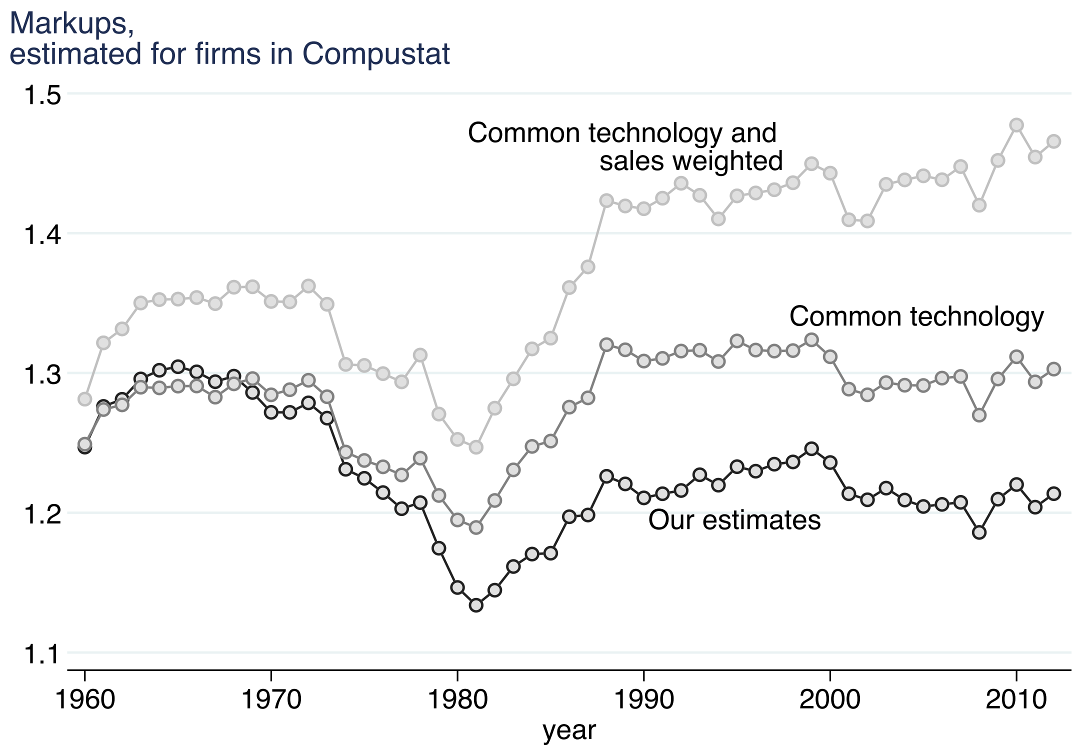

FIGURE A9: MARKUPS. The figure presents the aggregate markup for firms in Compustat. Our estimates are obtained as as a sales-weighted *harmonic* mean of firm-level markups. The figure also reports the aggregate markup that would result under the assumption of common output elasticities across firms in the same industry, and a version of these estimates that aggregates firms' markups using a sales-weighted arithmetic mean.

For comparison, we provide an alternative estimate of the aggregate markup obtained under the assumption that all firms in an industry operated technologies with the same capital intensity (as opposed to letting it vary by size class). This series reveals a mild secular increase in the aggregate markup from 1.25 in 1960 and 1.2 in 1980 to 1.3 in recent years, which aligns with the harmonic-mean estimates in Edmond, Midrigan and Xu (2022). We also provide estimates for an *arithmetic* mean of sales-weighted markups obtained under the assumption that all firms in a given industry operate technologies with the same capital intensity, which coincide with the series in De Loecker, Eeckhout and Unger (2020). Despite its increasing trend over time, this series is inappropriate for understanding the contribution of markups to the decline in the labor share because it ignores differences in technology across firm-size classes and uses the wrong weights for aggregation.

E.4 Additional evidence from Compustat

This section provides additional descriptive statistics from Compustat that support the notion that large firms operate more capital-intensive technologies. In what follows, we estimate regression models of the form

$$\ln y_{tfi} = \alpha_{ti} + \beta_{tc(f)} + \varepsilon_{tfi}, \tag{A9}$$

where we explain different measures for the capital intensity y_{tfi} of firm f in industry i at time t as a function of industry and year fixed effects (the α_{ti}) and size class dummies ($\beta_{tc(f)}$) that are allowed to vary over time. In particular, we estimate different size-class dummies for the periods of 1960–1980, 1980–2000 and 2000–2016. We treat firms in the smallest size class of an industry as the excluded category and report estimates weighted by firm sales.

Figure A10 plots estimates of equation (A9) for firms' investment rates, defined as their investment ($CAPX$ in Compustat) normalized by variable cost (top panel), employment (middle panel), and sales (bottom panel). The left panel provides estimates for manufacturing firms and the right panel for firms outside of manufacturing. For the 1980–2000 and 2000–2016 period, the largest manufacturing firms in each industry have had investment rates 60–140 log points higher than those of the smallest firms. Outside of manufacturing, the difference is less pronounced, with the largest firms having 10–90 log points higher investment rates than the smallest firms in their industries. In both cases, the gradient by size has become steeper over time.

Figure A11 plots estimates for firms' capital intensity, defined as their net capital stock ($PPENT$ in Compustat) normalized by variable cost (top panel), employment (middle panel), and sales (bottom panel). For the 1980–2000 and 2000–2016 period, the largest manufacturing firms in each industry had a 55 log point higher capital to variable cost ratio, a 120 log point higher capital per worker, and a 45 log point higher capital to sales ratio than the smaller firms in their industries. Here too, we see some evidence of the gradient by size becoming steeper over time, though the gradient and its rotation are less pronounced outside of manufacturing.

Finally, Figure A12 plots estimates for firms' reliance on capital services. Along a balanced growth path, the flow value of capital services used by a firm can be computed as[2]

$$\text{flow value of capital services} = (r - g) \cdot \text{net capital stock} + \text{capital expenditures}.$$

The figure provides estimates normalizing the flow value of capital services by variable costs (so that we get a measure of capital services relative to variable input services), employment, and sales (a measure of capital services in sales). In this exercise, we fix $r - g = 2.5\%$, which aligns with the calibration in Farhi and Gourio (2018). For the 1980–2000 and 2000–2016 period, the largest manufacturing firms in each industry had a 70 log point higher reliance on capital services vs. variable input services when compared to the smallest firms in their industries. Here too, we see some evidence of the gradient by size becoming steeper over time, with the gradient and its rotation being less pronounced outside of manufacturing.

[2]In particular, suppose the firm faces no adjustment costs. Then the PDV of capital services equal the PDV of capital costs. The PDV of capital costs are $(1+r) \cdot$ net capital stock $+ \frac{1+r}{r-g} \cdot$ capital expenditure, which gives the cost of the initially installed capital and of financing it plus the PDV of capital expenditures. The flow value of capital services is $\frac{r-g}{1+r} \cdot$ PDV of capital costs and we get the formula in the text.

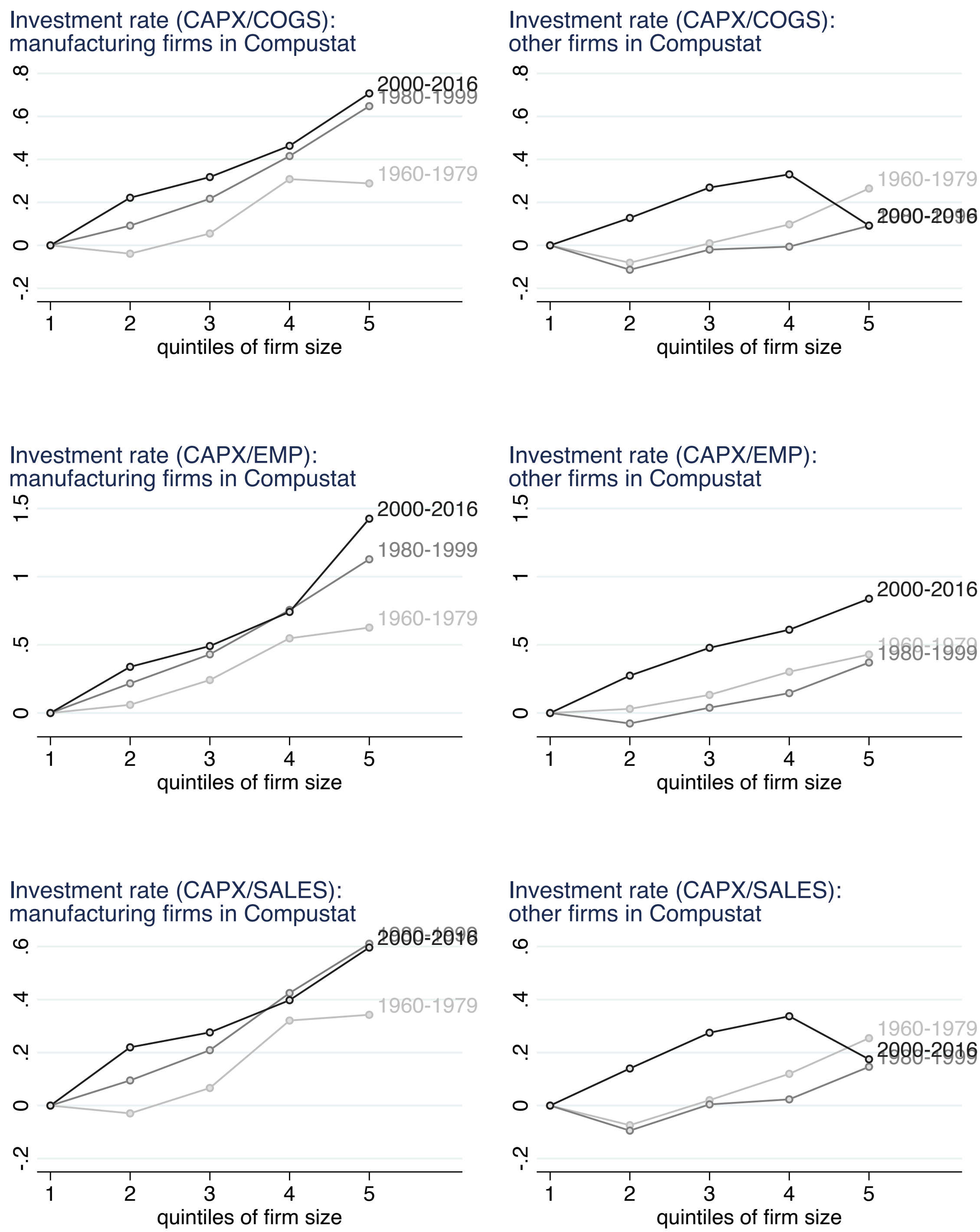

Figure A10: Investment rates, Compustat. The figure presents estimates of the relative difference in investment rates by firm-size class using Compustat. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms.

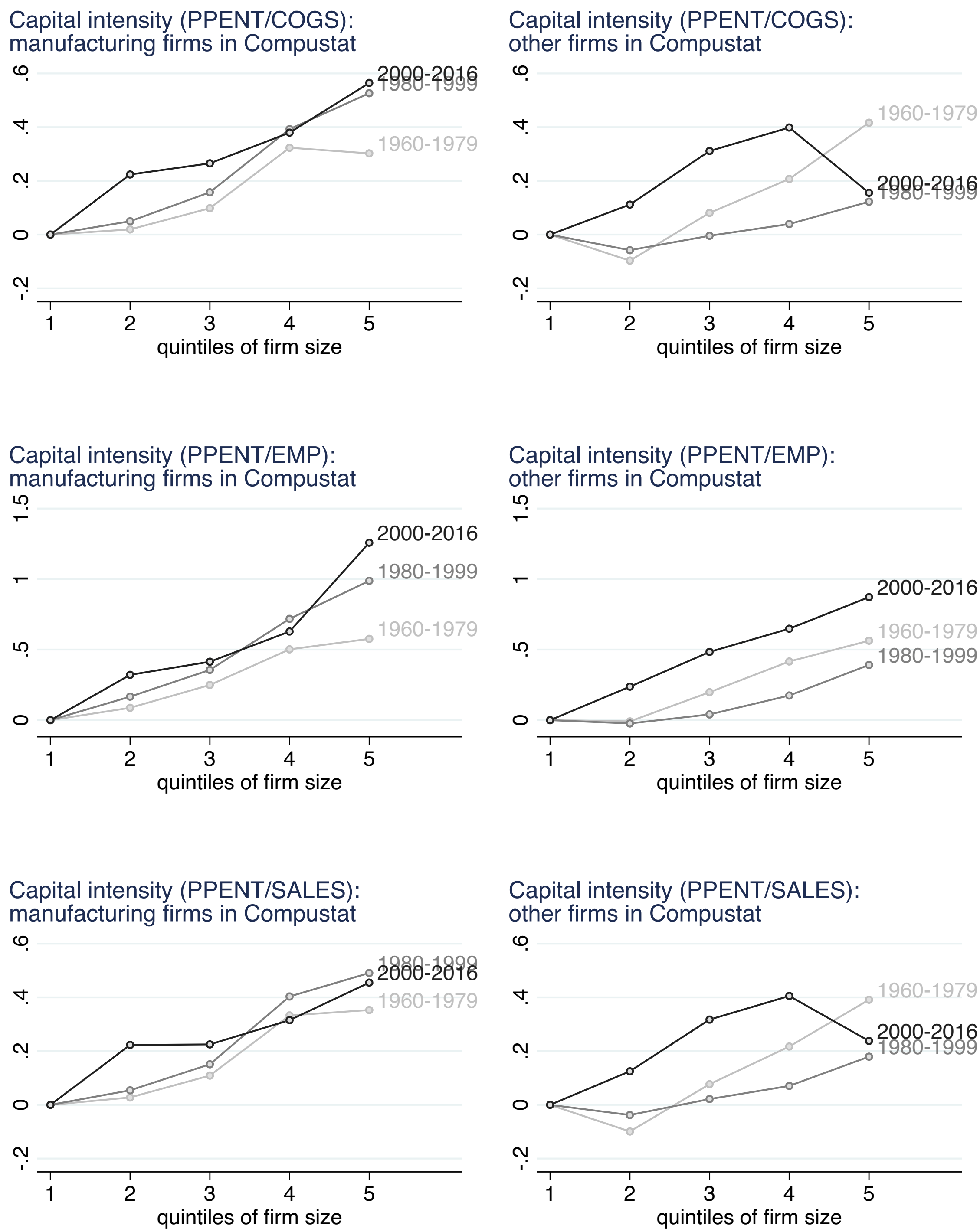

FIGURE A11: CAPITAL INTENSITY, COMPUSTAT. The figure presents estimates of the relative difference in capital intensity by firm-size class using Compustat. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms.

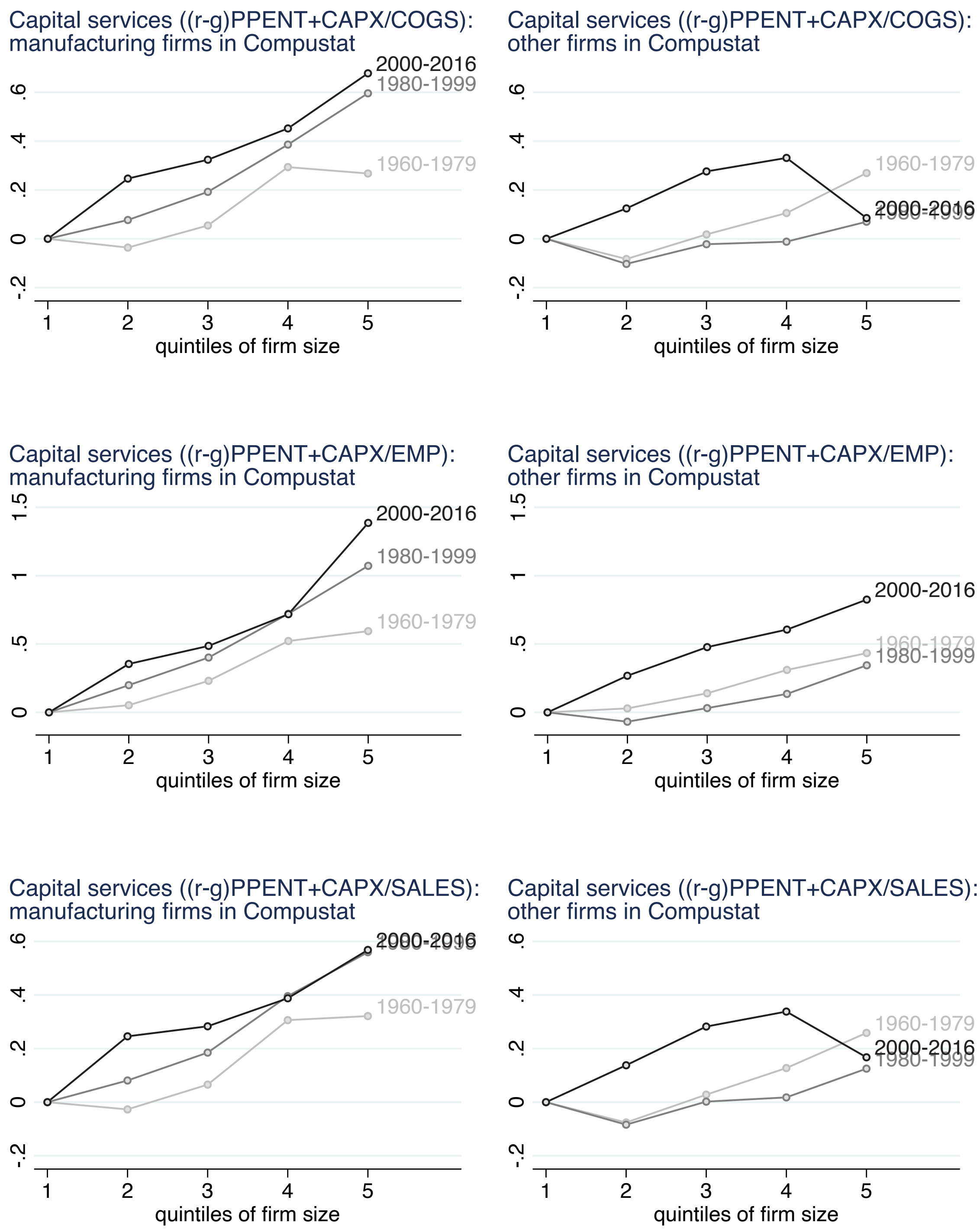

FIGURE A12: CAPITAL SERVICES, COMPUSTAT. The figure presents estimates of the relative difference in capital services by firm-size class using Compustat. The left panel presents estimates for Compustat manufacturing firms. The right panel presents estimates for Compustat non-manufacturing firms.

F Measurement of capital prices

We create measures of quality-adjusted capital prices at the sectoral level building on DiCecio (2009), Cummins and Violante (2002), and Gordon (1990).

In the first step, we obtain data on nonresidential asset prices and quantities from the BEA Fixed Asset Tables. These data cover 39 types of equipment, 32 types of structures, 3 types of software, and 22 types of other intellectual property products. We exclude other intellectual property products from our analysis since these would be considered part of the fixed cost of adopting new technologies in our model. The data cover the period from 1947 to 2020 and include information on investment at current nominal prices, investment at constant 2012 prices, stocks, and depreciation.

Using these series, we construct a price index for each detailed asset a as

$$p_{a,t} = \frac{\text{investment at current nominal prices}_{a,t}}{\text{investment at constant 2012 prices}_{a,t}}.$$

We let $\Delta \ln p_{a,t}$ denote the percent change in asset prices between time t and $t+1$.

Our second step involves adjusting the BEA prices for quality. We follow the work by DiCecio (2009) and Cummins and Violante (2002). These authors use the series for quality-adjusted investment prices from Gordon (1990), and which covered the postwar period up to 1983, and extend it from 1947 to 2011. Cummins and Violante (2002) estimate a statistical model explaining Gordon's quality-adjusted price indices as a function of those by the BEA/NIPA, their lags, and time trends. They then extrapolate this model to produce quality-adjusted price indices for 1947–2000. DiCecio (2009) follows the same procedure and creates an updated series up to 2011 for equipment and software. We use the estimates from DiCecio (2009) on the quality adjusted price of equipment and software, obtained via FRED (variable code PERICD), and denote the percent change in the quality-adjusted price of software and equipment as $\Delta \ln p^*_{t,\text{E\&S}}$. We then compute a user-cost weighted price index for software and equipment from the BEA data using a Törnqvist index

$$\Delta \ln p_{t,\text{E\&S}} = \sum_{a\in\text{E\&S}} \frac{1}{2} \cdot \left(s^{\text{E\&S}}_{t+1,a} + s^{\text{E\&S}}_{a,t} \right) \cdot \Delta \ln p_{a,t},$$

where $s^{\text{E\&S}}_{a,t}$ denotes the share of asset a in equipment and software capital services.[3] The

[3] We compute capital services derived from an asset a as

$$\text{capital services}_{a,t} = (r + \delta_a - \Delta \ln p_{a,t}) \cdot \text{stock asset}_{a,t},$$

where we take a required rate of return $r = 4\%$, and use the depreciation rate and change in capital prices from the BEA.

implied quality adjustment for equipment and software is therefore equal to

$$\Delta\text{quality adjustment}_{\text{E\&S},t} = \Delta \ln p_{E\&S,t} - \Delta \ln p^*_{\text{E\&S},t}.$$

In the BEA data, the price of equipment and software declined by an average of 3.1% per year in 1980-2011. The quality-adjusted series from DiCecio (2009) shows a decline on 5.7% per year, which implies an improvement in the quality of equipment and software of 2.6% per year.

We then compute a quality adjusted series for the detailed equipment and software products in the BEA data as

$$\Delta \ln p^*_{a,t} = \Delta \ln p_{a,t} - \Delta\text{quality adjustment}_{\text{E\&S},t}.$$

This assumes a common quality adjustment for all types of software and equipment. For structures, we do not perform quality adjustment.

In the third step, we account for changes in taxation using the estimates in Acemoglu, Manera and Restrepo (2020) of effective taxes on equipment, software, and structures. These authors estimate that the effective tax on equipment decreased from 12.4% to 4.7% during 1981–2018, the effective tax on software decreased from 14.6% to 4.7% during 1981–2018, and the effective tax on structures increased from 8.3% to 9% during 1981–2018. These changes in taxes imply a further reduction in the cost of producing tasks with capital of close to 10% during the whole 1981–2018 period.

In the fourth step, we compute a measure for the relative price of capital by asset, $\Delta \ln p^{*,r}_{a,t}$ by taking our quality-adjusted price indices adjusted for taxes and subtracting changes in the BEA price of consumption expenditures index.

In the final step, we construct a sector-specific measure of capital prices using a user-cost weighted Törnqvist index

$$\Delta \ln p_{i,t} = \sum_a \frac{1}{2} \cdot \left(s^{i,k}_{a,t+1} + s^{i,k}_{a,t} \right) \cdot \Delta \ln p^{*,r}_{a,t},$$

where $s^{i,k}_{a,t}$ denotes the share of asset a in total capital services in sector i, computed also from the industry-level version of the BEA Fixed Asset Tables. This index provides the average decline in capital prices used in sector i over the 1980–2011 period.

Our resulting sectoral price indices imply that the average price of capital used in manufacturing declined by 108 log points from 1980 to 2011. For retail, the average decline was of 86 log points, for utilities 68 log points, and for wholesale of 159 log points. These differences across sectors reflect the different bundles of capital goods used, with manufacturing and wholesale investing more heavily in equipment and software.

F.1 Separating intensive and extensive margins

As discussed in the text, there is no straightforward method for separating advances in capital at the intensive and extensive margin. The problem is that data on capital prices are available for coarse categories and do not distinguish between investments to replace old equipment (intensive margin) and investment in capital used in tasks previously assigned to labor (extensive margin).

Despite this limitation, the available data does point to a more limited decline in capital prices at the intensive margin, in line with our inferred q shocks for manufacturing and retail.

There are two ways of illustrating this point. The first is model dependent. Imagine that all advances in capital take place at the intensive margin. Our model implies that, in this case, one can compute an aggregate price index as an exact CES index of all capital price declines with an elasticity of substitution of η, which governs the substitution across the tasks benefiting from these advances. This is the correct way of aggregating all capital price declines taking place at the intensive margin. This index can be computed using "exact hat algebra" as

$$\frac{p_{i,t}^{CES}}{p_{i,t_0}^{CES}} = \left(\sum_a s_{a,t_0}^{i,k} \cdot \left(\frac{p_{a,t}^*}{p_{a,t_0}^*} \right)^{1-\eta} \right)^{\frac{1}{1-\eta}}.$$

Using a value of $\eta = 0.5$ from Humlum (2019), we estimate a 76 log point decrease in capital prices at the intensive margin for manufacturing and a 31 log point decrease in capital prices at the intensive margin for retail. In both cases, the data points to a minor share of capital advances at the intensive margin.

The second strategy involves classifying assets into "established" and "new" types of capital. We classify an asset as "established" in a sector if its net investment rate in the 70s was below the average sectoral rate of capital formation. We view a below-average net investment rate as an indication of an asset whose stock has already reached a high enough level. We classify an asset as "new" if its net investment rate in the 70s was above average. We view this as an indication of an asset whose stock was only being built.

In manufacturing, the price of established assets decreased by 48.6 log points and the price of new assets decreased by 166 log points. In retail, the price of established assets decreased by 36 log points and the price of new assets decreased by 117 log points.

The intuitive (but imperfect) idea is that price declines for established assets provide an indication of the size of advances at the intensive margin, and price declines for new assets provide an indication of the potential magnitude of advances at the extensive margin.

This mapping is necessarily imperfect, especially for the extensive margin. For example, industrial furnaces might have been in use in the 1980s while robots were not. Yet both would be aggregated into the BEA asset class "Special industrial machinery," causing us to mislabel advances in robotics as taking place at the intensive margin. This would cause us to over-estimate the extent of advances in capital at the intensive margin. Conversely, new types of capital and assets might be associated with new industries and products and are not necessarily used to automate tasks as in our model. For example, the stock of solar panels is a new asset that has nothing to do with automation. This would cause us to over-estimate the extent of advances in capital at the extensive margin.

For these reasons, our preferred interpretation of these estimates is as a reality check. The estimates support the idea that capital advances at the intensive margin were relatively modest. The estimates also point to larger capital advances in "new" types of capital, driven in part by software and computers. Both facts are necessary if we believe extensive margin advances in capital were a dominant force during this period. However, the missing pieces are that we do not know if in practice the "new" types of capital facilitated the substitution of capital for labor at the extensive margin, as in our model, or if these new types of capital had other uses. We also do not know if all forms of capital inside an established asset class operated at the intensive margin, even though this seems more plausible.

Both calculations reflect the fact that the decline in capital prices has been far from uniform, even across broad asset classes. Figure A13 illustrates this point. It plots the price decline per year for all non-residential fixed assets in the BEA Fixed Asset Tables. The figure shows a more pronounced decline for software and computer equipment.

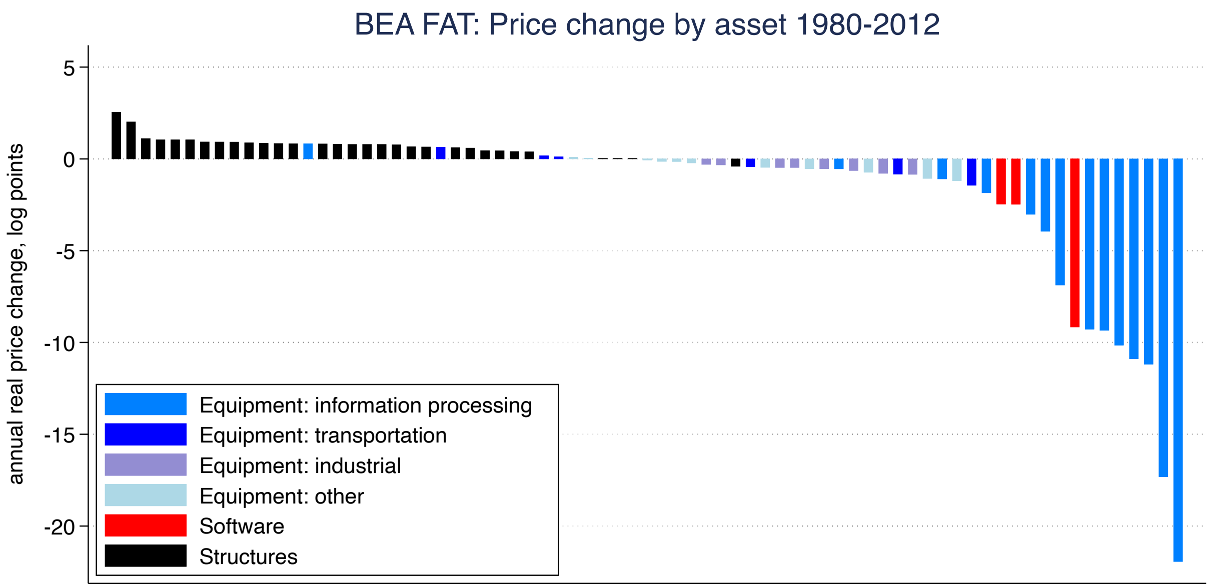

FIGURE A13: CAPITAL PRICE DECLINES BY BEA FIXED ASSET CLASS. The figure plots the average annual real quality-adjusted capital price change over 1980–2012, for private nonresidential fixed assets in the BEA Fixed Asset Tables.

G Historical behavior of labor shares

This section provides additional motivation for our focus on the 1982–2012 period. As a starting point, Figure A14 provide data on payroll shares by sector for 1947–1987 and 1987–2016 from the BEA industry accounts. We split the data into these two periods due to changes in industry definitions introduced by the BEA in 1987, as it switched from the *Standard Industry Classification* to the *North American Industry Classification System*. As discussed in the main text, Figure A14 shows that payroll shares were constant or increasing up to 1982, and then started a sharp decline both in manufacturing, retail and wholesale. Payroll shares differ from labor shares in that they exclude compensation and self employment. But looking at payroll shares has the advantage of allowing us to go back further in time.

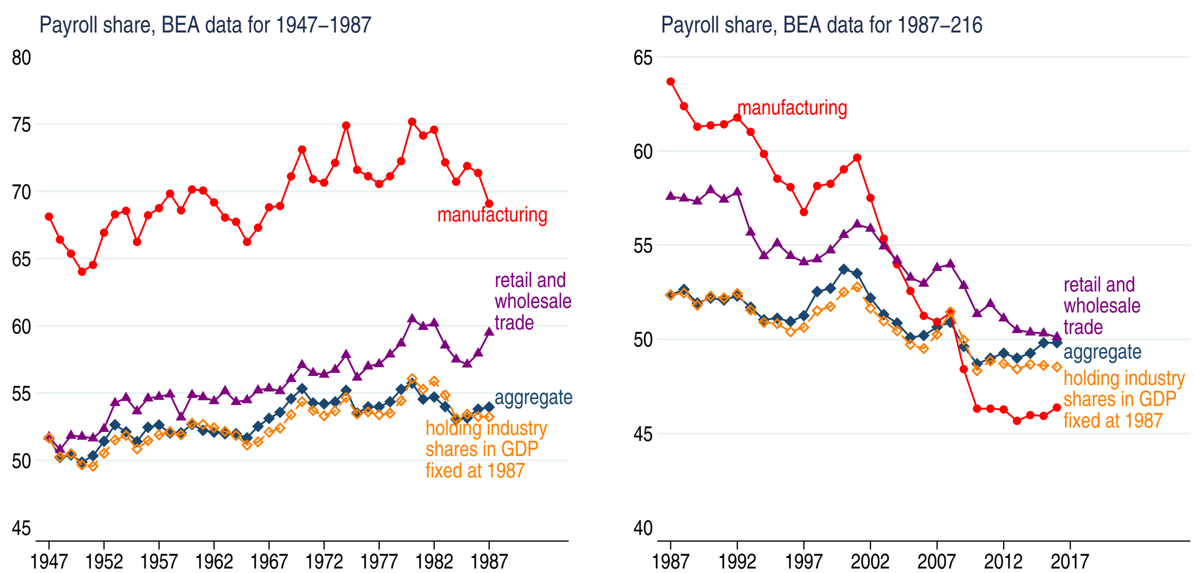

Figure A14: Payroll share in the US for 1947–2016. The figure plots the payroll share of value added, both for some specific sectors and the economy as a whole. Data from the BEA industry accounts. Industry definitions based on SIC in left panel, NAICS in right panel.

Labor shares (which also include non-wage compensation) are available starting in 1963 from the BEA-BLS integrated industry-level production account. Figure A15 confirms that labor shares exhibit the same trend behavior with a flat or slightly increasing trend until 1982 and a subsequent decline. This motivates our focus on the 1982–2012 period and supports our choice of 1982 as the steady state of the model.

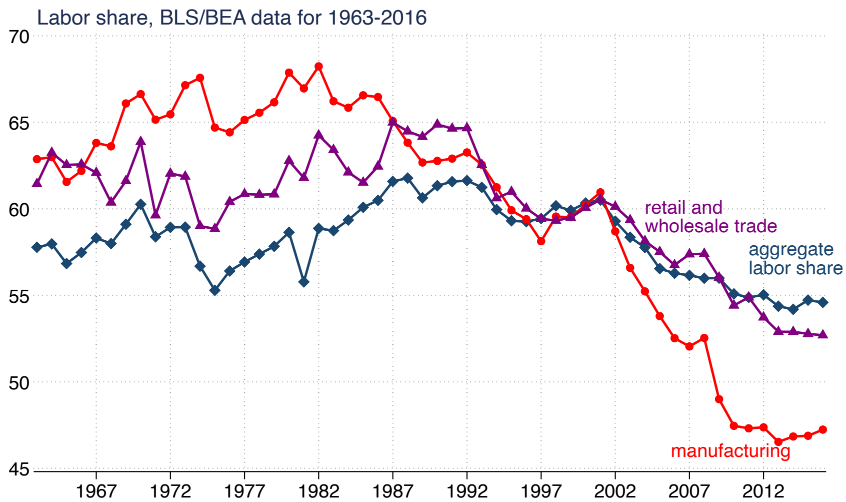

Figure A15: Labor share in the US for 1963–2016 The figure plots the labor share of value added, both for some specific sectors and the economy as a whole. Data from the BEA-BLS integrated industry-level production account.

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