

Nonparametric Counterfactual Predictions
in Neoclassical Models of International Trade
ONLINE APPENDIX

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A Proofs

A.1 Proposition 1

Proof of Proposition 1. (\Rightarrow) Suppose that $(\mathbf{q}, \mathbf{L}, \mathbf{p}, \mathbf{w})$ is a competitive equilibrium. For any country i , let us construct $L_i \equiv \{L_{ji}^n\}$ such that

$$L_{ji}^n = \sum_k l_{ji}^{nk} \text{ for all } i, j, \text{ and } n.$$

Together with the factors market clearing condition (5), the previous expression immediately implies

$$\sum_j L_{ij}^n = v_i^n \text{ for all } i \text{ and } n.$$

In order to show that (\mathbf{L}, \mathbf{w}) is a reduced equilibrium, we therefore only need to show

$$\begin{aligned} L_i &\in \operatorname{argmax}_{\tilde{L}_i} U_i(\tilde{L}_i) \\ \sum_{j,n} w_j^n \tilde{L}_{ji}^n &\leq \sum_n w_i^n v_i^n \text{ for all } i. \end{aligned} \quad (39)$$

We proceed by contradiction. Suppose that there exists a country i such that condition (39) does not hold. Since profits are zero in a competitive equilibrium with constant returns to scale, we must have $\sum_{j,k} p_{ji}^k q_{ji}^k = \sum_{j,n} w_j^n L_{ji}^n$. The budget constraint of the representative agent in the competitive equilibrium, in turn, implies $\sum_{j,n} w_j^n L_{ji}^n = \sum_n w_i^n v_i^n$. Accordingly, if condition (39) does not hold, there must be L'_i such that $U_i(L'_i) > U_i(L_i)$ and $\sum_{j,n} w_j^n (L'_{ji})' \leq \sum_n w_i^n v_i^n$. Now consider (\mathbf{q}'_i, L'_i) such that

$$\begin{aligned} (\mathbf{q}'_i, L'_i) &\in \operatorname{argmax}_{\tilde{\mathbf{q}}_i, \tilde{L}_i} u_i(\tilde{\mathbf{q}}_i) \\ \sum_k \tilde{l}_{ji}^{nk} &\leq (L'_{ji})' \text{ for all } j \text{ and } n, \\ \tilde{q}_{ji}^k &\leq f_{ji}^k(\tilde{l}_{ji}^k) \text{ for all } j \text{ and } k. \end{aligned}$$

We must have

$$u_i(\mathbf{q}'_i) = U_i(L'_i) > U_i(L_i) \geq u_i(\mathbf{q}_i),$$

where the last inequality derives from the fact that, by construction, L_i is sufficient to produce \mathbf{q}_i . Utility maximization in the competitive equilibrium therefore implies

$$\sum_{j,k} p_{ji}^k (q'_{ji})' > \sum_n w_i^n v_i^n.$$

Combining this inequality with $\sum_{j,n} w_j^n (L_{ji}^n)' \leq \sum_n w_i^n v_i^n$, we obtain

$$\sum_{j,k} p_{ji}^k (q_{ji}^k)' > \sum_{j,n} w_j^n (L_{ji}^n)'.$$

Hence, firms could make strictly positive profits by using L_i' to produce q_i' , which cannot be true in a competitive equilibrium. This establishes that (L, w) is a reduced equilibrium with the same factor prices and the same factor content of trade as the competitive equilibrium. The fact that $U_i(L_i) = u_i(q_i)$ can be established in a similar manner. If there were q_i' such that $u_i(q_i') = U_i(L_i) > u_i(q_i)$, then utility maximization would imply

$$\sum_{j,k} p_{ji}^k (q_{ji}^k)' > \sum_n w_i^n v_i^n = \sum_{j,n} w_j^n L_{ji}^n,$$

which would in turn violate profit maximization in the competitive equilibrium.

(\Leftarrow) Suppose that (L, w) is a reduced equilibrium. For any positive vector of output delivered in country i , $q_i \equiv \{q_{ji}^k\}$, let $C_i(w, q_i)$ denote the minimum cost of producing q_i ,

$$C_i(w, q_i) \equiv \min_{\tilde{l}_i} \sum_{j,k,n} w_j^n \tilde{l}_{ji}^{nk} \quad (40)$$

$$q_{ji}^k \leq f_{ji}^k(\tilde{l}_{ji}^k) \text{ for all } j \text{ and } k. \quad (41)$$

The first step of our proof characterizes basic properties of C_i . The last two steps use these properties to construct a competitive equilibrium that replicates the factor content of trade and the utility levels in the reduced equilibrium.

Step 1. For any country i , there exists $p_i \equiv \{p_{ji}^k\}$ positive such that the two following conditions hold: (i)

$$C_i(w, q_i) = \sum_{j,k} p_{ji}^k q_{ji}^k, \text{ for all } q_i > 0, \quad (42)$$

and (ii) if \tilde{l}_i solves (40), then \tilde{l}_i solves

$$\max_{\tilde{l}_{ji}^k} p_{ji}^k f_{ji}^k(\tilde{l}_{ji}^k) - \sum_n w_j^n \tilde{l}_{ji}^{nk} \text{ for all } j \text{ and } k. \quad (43)$$

For any i, j , and k , let us construct p_{ji}^k such that

$$p_{ji}^k = \min_{\tilde{l}_{ji}^k} \left\{ \sum_n w_j^n \tilde{l}_{ji}^{nk} \mid f_{ji}^k(\tilde{l}_{ji}^k) \geq 1 \right\}. \quad (44)$$

Take $\tilde{l}_{ji}^k(1)$ that solves the previous unit cost minimization problem. Since f_{ji}^k is homogeneous of degree one, we must have $f_{ji}^k(q_{ji}^k \tilde{l}_{ji}^k(1)) \geq q_{ji}^k$. By definition of C_i , we must also have $C_i(w, q_i) \leq \sum_{j,k,n} q_{ji}^k w_j^n \tilde{l}_{ji}^{nk}(1) = \sum_{j,k} p_{ji}^k q_{ji}^k$. To show that equation (42) holds, we therefore only need to show

that $C_i(\mathbf{w}, \mathbf{q}_i) \geq \sum_{j,k} p_{ji}^k q_{ji}^k$. We proceed by contradiction. Suppose that $C_i(\mathbf{w}, \mathbf{q}_i) < \sum_{j,k} p_{ji}^k q_{ji}^k$. Then there must be $q_{ji}^k > 0$ such that

$$\sum_n w_j^n l_{ji}^{nk} < q_{ji}^k \sum_n w_j^n l_{ji}^{nk}(1),$$

where l_{ji}^k is part of the solution of (40). Since f_{ji}^k is homogeneous of degree one, l_{ji}^k / q_{ji}^k would then lead to strictly lower unit cost than $l_{ji}^k(1)$, which cannot be. This establishes condition (i).

To establish condition (ii), we proceed again by contradiction. Suppose that there exists $(l_{ji}^k)'$ such that

$$p_{ji}^k f_{ji}^k((l_{ji}^k)') - \sum_n w_j^n (l_{ji}^{nk})' > p_{ji}^k f_{ji}^k(l_{ji}^k) - \sum_n w_j^n l_{ji}^{nk}. \quad (45)$$

Take the vector of output \mathbf{q}_i such that $q_{ji}^k = f_{ji}^k(l_{ji}^k)$ and zero otherwise. Condition (i) applied to that vector immediately implies

$$p_{ji}^k f_{ji}^k(l_{ji}^k) = \sum_n w_j^n l_{ji}^{nk}.$$

Combining this observation with inequality (45), we get $p_{ji}^k > \sum_n w_j^n (l_{ji}^{nk})' / f_{ji}^k((l_{ji}^k)')$, which contradicts the fact that p_{ji}^k is the minimum unit cost.

Step 2. Suppose that $(\mathbf{q}_i, \mathbf{l}_i)$ solves

$$\begin{aligned} \max_{\tilde{\mathbf{q}}_i, \tilde{\mathbf{l}}_i} u_i(\tilde{\mathbf{q}}_i) & \quad (46) \\ \tilde{q}_{ji}^k & \leq f_{ji}^k(\tilde{l}_{ji}^k) \text{ for all } j \text{ and } k, \\ \sum_{j,k,n} w_j^n \tilde{l}_{ji}^{nk} & \leq \sum_n w_i^n v_i^n. \end{aligned}$$

Then \mathbf{q}_i solves

$$\begin{aligned} \max_{\tilde{\mathbf{q}}_i} u_i(\tilde{\mathbf{q}}_i) & \quad (47) \\ \sum_{j,k} p_{ji}^k \tilde{q}_{ji}^k & \leq \sum_n w_i^n v_i^n, \end{aligned}$$

and \mathbf{l}_i solves

$$\max_{\tilde{l}_{ji}^k} p_{ji}^k f_{ji}^k(\tilde{l}_{ji}^k) - \sum_n w_j^n \tilde{l}_{ji}^{nk} \text{ for all } j \text{ and } k. \quad (48)$$

If $(\mathbf{q}_i, \mathbf{l}_i)$ solves (46), then

$$\begin{aligned} \mathbf{q}_i & \in \operatorname{argmax}_{\tilde{\mathbf{q}}_i} u_i(\tilde{\mathbf{q}}_i) \\ C_i(\mathbf{w}, \mathbf{q}_i) & \leq \sum_n w_i^n v_i^n. \end{aligned}$$

Combining this observation with Step 1 condition (i), we obtain that \mathbf{q}_i solves (47). Likewise, if

$(\mathbf{q}_i, \mathbf{l}_i)$ solves (46), then

$$\begin{aligned} \mathbf{l}_i &\in \operatorname{argmin}_{\bar{\mathbf{l}}} \sum_{j,k,n} \omega_j^n \bar{l}_{ji}^{nk}, \\ q_{ji}^k &\leq f_{ji}^k(\bar{\mathbf{l}}_{ji}^k) \text{ for all } j \text{ and } k. \end{aligned}$$

Combining this observation with Step 1 condition (ii), we obtain that \mathbf{l}_i solves (48).

Step 3. For all i , take $(\mathbf{q}_i, \mathbf{l}_i)$ that solves

$$\begin{aligned} \max_{\bar{\mathbf{q}}_i, \bar{\mathbf{l}}_i} u_i(\bar{\mathbf{q}}_i) & \tag{49} \\ \bar{q}_{ji}^k &\leq f_{ji}^k(\bar{\mathbf{l}}_{ji}^k) \text{ for all } j \text{ and } k, \\ \sum_k \bar{l}_{ji}^{nk} &\leq L_{ji}^n \text{ for all } j \text{ and } n, \end{aligned}$$

and set $\mathbf{q} = \sum_i \mathbf{q}_i$ and $\mathbf{l} = \sum_i \mathbf{l}_i$. Then $(\mathbf{q}, \mathbf{l}, \mathbf{p}, \boldsymbol{\omega})$ is a competitive equilibrium with the same factor prices, $\boldsymbol{\omega}$; (ii) the same factor content of trade, $L_{ji}^n = \sum_k l_{ji}^{nk}$ for all i, j , and n ; and (iii) the same welfare levels, $U_i(\mathbf{L}_i) = u_i(\mathbf{q}_i)$ for all i .

Since $(\mathbf{L}, \boldsymbol{\omega})$ is a reduced equilibrium, if $(\mathbf{q}_i, \mathbf{l}_i)$ solves (49), then $(\mathbf{q}_i, \mathbf{l}_i)$ solves (46). By Step 2, \mathbf{q}_i and \mathbf{l}_i must therefore solve (47) and (48), respectively. Hence, the utility maximization and profit maximization conditions (1) and (3) are satisfied. Since the constraint $\bar{q}_{ji}^k \leq f_{ji}^k(\bar{\mathbf{l}}_{ji}^k)$ must be binding for all j and k in any country i , the good market clearing condition (4) is satisfied as well. The factor market clearing condition directly derives from the fact that $(\mathbf{L}, \boldsymbol{\omega})$ is a reduced equilibrium and the constraint, $\sum_k \bar{l}_{ji}^{nk} \leq L_{ji}^n$, must be binding for all j and n in any country i . By construction, conditions (i)-(iii) necessarily hold. \square

A.2 Lemma 1

Proof of Lemma 1. We proceed in two steps.

Step 1. In a Ricardian economy, if good expenditure shares satisfy the connected substitutes property, then factor expenditure shares satisfy the connected substitutes property.

Our goal is to establish that factor demand, χ_i , satisfies the connected substitutes property—expressed in terms of the effective prices of the composite factors, $\boldsymbol{\omega}_i \equiv \{\tau_{ji} c_j\}$ —if good demand, $\boldsymbol{\sigma}_i$, satisfies the connected substitutes property, with

$$\boldsymbol{\sigma}_i(\mathbf{p}_i) \equiv \{s_i^k\} | s_i^k = p_i^k q_i^k / y_i \text{ for some } \mathbf{q}_i \in \operatorname{argmax}_{\bar{\mathbf{q}}} \{u_i(\bar{\mathbf{q}}) | \sum_k p_i^k \bar{q}_i^k \leq y_i\}.$$

Note that since \bar{u}_i is homothetic, $\boldsymbol{\sigma}_i$ does not depend on income in country i .

Consider a change in effective factor prices from $\boldsymbol{\omega}_i$ to $\boldsymbol{\omega}'_i$ and a partition of countries $\{M_1, M_2\}$ such that $\omega'_{ji} > \omega_{ji}$ for all $j \in M_1$ and $\omega'_{ji} = \omega_{ji}$ for all $j \in M_2$. Now take $x_i, x'_i > 0$ such that

$x_i \in \chi_i(\omega_i)$ and $x'_i \in \chi_i(\omega'_i)$. For each exporting country j , we can decompose total expenditure shares into the sum of expenditure shares across all sectors k ,

$$x_{ji} = \sum_k s_i^k x_{ji}^k,$$

where s_i^k denotes the share of expenditure on good k in in country i at the initial prices,

$$\begin{aligned} \{s_i^k\} &\in \sigma_i(\{p_i^k(\omega_i)\}), \\ p_i^k(\omega_i) &= \min_j \{\omega_{ji} / \alpha_{ji}^k\}. \end{aligned}$$

For any good k , there are two possible cases. If no country $j \in M_2$ has the minimum cost for good k at the initial factor prices, ω_i , then

$$\sum_{j \in M_2} x_{ji}^k = 0, \quad (50)$$

$$p_i^k(\omega) < p_i^k(\omega'). \quad (51)$$

Let us call this set of good K_1 . If at least one country $i \in M_2$ has the minimum cost for good k , then

$$\sum_{j \in M_2} (x_{ji}^k)' = 1, \quad (52)$$

$$p_i^k(\omega_i) = p_i^k(\omega'). \quad (53)$$

Let us call this second set of good K_2 . Since $x_i, x'_i > 0$, we know that both K_1 and K_2 are non-empty.

Now consider the total expenditure in country i on factors from countries $j \in M_2$ when factor prices are equal to ω'_i . It must satisfy

$$\sum_{j \in M_2} (x_{ji})' \geq \sum_{j \in M_2} \sum_{k \in K_2} (s_i^k)' (x_{ji}^k)' = \sum_{k \in K_2} (s_i^k)' \left[\sum_{j \in M_2} (x_{ji}^k)' \right].$$

Combining the previous inequality with (52), we obtain

$$\sum_{j \in M_2} (x_{ji})' \geq \sum_{k \in K_2} (s_i^k)'.$$

By the Inada conditions, all goods are consumed. Thus, we can invoke the connected substitutes property for goods in K_1 and K_2 . Conditions (51) and (53) imply

$$\sum_{k \in K_2} (s_i^k)' > \sum_{k \in K_2} s_i^k.$$

Since $\sum_{j \in M_2} x_{ji}^k \leq 1$, the two previous inequalities further imply

$$\sum_{j \in M_2} (x_{ji})' > \sum_{k \in K_2} s_i^k \left[\sum_{j \in M_2} x_{ji}^k \right] = \sum_{j \in M_2} \sum_{k \in K_2} s_i^k x_{ji}^k.$$

Finally, using (50) and the fact that $\{K_1, K_2\}$ is a partition, we get

$$\sum_{j \in M_2} (x_{ji})' > \sum_{j \in M_2} \sum_{k \in K_1} s_i^k x_{ji}^k + \sum_{j \in M_2} \sum_{k \in K_2} s_i^k x_{ji}^k = \sum_{j \in M_2} x_{ji}.$$

This establishes that χ_i satisfies the connected substitutes property.

Step 2. If factor demand χ_i satisfies the connected substitutes property, then for any vector of factor expenditure shares, $x > 0$, there is at most one vector (up to a normalization) of effective factor prices, ω , such that $x \in \chi_i(\omega)$.

We proceed by contradiction. Suppose that there exist ω , ω' , and $x_0 > 0$ such that $x_0 \in \chi_i(\omega)$, $x_0 \in \chi_i(\omega')$, and ω and ω' are not collinear. Since χ_i is homogeneous of degree zero in all factor prices, we can assume without loss of generality that $\omega_j \geq \omega'_j$ for all j , with at least one strict inequality and one equality. Now let us partition all countries into two groups, M_1 and M_2 , such that

$$\omega'_j > \omega_j \text{ if } j \in M_1, \quad (54)$$

$$\omega'_j = \omega_j \text{ if } j \in M_2. \quad (55)$$

Since χ_i satisfies the connected substitutes property, conditions (54) and (55) imply that for any $x, x' > 0$ such that $x \in \chi_i(\omega)$ and $x' \in \chi_i(\omega')$, we must have

$$\sum_{j \in M_2} x'_j > \sum_{j \in M_2} x_j,$$

which contradicts the existence of $x_0 \in \chi_i(\omega) \cap \chi_i(\omega')$. Lemma 1 follows from Steps 1 and 2. \square

A.3 Lemma 2

Proof of Lemma 2. We proceed by contradiction. Suppose that there exist two equilibrium vectors of factor prices, $c \equiv (c_1, \dots, c_I)$ and $c' \equiv (c'_1, \dots, c'_I)$, that are not collinear. By Proposition 1, we know that c and c' must be equilibrium vectors of the reduced exchange model. So they must satisfy

$$\sum_i L_{ji} = \bar{f}_j(v_j), \text{ for all } j, \quad (56)$$

$$\sum_i L'_{ji} = \bar{f}_j(v_j), \text{ for all } j, \quad (57)$$

where $\{L_{ji}\}$ and $\{L'_{ji}\}$ are the optimal factor demands in the two equilibria,

$$\{L_{ji}\} \in L_i(\omega_i), \text{ for all } i,$$

$$\{L'_{ji}\} \in L_i(\omega'_i), \text{ for all } i,$$

where $\omega_i \equiv \{\tau_{ji}c_j\}$ and $\omega'_i \equiv \{\tau_{ji}c'_j\}$ are the associated vectors of effective factor prices.

We can follow the same strategy as in Step 2 of the proof of Lemma A.3. Without loss of generality, let us assume that $c'_j \geq c_j$ for all j , with at least one strict inequality and one equality. We can again partition all countries into two groups, M_1 and M_2 , such that

$$c'_j > c_j \text{ if } j \in M_1, \quad (58)$$

$$c'_j = c_j \text{ if } j \in M_2. \quad (59)$$

The same argument then implies that in any country i ,

$$\sum_{j \in M_2} x'_{ji} > \sum_{j \in M_2} x_{ji},$$

where $\{x_{ji}\}$ and $\{x'_{ji}\}$ are the expenditure shares associated with $\{L_{ji}\}$ and $\{L'_{ji}\}$, respectively. By definition of the factor expenditure shares, the previous inequality can be rearranged as

$$\sum_{j \in M_2} c'_j L'_{ji} / (c'_i \bar{f}_i(\mathbf{v}_i)) > \sum_{j \in M_2} c_j L_{ji} / (c_i \bar{f}_i(\mathbf{v}_i)).$$

Since $c'_i \geq c_i$ for all i , this implies

$$\sum_{j \in M_2} c'_j L'_{ji} > \sum_{j \in M_2} c_j L_{ji}.$$

Summing across all importers i , we therefore have

$$\sum_{j \in M_2} c'_j \sum_i L'_{ji} > \sum_{j \in M_2} c_j \sum_i L_{ji}.$$

By equations (56) and (57), this further implies

$$\sum_{j \in M_2} c'_j \bar{f}_j(\mathbf{v}_j) > \sum_{j \in M_2} c_j \bar{f}_j(\mathbf{v}_j),$$

which contradicts (59). □

B Estimation

In this section we discuss further details of the estimation procedure outlined in Section 6.2.

B.1 GMM Estimator

As in Section 6.2, define the stacked matrix of instruments, \mathbf{Z} , and the stacked vector of errors, $\mathbf{e}(\boldsymbol{\theta})$. The GMM estimator is

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \mathbf{e}(\boldsymbol{\theta})' \mathbf{Z} \Phi \mathbf{Z}' \mathbf{e}(\boldsymbol{\theta}).$$

where Φ is the GMM weight. We confine attention to the consistent one-step procedure by setting $\Phi = (\mathbf{Z}'\mathbf{Z})^{-1}$.

B.2 Standard Errors

In our baseline specification, we allow for the possibility of autocorrelation in the error term. Specifically, we assume that observations are independent across exporter-importer pairs, but do not impose any restriction on the autocorrelation across periods for the same pair. Following [Cameron and Miller \(2010\)](#), we have that

$$\sqrt{M} (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}) \rightarrow N \left[0, (B' \Phi B)^{-1} (B' \Phi \Lambda \Phi B) (B' \Phi B)^{-1} \right]$$

where $B \equiv E \left[Z'_{ji,t} \nabla_{\boldsymbol{\theta}} e_{ji,t}(\boldsymbol{\theta}) \right]$ and $\Lambda \equiv E[(Z'_{ji} e_{ji})(Z'_{ji} e_{ji})']$, with $Z_{ji} = [Z_{ji,t}]_{t=1}^T$ and $e_{ji} \equiv [e_{ji,t}]_{t=1}^T$ being matrices of stacked periods for exporter-importer pair (j, i) .

The covariance matrix can be consistently estimated using

$$\widehat{Avar}(\hat{\boldsymbol{\theta}}) \equiv (\hat{B}' \Phi \hat{B})^{-1} (\hat{B}' \Phi \hat{\Lambda} \Phi \hat{B}) (\hat{B}' \Phi \hat{B})^{-1} \quad (60)$$

where $\hat{B} \equiv (\mathbf{Z}' \nabla_{\boldsymbol{\theta}} \mathbf{e}(\hat{\boldsymbol{\theta}}))$, $\nabla_{\boldsymbol{\theta}} \mathbf{e}(\hat{\boldsymbol{\theta}}) \equiv [D_{\boldsymbol{\theta}_2} \mathbf{e}(\hat{\boldsymbol{\theta}}) \mid -\mathbf{Z}^1]$, and $\hat{\Lambda} \equiv \Gamma' \Gamma$ such that $\Gamma \equiv [e_{ji}(\hat{\boldsymbol{\theta}})' Z_{ji}]_{ji}$.

This analysis ignored the fact that we take draws of (α_s, ϵ_s) to compute simulated moment conditions in the algorithm described below. Although this simulation step affects standard errors, the asymptotic distribution of the estimator is the same as the number of simulated draws goes to infinite. Thus, we compute the covariance matrix according to expression (60) which is assumed to be an appropriate approximation for the large number of simulations (discussed below) used in the empirical implementation.

B.3 Estimation Algorithm

In order to estimate the model, it is convenient to focus on the following log-transformation of effective factor prices, $\delta_{ji,t} \equiv -\bar{e} \ln(\mu_{ji} \omega_{ji,t} / \mu_{1i} \omega_{1i,t})$. Define $\chi(\delta_{i,t} | \boldsymbol{\theta}_2)$ as the demand system in equation (28) expressed in terms of $\delta_{i,t} \equiv \{\delta_{ji,t}\}$, so that

$$\chi_j(\delta_{i,t} | \boldsymbol{\theta}_2) = \int \frac{\exp(\alpha \sigma_{\alpha} \ln \kappa_j + \epsilon^{\sigma_{\epsilon}} \delta_{ji,t})}{1 + \sum_{l=2}^N \exp(\alpha \sigma_{\alpha} \ln \kappa_l + \epsilon^{\sigma_{\epsilon}} \delta_{li,t})} dF(\alpha, \epsilon) \quad (61)$$

with $\theta_2 \equiv (\sigma_\alpha, \sigma_\epsilon)$. As described in Section 6.2, we can write

$$e_{ji,t}(\theta) \equiv \ln \chi_j^{-1}(x_{i,t}|\theta_2) - \ln \chi_j^{-1}(x_{1,t}|\theta_2) - \mathbf{Z}_{ji,t}^1 \cdot \theta_1,$$

with $\theta_1 = (-\bar{\epsilon}, \{\zeta_{ji}\})$ and $\mathbf{Z}_{ji,t}^1 \equiv (\Delta \ln(z^\tau)_{ji,t} - \Delta \ln(z^\tau)_{j1,t}, \mathbf{d}_{ji,t})$.

The simulated GMM procedure is implemented with the following steps.

Step 0. Draw S simulated pairs $(\alpha_s, \ln \epsilon_s) \sim N(0, I)$. We set $S = 4,000$ and use the same draws for all markets.

Step 1. Conditional on θ_2 , compute the vector $\chi^{-1}(x_{i,t}|\theta_2) \equiv \{\delta_{ji,t}\}_{j=2}^N$ that solves the following system:

$$\{\chi_j(\delta_{i,t}|\theta_2)\}_{j=2}^N = \{x_{ji,t}\}_{j=2}^N$$

where $x_{ji,t}$ is the expenditure share of importer i on exports of j at year t and

$$\chi_j(\delta_{i,t}|\theta_2) = \frac{1}{S} \sum_{s=1}^S \frac{\exp[\alpha_s \sigma_\alpha \ln \kappa_j + (\epsilon_s)^{\sigma_\epsilon} \delta_{ji,t}]}{1 + \sum_{l=2}^N \exp[\alpha_s \sigma_\alpha \ln \kappa_l + (\epsilon_s)^{\sigma_\epsilon} \delta_{li,t}]}.$$

Uniqueness and existence of the solution is guaranteed by the fixed point argument in [Berry, Levinsohn and Pakes \(1995\)](#). To solve the system, consider the fixed point of the following function:

$$G(\delta_{i,t}) = [\delta_{ji,t} + \lambda (\ln x_{ji,t} - \ln \chi_j(\delta_{i,t}|\theta_2))]_{j=2}^N$$

where λ is a parameter controlling the adjustment speed. This fixed point is obtained as the limit of the sequence: $\delta_{i,t}^{n+1} = G(\delta_{i,t}^n)$. Numerically, we compute the sequence until $\max_j |\ln x_{ji,t} - \ln \chi_j(\delta_{i,t}|\theta_2)| < tol$, where tol is some small number that we discuss further below.

This step is implemented as follows. First, the initial guess $\delta_{ij,t}^0$ in the initial iteration is set to be the logit solution $\delta_{ji,t}^0 = \ln x_{ji,t} - \ln x_{1i,t}$. In subsequent iterations, we use the following rule. If θ_2 is close to the parameter vector of the previous iteration, we use the system solution in the last iteration. Otherwise, we use the vector that solved the system for the same importer in the previous year (if it is the first year, we use the logit solution). Second, the speed of adjustment is initially set to $\lambda = 3$. If distance increases in iteration n , then we reduce λ by 5% and compute $\delta_{i,t}^{n+1}$ again until distance decreases in the step and use the new value of λ until the solution is found. If λ falls below a minimum ($\underline{\lambda} = .001$), then we assume no solution for the system and set the objective function to a high value. Lastly, we set $tol = 10^{-8}$ and, every 20,000 iterations, we increase tolerance by a factor of two. This guarantees that the algorithm does not waste time on convergence for parameter values far away from the real ones, as suggested by [Nevo \(2000\)](#).

Step 2. Conditional on θ_2 , solve analytically for linear parameters directly from the minimization problem: $\hat{\theta}_1(\theta_2) = (\mathbf{Z}^1 \mathbf{Z} \Phi \mathbf{Z}^1)^{-1} \mathbf{Z}^1 \mathbf{Z} \Phi \mathbf{Z}^1 \mathbf{X}$, with $\mathbf{X} \equiv [\ln \chi_j^{-1}(x_{i,t}|\theta_2) - \ln \chi_j^{-1}(x_{1,t}|\theta_2)]$.

Step 3. Conditional on θ_2 , compute the vector of structural errors using $e_{ji,t}(\theta_2) = \ln \chi_j^{-1}(x_{i,t}|\theta_2) - \ln \chi_j^{-1}(x_{1,t}|\theta_2) - \mathbf{Z}_{ji,t}^1 \cdot \hat{\theta}_1(\theta_2)$.

Step 4. Numerically minimize the objective function to obtain estimates of θ_2 :

$$\hat{\theta}_2 \equiv \arg \min_{\theta_2} H(\theta_2) \equiv \mathbf{e}(\theta_2)' \mathbf{Z} \Phi \mathbf{Z}' \mathbf{e}(\theta_2).$$

The numerical minimization is implemented using the “trust-region-reflective” algorithm that requires an analytical gradient of the objective function (described below). This algorithm is intended to be more efficient in finding the local minimum within a particular attraction region. First, we solve the minimization problem using a grid of ten initial conditions randomly drawn from a uniform distribution in the parameter space. Second, we solve a final minimization problem using as initial condition the minimum solution obtained from the first-round minimization. Here, we impose a stricter convergence criteria and we reduce the tolerance level of the system solution in Step 1 to $tol = 10^{-12}$.

Objective Function Gradient. The Jacobian of $H(\theta_2)$ is $\nabla H(\theta_2) = 2 \cdot D\mathbf{e}(\theta_2)' \mathbf{Z} \Phi \mathbf{Z}' \mathbf{e}(\theta_2)$ where $D\mathbf{e}(\theta_2) = \left[\frac{\partial e_{ji,t}}{\partial \theta_{21}} \dots \frac{\partial e_{ji,t}}{\partial \theta_{2L}} \right]_{ijt}$ is the stacked matrix of Jacobian vectors of the structural error from Step 3. By the envelope theorem, the Jacobian is $D\mathbf{e}_{i,t}(\theta_2) = D\delta_{i,t}(\theta_2) - D\delta_{1,t}(\theta_2)$ because $\hat{\theta}_1(\theta_2)$ is obtained from the analytical minimization of the inner problem restricted to a particular level of θ_2 . For each importer-year, the implicit function theorem implies that

$$D\delta_{i,t}(\theta_2) = \begin{bmatrix} \frac{\partial \delta_{2i,t}}{\partial \theta_{21}} & \dots & \frac{\partial \delta_{2i,t}}{\partial \theta_{2L}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{Ni,t}}{\partial \theta_{21}} & \dots & \frac{\partial \delta_{Ni,t}}{\partial \theta_{2L}} \end{bmatrix} = - \begin{bmatrix} \frac{\partial \bar{\chi}_2}{\partial \delta_{2i,t}} & \dots & \frac{\partial \bar{\chi}_2}{\partial \delta_{Ni,t}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \bar{\chi}_N}{\partial \delta_{2i,t}} & \dots & \frac{\partial \bar{\chi}_N}{\partial \delta_{Ni,t}} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial \bar{\chi}_2}{\partial \theta_{21}} & \dots & \frac{\partial \bar{\chi}_2}{\partial \theta_{2L}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \bar{\chi}_N}{\partial \theta_{21}} & \dots & \frac{\partial \bar{\chi}_N}{\partial \theta_{2L}} \end{bmatrix}$$

where

$$\frac{\partial \chi_j}{\partial \delta_{li,t}} = \begin{cases} -\frac{1}{S} \sum_{s=1}^S (\epsilon_s)^{\sigma_\epsilon} \cdot x_{ji,t}(\alpha_s, \epsilon_s) x_{li,t}(\alpha_s, \epsilon_s) & \text{if } l \neq j \\ \frac{1}{S} \sum_{s=1}^S (\epsilon_s)^{\sigma_\epsilon} \cdot x_{ji,t}(\alpha_s, \epsilon_s) (1 - x_{ji,t}(\alpha_s, \epsilon_s)) & \text{if } l = j \end{cases}$$

$$\frac{\partial \chi_j}{\partial \sigma_\epsilon} = \frac{1}{S} \sum_{s=1}^S (\ln \epsilon_s) (\epsilon_s)^{\sigma_\epsilon} \cdot x_{ji,t}(\alpha_s, \epsilon_s) \cdot \left[\delta_{ji,t} - \sum_{l=2}^N x_{li,t}(\alpha_s, \epsilon_s) \cdot \delta_{li,t} \right]$$

$$\frac{\partial \chi_j}{\partial \sigma_\alpha} = \frac{1}{S} \sum_{s=1}^S \alpha_s \cdot x_{ji,t}(\alpha_s, \epsilon_s) \cdot \left[\ln \kappa_i - \sum_{l=2}^N x_{li,t}(\alpha_s, \epsilon_s) \cdot \ln \kappa_l \right].$$

C Sample of Countries

Table A1: List of exporting countries

Abbreviation	Exporter	log(p.c. GDP) [USA=0]
AUS	Australia	-0.246
AUT	Austria	-0.249
BLX	Belgium-Luxembourg	-0.261
BRA	Brazil	-1.666
BGR	Bulgaria	-1.603
CAN	Canada	-0.211
CHN	China	-2.536
CZE	Czech Republic	-0.733
DNK	Denmark	-0.303
BAL	Estonia-Latvia	-1.475
FIN	Finland	-0.522
FRA	France	-0.398
DEU	Germany	-0.290
GRC	Greece	-0.760
HUN	Hungary	-1.121
IND	India	-3.214
IDN	Indonesia	-2.284
IRL	Ireland	-0.574
ITA	Italy	-0.332
JPN	Japan	-0.183
LTU	Lithuania	-1.526
MEX	Mexico	-1.263
NLD	Netherlands	-0.352
POL	Poland	-1.428
PRT	Portugal	-0.830
KOR	Republic of Korea	-0.823
RoW	Rest of the World	-2.286
ROU	Romania	-1.816
RUS	Russia	-0.954
SVK	Slovak Republic	-1.102
SVN	Slovenia	-0.728
ESP	Spain	-0.644
SWE	Sweden	-0.367
TWN	Taiwan	-0.584
TUR	Turkey	-1.305
GBR	United Kingdom	-0.436
USA	United States	0.000

D Counterfactual Analysis

D.1 Preliminaries

In the counterfactual analysis of Section 7, we use the complete trade matrix for the 37 exporters listed in Table A1. In order to reconcile theory and data, we incorporate trade imbalances as follows. For each country, we define $\rho_{j,t}$ as the difference between aggregate gross expenditure and aggregate gross production. We proceed under the assumption that trade imbalances remain constant at their observed level in terms of the factor price of the reference country. Here, the reference country is the United States ($j = 1$) such that its factor price is normalized to one, $\hat{w}_1 = 1$. In particular, the market clearing condition in (15) becomes

$$\sum_{i=1}^N \hat{x}_{ji,t} x_{ji,t} ((\hat{w}_i \hat{\nu}_i) y_{i,t} + \rho_{i,t}) = (\hat{w}_j \hat{\nu}_j) y_{j,t}, \quad \text{for } j = 2, \dots, N \quad (62)$$

where

$$\hat{x}_{ji,t} x_{ji,t} = \frac{1}{S} \sum_{s=1}^S \frac{\exp[\alpha_s \sigma_\alpha \ln \kappa_j + (\epsilon_s)^{\sigma_\epsilon} (\chi_j^{-1}(\mathbf{x}_{i,t} | \boldsymbol{\theta}_2) - \bar{\epsilon} \ln(\hat{w}_j \hat{\nu}_{ji}))]}{1 + \sum_{l=2}^N \exp[\alpha_s \sigma_\alpha \ln \kappa_l + (\epsilon_s)^{\sigma_\epsilon} (\chi_l^{-1}(\mathbf{x}_{i,t} | \boldsymbol{\theta}_2) - \bar{\epsilon} \ln(\hat{w}_l \hat{\nu}_{li}))]}. \quad (63)$$

Notice that, by construction, $\sum_{i=1}^N \rho_{i,t} = 0$. Thus, the solution of the system of $N - 1$ equations above implies that the market clearing condition for the reference country is automatically satisfied.

D.2 Algorithm

To compute the vector $\hat{\mathbf{w}} = \{\hat{w}_j\}_{j=2}^N$ that solves system (62), we use the same algorithm as in [Alvarez and Lucas \(2007\)](#).

Step 0. Initial guess: $\hat{\mathbf{w}}^k = [1, \dots, 1]$ if $k = 0$.

Step 1. Conditional on $\hat{\mathbf{w}}^k$, compute $\hat{x}_{ji,t} x_{ji,t}$ according to (63).

Step 2. Compute the excess labor demand as

$$F_j(\hat{\mathbf{w}}^k) \equiv \frac{1}{y_{j,t}} \left[-(\hat{w}_j \hat{\nu}_j) y_{j,t} + \sum_{i=1}^N \hat{x}_{ji,t} x_{ji,t} ((\hat{w}_i \hat{\nu}_i) y_{i,t} + \rho_{i,t}) \right]$$

where we divide by $y_{j,t}$ to scale excess demand by country size.

Step 3. If $\max_j |F_j(\hat{\mathbf{w}}^k)| < tol$, then stop the algorithm. (In practice we set $tol = 10^{-8}$ here.)

Otherwise, return to Step 1 with new factor prices computed as

$$\hat{w}_j^{k+1} = \hat{w}_j^k + \mu F_j(\hat{w}^k)$$

where μ is a positive constant. Intuitively, this updating rule increases the price of those factors with a positive excess demand.

D.3 Welfare

By Proposition 3, we can compute welfare changes in any country i by solving for $e(\cdot, U_i')$. To do so, we guess that for all $\omega \equiv \{\omega_l\}$,

$$e(\omega, U_i') = (y_i') \frac{\exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} (\omega_l)^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))}{\exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} ((\omega_{li,t})')^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))}. \quad (64)$$

We then check that our guess satisfies (17) and (18) if χ satisfies (28). By equations (19) and (64), welfare changes must therefore satisfy

$$\Delta W_i = \frac{(y_i') / \exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} ((\omega_{li,t})')^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))}{y_i / \exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} (\chi_l^{-1}(\mathbf{x}_{i,t}))^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))} - 1.$$

Using the fact that $(y_i)' / y_i = \hat{w}_i$ and $(\omega_{li,t})' = \hat{w}_l \hat{\tau}_{li} \chi_l^{-1}(\mathbf{x}_{i,t})$, this finally leads to

$$\Delta W_i = (\hat{w}_i) \frac{\exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} (\chi_l^{-1}(\mathbf{x}_{i,t}))^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))}{\exp(\int \frac{1}{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})} \ln[\sum_{l=1}^N (\kappa_l)^{\sigma_{\alpha\alpha}} (\hat{w}_l \hat{\tau}_{li} \chi_l^{-1}(\mathbf{x}_{i,t}))^{-(\bar{\epsilon}\epsilon^{\sigma\epsilon})}] dF(\alpha, \epsilon))} - 1,$$

with $\{\hat{w}_l\}$ obtained from the algorithm in Section D.2.

D.4 Confidence Intervals

The confidence intervals for the counterfactual analysis are computed with the following bootstrap procedure. First, draw parameter values from the asymptotic distribution of the GMM estimator: $\theta(b) \sim N(\hat{\theta}, \widehat{AVar}(\hat{\theta}))$. Second, compute $\chi^{-1}(\mathbf{x}_{i,t} | \theta_2(b))$ using the algorithm described in Step 1 of Section B.3. Third, compute the counterfactual exercise with $\theta(b)$ and $\chi^{-1}(\mathbf{x}_{i,t} | \theta_2(b))$ using the algorithm described in Section D.2. Lastly, repeat these three steps for $b = 1, \dots, 200$. The bootstrap confidence interval corresponds to $[EV^{(.025)}, EV^{(.975)}]$ where $EV^{(\alpha)}$ denotes the α -th quantile value of the equivalent variation obtained across the set of 200 parameter draws.

D.5 Additional Results

Table A2: Welfare gains from Chinese integration since 1995: all countries, 2007

Exporter	CES (standard gravity)		Mixed CES	
	Welfare Gains	95% Confidence Interval	Welfare Gains	95% Confidence Interval
Australia	0.144	(0.109, 0.243)	0.225	(0.136, 0.598)
Austria	0.058	(0.043, 0.100)	0.102	(0.055, 0.296)
Belgium-Luxembourg	0.056	(0.042, 0.097)	0.108	(0.044, 0.312)
Brazil	0.071	(0.054, 0.121)	0.058	(0.049, 0.191)
Bulgaria	0.061	(0.045, 0.106)	-0.005	(-0.077, 0.078)
Canada	0.053	(0.039, 0.092)	0.098	(0.044, 0.301)
China	1.039	(0.788, 1.740)	1.544	(1.006, 4.284)
Czech Republic	0.151	(0.112, 0.262)	0.209	(0.140, 0.570)
Denmark	0.014	(0.010, 0.026)	0.034	(-0.009, 0.137)
Estonia-Latvia	0.081	(0.061, 0.140)	0.043	(0.033, 0.190)
Finland	0.100	(0.075, 0.171)	0.154	(0.092, 0.437)
France	0.030	(0.023, 0.052)	0.057	(0.029, 0.214)
Germany	0.122	(0.092, 0.208)	0.201	(0.117, 0.519)
Greece	0.004	(0.003, 0.006)	0.018	(-0.003, 0.114)
Hungary	0.214	(0.161, 0.370)	0.208	(0.169, 0.555)
India	0.126	(0.094, 0.218)	0.022	(-0.141, 0.185)
Indonesia	0.026	(0.019, 0.047)	-0.061	(-0.415, 0.016)
Ireland	0.135	(0.101, 0.234)	0.150	(0.116, 0.379)
Italy	0.008	(0.006, 0.015)	0.035	(0.002, 0.161)
Japan	0.095	(0.072, 0.162)	0.186	(0.093, 0.599)
Lithuania	0.065	(0.049, 0.110)	0.022	(-0.003, 0.114)
Mexico	0.121	(0.090, 0.211)	0.099	(0.082, 0.360)
Netherlands	0.043	(0.032, 0.076)	0.068	(0.019, 0.157)
Poland	0.086	(0.064, 0.151)	0.040	(0.030, 0.210)
Portugal	0.050	(0.038, 0.081)	0.055	(0.043, 0.141)
Republic of Korea	0.298	(0.226, 0.500)	0.399	(0.273, 0.951)
Rest of the World	0.293	(0.221, 0.493)	0.105	(-0.160, 0.384)
Romania	-0.005	(-0.009, -0.004)	-0.077	(-0.367, -0.013)
Russia	0.105	(0.079, 0.180)	0.103	(0.085, 0.221)
Slovak Republic	0.116	(0.087, 0.200)	0.120	(0.093, 0.343)
Slovenia	0.012	(0.008, 0.022)	0.020	(0.007, 0.078)
Spain	0.075	(0.056, 0.127)	0.112	(0.071, 0.331)
Sweden	0.076	(0.057, 0.130)	0.113	(0.072, 0.315)
Taiwan	0.695	(0.531, 1.140)	0.946	(0.651, 2.146)
Turkey	0.024	(0.018, 0.043)	0.019	(0.015, 0.086)
United Kingdom	0.014	(0.010, 0.024)	0.022	(0.002, 0.094)
United States	0.034	(0.025, 0.062)	0.071	(0.035, 0.237)

Notes: Estimates of welfare changes (computed as the minus of the equivalent variation) from replacing China's trade costs to all other countries in 2007 at their 1995 levels. "CES (standard gravity)" and "Mixed CES" report these welfare changes obtained using the factor demand system in Panels A and C, respectively, of Table 2. 95% confidence intervals computed using the bootstrap procedure documented in Appendix D.