Web appendix for "The scope for ideological bias..."

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1 Derivation of the autocoherence conditions in model 1

For convenience let us replicate Table 2 from the text:

Observable	Expression
Output	$y = \hat{a}_{yu}\hat{u} + \hat{a}_{y\varepsilon}\hat{\varepsilon} + \hat{a}_{yv}\hat{v}$
Price	$\pi = \hat{a}_{\pi u}\hat{u} + \hat{a}_{\pi\varepsilon}\hat{\varepsilon} + \hat{a}_{\pi v}\hat{v}$
Coefficients	Expression
$\hat{a}_{yu}$	$\hat{b} + \frac{\hat{a}\hat{\omega}\gamma}{\hat{\sigma}_u^2}$
$\hat{a}_{yv}$	$\hat{\theta}\hat{b}$
$\hat{a}_{y\varepsilon}$	$\gamma\hat{a}$
$\hat{a}_{\pi u}$	$\hat{\rho}\hat{b} + \frac{\hat{\rho}\hat{a}\hat{\omega}\gamma}{\hat{\sigma}_u^2}$
$\hat{a}_{\pi v}$	$\hat{\rho}(\hat{b}\hat{\theta} - 1)$
$\hat{a}_{\pi\varepsilon}$	$\hat{\rho}\gamma\hat{a}$

Table 2

Table 1 is obtained trivially from the above by removing hats. The six autocoherence conditions are $Ez^2 = \hat{E}z^2$, $Eyz = \hat{E}yz, E\pi z = \hat{E}\pi z, Ey^2 = \hat{E}y$, and $E\pi^2 = \hat{E}\pi^2$. Clearly, we can replace the second and third conditions by the simpler ones $E(y \mid z) = \hat{E}(y \mid z)$ and $E(\pi \mid z) = \hat{E}(\pi \mid z)$.

In what follows we assume, as in the paper, that ω is common knowledge: $\hat{\omega} = \omega$.

We get the following formulas

1. Variance of z. For this we only have to use the definition of z.

$$
\begin{aligned}
Ez^2 &= 1 = \omega^2/\sigma_u^2 + \sigma_\varepsilon^2 \\
&= \hat{E}z^2 = \omega^2/\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2.
\end{aligned} \tag{1}
$$

Because (1) has to hold, we can simplify the following expressions:

$$
\hat{E}(u \mid z) = \frac{\hat{\omega} z}{\hat{\omega}^2/\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2} = \omega z; \tag{2}
$$

$$
\hat{E}(\varepsilon \mid z) = \frac{\hat{\sigma}_\varepsilon^2 z}{\hat{\omega}^2/\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2} = \hat{\sigma}_\varepsilon^2 z; \tag{3}
$$

$$
\gamma = -\frac{\hat{a}\hat{b}\hat{\omega}}{(\varphi + \hat{a}^2)(\hat{\omega}^2/\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2)} = -\frac{\hat{a}\hat{b}\omega}{\varphi + \hat{a}^2}. \tag{4}
$$

2. Expectation of y conditional on z. This is easily obtained from Table 1 and (4):

$$
\begin{aligned}
E(y \mid z) &= a_{yu}\omega + a_{y\varepsilon}\sigma_\varepsilon^2 = \hat{E}(y \mid z) = \hat{a}_{yu}\omega + \hat{a}_{y\varepsilon}\hat{\sigma}_\varepsilon^2 \\
\iff & \omega\hat{b} + \hat{a}\gamma\frac{\omega^2}{\hat{\sigma}_u^2} + \hat{a}\gamma\hat{\sigma}_\varepsilon^2 = \omega\hat{b} + \hat{a}\gamma = \omega b + a\gamma \\
\iff & \gamma(\hat{a} - a) = \omega(b - \hat{b}).
\end{aligned} \tag{5}
$$

3. Expectation of π conditional on z. From Table 2,

$$
E(\pi \mid z) = a_{\pi u}\omega + a_{\pi\varepsilon}\sigma_\varepsilon^2 = \rho E(y \mid z) = \hat{\rho}\hat{E}(y \mid z),
$$

which, since $E(y \mid z) = \hat{E}(y \mid z)$ from the preceding autocoherence condition, is equivalent to

$$
\hat{\rho} = \rho. \tag{6}
$$

Since the steps in proving (6) do not hinge on the assumption that ω is known, **this proves Proposition 1.**

4. Covariance between y and π

$$
\begin{aligned}
E\pi y &= a_{yu}a_{\pi u}\sigma_u^2 + a_{y\varepsilon}a_{\pi\varepsilon}\sigma_\varepsilon^2 + a_{yv}a_{\pi v}\sigma_v^2 \\
&= \hat{E}\pi y = \hat{a}_{yu}\hat{a}_{\pi u}\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}\hat{a}_{\pi\varepsilon}\hat{\sigma}_\varepsilon^2 + \hat{a}_{yv}\hat{a}_{\pi v}\hat{\sigma}_v^2
\end{aligned} \tag{7}
$$

This is equivalent to, using Table 2 and (6),

$$
a_{yu}^2\sigma_u^2 + a_{y\varepsilon}^2\sigma_\varepsilon^2 + a_{yv}(a_{yv} - 1)\sigma_v^2 = \hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}^2\sigma_\varepsilon^2 + \hat{a}_{yv}(\hat{a}_{yv} - 1)\hat{\sigma}_v^2 \tag{8}
$$

5. Variance of y

$$\begin{aligned} Ey^2 &= a_{yu}^2\sigma_u^2 + a_{y\varepsilon}^2\sigma_\varepsilon^2 + a_{yv}^2\sigma_v^2 \\ &= \hat{E}y^2 = \hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}^2\hat{\sigma}_\varepsilon^2 + \hat{a}_{yv}^2\hat{\sigma}_v^2. \end{aligned} \tag{9}$$

6. Variance of π

$$\begin{aligned} E\pi^2 &= a_{\pi u}^2\sigma_u^2 + a_{\pi\varepsilon}^2\sigma_\varepsilon^2 + a_{\pi v}^2\sigma_v^2 \\ &= \hat{E}\pi^2 = \hat{a}_{\pi u}^2\hat{\sigma}_u^2 + \hat{a}_{\pi\varepsilon}^2\hat{\sigma}_\varepsilon^2 + \hat{a}_{\pi v}^2\hat{\sigma}_v^2. \end{aligned} \tag{10}$$

This is equivalent to

$$a_{yu}^2\sigma_u^2 + a_{y\varepsilon}^2\sigma_\varepsilon^2 + (a_{yv}-1)^2\sigma_v^2 = \hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{y\varepsilon}^2\hat{\sigma}_\varepsilon^2 + (\hat{a}_{yv}-1)^2\hat{\sigma}_v^2 \tag{11}$$

Now, combining (9) and (8) we find that (8) can be replaced by

$$a_{yv}\sigma_v^2 = \hat{a}_{yv}\hat{\sigma}_v^2. \tag{12}$$

Combining (9) and (11) we find that (11) can be replaced by

$$\sigma_v^2(1-2a_{yv}) = \hat{\sigma}_v^2(1-2\hat{a}_{yv}). \tag{13}$$

In turn, (12) and (13) are equivalent to

$$\begin{aligned} \sigma_v^2 &= \hat{\sigma}_v^2; \\ \theta b &= \hat{\theta}\hat{b}, \end{aligned} \tag{14}$$

where I have used the definitions in Tables 1 and 2 for a_{yv} and $\hat{a}_{yv}$.

Finally, using these same tables, as well as (1), we can rewrite condition (9) as follows:

$$b^2\sigma_u^2 + a^2\gamma^2 + 2ab\gamma\omega = \hat{b}^2\hat{\sigma}_u^2 + \hat{a}^2\gamma^2 + 2\hat{a}\hat{b}\gamma\omega. \tag{15}$$

The following table summarizes the 6 autocoherent conditions, in the simplified forms we have just derived:

$\omega^2/\hat{\sigma}_u^2 + \hat{\sigma}_\varepsilon^2 = 1$
$\gamma(\hat{a}-a) = \omega(b-\hat{b})$.
$\hat{\rho} = \rho$
$\hat{\sigma}_v^2 = \sigma_v^2$
$\hat{\theta}\hat{b} = \theta b$
$\hat{b}^2\hat{\sigma}_u^2 + \hat{a}^2\gamma^2 + 2\hat{a}\hat{b}\gamma\omega = b^2\sigma_u^2 + a^2\gamma^2 + 2ab\gamma\omega$

Table A1 – The autocoherence conditions.

Proof of Proposition 2 – Table A1 proves claim (ii) in Proposition 2. Claim (i) then derives from the formula for γ and from (1). Claim (iii) comes from the equalities in Table A1 and the definitions of $\hat{a}_{yv}$ and $\hat{a}_{\pi v}$ in Table 2.

2 Proof of Proposition 3

The autocoherent model picked by the expert must achieve $\gamma = \gamma_E$, or equivalently

$$-\frac{\hat{a}\hat{b}\omega}{\varphi + \hat{a}^2} = -\frac{ab\omega}{\varphi_E + a^2}.$$

As seen in the text (Equation (16) in the text), $\hat{a}$ and $\hat{b}$ are linked by the following autocoherence condition:

$$\hat{b} = b\frac{\varphi + \hat{a}^2}{\varphi + a\hat{a}}. \tag{16}$$

Solving for these two equations we get

$$\begin{aligned} \hat{a} &= a\frac{\varphi}{\varphi_E}; \\ \hat{b} &= b\frac{\varphi_E^2 + a^2\varphi}{\varphi_E^2 + a^2\varphi_E}. \end{aligned}$$

This proves the first two conditions in Proposition 3. Conditions (iv) and (v) are already known, and the value of $\hat{\theta}$ in condition (iii) is straightforward from (14). Finally, it can be checked that Equation (17) in the text can be obtained from substituting (i) and (ii) of Proposition 3 into (15).

Conversely, it is straightforward to check that if the perceived model has the parameter values of Proposition 3, all the autocoherence conditions hold for $\gamma = \gamma_E$, which is the stabilization level that the government will choose.

3 Proof of Proposition 4

First, recall that $\hat{a} = \frac{\hat{\alpha}}{1+\hat{\mu}\eta}$ and $\hat{b} = \frac{1}{1+\hat{\mu}\eta}$. From (i) and (ii) in Proposition 3 we get that

$$\hat{\alpha} = \alpha\varphi\frac{\varphi_E + a^2}{\varphi_E^2 + a^2\varphi}.$$

Therefore,

$$\frac{d\hat{\alpha}}{d\varphi_E} \propto a^2\varphi - 2a^2\varphi_E - \varphi_E^2.$$

This expression is negative for $\varphi_E > -a^2 + \sqrt{a^4 + a^2\varphi}$, which is smaller than $\varphi/2$.

Using (i) and (ii) again, we have that

$$\hat{\mu} = \mu \frac{\varphi_E^2 + a^2 \varphi_E}{\varphi_E^2 + a^2 \varphi} + \frac{1}{\eta} \frac{a^2(\varphi_E - \varphi)}{\varphi_E^2 + a^2 \varphi}.$$

It is easy to check that

$$\frac{d\hat{\mu}}{d\varphi_E} \propto -\varphi_E^2 + a^2 \varphi + 2\varphi\varphi_E,$$

which, from the discussion in the text following Proposition 3, is clearly positive for $\varphi < \varphi_m$.

Q.E.D.

4 Correct model equilibrium with inflation inertia

PROPOSITION A1 – A correct model equilibrium exists such that

$$\begin{aligned} 0 &< c_\pi < 1; \\ c_y &< 0, \\ \gamma &> 0. \end{aligned}$$

PROOF – To construct such an equilibrium, we have to show that there exists a solution to these three equations:

$$c_y = -\mu(h-1)c_\pi^2 + c_y c_\pi + \alpha\gamma - \alpha\gamma c_\pi, \tag{17}$$

$$c_\pi = \rho c_y + \beta c_\pi^2 + 1 - \beta. \tag{18}$$

$$\gamma = -\frac{mn}{\varphi + m^2}.$$

Where by definition

$$m = \frac{\alpha(1 - \beta c_\pi)}{1 - \beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma}, \tag{19}$$

$$n = \frac{(1-\beta)(c_y - \mu(h-1)c_\pi - \alpha\gamma)}{1 - \beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma}. \tag{20}$$

From (17), we have that

$$c_y - \alpha\gamma = -\frac{\mu(h-1)c_\pi^2}{1-c_\pi}, \tag{21}$$

which is < 0 for $c_\pi < 1$. It follows that for $c_\pi < 1$, the denominator of (19) and (20) is positive.

Note that for any c_π the values of c_y and γ can be solved uniquely from (17) and (18). Let us denote these solutions by $c_y(c_\pi)$ and $\gamma(c_\pi)$. We have that

$$c_y(c_\pi) = \frac{1}{\rho}(c_\pi(1-\beta c_\pi) + \beta - 1) \tag{22}$$

and

$$\gamma(c_\pi) = \frac{c_y(c_\pi)(1-c_\pi) + \mu(h-1)c_\pi^2}{\alpha(1-c_\pi)} \tag{23}$$

Clearly, $c_y()$ is continuous over $\mathbb{R}$ and $\gamma()$ is continuous over $\mathbb{R} - \{1\}$. We can also compute the corresponding values for m and n denoted by $m(c_\pi)$ and $n(c_\pi)$. These also are continuous functions of c_π over $[0,1)$. An equilibrium obtains if there exists a value of c_π for which

$$\gamma(c_\pi) = -\frac{m(c_\pi)n(c_\pi)}{\varphi + m(c_\pi^2)} \equiv \tilde{\gamma}(c_\pi).$$

From the above equations it is easy to check that

$$c_y(0) = -\frac{1-\beta}{\rho} < 0, \tag{24}$$

$$\gamma(0) = -\frac{1-\beta}{\rho\alpha} < 0,$$

$$m(0) = \alpha > 0.$$

$$n(0) = 0. \tag{25}$$

Therefore,

$$\tilde{\gamma}(0) = 0 > \gamma(0).$$

Furthermore, from (23), (19) and (20),

$$\begin{aligned}\lim_{c_\pi \longrightarrow 1} \gamma(c_\pi) &= +\infty, \\ \lim_{c_\pi \longrightarrow 1} m &= 0, \\ \lim_{c_\pi \longrightarrow 1} n &= -1/\rho.\end{aligned}$$

Therefore,

$$\lim_{c_\pi \longrightarrow 1} \tilde{\gamma}(c_\pi) = 0 < \lim_{c_\pi \longrightarrow 1} \gamma(c_\pi).$$

By continuity, there exists $c_\pi \in (0,1)$ for which $\gamma(c_\pi) = \tilde{\gamma}(c_\pi)$, which proves that there exists an equilibrium.

In any such equilibrium, one must have $\gamma > 0$. To see this, note that, $m > 0$ since its denominator is > 0 and both β and c_π are between 0 and 1. Furthermore, substituting (21) into the numerator of (20) implies that $n < 0$. Consequently, $\tilde{\gamma} = \gamma > 0$.

Consider now the sign of c_y. By (18), it is the same as that of $c_\pi - \beta c_\pi^2 - (1-\beta)$. The two roots of this polynomial are equal to 1 and $\frac{1-\beta}{\beta}$. If $\frac{1-\beta}{\beta} > 1$, this expression is negative over $(0,1)$, implying that $c_y < 0$. Assume that $\frac{1-\beta}{\beta} < 1$. We show that we can then pick $c_\pi < \frac{1-\beta}{\beta}$, implying again that $c_y < 0$. To see this, compute

$$c_y(\frac{1-\beta}{\beta}) = 0, \tag{26}$$

$$\gamma(\frac{1-\beta}{\beta}) = \frac{\mu(h-1)}{\alpha}\frac{(1-\beta)^2}{\beta(2\beta-1)} > 0, \tag{27}$$

$$m(\frac{1-\beta}{\beta}) = \frac{\alpha\beta}{\beta + \mu\rho(h-1)\frac{1-\beta}{2\beta-1}} > 0, \tag{28}$$

$$n(\frac{1-\beta}{\beta}) = -\frac{(1-\beta)^2}{2\beta-1}\frac{\mu(h-1)}{\beta + \mu\rho(h-1)\frac{1-\beta}{2\beta-1}} < 0. \tag{29}$$

Hence

$$\begin{aligned}\tilde{\gamma}(\frac{1-\beta}{\beta}) &= \frac{\beta}{2\beta-1}\frac{\alpha\mu(1-\beta)^2(h-1)}{\varphi\left(\beta + \mu\rho(h-1)\frac{1-\beta}{2\beta-1}\right)^2 + \alpha^2\beta^2} \\ &< \gamma(\frac{1-\beta}{\beta}).\end{aligned}$$

Therefore in this case we can pick an equilibrium such that $c_\pi < \frac{1-\beta}{\beta}$, implying that $c_y < 0$.

QED.

5 Linearization of equilibrium conditions

I now derive the formulas that form the basis of the numerical results for the model with inflation inertia. Let

$$
\begin{aligned}
c_y &= c_{yc} + \Delta c_\pi, \\
c_\pi &= c_{\pi c} + \Delta c_y, \\
\hat{\beta} &= \beta + \Delta \hat{\beta}, \\
\hat{\rho} &= \rho + \Delta \hat{\rho}, \\
\gamma &= \gamma_c + \Delta \gamma, \\
\varphi_E &= \varphi + \Delta \varphi, \\
m &= m_c + \Delta m, \\
n &= n_c + \Delta n, \\
\hat{m} &= m + \Delta \hat{m}, \\
\hat{n} &= n + \Delta \hat{n}, \\
\hat{\sigma}_u^2 &= \sigma_u^2 + \Delta \hat{\sigma}_u^2, \\
\hat{\sigma}_v^2 &= \sigma_v^2 + \Delta \hat{\sigma}_v^2.
\end{aligned}
$$

The subscript c refers to the equilibrium value in the correct model (CM) equilibrium. For any variable x, $\Delta \hat{x}$ is the difference between its perceived and actual value *in the autocoherent model (ACM) equilibrium associated with* $\varphi = \varphi_E$. On the other hand, Δx is the difference between its actual value in the ACM equilibrium and its actual value in the CM equilibrium. The effect of $\Delta\varphi$, the preference gap between the expert and the government, on the gap between the perceived and actual value of any variable x, $\Delta \hat{x}/\Delta\varphi$, is called its *ideological sensitivity.*

The set of equilibrium conditions is summarized in the following tables:

Equation	Meaning
$c_y = -\mu(h-1)c_\pi^2 + c_y c_\pi + \alpha\gamma - \alpha\gamma c_\pi$	Equilibrium condition (17)
$c_\pi = \rho c_y + \beta c_\pi^2 + 1 - \beta.$	Equilibrium condition (18)
$m = \frac{\alpha(1-\beta c_\pi)}{1-\beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma}$	Definition of m, (19)
$n = \frac{(1-\beta)(c_y - \mu(h-1)c_\pi - \alpha\gamma)}{1-\beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma}$	Definition of n, (20)
$\gamma = -\frac{\hat{m}\hat{n}}{\varphi + \hat{m}^2}$	Government's choice of γ
$\hat{m} = \frac{\hat{\alpha}(1-\hat{\beta}\hat{c}_\pi)}{1+\gamma\hat{\alpha}\hat{\rho} - \hat{\beta}\hat{c}_\pi - \hat{\rho}\hat{c}_y + \hat{\mu}\hat{\rho}(h-1)\hat{c}_\pi}$	Definition of $\hat{m}$, from Equation (32) in the text
$\hat{n} = \frac{(1-\hat{\beta})(\hat{c}_y - \hat{\mu}(h-1)\hat{c}_\pi - \hat{\alpha}\gamma)}{1+\gamma\hat{\alpha}\hat{\rho} - \hat{\beta}\hat{c}_\pi - \hat{\rho}\hat{c}_y + \hat{\mu}\hat{\rho}(h-1)\hat{c}_\pi}$	Definition of $\hat{n}$, from Equation (33) in the text
$c_\pi = \hat{\rho} c_y + \hat{\beta} c_\pi^2 + 1 - \hat{\beta}.$	AC condition (41) in the text[1]
$\gamma = -\frac{mn}{\varphi_E + m^2}$	Expert's choice of γ

Table A2 – Model's solution for VAR coefficients and autocoherence condition for c_π

$\hat{a}_{yu} = \frac{\hat{m}}{\alpha}$	Impact effect of demand shock on output
$\hat{a}_{yv} = \hat{\theta}\frac{\hat{m}}{\alpha} - \frac{\hat{\rho}\hat{n}}{1-\hat{\beta}}$	Impact effect of supply shock on output
$\hat{a}_{\pi u} = \frac{\hat{\rho}\hat{m}}{\alpha(1-\hat{\beta}c_\pi)}$	Impact effect of demand shock on inflation
$\hat{a}_{\pi v} = \frac{\hat{\rho}(\hat{\theta}-1)\hat{m}}{\alpha(1-\hat{\beta}c_\pi)}$	Impact effect of supply shock on inflation
$a_{yu}^2\sigma_u^2 + a_{yv}^2\sigma_v^2 = \hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{yv}^2\hat{\sigma}_v^2,$	Variance of output innovations matched
$a_{\pi u}^2\sigma_u^2 + a_{\pi v}^2\sigma_v^2 = \hat{a}_{\pi u}^2\hat{\sigma}_u^2 + \hat{a}_{\pi v}^2\hat{\sigma}_v^2$	Variance of inflation innovation matched
$a_{yu}a_{\pi u}\sigma_u^2 + a_{yv}a_{\pi v}\sigma_v^2 = \hat{a}_{yu}\hat{a}_{\pi u}\hat{\sigma}_u^2 + \hat{a}_{yv}\hat{a}_{\pi v}\hat{\sigma}_v^2.$	Covariance of innovations matched

Table A3 – Aurocoherence conditions on VAR innovations

The model can be solved as follows.

First, There are 5 equations that define a real block, i.e. which characterizes the actual behavior of the economy. These equations are

$$c_y = -\mu(h-1)c_\pi^2 + c_y c_\pi + \alpha\gamma - \alpha\gamma c_\pi \tag{30}$$

$$c_\pi = \rho c_y + \beta c_\pi^2 + 1 - \beta. \tag{31}$$

$$m = \frac{\alpha(1-\beta c_\pi)}{1-\beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma} \tag{32}$$

$$n = \frac{(1-\beta)(c_y - \mu(h-1)c_\pi - \alpha\gamma)}{1-\beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma} \tag{33}$$

$$\gamma = -\frac{mn}{\varphi_E + m^2}. \tag{34}$$

[1] Recall that the autocoherence condition for c_y holds, i.e. eq. (40) in the text, holds given our assumption that α and β are common knowledge.

Given φ_E, these equations allow to compute c_y, c_π, m, n, and γ. When linearized around a correct model equilibrium, they deliver $\Delta c_y, \Delta c_\pi, \Delta m, \Delta n$, and $\Delta\gamma$ as a function of $\Delta\varphi_E$. In particular, from the last equation we have that

$$(\varphi + m_c^2)\Delta\gamma + \gamma_c\Delta\varphi + 2\gamma_c m_c \Delta m = -m_c\Delta n - n_c\Delta m. \tag{35}$$

The remaining 4 equations of Table A2 define a perceived block, given by

$$\hat{m} = \frac{\alpha(1-\hat{\beta}c_\pi)}{1+\gamma\alpha\hat{\rho} - \hat{\beta}c_\pi - \hat{\rho}c_y + \mu\hat{\rho}(h-1)c_\pi} \tag{36}$$

$$\hat{n} = \frac{(1-\hat{\beta})\left(c_y - \mu(h-1)c_\pi - \alpha\gamma\right)}{1+\gamma\alpha\hat{\rho} - \hat{\beta}c_\pi - \hat{\rho}c_y + \mu\hat{\rho}(h-1)c_\pi} \tag{37}$$

$$c_\pi = \hat{\rho}c_y + \hat{\beta}c_\pi^2 + 1 - \hat{\beta} \tag{38}$$

$$\gamma = -\frac{\hat{m}\hat{n}}{\varphi + \hat{m}^2}. \tag{39}$$

We have used the autocoherence conditions for c_y and c_π and the assumption that α and μ are common knowledge. These equations solve for $\hat{\rho}, \hat{\beta}, \hat{m}$ and $\hat{n}$ for any γ delivered by the real block. I now show that they imply proposition 5.

Proof of Proposition 5 – Linearizing Equation (39) yields

$$(\varphi + m_c^2)\Delta\gamma + 2\gamma_c m_c(\Delta\hat{m} + \Delta m) = -m_c(\Delta\hat{n} + \Delta n) - n_c(\Delta\hat{m} + \Delta m). \tag{40}$$

Subtracting (35) from (40) we get

$$-\gamma_c\Delta\varphi + 2\gamma_c m_c\Delta\hat{m} + m_c\Delta\hat{n} + n_c\Delta\hat{m} = 0. \tag{41}$$

By construction, from (31) and (35) in the text, it must be that $\gamma m + n = c_y = \gamma\hat{m} + \hat{n}$, implying that

$$\Delta\hat{n} = -\gamma_c\Delta\hat{m}. \tag{42}$$

Substituting into the preceding formula, we get that

$$\Delta\hat{m} = \frac{\gamma_c\Delta\varphi}{c_{yc}}, \tag{43}$$

which proves point (iii) and, together with (42), point (iv).

Next, from (36) and (37), we get that

$$\frac{\hat{m}}{\hat{n}} = \frac{\alpha(1-\hat{\beta}c_\pi)}{(1-\hat{\beta})(c_y - \mu(h-1)c_\pi - \alpha\gamma)}.$$

Consequently,

$$\frac{\Delta\hat{m}}{m_c} - \frac{\Delta\hat{n}}{n_c} = -\frac{c_\pi \Delta\hat{\beta}}{1-\beta c_\pi} + \frac{\Delta\hat{\beta}}{1-\beta}.$$

Substituting (42), we get that

$$\frac{(m_c\gamma_c + n_c)\Delta\hat{m}}{m_c n_c} = \frac{1-c_{\pi c}}{(1-\beta)(1-\beta c_\pi)}\Delta\hat{\beta}.$$

Replacing $m_c\gamma_c + n_c$ with c_{yc} and substituting in (43), we get (ii).

Linearizing (38) and (31) and taking differences, we get the trade-off between $\Delta\hat{\rho}$ and $\Delta\hat{\beta}$:

$$c_{yc}\Delta\hat{\rho} - \Delta\hat{\beta}(1-c_{\pi c}^2) = 0. \tag{44}$$

Substituting into (ii), we get (i).

QED

The rest of the equilibrium perceived model is determined by Table A3, which can be labelled the "residual block". It determines $(\hat{a}_{yu}, \hat{a}_{yv}, \hat{a}_{\pi u}, \hat{a}_{\pi v}, \hat{\theta}, \hat{\sigma}_u^2, \hat{\sigma}_v^2)$. This block can be linearized and then solved numerically. Let $v = (\Delta\hat{m}, \Delta\hat{n}, \Delta\hat{\rho}, \Delta\hat{\beta})'$ and $w = (\Delta\hat{a}_{yu}, \Delta\hat{a}_{yv}, \Delta\hat{a}_{\pi u}, \Delta\hat{a}_{\pi v}, \Delta\hat{\theta}, \Delta\hat{\sigma}_u^2, \Delta\hat{\sigma}_v^2)'$, where again $\Delta\hat{a}_{yu} = \hat{a}_{yu} - a_{yu}$, etc. We have that

$$Pw + Qv = 0,$$

where P is a 7 x 7 matrix and Q a 7 x 4 matrix. The nonzero coefficients are:

$$\begin{aligned}
P_{11} &= 1, P_{22} = 1, P_{25} = -m_c/\alpha, P_{33} = 1/a_{\pi uc}, \\
P_{44} &= 1/a_{\pi vc}, P_{43} = -1/a_{\pi uc}, P_{45} = 1/(1-\theta). \\
P_{51} &= 2a_{yuc}\sigma_u^2, P_{52} = 2a_{yvc}\sigma_v^2, P_{56} = a_{yuc}^2, P_{57} = a_{yvc}^2; \\
P_{63} &= 2a_{\pi uc}\sigma_u^2, P_{64} = 2a_{\pi vc}\sigma_v^2, P_{66} = a_{\pi uc}^2, P_{67} = a_{\pi vc}^2; \\
P_{71} &= a_{\pi uc}\sigma_u^2, P_{72} = a_{\pi vc}\sigma_v^2, P_{73} = a_{yuc}\sigma_u^2, P_{74} = a_{yvc}\sigma_v^2, P_{76} = a_{yuc}a_{\pi uc}, P_{77} = a_{yvc}a_{\pi vc}.
\end{aligned}$$

and

$$\begin{aligned}
Q_{11} &= -1/\alpha, Q_{21} = -\theta/\alpha, Q_{22} = \rho/(1-\beta), \\
Q_{23} &= n_c/(1-\beta), Q_{24} = \frac{\rho n_c}{(1-\beta)^2}, \\
Q_{31} &= -1/m_c, Q_{33} = -1/\rho, Q_{34} = -\frac{c_\pi}{1-\beta c_\pi}.
\end{aligned}$$

Then, $w = -P^{-1}Qv$. These formulas allow to numerically compute the ideological sensitivity of the structural and reduced form parameters. This has been done for 19683 different set of parameters, defined by the following Table:

h	1.2,1.5,1
β	0.2,0.6,0.8
ρ	0.2,1,3
α	0.1,0.3,1
μ	0.02,0.1,0.6
θ	0.1,0.2,0.5
σ_u^2	0.00004,0.0004,0.004
σ_v^2	0.00004,0.0004,0.004
φ	0.1,1,5

Table A4 – Parameter sets

In all these simulations, without exception, the ideological sensitivities $\Delta\hat{\sigma}_u^2/\Delta\varphi$ and $\Delta\hat{\sigma}_v^2/\Delta\varphi$ are negative. Furthermore, that of the perceived share of output fluctuations due to demand shocks, defined by

$$\hat{s}_u = \frac{\hat{a}_{yu}^2\hat{\sigma}_u^2}{\hat{a}_{yu}^2\hat{\sigma}_u^2 + \hat{a}_{yv}^2\hat{\sigma}_v^2} = \frac{\hat{a}_{yu}^2\hat{\sigma}_u^2}{a_{yu}^2\sigma_u^2 + a_{yv}^2\sigma_v^2},$$

is always negative in all cases.

The real block can also be solved by linearization. We get that $Ax + B\Delta\varphi = 0$, where $x \equiv (\Delta c_y, \Delta c_\pi, \Delta m, \Delta n, \Delta\gamma)'$, A is a 5x5 matrix and B is a 5x1 vector with the following nonzero coefficients (here (33) has been replaced by the simpler equation $\gamma m + n = c_y$):

$$\begin{aligned}
A_{11} &= 1 - c_{\pi c}, A_{12} = 2\mu(h-1)c_{\pi c} - c_{yc} + \alpha\gamma_c, A_{15} = -\alpha(1 - c_{\pi c}), \\
A_{22} &= 1 - 2\beta c_{\pi c}, A_{21} = -\rho, \\
A_{31} &= -\rho m_c, A_{32} = (\alpha - m_c)\beta + m_c\mu\rho(h-1), A_{33} = \frac{\alpha(1-\beta c_{\pi c})}{m_c}, A_{35} = \alpha\rho m_c \\
A_{41} &= -1, A_{43} = \gamma_c, A_{44} = 1, A_{45} = m_c, \\
A_{53} &= 2m_c\gamma_c + n_c, A_{54} = m_c, A_{55} = \varphi + m_c^2 \\
B_{51} &= \gamma_c.
\end{aligned}$$

These formulas allow to compute the response of x to $\Delta\varphi$. In particular, it has been checked that in all the simulations above, one has $\frac{\Delta\gamma}{\Delta\varphi} < 0$.

6 Non-myopic government and expert

Assume the government is non myopic and cannot commit on its fiscal policy rule. At each date it sets g_t, so as to maximize

$$\hat{V}(\pi_{t-1}, g_t) = \max_{g_t} \hat{E}(-\varphi g_t^2 - (y_t - v_t)^2 + \delta \hat{V}(\pi_t, g(\pi_t))),$$

where δ is the discount factor, $g_t = g(\pi_{t-1})$ is the equilibrium policy rule and expectations are conditional on π_{t-1} *and* g_t. That is, g_t is freely chosen by the government at t. On the other hand, in the absence of commitment, the government rationally anticipates that it will follow the equilibrium policy rule at any future date $s > t$.

The FOC is, using the fact that $\hat{V}_g(\pi_t, g(\pi_t)) = 0$,

$$\begin{aligned} 0 &= \hat{V}_g(\pi_{t-1}, g_t) \\ &= -2(\varphi g_t + \frac{\hat{d}y_t}{\hat{d}g_t}\hat{E}y_t) + \delta\frac{\hat{d}\pi_t}{\hat{d}g_t}\hat{E}\hat{V}_\pi(\pi_t, g(\pi_t)). \end{aligned} \tag{45}$$

By the envelope theorem, we have that

$$\hat{V}_\pi(\pi_{t-1}, g_t) = -2\frac{\hat{d}y_t}{\hat{d}\pi_{t-1}}\hat{E}y_t + \delta\frac{\hat{d}\pi_t}{\hat{d}\pi_{t-1}}\hat{E}\hat{V}_\pi(\pi_t, g(\pi_t)). \tag{46}$$

As above, we look for an equilibrium where the optimal policy is a linear function of the state variable,

$$g_t = \gamma\pi_{t-1}. \tag{47}$$

From the two perceived model equations in the text (29)-(30), which I rewrite here for convenience,

$$y_t = -\hat{\mu}(h-1)\hat{c}_\pi\pi_t + \hat{c}_y\pi_t + \hat{\alpha}g_t + \hat{u}_t + \hat{\theta}\hat{v}_t - \hat{\alpha}\gamma\pi_t, \tag{48}$$

$$\pi_t = \hat{\rho}y_t + \hat{\beta}\hat{c}_\pi\pi_t + (1-\hat{\beta})\pi_{t-1} - \hat{\rho}\hat{v}_t, \tag{49}$$

we get that

$$\hat{E}y_t = \hat{m}g_t + \hat{n}\pi_{t-1}, \tag{50}$$

as before, and

$$\hat{E}\pi_t = \hat{q}g_t + \hat{r}\pi_{t-1}, \tag{51}$$

where

$$\hat{q} = \frac{\alpha\hat{\rho}}{1-\hat{\beta}c_\pi - \hat{\rho}c_y + \mu\hat{\rho}(h-1)c_\pi + \hat{\rho}\alpha\gamma}, \tag{52}$$

$$\hat{r} = \frac{1-\hat{\beta}}{1-\hat{\beta}c_\pi - \hat{\rho}c_y + \mu\hat{\rho}(h-1)c_\pi + \hat{\rho}\alpha\gamma}. \tag{53}$$

In those formulas, we readily make use of the assumption that α and μ are common knowledge and of the autocoherence conditions $c_\pi = \hat{c}_\pi$ and $c_y = \hat{c}_y$.

Furthermore, since (48)-(49) is additive in the shocks $\hat{u}_t$ and $\hat{v}_t$, it is also true that $\frac{\hat{d}y_t}{\hat{d}g_t} = \hat{m}_t$, $\frac{\hat{d}y_t}{\hat{d}\pi_{t-1}} = \hat{n}$, $\frac{\hat{d}\pi_t}{\hat{d}g_t} = \hat{q}$, and $\frac{\hat{d}\pi_t}{\hat{d}\pi_{t-1}} = \hat{r}$.

Elimination of $\hat{E}\hat{V}_\pi$ between (45) and (46), shows that in equilibrium $\hat{V}_\pi(\pi_{t-1}, g_t)$ must be linear in (π_{t-1}, g_t). Consequently, it must be that $\hat{V}_\pi(\pi_t, g(\pi_t)) = \hat{V}_\pi(\pi_t, \gamma\pi_t) = e\pi_t$, where e is a coefficient which remains to be determined. From (51), it follows that

$$\hat{E}\hat{V}_\pi(\pi_t, g(\pi_t)) = e.(\hat{q}g_t + \hat{r}\pi_{t-1}). \tag{54}$$

To compute e, we apply (46) at $g_t = g(\pi_{t-1}) = \gamma\pi_{t-1}$, which yields

$$e = -2\hat{n}(\hat{m}\gamma + \hat{n}) + \delta e\hat{r}(\hat{q}\gamma + \hat{r}),$$

and noting that from (50) and (51) it must be that $\hat{m}\gamma + \hat{n} = c_y$ and $\hat{q}\gamma + \hat{r} = c_\pi$, it follows that

$$e = \frac{-2\hat{n}c_y}{1-\delta\hat{r}c_\pi}. \tag{55}$$

Substituting (54) and (50) into the FOC (45), and then using (55), we get a formula for the equilibrium γ :

$$\gamma\left(\varphi + \hat{m}^2 + \delta\hat{q}^2\frac{\hat{n}c_y}{1-\delta\hat{r}c_\pi}\right) = -\hat{m}\hat{n} - \delta\hat{q}\hat{r}\frac{\hat{n}c_y}{1-\delta\hat{r}c_\pi}. \tag{56}$$

This expression should replace (39). The expert will equate γ with the policy parameter he would pick on the basis of his own preferences, therefore

$$\gamma\left(\varphi_E + m^2 + \delta_E q^2\frac{nc_y}{1-\delta_E rc_\pi}\right) = -mn - \delta_E qr\frac{nc_y}{1-\delta_E rc_\pi}. \tag{57}$$

where obviously

$$q = \frac{\alpha\rho}{1-\beta c_\pi - \rho c_y + \mu\rho(h-1)c_\pi + \rho\alpha\gamma}, \tag{58}$$

and

$$r=\frac{1-\beta}{1-\beta c_{\pi}-\rho c_{y}+\mu\rho(h-1)c_{\pi}+\rho\alpha\gamma}. \tag{59}$$

Equation (57) replaces (34). It is the only equation of the model where the expert's preferences appear.

It is again possible to construct a correct model equilibrium, as summarized by the following proposition

PROPOSITION A2 – A correct model equilibrium exists such that

$$\begin{aligned} 0 &< c_{\pi}<1; \\ c_{y} &< 0. \end{aligned}$$

PROOF – The steps are the same as in Proposition A1, but the $\tilde{\gamma}$ function in its proof now has to be replaced with a new formula from (57), that is,

$$\breve{\gamma}(c_{\pi})=\frac{-m(c_{\pi})n(c_{\pi})-\delta q(c_{\pi})r(c_{\pi})\frac{n(c_{\pi})c_{y}(c_{\pi})}{1-\delta r(c_{\pi})c_{\pi}}}{\varphi+m(c_{\pi})^{2}+\delta q(c_{\pi})^{2}\frac{n(c_{\pi})c_{y}(c_{\pi})}{1-\delta r(c_{\pi})c_{\pi}}}. \tag{60}$$

The functions $q()$ and $r()$ are obviously defined by expressing the RHS of (58) and (59) as functions of c_{π} by using (22).

Next, note that from (25), $\breve{\gamma}(0)=0>\gamma(0)=-\frac{1-\beta}{\rho\alpha}$.

Second,

$$\lim_{c_{\pi}\longrightarrow 1}\breve{\gamma}(c_{\pi})=0<\lim_{c_{\pi}\rightarrow 1}\gamma=+\infty,$$

since $c_{y}(1)=0=\lim m=\lim r=\lim q$.

By continuity, again, there exists an equilibrium such that $c_{\pi}\in(0,1)$.

To complete the proof, we again prove that we can choose the equilibrium such that $c_{y}<0$. Note that (22) still holds. Clearly, then, if $(1-\beta)/\beta\geq 1$, $c_{y}<0$. Assume that $(1-\beta)/\beta<1$. Since $c_{y}((1-\beta)/\beta)=0$, from (60), (58), and (26)-(29) we have that

$$\breve{\gamma}((1-\beta)/\beta)=\frac{\mu(h-1)\alpha\beta(1-\beta)^{2}}{(2\beta-1)\left[\varphi\left(\beta+\mu\rho(h-1)\frac{1-\beta}{2\beta-1}\right)^{2}+\alpha^{2}\beta^{2}\right]}<\frac{\mu(h-1)}{\alpha}\frac{(1-\beta)^{2}}{\beta(2\beta-1)}=\gamma(\frac{1-\beta}{\beta}).$$

As in Proposition A1, by continuity, there exists a solution such that $c_{\pi}\in\left(0,\frac{1-\beta}{\beta}\right)$ and therefore $c_{y}<0$.

This completes the proof of Proposition A1. Note that now we cannot establish an analytical result for the sign of γ.

QED

Proof of Proposition 6 – The system is recursive in the sense that $\Delta\varphi$ and $\Delta\delta$ only appear through Equation (57). Consequently, any equilibrium quantity, whether it is perceived or real, will depend on $\Delta\varphi$ and $\Delta\delta$ only through $\omega_\varphi\Delta\varphi + \omega_\delta\Delta\delta$, where ω_φ and ω_δ are the coefficients that multiply $\Delta\varphi$ and $\Delta\delta$ in a linearization of (57). Next, observe that since $c_y < 0$ and, from (21), $c_y - \alpha\gamma < 0$, we have that $q > 0$ and $0 < r < 1$ in the CME, as well as $m > 0$ and $n < 0$. From this it follows that (i) an increase in φ_E raises the LHS of (57) if $\gamma > 0$, (ii), an increase in δ_E raises the LHS of (57) if $\gamma > 0$, and (iii) an increase in δ_E reduces the RHS of (57). Therefore, $\omega_\varphi\omega_\delta > 0$ if $\gamma > 0$, from which the statement in Proposition 6 follows trivially.

QED

Under this extension, it is no longer possible to prove analytical results regarding the perceived model. The entire system has to be linearized.

First, linearizing both (56) and (57) and subtracting one from the other yields:

$$\begin{aligned} & \gamma_c \left[\begin{array}{c} 2m_c(1-\delta r_c c_{\pi c})\Delta\hat{m} - \delta(\varphi + m_c^2)c_{\pi c}\Delta\hat{r} + 2\delta q_c \Delta\hat{q} n_c c_{yc} + \delta q_c^2 c_{yc}\Delta\hat{n} \\ -(1-\delta r_c c_{\pi c})\Delta\varphi + (\varphi + m_c^2) r_c c_{\pi c}\Delta\delta - q_c^2 n_c c_{yc}\Delta\delta \end{array} \right] \\ = \; & -n_c(1-\delta r_c c_{\pi c})\Delta\hat{m} - m_c(1-\delta r_c c_{\pi c})\Delta\hat{n} + \delta m_c n_c c_{\pi c}\Delta\hat{r} - \delta r_c n_c c_{yc}\Delta\hat{q} - \delta q_c n_c c_{yc}\Delta\hat{r} \\ & -\delta q_c r_c c_{yc}\Delta\hat{n} + \Delta\delta q_c r_c c_{yc} n_c - \Delta\delta m_c n_c c_{\pi c} r_c. \end{aligned} \tag{61}$$

where $\Delta\hat{q}$ is implictly defined by linearizing (52), or equivalently the simpler relationship $\hat{q} = \hat{\rho}\hat{m}/(1-\hat{\beta}c_\pi)$, yielding

$$\frac{\Delta\hat{q}}{q_c} = \frac{\Delta\hat{\rho}}{\rho} + \frac{\Delta\hat{m}}{m_c} + \frac{c_{\pi c}\Delta\hat{\beta}}{1-\beta c_{\pi c}},$$

and similarly from (53), which is equivalent to $\hat{r} = (1-\hat{\beta})\hat{m}/\left[\alpha(1-\hat{\beta}c_\pi)\right]$, we have that

$$\frac{\Delta\hat{r}}{r_c} = \frac{\Delta\hat{m}}{m_c} + \frac{(c_{\pi c}-1)\Delta\hat{\beta}}{(1-\beta)(1-\beta c_{\pi c})}.$$

Equation (61) replaces (41) and is completed by three equations that are unchanged from the myopic model: (42), (44), and the linearization of (36) minus that of (32), which boils down to

$$\frac{\Delta\hat{m}}{m_c} + \frac{1}{D_1}\left(-c_{\pi c}\Delta\hat{\beta} - c_{yc}\Delta\hat{\rho} + \mu(h-1)c_{\pi c}\Delta\hat{\rho} + \alpha\gamma_c\Delta\hat{\rho}\right) = -\frac{c_{\pi c}\Delta\hat{\beta}}{1-\beta c_{\pi c}},$$

where

$$D_1 = 1 - \beta c_{\pi c} - \rho c_{yc} + \mu\rho(h-1)c_{\pi c} + \alpha\rho\gamma_c.$$

These four equations allow to solve for $v = (\Delta\hat{m}, \Delta\hat{n}, \Delta\hat{\rho}, \Delta\hat{\beta})'$ as they can be written as $Gv + H(\Delta\varphi, \Delta\delta)' = 0$, where G is a 4x4 matrix and H a 4x2 matrix with the following nonzero coefficients:

$$
\begin{aligned}
G_{11} &= \frac{1}{m_c}, G_{13} = \frac{-c_{yc} + \mu(h-1)c_{\pi c} + \alpha\gamma_c}{D_1}, G_{14} = -\frac{c_{\pi c}}{D_1} + \frac{c_{\pi c}}{1-\beta c_{\pi c}}, \\
G_{21} &= \gamma_c, G_{22} = 1, \\
G_{33} &= c_{yc}, G_{34} = c_{\pi c}^2 - 1, \\
G_{41} &= 2m_c\gamma_c(1-\delta r_c c_{\pi c}) - \delta(\varphi + m_c^2)c_{\pi c}\gamma_c\frac{r_c}{m_c} + 2\delta q_c^2 n_c \frac{c_{yc}}{m_c}\gamma_c \\
&\quad + n_c(1-\delta r_c c_{\pi c}) - \delta n_c c_{\pi c} r_c + 2\delta r_c n_c c_{yc}\frac{q_c}{m_c} \\
G_{42} &= \delta\gamma_c q_c^2 c_{yc} + m_c(1-\delta r_c c_{\pi c}) + \delta r_c q_c c_{yc} \\
G_{43} &= 2\delta\gamma_c q_c^2 c_{yc} n_c/\rho + \delta r_c q_c c_{yc} n_c/\rho \\
G_{44} &= -\frac{\delta r_c c_{\pi c}\gamma_c(\varphi + m_c^2)(c_{\pi c}-1)}{(1-\beta)(1-\beta c_{\pi c})} + \frac{2\delta q_c^2 n_c \gamma_c c_{yc} c_{\pi c}}{1-\beta c_{\pi c}} \\
&\quad - \frac{\delta m_c n_c c_{\pi c} r_c (c_{\pi c}-1)}{(1-\beta)(1-\beta c_{\pi c})} + \frac{\delta r_c q_c c_{yc} n_c c_{\pi c}}{1-\beta c_{\pi c}} + \frac{\delta r_c q_c c_{yc} n_c (c_{\pi c}-1)}{(1-\beta)(1-\beta c_{\pi c})} \\
H_{41} &= -\gamma_c(1-\delta r_c c_{\pi c}) \\
H_{42} &= \gamma_c r_c c_{\pi c}(\varphi + m_c^2) - \gamma_c q_c^2 n_c c_{yc} - q_c r_c c_{yc} n_c + m_c n_c c_{\pi c} r_c
\end{aligned}
$$

Clearly, then, $v = -G^{-1}H(\Delta\varphi, \Delta\delta)'$. The residual block is unchanged compared to the myopic model, therefore w can again be computed as $w = -P^{-1}Qv$.

Relative to the myopic case, the real block is defined as follows: (30)-(32) are unchanged, as well as the condition $\gamma m + n = c_y$ which is used instead of (33). The definition of q (58), in the form $q = \rho m/(1-\beta c_\pi)$ is added to the system, while r is replaced by the RHS of (59), expressed as $(1-\beta)m/[\alpha(1-\beta c_\pi)]$, and the optimality condition (34) has to be replaced by (57). As a result, when linearized, the real block is now expressed as $Ax + B(\Delta\varphi, \Delta\delta)' = 0$, where now $x \equiv (\Delta c_y, \Delta c_\pi, \Delta m, \Delta n, \Delta q, \Delta\gamma)'$ and the matrices A and B are 6x6 and 6x2 respectively, and their nonzero coefficients are now defined as

$$
\begin{aligned}
A_{11} &= 1-c_{\pi c}, A_{12}=2\mu(h-1)c_{\pi c}-c_{yc}+\alpha\gamma_c, A_{16}=-\alpha(1-c_{\pi c}),\\
A_{22} &= 1-2\beta c_{\pi c}, A_{21}=-\rho,\\
A_{31} &= -\rho m_c, A_{32}=(\alpha-m_c)\beta+m_c\mu\rho(h-1), A_{33}=\frac{\alpha(1-\beta c_{\pi c})}{m_c}, A_{36}=\alpha\rho m_c\\
A_{41} &= -1, A_{43}=\gamma_c, A_{44}=1, A_{46}=m_c,\\
A_{52} &= -\beta/(1-\beta c_{\pi c}), A_{53}=-1/m_c, A_{55}=1/q_c,\\
A_{61} &= \gamma_c\delta q_c^2 n_c+\delta q_c n_c r_c, A_{62}=-\gamma_c\delta r_c(\varphi+m_c^2)-m_c n_c\delta r_c+\tilde{A}\frac{r_c\beta}{1-\beta c_{\pi c}},\\
A_{63} &= (2\gamma_c m_c+n_c)(1-\delta r_c c_{\pi c})+\tilde{A}r_c/m_c,\\
A_{64} &= \delta\gamma_c q_c^2 c_{yc}+m_c(1-\delta r_c c_{\pi c})+\delta q_c r_c c_{yc}, A_{65}=2\gamma_c\delta q_c n_c c_{yc}+n_c\delta r_c c_{yc},\\
A_{66} &= \left(\varphi+m_c^2\right)(1-\delta r_c c_{\pi c})+\delta q_c^2 n_c c_{yc},\\
B_{61} &= \gamma_c(1-\delta r_c c_{\pi c}), B_{62}=-\gamma_c(\varphi+m_c^2)r_c c_{\pi c}+\gamma_c q_c^2 n_c c_{yc}-m_c n_c r_c c_{\pi c}+q_c r_c n_c c_{yc}.
\end{aligned}
$$

and the quantity $\tilde{A}$ stands for $-\gamma_c\delta c_{\pi c}(\varphi+m_c^2)+\delta q_c n_c c_{yc}-\delta c_{\pi c}m_c n_c$.

The simulations above have been run for the entire alternative sets of parameters and for $\delta = 0.5, 0.9$, and 0.99. In all these simulations, without exceptions, we get that $\Delta\hat{\rho}/\Delta\varphi > 0, \Delta\hat{\beta}/\Delta\varphi < 0, \Delta\hat{m}/\Delta\varphi < 0, \Delta\hat{n}/\Delta\varphi > 0, \Delta\hat{\sigma}_u^2/\Delta\varphi < 0, \Delta\hat{\sigma}_v^2/\Delta\varphi < 0$, $\frac{\Delta\hat{s}_u}{\Delta\varphi} < 0$, and $\frac{\Delta\gamma}{\Delta\varphi} < 0$, as in Proposition 5 and the simulations that follow it. In all those simulations, $\gamma_c > 0$, so that Proposition 6 applies. Consequently, we also have that $\Delta\hat{\rho}/\Delta\delta > 0, \Delta\hat{\beta}/\Delta\delta < 0, \Delta\hat{m}/\Delta\delta < 0, \Delta\hat{n}/\Delta\delta > 0, \Delta\hat{\sigma}_u^2/\Delta\delta < 0, \Delta\hat{\sigma}_v^2/\Delta\delta < 0$, $\frac{\Delta\hat{s}_u}{\Delta\delta} < 0$, and $\frac{\Delta\gamma}{\Delta\delta} < 0$.

7 Scilab source code for the simulations reported in section II.B.

```
h=1.5
   mu=0.6
   ro=1
   al=0.3
   be=0.1
   th=0.2
   siu=0.0004
   siv=0.0004
   phi=0.1
   fpos=zeros(4,1)
```

```
nsim=3*3*3*3*3*3*3*3*3
avg=fpos
for h=[1.2 1.5 2]
for mu = [0.2 0.6 0.8]
for ro = [0.2 1 3]
for al = [0.1 0.3 1]
for be = [0.02 0.1 0.6]
for th = [0.1 0.2 0.5]
for siu=[0.00004 0.0004 0.004]
for siv =[0.00004 0.0004 0.004]
for phi = [0.2 1 5]
// First we compute benchmark correct model equilibrium
cpimin=0
cpimax=min(1,(1-mu)/mu)
while cpimax-cpimin>0.0001
cpi=(cpimin+cpimax)/2
cy=(cpi-mu*cpi^2+mu-1)/ro
gal=(cy+be*(h-1)*cpi^2-cy*cpi)/al/(1-cpi)
m=al*(1-mu*cpi)/(1-mu*cpi-ro*cy+be*ro*(h-1)*cpi+ro*al*gal)
n=(cy-be*(h-1)*cpi-al*gal)*(1-mu)/(1-mu*cpi-ro*cy+be*ro*(h-1)*cpi+ro*al*gal)
gar=-m*n/(phi+m^2)
dif=gar-gal
if dif>0 then
cpimin=cpi
else
cpimax=cpi
end
end
ga=gar
ayu=m/al
ayv=(th*m/al-ro*n/(1-mu))
apiu=ro*m/al/(1-mu*cpi)
apiv=ro*(th-1)*m/al/(1-mu*cpi)
dmh=ga/cy
dnh=-ga^2/cy
dmuh=-(1-mu)*(1-mu*cpi)/(1-cpi)/(phi+m^2)
droh=-(1-mu)*(1-mu*cpi)/(phi+m^2)*(1+cpi)/cy
v=[dmh dnh droh dmuh]'
pp=zeros(7,7)
qq=zeros(7,4)
pp(1,1)=1
```

```
pp(2,2)=1
pp(2,5)=-m/al
pp(3,3)=1/apiu
pp(4,4)=1/apiv
pp(4,3)=-1/apiu
pp(4,5)=1/(1-th)
pp(5,1)=2*ayu*siu
pp(5,2)=2*ayv*siv
pp(5,6)=ayu^2
pp(5,7)=ayv^2
pp(6,3)=2*apiu*siu
pp(6,4)=2*apiv*siv
pp(6,6)=apiu^2
pp(6,7)=apiv^2
pp(7,1)=apiu*siu
pp(7,2)=apiv*siv
pp(7,3)=ayu*siu
pp(7,4)=ayv*siv
pp(7,6)=ayu*apiu
pp(7,7)=ayv*apiv
qq(1,1)=-1/al
qq(2,1)=-th/al
qq(2,2)=ro/(1-mu)
qq(2,3)=n/(1-mu)
qq(2,4)=ro*n/(1-mu)^2
qq(3,1)=-1/m
qq(3,3)=-1/ro
qq(3,4)=-cpi/(1-mu*cpi)
ww=-inv(pp)*qq*v
chdemprop=2*ww(1,:)/ayu+ww(6,:)/siu
//for chdemprop this gives the sign, not the exact value
fpos(1:2,1)=fpos(1:2,1)+(ww(6:7,1)>=0)
fpos(3,1)=fpos(3,1)+(chdemprop>=0)
avg(1:2,1)=avg(1:2,1)+ww(6:7,1)
avg(3,1)=avg(3,1)+chdemprop
// Now we numerically compute the changes in the real economy
aa=zeros(5,5)
bb=zeros(5,1)
aa(1,1)=1-cpi
aa(1,2)=2*be*(h-1)*cpi-cy+al*ga
aa(1,5)=-al*(1-cpi)
```

```
aa(2,2)=1-2*mu*cpi
aa(2,1)=-ro
aa(3,1)=-ro*m
aa(3,2)=-mu*m+al*mu+m*be*ro*(h-1)
aa(3,3)=al*(1-mu*cpi)/m
aa(3,5)=al*ro*m
aa(4,1)=-1
aa(4,3)=ga
aa(4,4)=1
aa(4,5)=m
aa(5,3)=2*m*ga+n
aa(5,4)=m
aa(5,5)=phi+m^2
bb(5,1)=ga
xx=-inv(aa)*bb
fpos(4,1)=fpos(4,1)+(xx(5,1)>0)
avg(4,1)=avg(4,1)+xx(5,1)
end
end
end
end
end
end
end
end
end
avg=avg/nsim
```

8 Scilab source code for the simulations reported in Section II.C.

```
h=1.5
mu=0.6
ro=1
al=0.3
be=0.1
th=0.2
siu=0.0004
siv=0.0004
```

```
phi=0.1
de=0.5
fpos=zeros(4,2)
nsim=3*3*3*3*3*3*3*3*3
avg=fpos
gpos=zeros(4,2)
gavg=gpos
gasgn=0
fpos2=zeros(3,1)
gpos2=zeros(4,1)
n1=0
mu1mean=0
ro1mean=0
al1mean=0
be1mean=0
th1mean=0
phi1mean=0
siu1mean=0
siv1mean=0
for h=[1.2 1.5 2]
for mu = [0.2 0.6 0.8]
for ro = [0.2 1 3]
for al = [0.1 0.3 1]
for be = [0.02 0.1 0.6]
for th = [0.1 0.2 0.5]
for siu=[0.00004 0.0004 0.004]
for siv =[0.00004 0.0004 0.004]
for phi = [0.2 1 5]
// First we compute benchmark correct model equilibrium
cpimin=0
cpimax=min(1,(1-mu)/mu)
while cpimax-cpimin>0.000001
cpi=(cpimin+cpimax)/2
cy=(cpi-mu*cpi^2+mu-1)/ro
gal=(cy+be*(h-1)*cpi^2-cy*cpi)/al/(1-cpi)
m=al*(1-mu*cpi)/(1-mu*cpi-ro*cy+be*ro*(h-1)*cpi+ro*al*gal)
n=(cy-be*(h-1)*cpi-al*gal)*(1-mu)/(1-mu*cpi-ro*cy+be*ro*(h-1)*cpi+ro*al*gal)
q=ro*m/(1-mu*cpi)
r=(1-mu)*m/al/(1-mu*cpi)
z=de*q*n*cy/(1-de*r*cpi)
gar=(-m*n-r*z)/(phi+m^2+q*z)
```

```
    dif=gar-gal
    if dif>0 then
    cpimin=cpi
    else
    cpimax=cpi
    end
    end
    ga=gar
    ayu=m/al
    ayv=(th*m/al-ro*n/(1-mu))
    apiu=ro*m/al/(1-mu*cpi)
    apiv=ro*(th-1)*m/al/(1-mu*cpi)
    ee=zeros(4,4)
    ff=zeros(4,2)
    d1=1-mu*cpi-ro*cy+be*ro*(h-1)*cpi+al*ro*ga
    ee(1,1)=1/m
    ee(1,3)=(-cy+be*(h-1)*cpi+al*ga)/d1
    ee(1,4)=-cpi/d1+cpi/(1-mu*cpi)
    ee(2,1)=ga
    ee(2,2)=1
    ee(3,3)=cy
    ee(3,4)=cpi^2-1
    ee(4,1)=2*ga*m*(1-de*r*cpi)-de*(phi+m^2)*cpi*ga*r/m+2*de*q^2*n*cy/m*ga+n*(1-
de*r*cpi)-de*n*r*cpi+2*de*r*n*cy*q/m
    ee(4,2)=de*ga*q^2*cy+m*(1-de*r*cpi)+de*r*q*cy
    ee(4,3)=2*de*ga*q^2*cy*n/ro+de*r*q*cy*n/ro
    ee(4,4)=-de*r*cpi*ga*(phi+m^2)*(cpi-1)/(1-mu)/(1-mu*cpi)+2*de*q^2*n*cy*cpi*ga/(1-
mu*cpi)-de*m*n*cpi*r*(cpi-1)/(1-mu)/(1-mu*cpi)+de*r*q*cy*n*cpi/(1-mu*cpi)+de*r*q*cy*n*
1)/(1-mu)/(1-mu*cpi)
    ff(4,1)=-ga*(1-de*r*cpi)
    ff(4,2)=ga*r*cpi*(phi+m^2)-ga*q^2*n*cy-q*r*cy*n+m*n*cpi*r
    //The following lines compute v=-inv(ee)*ff in a way which is robust to
singularities in ee, due to the use of intermediate expressions m and n that
may turn up to be colinear for some parameter values
    v2=-inv(ee(3:4,3:4)-ee(3:4,1:2)*inv(ee(1:2,1:2))*ee(1:2,3:4))*ff(3:4,:)
    v1=-inv(ee(1:2,1:2))*ee(1:2,3:4)*v2
    v=cat(1,v1,v2)
    gpos=gpos+(v>0)
    gavg=gavg+v
    gasgn=gasgn+(ga>0)
    pp=zeros(7,7)
```

```
qq=zeros(7,4)
pp(1,1)=1
pp(2,2)=1
pp(2,5)=-m/al
pp(3,3)=1/apiu
pp(4,4)=1/apiv
pp(4,3)=-1/apiu
pp(4,5)=1/(1-th)
pp(5,1)=2*ayu*siu
pp(5,2)=2*ayv*siv
pp(5,6)=ayu^2
pp(5,7)=ayv^2
pp(6,3)=2*apiu*siu
pp(6,4)=2*apiv*siv
pp(6,6)=apiu^2
pp(6,7)=apiv^2
pp(7,1)=apiu*siu
pp(7,2)=apiv*siv
pp(7,3)=ayu*siu
pp(7,4)=ayv*siv
pp(7,6)=ayu*apiu
pp(7,7)=ayv*apiv
qq(1,1)=-1/al
qq(2,1)=-th/al
qq(2,2)=ro/(1-mu)
qq(2,3)=n/(1-mu)
qq(2,4)=ro*n/(1-mu)^2
qq(3,1)=-1/m
qq(3,3)=-1/ro
qq(3,4)=-cpi/(1-mu*cpi)
ww=-inv(pp)*qq*v
chdemprop=2*ww(1,:)/ayu+ww(6,:)/siu
//for chdemprop this gives the sign, not the exact value
fpos(1:2,:)=fpos(1:2,:)+(ww(6:7,:)>=0)
fpos(3,:)=fpos(3,:)+(chdemprop>=0)
avg(1:2,:)=avg(1:2,:)+ww(6:7,:)
avg(3,:)=avg(3,:)+chdemprop
//Now we numerically compute the changes in the real economy
aa=zeros(6,6)
bb=zeros(6,2)
atilda=-ga*de*cpi*(phi+m^2)+de*q*n*cy-de*cpi*m*n
```

```
aa(1,1)=1-cpi
aa(1,2)=2*be*(h-1)*cpi-cy+al*ga
aa(1,6)=-al*(1-cpi)
aa(2,2)=1-2*mu*cpi
aa(2,1)=-ro
aa(3,1)=-ro*m
aa(3,2)=-mu*m+al*mu+m*be*ro*(h-1)
aa(3,3)=al*(1-mu*cpi)/m
aa(3,6)=al*ro*m
aa(4,1)=-1
aa(4,3)=ga
aa(4,4)=1
aa(4,6)=m
aa(5,2)=-mu/(1-mu*cpi)
aa(5,3)=-1/m
aa(5,5)=1/q
aa(6,1)=ga*de*q^2*n+de*q*n*r
aa(6,2)=-ga*de*r*(phi+m^2)-m*n*de*r+atilda*r*mu/(1-mu*cpi)
aa(6,3)=(2*ga*m+n)*(1-de*r*cpi)+atilda*r/m
aa(6,4)=de*ga*q^2*cy+m*(1-de*r*cpi)+de*q*r*cy
aa(6,5)=2*ga*de*q*n*cy+de*n*r*cy
aa(6,6)=(phi+m^2)*(1-de*r*cpi)+de*q^2*n*cy
bb(6,1)=ga*(1-de*r*cpi)
bb(6,2)=-ga*(phi+m^2)*r*cpi+ga*q^2*n*cy-m*n*r*cpi+q*r*n*cy
xx=-inv(aa)*bb
fpos(4,:)=fpos(4,:)+(xx(6,:)>0)
avg(4,:)=avg(4,:)+xx(6,:)
//gpos2 and fpos2 check that phi and de have opposite effect on perceived parameter iff dga/dde>0
end
end
end
end
end
end
end
end
end
avg=avg/nsim
gavg=gavg/nsim
```