Investment strategy and selection bias: An equilibrium perspective on overoptimism ONLINE APPENDIX

Philippe Jehiel*

Convergence to steady state

The main model assumes steady state. Embedding the above framework into a dynamic setting in which new cohorts of investors sample from previous cohorts of investors naturally leads to asking when we should expect to see convergence to steady state as considered in the main analysis. Another legitimate concern is whether the overoptimism bias identified in the main analysis would still arise in case there would be no convergence. To model the dynamics most simply, consider within the MLRP scenario discussed in the main model a sequence of time periods $t = 1, 2, ...$ Assume that in every period $t > 1$ there is a new cohort of investors of the same mass who sample from the implemented projects handled by the cohort of investors living in period $t-1$, and assume to fix ideas that in the first period investors choose to invest whatever signal they observe.

In such a dynamic setting, investors in period t would adopt a threshold strategy z_t specifying to invest if the observed signal realization a is above z_t and to not invest otherwise where the sequence of z_t would be characterized inductively by $z_1 = \underline{a}$ (since the first generation of investors was assumed to invest always) and for all $t > 1$, the threshold z_{t+1} would be uniquely defined by $H(z_{t+1}, z_t) = c$ (assuming $H(\underline{a}, z) < c < H(\overline{a}, z)$ for all z) where $H(\cdot, \cdot)$ is the function defined in Section 3 of the paper. It appears that z_2 coincides with a^R, and using the monotonicity of H, it can be shown by induction that the sequence $(z_{2k+1})_{k\geq 1}$ is weakly decreasing and satisfies $z_{2k+1} \geq a^S$ for all k while the sequence $(z_{2k})_{k\geq 1}$ is weakly increasing and satisfies $z_{2k} \leq a^S$ for all k where a^S is the equilibrium threshold defined in Proposition 1. Thus, $(z_{2k+1})_{k\geq 1}$ converges to z^* and $(z_{2k})_{k\geq 1}$ converges to z_* with $z_* \leq a^S \leq z^*$. If $z_* = z^* = a^S$ the system converges to the

*PSE, 48 boulevard Jourdan, 75014 Paris, France and University College London ; jehiel@enpc.fr.

steady state described in Proposition 1. If $z_* < a^S < z^*$, the system converges to a limit two-period cycle in which in odd periods there is less activity as dictated by the threshold strategy z^* and in even periods there is more activity as dictated by the threshold strategy z_*. Whether the system converges or cycles depends on how the slope $(\partial H/\partial z)(\partial H/\partial a)$ compares to 1. When it is uniformly lower than 1, (as is the case for the leading example with variance $\sigma = 1$), there is convergence. When it is larger than 1 in the neighborhood of $a = z = a^S$, the two-period limit cycle prevails.[1]

It should be noted that in the above dynamics whether or not there is convergence, the overoptimism and overinvestment biases hold in every period (this follows from the monotonicity of H and the observation that $H(a^R, \underline{a}) = c$). Moreover, since $z_t \leq a^R$ for all t and $z_2 = a^R$, the monotonicity of H implies that the smallest z_t which corresponds to the most biased investment strategy is obtained in period 3 when the samples considered by the current cohort consist of projects handled by rational investors. In all subsequent periods, because sampled investors adopt suboptimal strategies, the sampling heuristic leads to less severe biases.

Cycling with heterogeneous investors

It is natural to combine dynamics as just considered with the possibility that investors could vary in their degree of sophistication, some of them being rational and others being subject to selection neglect as proposed in the main model. A full-fledged dynamic model along these lines would aim at endogenizing entry and exit of entrepreneurs, assuming for example entrepreneurs' sophistication vary with their experience. Analyzing such a model is clearly beyond the scope of this online appendix. Yet, in order to illustrate that some rich dynamics can be expected, consider the following stylized setting. In each period $t = 1, 2, ...$ a new cohort of agents decides whether or not to become entrepreneur. Every entrepreneur faces the same distribution of projects as described above but agents may have different outside options assumed to be drawn independently across agents from a distribution with cumulative G. In every period, the share of rational agents is λ and the share of sampling agents is $1 - \lambda$. Let w^R denote the expected payoff a rational investor gets by becoming an entrepreneur (i.e., $w^R = E(\max v^R(a) - c, 0)$), and let $w^S(\lambda)$ denote the expected payoff a sampling investor subjectively expects to get when facing

[1] If investors were sampling from all previous cohorts rather than just the most recent one, I suspect the convergence scenario would be made more likely (because such a sampling device would smoothen the reaction to previous behaviors), but more work is needed to establish this formally.

a share λ (resp. $1-\lambda$) of rational (resp. sampling) investors.[2] Rational agents become entrepreneur whenever their outside option falls below w^R, i.e. with probability $G(w^R)$. Sampling agents who would sample from a mix λ of rational investors and $1-\lambda$ of sampling investors would become entrepreneur with probability $G(w^S(\lambda))$. Thus assuming the cohort of (sampling) agents in period t samples from the implemented projects in period $t-1$, the share λ_t of rational investors in period t would follow the dynamic:

$$\lambda_t = \frac{\mu G(w^R)}{\mu G(w^R) + (1-\mu)G(w^S(\lambda_{t-1}))}.$$

As can be inferred from the above analysis, $w^S(\cdot)$ is increasing in λ. Thus, a higher share of rational investors in period t would lead more sampling agents to become entrepreneurs in period $t+1$, which would result in a lower share of rational investors in period $t+1$. Depending on the shape of G, such a dynamic system may either converge to a limit share λ^* of rational investors or lead to long term cycling between high and low shares (away and respectively above and below λ^*) of rational investors, corresponding respectively to low and high levels of entrepreneurial activity.[3]

[2] With the notation previously introduced, $w^S(\lambda) = E[\max(H(a, a^S(\lambda)) - c, 0)]$ where the density of a is $f_\lambda(a) = \frac{\sum_{x\in X} f(a|x)[(1-\lambda)(1-F(a^S(\lambda)|x))+\lambda(1-F(a^R|x))]l(x)}{\sum_{x\in X}([1-\lambda)(1-F(a^S(\lambda)|x))+\lambda(1-F(a^R|x))]l(x)}$.

[3] λ^* is a solution to $\lambda^* = \frac{\mu G(w^R)}{\mu G(w^R)+(1-\mu)G(w^S(\lambda^*))}$ and if G has sufficient mass around $w^S(\lambda^*)$ one should expect cycling to emerge.