Competing for Loyalty: The Dynamics of Rallying Support

Matias Iaryczower and Santiago Oliveros

Online Appendix

Appendix A. Proofs

Proof of Proposition 4.1. We begin allowing a MPE in mixed strategies. When the leader meets follower i in state m, she makes an offer $p(m)$ with probability $\gamma_m \in [0,1]$. The follower accepts the offer with probability $\alpha_m \in [0,1]$. Note that the follower i meeting the leader in state m accepts only if $\delta w_{out}(m-1)+p(m) \geq \delta w(m)$, and accepts with probability one if this inequality holds strictly. Note that since i accepts offers $p(m) > -\delta[w_{out}(m-1) - w(m)]$ with probability one, then any such proposal cannot be offered in equilibrium, for L could make a lower offer and still get accepted. Thus, whenever L meets a follower i in state m, she offers

$$p(m) = \begin{cases} -\delta[w_{out}(m-1) - w(m)] & \text{if (8) holds} \\ -\infty & \text{otherwise.} \end{cases} \tag{7}$$

L is willing to make the offer in state m if

$$\alpha_m[\delta v(m-1) - p(m)] + (1-\alpha_m)\delta v(m) \geq \delta v(m),$$

which boils down to

$$p(m) \leq \delta[v(m-1) - v(m)],$$

as before. Thus the leader obtains a non-negative payoff from making an offer if and only if

$$s(m) \equiv [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] \geq 0 \tag{8}$$

Now suppose that in equilibrium (8) holds strictly in state m. Then the follower meeting the leader in state m must accept all such offers; i.e., $\alpha_m = 1$. This is because since the follower accepts any offer higher than $-\delta[w_{out}(m-1) - w(m)]$, if $\alpha_m < 1$ the leader would increase the offer slightly, getting a discrete gain in payoffs. Thus, if in equilibrium the follower rejects the leader's offer with positive probability in state m, (8) must hold with equality in state m; i.e., if $\alpha_m < 1$, then

$$s(m) = [v(m-1) - v(m)] + [w_{out}(m-1) - w(m)] = 0$$

The value of an uncommitted follower in state m is

$$w(m) = \left(\frac{2}{n+2m-1}\right)\delta w(m) + \left(\frac{n+2m-3}{n+2m-1}\right)\delta[\gamma_m\alpha_m w\,(m-1) + (1-\gamma_m\alpha_m)w\,(m)],$$

or equivalently,

$$w(m) = H_m \delta w\,(m-1)\,, \tag{9}$$

where

$$H_m \equiv \left(\frac{(n+2m-3)\gamma_m\alpha_m}{n+2m-1-2\delta-(n+2m-3)\delta(1-\gamma_m\alpha_m)}\right)$$

Thus

$$w(m) = \left[\prod_{k=1}^{m} H_k\right] \delta^m w \tag{10}$$

The value of a committed follower in state m is

$$w_{out}(m) = \gamma_m \alpha_m \delta w_{out}(m-1) + (1-\gamma_m\alpha_m)\delta w_{out}(m)$$

or

$$w_{out}(m) = \left(\frac{\gamma_m \alpha_m \delta}{1-\delta(1-\gamma_m\alpha_m)}\right) w_{out}(m-1)$$

so that

$$w_{out}(m) = \left[\prod_{k=1}^{m}\left(\frac{\gamma_k \alpha_k}{1-\delta(1-\gamma_k\alpha_k)}\right)\right]\delta^m w. \tag{11}$$

The value for the leader in state m is

$$v(m) = \gamma_m\alpha_m(\delta v(m-1) - p(m)) + (1-\gamma_m\alpha_m)\delta v(m)$$

or

$$v(m) = \left(\frac{\gamma_m \alpha_m \delta}{1-\delta(1-\gamma_m\alpha_m)}\right)(v(m-1) + w_{out}(m-1) - w(m)), \tag{12}$$

Now suppose that in equilibrium L makes a relevant offer in every $m > 1$. We will solve for the equilibrium values and then come back and verify that (8) holds for all m to check that this is an equilibrium. First, note that since L makes a relevant offer in every meeting, (10) boils down to

$$w(m) = \left[\prod_{k=1}^{m}\left(\frac{n+2k-3}{n+2k-1-2\delta}\right)\right]\delta^m w = \left[\prod_{k=1}^{m} r(k)\right]\delta^m w \tag{13}$$

and (11) boils down to

$$w_{out}(m) = \delta^m w. \tag{14}$$

Substituting (13) and (14) in (12), we have

$$v(m) = \delta v(m-1) + \left(1-\delta\prod_{k=1}^{m} r(k)\right)\delta^m w$$

Recursively we have that

$$v(m) = \delta^m v + \left[\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l} r(k)\right)\right]\delta^m w \tag{15}$$

Then note that

$$v(m-1) - v(m) = \delta^{m-1}(1-\delta)v + \delta^{m-1}w\left\{(1-\delta)\sum_{l=1}^{m-1}\left(1-\delta\prod_{k=1}^{l} r(k)\right) - \delta\left(1-\delta\prod_{k=1}^{m} r(k)\right)\right\}$$

and

$$w(m) - w_{out}(m-1) = -\left[1 - \delta \prod_{k=1}^{m} r(k)\right] \delta^{m-1} w$$

so substituting, (8) is

$$s^*(m) = (1-\delta)\delta^{m-1}\left[v + w\sum_{l=1}^{m}\left(1 - \delta\prod_{k=1}^{l} r(k)\right)\right] \geq 0$$

which is satisfied if and only if

$$v + w\sum_{l=1}^{m}\left(1 - \delta\prod_{k=1}^{l} r(k)\right) \geq 0$$

Because this always holds for $v > 0$ and $w > 0$, it follows that this is an equilibrium.

Next we show that this is the unique equilibrium with an induction argument. First note from (10) and (12) that for all $m \geq 1$, $v(m)$ and $w(m)$ are maximized when $\gamma_m = \alpha_m = 1$. Then $s^*(1) \geq 0$ implies $s(1) = [v - v(1)] + [w - w(1)] > 0$ whenever $\gamma_1\alpha_1 < 1$. It follows that in state $m = 1$ the leader makes a proposal with probability one; i.e., $\gamma_1 = 1$. But then $\alpha_1 = 1$ as well. For suppose $\alpha_1 \in (0,1)$. Then $s(1) > 0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one. Now suppose that in equilibrium $\gamma_t = \alpha_t = 1$ for all $t < m$. Consider the surplus in state m. Note that $v(m-1)$ and $w_{out}(m-1)$ are exactly as in the equilibrium characterized above. Since $v(m)$ and $w(m)$ are maximized when $\gamma_m = \alpha_m = 1$, then $s^*(m) \geq 0$ implies $s(m) > 0$ whenever $\gamma_m\alpha_m < 1$. Thus $\gamma_m = 1$. As before, then also $\alpha_1 = 1$, for otherwise $s(m) > 0$ and the leader would gain by increasing the offer slightly, getting it accepted with probability one. □

Proof of Proposition 5.1. Part (i) follows from standard arguments (click here for details). Next consider part (ii). Let $\beta(\vec{m})$ denote the probability that any given uncommitted follower meets with one of the leaders. Note if ℓ has to secure the support of m_ℓ more followers there are $(n+1) - m_A - m_B$ committed followers, and $m_A + m_B - 1$ uncommitted followers. Then $\beta(\vec{m}) = 1/(m_A + m_B - 1)$. As in the examples, eq. (1) implies that the expected payoff of a follower after meeting one of the leaders is $\delta W(\vec{m})$ independently of whether he accepts the proposal or not. This is a crucial property, for it allows us to decouple the system of partial difference equations for $W(\vec{m})$ and $V_\ell(\vec{m})$, $\ell = A, B$. Then

$$W(\vec{m}) = \left(\frac{1}{m_A + m_B - 1}\right)\delta W(\vec{m}) + \left(\frac{m_A + m_B - 2}{m_A + m_B - 1}\right)\delta\sum_{\ell}\pi_\ell W(\vec{m}^\ell), \tag{16}$$

so that letting $C(k) \equiv \frac{k-2}{k-(1+\delta)}$, we have

$$W(\vec{m}) = C(m_A + m_B)\delta\sum_{\ell}\pi_\ell W(\vec{m}^\ell). \tag{17}$$

Equation (17) is a partial difference equation with end points $W(m_A, 0) = w_B$ for $m_A > 0$ and $W(0, m_B) = w_A$ for $m_B > 0$, and $W(1,1) = 0$, which we can solve to obtain (3).

Having obtained the expression for $W(\vec{m})$ in terms of the fundamentals, we can write down equilibrium transfers. Note that once a follower is committed, all strategic considerations are brushed aside, as a committed follower just needs to wait for a leader to form a majority. Thus

$$W_{out}(\vec{m}) = \sum_{j=A,B} (\delta\pi_j)^{m_j} \times \left(\sum_{l=0}^{m_{-j}-1} \binom{m_j - 1 + l}{l} \times (\delta\pi_{-j})^l \right) \times w_j \tag{18}$$

From equation (1), expressions (3) and (18) pin down equilibrium transfers $p_\ell(\vec{m})$ in terms of the fundamentals. This in turn allows us to solve for the value of the leaders, which is given, in recursive form, by

$$V_\ell(\vec{m}) = \pi_\ell \left(\delta V_\ell \left(\vec{m}^\ell \right) - p_\ell(\vec{m}) \right) + (1 - \pi_\ell)\delta V_\ell \left(\vec{m}^{-\ell} \right) \tag{19}$$

Once we write transfers in terms of the primitives, (19) becomes a stand alone difference equation, which we can solve. This allows us to prove the next result.

We will show that for any $j = A, B$ there is a $v^* \in \mathbb{R}_+$ such that if $\overline{v}_j \geq v^*$, when all players play the proposed equilibrium strategies, $S_j(\vec{m}) \geq 0$ for all $\vec{m}$.

Consider the surplus expression (2). Note that (3) and (18) imply that $W_{out}(\vec{m}^j) - W(\vec{m})$ does not depend on $(\overline{v}_A, \underline{v}_A, \overline{v}_B, \underline{v}_B)$. It follows that $\overline{v}_{-j}$ and $\underline{v}_{-j}$ do not affect $S_j(\vec{m})$, and $\overline{v}_j$ and $\underline{v}_j$ enter $S_j(\vec{m})$ only through the term $V_j(\vec{m}^j) - V_j(\vec{m})$. Now, note that having expressed $p_j(\vec{m})$ in terms of the primitives of the model, we can solve (19) as a stand alone partial difference equation, to obtain

$$\begin{aligned} V_j(\vec{m}) &= (\delta\pi_j)^{m_j} \left[\sum_{l=0}^{m_{-j}-1} \binom{m_j - 1 + l}{l} (\delta\pi_{-j})^l \right] \overline{v}_j \\ &+ (\delta\pi_{-j})^{m_{-j}} \left[\sum_{l=0}^{m_j-1} \binom{m_{-j} - 1 + l}{l} (\delta\pi_j)^l \right] \underline{v}_j - H(\vec{m}). \end{aligned} \tag{20}$$

where $H(\vec{m})$ is a function of prices $p_j(r, s)$ for $r \leq m_j, s \leq m_{-j}$, which are constant in $(\overline{v}_j, \underline{v}_j, \overline{v}_{-j}, \underline{v}_{-j})$ by (3) and (18). Thus $V_j(\vec{m}^j) - V_j(\vec{m})$ is given by

$$\left\{ (\delta\pi_j)^{m_j-1} \left[\sum_{l=0}^{m_{-j}-1} \binom{m_j - 2 + l}{l} (\delta\pi_{-j})^l \right] - (\delta\pi_j)^{m_j} \left[\sum_{l=0}^{m_{-j}-1} \binom{m_j - 1 + l}{l} (\delta\pi_{-j})^l \right] \right\} \overline{v}_j$$

$$- (\delta\pi_{-j})^{m_{-j}} \binom{m_A + m_B - 2}{m_j - 1} (\delta\pi_j)^{m_j-1} \underline{v}_j + H(\vec{m}) - H(\vec{m}^j).$$

We will show that this expression can be made arbitrarily large by increasing $\overline{v}_j$ or reducing $\underline{v}_j$. From the second line it follows that all else equal, there is a $\underline{v}^*$ such that if $\underline{v}_j < \underline{v}^*$, then

$S_j(\vec{m}) > 0$. Next, after some algebra, the bracket in the first line can be written as

$$(\delta\pi_j)^{m_j-1}\left[(1-\delta)\sum_{l=0}^{m_{-j}-1}\binom{m_j-1+l}{l}(\delta\pi_{-j})^l+\binom{m_A+m_B-2}{m_j-1}(\delta\pi_{-j})^{m_{-j}}\right]>0.$$

Thus, all else equal, there is a $\overline{v}^*$ such that if $\overline{v}_j > \overline{v}^*$, then $S_j(\vec{m}) > 0$. □

Proof of Theorem 5.3. Let $\gamma_j(\vec{m})$ be the probability that leader $j = A, B$ makes an offer in state $\vec{m}$, $\alpha_j(\vec{m})$ be the probability that an uncommitted follower accepts an offer from leader $j = A, B$ in state $\vec{m}$, and $\mu_j(\vec{m}) \equiv \gamma_j(\vec{m})\,\alpha_j(\vec{m})$. Then

$$W(\vec{m}) = \left(\frac{1}{m_A+m_B-1}\right)\delta W(\vec{m}) + \left(\frac{m_A+m_B-2}{m_A+m_B-1}\right)\sum_{j=A,B}\pi_j\begin{pmatrix}\mu_j(\vec{m})\,\delta W(\vec{m}^j)\\ +(1-\mu_j)\,\delta W(\vec{m})\end{pmatrix}. \tag{21}$$

For $j = A, B$, define

$$\xi_j(\vec{m}) \equiv \frac{\delta\pi_j\mu_j(\vec{m})}{\left(\frac{m_A+m_B-1}{m_A+m_B-2}\right)(1-\delta)+\delta\sum_{j=A,B}\pi_j\mu_j(\vec{m})}$$

whenever $\vec{m} \neq (1,1)$, and $\xi_j(1,1) \equiv 0$. Then we can write (21) as

$$W(\vec{m}) = \sum_{j=A,B}\xi_j(\vec{m})\,W(\vec{m}^j) \tag{22}$$

for all $\vec{m}$ and $j = A, B$. Note in particular that the recursion (22) implies that if $w_A, w_B > 0$ (as we are assuming here), then $W(\vec{m}) \geq 0$ for all $\vec{m}$.

We need to show that $W(q,q) < \max\{w_A(q), w_B(q)\}$. The proof follows from three lemmas. Lemma A.2 establishes the result for $q = 1$ and shows an additional result for all boundary states which is used in Lemma A.3. The proof for interior states is by induction. Lemma A.3 establishes the base case, and Lemma A.4 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma A.1, which establishes an intermediate result that is used in the proof of Lemmas A.2 and A.4. □

Lemma A.1 (Bound)**.** *In any MPE of the game $\Gamma(\vec{m})$,*

$$W(\vec{m}) \leq \max_{j\in\{A,B\}}\left\{\delta r(m_j)\,W(\vec{m}^j)\right\}$$

Proof of Lemma A.1. Note that for all $m_A \geq 2, m_B \geq 2$ we have

$$W(\vec{m}) = \xi_A(\vec{m})\,W(\vec{m}^A) + \xi_B(\vec{m})\,W(\vec{m}^B)$$

Thus we need to show that

$$\sum_{j=A,B}\xi_j(\vec{m})\,W(\vec{m}^j) \leq \delta\max\left\{r(m_A)\,W(\vec{m}^A), r(m_B)\,W(\vec{m}^B)\right\}$$

Without loss of generality assume that $W\left(\vec{m}^A\right) \geq W\left(\vec{m}^B\right)$ so it is sufficient if

$$\left[\sum_{j=A,B} \xi_j\left(\vec{m}\right)\right] W\left(\vec{m}^A\right) \leq \delta \max\left\{r\left(m_A\right) W\left(\vec{m}^A\right), r\left(m_B\right) W\left(\vec{m}^B\right)\right\}$$

Note that since $r\left(m\right) = \frac{n+2m-3}{n+2m-(1+2\delta)}$, then

$$\left[\sum_{j=A,B} \xi_j\left(\vec{m}\right)\right] = \frac{\delta\left[\pi_A\mu_A\left(\vec{m}\right) + \pi_B\mu_B\left(\vec{m}\right)\right]}{\frac{m_A+m_B-1}{m_A+m_B-2}\left(1-\delta\right) + \delta\left[\pi_A\mu_A\left(\vec{m}\right) + \pi_B\mu_B\left(\vec{m}\right)\right]} \leq \delta \min\left\{r\left(m_A\right), r\left(m_B\right)\right\}. \tag{23}$$

Then it is sufficient if

$$\min\left\{r\left(m_A\right), r\left(m_B\right)\right\} W\left(\vec{m}^A\right) \leq \max\left\{r\left(m_A\right) W\left(\vec{m}^A\right), r\left(m_B\right) W\left(\vec{m}^B\right)\right\}$$

which is true when either $r\left(m_A\right) W\left(\vec{m}^A\right) \geq r\left(m_B\right) W\left(\vec{m}^B\right)$ or the opposite holds. □

Lemma A.2 (Boundaries)**.**

$$W\left(m_A, 1\right) < w_B\left(1\right) \textit{ for all } m_A \geq 1 \quad \textit{and} \quad W\left(1, m_B\right) < w_A\left(1\right) \textit{ for all } m_B \geq 1$$

Proof of Lemma A.2. The result for state $\vec{m} = (1,1)$ follows immediately from the fact that $W(1,1) = 0$. Now consider the remaining boundary states (states adjacent to terminal states). Solving the recursion (22) for the boundaries, we obtain

$$W\left(m_A, 1\right) = \left(\sum_{l=1}^{m_A-1} \xi_B\left(m_A - l, 1\right) \prod_{k=0}^{l-1} \xi_A\left(m_A - k, 1\right) + \xi_B\left(m_A, 1\right)\right) w_B \tag{24}$$

$$W\left(1, m_B\right) = \left(\sum_{l=1}^{m_B-2} \xi_A\left(1, m_B - l\right) \prod_{k=0}^{l-1} \xi_B\left(1, m_B - k\right) + \xi_A\left(1, m_B\right)\right) w_A \tag{25}$$

for all $m_A, m_B \geq 1$.

Consider $\vec{m} = (2,1)$. Note that since

$$W\left(2,1\right) = \frac{\delta\pi_B\mu_B\left(2,1\right)}{2\left(1-\delta\right) + \delta\left(\pi_A\mu_A\left(2,1\right) + \pi_B\mu_B\left(2,1\right)\right)} w_B$$

for $n \geq 3$,

$$W\left(2,1\right) < \frac{\delta}{2-\delta} w_B < \frac{(n-1)\delta}{n+1-2\delta} w_B = r(1)\delta w_B = w_B(1)$$

By the same argument, $W\left(1,2\right) < w_A\left(1\right)$. Next, consider $W\left(m_A, 1\right)$ for $m_A \geq 3$. We have

$$\begin{aligned} W\left(m_A, 1\right) &= \left(\sum_{l=0}^{m_A-3} \xi_B\left(m_A - (1+l), 1\right) \prod_{k=0}^{l} \xi_A\left(m_A - k, 1\right) + \xi_B\left(m_A, 1\right)\right) w_B \\ &= \left(\begin{array}{c} \sum_{l=0}^{m_A-3}\left[\xi_B\left(m_A - (1+l), 1\right) + \xi_A\left(m_A - (1+l), 1\right) - 1\right] \prod_{k=0}^{l} \xi_A\left(m_A - k, 1\right) \\ - \prod_{k=0}^{m_A-2} \xi_A\left(m_A - k, 1\right) + \left(\xi_B\left(m_A, 1\right) + \xi_A\left(m_A, 1\right)\right) \end{array}\right) w_B, \end{aligned}$$

and since $\left(\xi_B\left(m_A-(1+l),1\right)+\xi_A\left(m_A-(1+l),1\right)\right)\leq 1$, it follows that
$$W\left(m_A,1\right)\leq\left(\xi_B\left(m_A,1\right)+\xi_A\left(m_A,1\right)\right)w_B<\delta r\left(1\right)w_B=w_B\left(1\right)$$
Analogously, we have that $W\left(1,m_B\right)<w_A\left(1\right)$. □

Lemma A.3 (Base Case)**.**
$$W\left(m_A,2\right)<\max\left\{w_A\left(m_A\right),w_B\left(2\right)\right\}\quad \text{for all } m_A\geq 2$$
and
$$W\left(2,m_B\right)<\max\left\{w_A\left(2\right),w_B\left(m_B\right)\right\}\quad \text{for all } m_B\geq 2$$

Proof of Lemma A.3. First, note that
$$\begin{aligned}
W\left(2,2\right) &\leq \xi\left(4\right)\max\left\{w_B\left(1\right),w_A\left(1\right)\right\}\\
&< \delta\left(\frac{2}{3-\delta}\right)\max\left\{w_B\left(1\right),w_A\left(1\right)\right\}\\
&< \delta r(2)\max\left\{w_B\left(1\right),w_A\left(1\right)\right\}=\max\left\{w_B\left(2\right),w_A\left(2\right)\right\}
\end{aligned}$$

Next consider $W\left(m_A,2\right)$. By successive application of Lemma A.1

$$\begin{aligned}
W\left(m_A,2\right) &\leq \max\left\{\delta r\left(m_A\right)W\left(m_A-1,2\right),\delta W\left(m_A,1\right)r\left(2\right)\right\}\\
&\leq \max\left\{\delta^2\prod_{j=0}^{1}r\left(m_A-j\right)W\left(m_A-2,2\right),\delta^2\prod_{j=0}^{0}r\left(m_A-j\right)W\left(m_A-1,1\right)r\left(2\right),\delta W\left(m_A,1\right)r\left(2\right)\right\}\\
&\ldots.\\
&\leq \max\left\{\delta^{m_A}\prod_{j=0}^{m_A-2}r\left(m_A-j\right)W\left(1,2\right),\max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1}r\left(m_A-j\right)W\left(m_A-k,1\right)r\left(2\right)\right\},\delta W\left(m_A,1\right)r\left(2\right)\right\}
\end{aligned}$$

Now, we have shown in Lemma A.2 that $W\left(m_A,1\right)<w_B\left(1\right)$ for all $m_A\geq 1$, and $W\left(1,m_B\right)<w_A\left(1\right)$ for all $m_B\geq 1$. Using these results in the RHS of the expression above, we get
$$W\left(m_A,2\right)<\max\left\{\delta^{m_A}\prod_{j=0}^{m_A-2}r\left(m_A-j\right)w_A(1),\ \max_{k\leq m_A-2}\left\{\ \delta^{k+1}\prod_{j=0}^{k-1}r\left(m_A-j\right)w_B(1)\ \right\}r\left(2\right)\ ,\delta r\left(2\right)w_B(1)\right\}$$

Using (13) we get that
$$\max_{k\leq m_A-2}\left\{\delta^{k+1}\prod_{j=0}^{k-1}r\left(m_A-j\right)w_B(1)\right\}=\delta^2 r\left(m_A\right)w_B(1),$$
so
$$W\left(m_A,2\right)<\max\left\{\delta^{m_A}\prod_{j=0}^{m_A-2}r\left(m_A-j\right)w_A(1),\delta r\left(2\right)w_B(1)\right\}.$$
Therefore, using equation (13) and Lemma A.2 one more time, we have
$$W\left(m_A,2\right)<\max\left\{w_A(m_A),w_B(2)\right\}.$$
By the same logic, $W(2,m_B)<\max\left\{w_B(m_B),w_A(2)\right\}$. □

Lemma A.4 (Induction Step)**.** *Consider any state $\vec{m} \geq (3,3)$. If*

$$W\left(\vec{m}^B\right) \leq \max\left\{w_A\left(m_A\right), w_B\left(m_B - 1\right)\right\} \tag{26}$$

and

$$W\left(\vec{m}^A\right) \leq \max\left\{w_A\left(m_A - 1\right), w_B\left(m_B\right)\right\} \tag{27}$$

then

$$W\left(\vec{m}\right) \leq \max\left\{w_A\left(m_A\right), w_B\left(m_B\right)\right\} \tag{28}$$

Proof of Lemma A.4. By Lemma A.1,

$$W\left(\vec{m}\right) \leq \max\left\{\delta r\left(m_A\right) W\left(\vec{m}^A\right), \delta r\left(m_B\right) W\left(\vec{m}^B\right)\right\} \tag{29}$$

Using (26) and (27), and then noting that $w_j(m_j) = \delta r\left(m_j\right) w_j\left(m_j - 1\right)$ for $j = A, B$, and substituting, (29) becomes

$$W\left(\vec{m}\right) \leq \max\left\{\begin{array}{c} \max\left\{w_A\left(m_A\right), \delta r\left(m_A\right) w_B\left(m_B\right)\right\}, \\ \\ \max\left\{\delta r\left(m_B\right) w_A\left(m_A\right), w_B\left(m_B\right)\right\} \end{array}\right\} \leq \max\left\{w_A\left(m_A\right), w_B\left(m_B\right)\right\}$$

□

Proof of Theorem 5.5. The first statement follows as a corollary of Theorem 5.3. Now consider the second part. From expression (3), we have

$$W(q,q) = \sum_{l=0}^{q-2}\left(\prod_{k=0}^{q-1+l} C(2q-k)\right) \times \binom{q-1+l}{l} \times \left[(\delta\pi_A)^q(\delta\pi_B)^l w_A + (\delta\pi_B)^q(\delta\pi_A)^l w_B\right] \tag{30}$$

On the other hand, with a single alternative, $w(q) = \left(\prod_{m=1}^{q} r(m)\right)\delta^q w$. Now, since $r(k) = \frac{n+2k-3}{n+2k-(1+2\delta)}$ by definition and $n = 2q-1$, we have $r(k) = C(q+k)$. Thus

$$w(q) = \left(\prod_{k=1}^{q} C(q+k)\right)\delta^q w = \left(\prod_{k=0}^{q-1} C(2q-k)\right)\delta^q w \tag{31}$$

Suppose without loss of generality that $w_A > w_B$. We want to show that for sufficiently large q the equilibrium payoff of an uncommitted follower in the game with a single alternative yielding value w_B is larger than his (competitive) equilibrium payoff in the game with two alternatives yielding value w_A and w_B. Suppose not. Then making $w = w_B$ in (31), and dividing (30) by (31),

$$U(q) \equiv \sum_{l=0}^{q-2}\left(\prod_{k=q}^{q-1+l} C(2q-k)\right) \times \binom{q-1+l}{l} \times \left[(\pi_A)^q(\delta\pi_B)^l + (\pi_B)^q(\delta\pi_A)^l\left(\frac{w_B}{w_A}\right)\right] \geq \frac{w_B}{w_A}$$

Now, since $\delta \leq 1$, $\prod_{k=q}^{q-1+l} C\left(2q-k\right) \leq 1$, and $w_B/w_A < 1$, for any integer q, we have

$$U(q) < \sum_{j=A,B}(\pi_j)^q \sum_{l=0}^{q-2} \frac{\Gamma\left(q+l\right)}{\Gamma\left(q\right)} \frac{\left(\delta\pi_j\right)^l}{l!} \equiv \overline{U}(q)$$

where for any integer k, we define $\Gamma(k) \equiv (k-1)!$. Now define the function

$$F(a,b,c,z) \equiv \sum_{l=0}^{\infty} \frac{\binom{a+l}{l}\binom{b+l}{l}}{\binom{c+l}{l}} z^l,$$

and note that we can write

$$\overline{U}(q) = \sum_{j=A,B} (\pi_j)^q \left(F(q,1,1,\delta\pi_{-j}) - \sum_{l=q-1}^{\infty} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta\pi_{-j})^l}{l!} \right) \tag{32}$$

$$= \sum_{j=A,B} \left\{ \left(\frac{\pi_j}{1-\delta\pi_{-j}} \right)^q - (\pi_j)^q \sum_{l=q-1}^{\infty} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta\pi_{-j})^l}{l!} \right\}.$$

where the equality follows from the fact that $F(a,b,b,z) = (1-z)^{-a}$ (see Property 15.1.8 for hypergeometric functions in Abramowitz and Stegun (2012; p.556)). Noting that

$$\left(\frac{\pi_j}{1-\delta\pi_{-j}} \right) = \left(\frac{1-\pi_{-j}}{1-\delta\pi_{-j}} \right) < 1$$

as long as $\delta < 1$, it follows that for any $\varepsilon > 0$ there is a Q such that if $q > Q$, then $\overline{U}(q) < \varepsilon$. Thus, for any π_B/π_A, there is a Q such that $U(q) < \pi_B/\pi_A$ whenever $q > Q$. □

Proof of Theorem 5.6. Let $\mathcal{L}$ be the set of leaders with $|\mathcal{L}| \geq 2$. Let $\gamma_j(\vec{m})$ be the probability that leader $j \in \mathcal{L}$ makes an offer in state $\vec{m}$, $\alpha_j(\vec{m})$ be the probability that an uncommitted follower accepts an offer from leader $j \in \mathcal{L}$ in state $\vec{m}$, and $\mu_j(\vec{m}) \equiv \gamma_j(\vec{m})\alpha_j(\vec{m})$. Then

$$\begin{aligned} W(\vec{m}) &= \left(\frac{1}{n - \sum_{j\in\mathcal{L}} \left(\frac{n+1}{2} - m_j\right)} \right) \delta W(\vec{m}) \\ &+ \left(1 - \frac{1}{n - \sum_{j\in\mathcal{L}} \left(\frac{n+1}{2} - m_j\right)} \right) \sum_{j\in\mathcal{L}} \pi_j \begin{pmatrix} \mu_j(\vec{m})\,\delta W\left(\vec{m}^j\right) \\ +(1-\mu_j)\,\delta W(\vec{m}) \end{pmatrix} \end{aligned} \tag{33}$$

Let $M^+ \equiv \{\vec{m} : n - \sum_{j\in\mathcal{L}} \left(\frac{n+1}{2} - m_j\right) > \min_{j\in\mathcal{L}} m_j\}$ denote the set of states in which not all uncommitted followers are critical. For $j \in \mathcal{L}$ and all $\vec{m} \in M^+$ define

$$\xi_j(\vec{m}) \equiv \frac{\delta\pi_j\mu_j(\vec{m})}{\frac{n - \sum_{j\in\mathcal{L}}\left(\frac{n+1}{2} - m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2} - m_j\right)}(1-\delta) + \delta\sum_{j\in\mathcal{L}} \pi_j\mu_j(\vec{m})}$$

For all $j \in \mathcal{L}$ and $\vec{m} \in M^+$ we can write (33) as

$$W(\vec{m}) = \sum_{j\in\mathcal{L}} \xi_j(\vec{m})\, W\left(\vec{m}^j\right) \tag{34}$$

Note in particular that the recursion (34) implies that if $w_j > 0$ for all $j \in \mathcal{L}$ (as we are assuming here), then $W(\vec{m}) \geq 0$ for all $\vec{m} \in M^+$.

We need to show that $W(\vec{q}) < \max_{j\in\mathcal{L}}\{w_j(q)\}$. The proof follows from three lemmas. Lemma A.6 establishes the result for critical states $\vec{m}$, in which at least one leader is exactly one step away from winning, and shows an additional result for all boundary states which is used in Lemma A.7. The proof for interior states is by induction. Lemma A.7 establishes the base case, and Lemma A.8 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma A.5, which establishes an intermediate result that is used in the proof of Lemmas A.6 and A.8. □

Lemma A.5 (Bound)**.** *In any MPE of the game* $\Gamma(\vec{m})$*,*

$$W(\vec{m}) \le \max_{j\in\mathcal{L}}\left\{\delta r(m_j) W\left(\vec{m}^j\right)\right\} \tag{35}$$

Proof of Lemma A.5. Note that

$$\sum_{j\in\mathcal{L}}\xi_j(\vec{m}) \le \frac{\delta}{\frac{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}(1-\delta)+\delta}$$

and since

$$\frac{\delta}{\frac{n-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}{n-1-\sum_{j\in\mathcal{L}}\left(\frac{n+1}{2}-m_j\right)}(1-\delta)+\delta} \le \delta\frac{n+2m_k-3}{n+2m_k-(1+2\delta)} = \delta r(m_k)$$

we have that for all $k\in\mathcal{L}$

$$\sum_{j\in\mathcal{L}}\xi_j(\vec{m}) \le \delta r(m_k) \tag{36}$$

Therefore for all $k\in\mathcal{L}$ we have

$$\begin{aligned}\sum_{j\in\mathcal{L}}\xi_j(\vec{m})\max_{j\in\mathcal{L}}\left\{W\left(\vec{m}^j\right)\right\} &\le \delta r(m_k)\max_{j\in\mathcal{L}}\left\{W\left(\vec{m}^j\right)\right\}\\ \sum_{j\in\mathcal{L}}\xi_j(\vec{m})W\left(\vec{m}^j\right) &\le \delta\max_{j\in\mathcal{L}}\left\{r(m_j)W\left(\vec{m}^j\right)\right\}\end{aligned}$$

□

Lemma A.6 (Boundaries)**.** *Let* $M^*(j) = \{\vec{m} : m_j = 1 \le m_k \text{ for } k\neq j \text{ and } k\in\mathcal{L}\}$ *denote the set of (critical) states in which leader* $j\in\mathcal{L}$ *is one step away from winning. Let* $\vec{m}^*(j)\in M^*(j)$ *be a generic element of this set and define* $|\vec{m}^*(j)|_h$ *as the number of followers that leader* $h = A, B, \ldots$ *needs to win. Then for all* $\vec{m}^*(j)\in M^*(j)$ *we have*

$$W(\vec{m}^*(j)) \le \max_{h\in\mathcal{L}}\{w_h(|\vec{m}^*(j)|_h)\} \tag{37}$$

and the inequality is strict if $W(\vec{m}^*(j)) > 0$ *is positive.*

Proof of Lemma A.6. Without loss of generality, let's focus on $\vec{m}^*(A)\in M^*(A)$. Note that if there is another leader k such that $|\vec{m}^*(j)|_k = 1$, we have two leaders that need only one follower to win. Therefore, we must have that there is only one remaining uncommitted follower and $W(\vec{m}^*(A)) = 0$, which trivially verifies (37).

Let's focus then on the set of states $\widehat{M}^*(A) = \{\vec{m}^*(A)\in M^*(A) : m_k \ge 2 \text{ for } k\in\{\mathcal{L}\setminus A\}\}$ in which A is the only leader that needs one supporter while the rest needs at least 2. Note that for any $\vec{m}^*(A) \in \widehat{M}^*(A)$, (34) is equivalent to

$$W(\vec{m}^*(A)) = \xi_A(\vec{m}^*(A))\, w_A + \sum_{j\in\{\mathcal{L}|\mathcal{A}\}} \xi_j(\vec{m}^*(A))\, W\left(\vec{m}^*(A)^j\right)$$

If $w_A \geq W\left(\vec{m}^*(A)^k\right)$ for all $j \in \{\mathcal{L}\setminus A\}$ we are done since

$$W(\vec{m}^*(A)) \leq \sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\, w_A < \delta r(1)\, w_A = w_A(1) \tag{38}$$

Then assume that $w_A < W\left(\vec{m}^*(A)^k\right)$ for some $k \in \{\mathcal{L}\setminus A\}$ so we have

$$W(\vec{m}^*(A)) \leq \sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\, W\left(\vec{m}^*(A)^k\right)$$

Note that there is no h such that $|\vec{m}^*(A)^h|_h = 1$ since this implies that $|\vec{m}^*(A)|_h = 2$, and therefore any trade leads to only one remaining uncommitted follower. This implies $W(\vec{m}^*(A)^k) = 0$, which contradicts the assumption that $w_A < W(\vec{m}^*(A)^k)$. Therefore, we must have that $\vec{m}^*(A)^k \in \widehat{M}^*(A)$.

Let's focus then on the state $\vec{m}^*(A)^k$. Using (34) we have

$$W\left(\vec{m}^*(A)^k\right) = \xi_A\left(\vec{m}^*(A)^k\right) w_A + \sum_{j\in\{\mathcal{L}\setminus\{A,k\}\}} \xi_j\left(\vec{m}^*(A)^k\right) W\left([\vec{m}^*(A)^k]^j\right)$$

Again we have that if $w_A \geq W\left([\vec{m}^*(A)^k]^j\right)$ we are done, since

$$W\left(\vec{m}^*(A)^k\right) \leq \sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right) w_A < \delta r(1)\, w_A = w_A(1)$$

so we assume again that $w_A < W\left([\vec{m}(A)^k]^{k'}\right)$ for some $k' \in \{\mathcal{L}\setminus A\}$, which implies that

$$W\left(\vec{m}^*(A)^k\right) \leq \sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right) W\left([\vec{m}^*(A)^k]^{k'}\right)$$

and therefore

$$W(\vec{m}^*(A)) \leq \left\{\sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times W\left([\vec{m}^*(A)^k]^{k'}\right) \tag{39}$$

Note that we can proceed in the same fashion until we reach a state in which another leader needs only one follower or there are only two remaining followers. In the first case we have that the value function is 0 because there is only one remaining uncommitted follower, so we focus in the second case. Call this state $\vec{m}_1^*(A)$ (there are many paths in which this state could be reached but only one path that maximizes it step by step) and note that

$$W(\vec{m}^*(A)) \leq \left\{\sum_{j\in\mathcal{L}} \xi_j(\vec{m}^*(A))\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left([\vec{m}^*(A)^k]^{k'}\right)\right\} \times \ldots \times W(\vec{m}_1(A))$$

Note also that the fact that there are only two remaining followers implies that $W\left(\vec{m}_1(A)^j\right)=0$ so (34)

$$\begin{aligned} W\left(\vec{m}_1^*(A)\right) &= \xi_A\left(\vec{m}_1^*(A)\right) w_A + \sum_{j\in\{\mathcal{L}|\mathcal{A}\}} \xi_j\left(\vec{m}_1^*(A)\right) W\left(\vec{m}_1^*(A)^j\right) \\ &= \xi_A\left(\vec{m}_1^*(A)\right) w_A \end{aligned}$$

and

$$W\left(\vec{m}_1^*(A)\right) = \xi_A\left(\vec{m}_1^*(A)\right) w_A < \delta r(1) w_A = w_A(1)$$

Therefore,

$$\begin{aligned} W\left(\vec{m}^*(A)\right) &< \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)\right)\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left(\vec{m}^*(A)^k\right)\right\} \times \left\{\sum_{j\in\mathcal{L}} \xi_j\left([\vec{m}^*(A)^k]^{k'}\right)\right\} \times \ldots \times w_A(1) \\ &< w_A(1) \end{aligned}$$

Recall that $\left|\vec{m}^*(A)\right|_A = 1$ and it is trivial to see that

$$w_A\left(\left|\vec{m}^*(A)\right|_A\right) \le \max_{j\in\mathcal{L}} \left\{ w_j\left(\left|\vec{m}^*(A)\right|_j\right)\right\}$$

which gives (37). □

Lemma A.7 (Base Case)**.** *Let*

$$M^{**}(j) = \{\vec{m} : m_j = 2 \le m_k \text{ for } k \in \{\mathcal{L}\,|j\}\}$$

be the set of states in which all leaders need at least 2 supporters to win and leader $j \in \mathcal{L}$ needs exactly 2 supporters, then

$$W\left(\vec{m}^{**}(j)\right) < \max_{h\in\mathcal{L}} \left\{ w_h\left(\left|\vec{m}^{**}(j)\right|_h\right)\right\} \tag{40}$$

Proof of Lemma A.7. Without loss of generaility assume that $j = A$ and let $L\left(\vec{m}^{**}(A)\right) = \{h \in \mathcal{L} : \left|\vec{m}^{**}(A)\right|_h = 2\}$ be the set of leaders such that if trading they are one supporter away from winning; note that $A \in L\left(\vec{m}^{**}(A)\right)$. Note that

$$W\left(\vec{m}^{**}(A)\right) = \sum_{k\in L(\vec{m}^{**}(A))} \xi_k\left(\vec{m}^{**}(A)\right) W\left(\vec{m}^{**}(A)^k\right) + \sum_{k\notin L(\vec{m}^{**}(A))} \xi_k\left(\vec{m}^{**}(A)\right) W\left(\vec{m}^{**}(A)^k\right)$$

If the state $\vec{m}^{**}(A)$ involves only one remaining uncommitted follower we have that $W\left(\vec{m}^{**}(A)\right) = 0$, so we are done. Assume then that there are more than one uncommitted follower. If there is some $h \in L\left(\vec{m}^{**}(j)\right)$ such that $W\left(\vec{m}^{**}(A)^h\right) \ge \max_{k\in\{\mathcal{L}|h\}} \left\{W\left(\vec{m}^{**}(A)^k\right)\right\}$, we have that

$$W\left(\vec{m}^{**}(A)\right) \le \sum_{k\in\mathcal{L}} \xi_k\left(\vec{m}^{**}(A)\right) W\left(\vec{m}^{**}(A)^h\right)$$

Using that $\sum_{k\in\mathcal{L}} \xi_k\left(\vec{m}^{**}(A)\right) < \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right)$ for all $k \in \mathcal{L}$ we have that

$$W\left(\vec{m}^{**}(A)\right) < \delta r\left(\left|\vec{m}^{**}(A)\right|_k\right) W\left(\vec{m}^{**}(A)^h\right)$$

for all $k \in \mathcal{L}$, and since $\left|\vec{m}^{**}(A)^h\right|_h = 1$ (since $h \in L(\vec{m}^{**}(A))$) and therefore $\vec{m}^{**}(A)^h \in M^*(h)$ we can apply the previous Lemma to get

$$W(\vec{m}^{**}(A)) < \delta r(|\vec{m}^{**}(A)|_k) \max_{l \in \mathcal{L}} \left\{ w_l \left(\left|\vec{m}^{**}(A)^h\right|_l \right) \right\}$$

for all $k \in \mathcal{L}$. Using now that $\left|\vec{m}^{**}(A)^h\right|_l = |\vec{m}^{**}(A)|_l$ for all $l \in \{\mathcal{L}\,|h\}$ and $\left|\vec{m}^{**}(A)^h\right|_h = |\vec{m}^{**}(A)|_h - 1$ we have

$$\begin{aligned}
W(\vec{m}^{**}(A)) &< \delta r(|\vec{m}^{**}(A)|_k) \max \left\{ \max_{l \in \{\mathcal{L} \setminus h\}} \{w_l(|\vec{m}^{**}(A)|_l)\}, w_h(|(\vec{m}^{**}(A)-1)|_h) \right\} \\
&\leq \max \left\{ \delta r(|\vec{m}^{**}(A)|_k) \max_{l \in \{\mathcal{L} \setminus h\}} \{w_l(|\vec{m}^{**}(A)|_l)\}, \delta r(|\vec{m}^{**}(A)|_k) w_h(|(\vec{m}^{**}(A)-1)|_h) \right\} \\
&\leq \max \left\{ \max_{l \in \{\mathcal{L} \setminus h\}} \{w_l(|\vec{m}^{**}(A)|_l)\}, w_h(|(\vec{m}^{**}(A))|_h) \right\} = \left\{ \max_{l \in \mathcal{L}} \{w_l(|\vec{m}^{**}(A)|_l)\} \right\}
\end{aligned}$$

If there is some $h \notin L(\vec{m}^{**}(j))$ such that $W\left(\vec{m}^{**}(A)^h\right) \geq \max_{k \in \{\mathcal{L} \setminus h\}} \left\{ W\left(\vec{m}^{**}(A)^k\right) \right\}$ we have that

$$W(\vec{m}^{**}(A)) \leq \delta r(|\vec{m}^{**}(A)|_k) W\left(\vec{m}^{**}(A)^h\right)$$

and since $\vec{m}^{**}(A)^h \in M^{**}(j)$ we have two options. Either there is only one remaining follower in which case we are trivially done or there is more than one follower, in which case we can repeat again the recursion until we reach (40). □

Lemma A.8 (Induction Step)**.** *Consider any state $\vec{m}$ such that $|\vec{m}|_j \geq 2$ for all $j \in \mathcal{L}$. If*

$$W\left(\vec{m}^j\right) \leq \max \left\{ \max_{h \in \{\mathcal{L}|j\}} \{w_h(|\vec{m}|_h)\}, w_j\left(|\vec{m}|_j - 1\right) \right\} \tag{41}$$

for all $j \in \mathcal{L}$, then

$$W(\vec{m}) \leq \max_{j \in \mathcal{L}} \left\{ w_j\left(|\vec{m}|_j\right) \right\} \tag{42}$$

Proof of Lemma A.8. By Lemma A.5,

$$W(\vec{m}) \leq \max_{j \in \mathcal{L}} \left\{ \delta r\left(|\vec{m}|_j\right) W\left(\vec{m}^j\right) \right\} \tag{43}$$

Using (41), and then noting that $w_j(|\vec{m}|_j) = \delta r\left(|\vec{m}|_j\right) w_j\left(|\vec{m}|_j - 1\right)$ for $j \in \mathcal{L}$, and substituting, (43) becomes

$$W(\vec{m}) \leq \max \left\{ \max_{h \in \{\mathcal{L} \setminus j\}} \left\{ \delta r\left(|\vec{m}|_j\right) w_h(|\vec{m}|_h) \right\}, w_j\left(|\vec{m}|_j\right) \right\}$$

which implies the result. □

Proof of Proposition 5.7. Let

$$\Pi^*(m) \equiv v + \left[\sum_{l=1}^{m} \left(1 - \delta \prod_{k=1}^{l} r(k) \right) \right] w \quad \text{for } m = 1, \ldots, q \tag{44}$$

In the proof of Proposition 4.1 we showed that the condition (8) for non-negative surplus in state m in a FTE is $s(m) = \delta^{m-1}(1-\delta)\Pi^*(m) \geq 0 \Leftrightarrow \Pi^*(m) \geq 0$. Note that when $w < 0$, for any $m, m' \leq q$ s.t. $m < m'$, $\Pi^*(m') \geq 0$ implies

$$v \geq \left[\sum_{l=1}^{m'}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right](-w) > \left[\sum_{l=1}^{m}\left(1-\delta\prod_{k=1}^{l}r(k)\right)\right](-w),$$

so that $\Pi^*(m) > \Pi^*(m') \geq 0$. Thus, in particular, $\Pi^*(q) \geq 0 \Rightarrow \Pi^*(m) > 0$ for all $m < q$. It follows that $\Pi^*(q) \geq 0$ is a necessary and sufficient condition for existence of a FTE.

To consider all equilibria, we split the analysis in two cases: (1) $\Pi^*(q) \geq 0$, and (2) $\Pi^*(q) < 0$.

Case 1: $\Pi^*(q) \geq 0$. We focus on two subcases: $\Pi^*(q-1) + w > 0$ and $\Pi^*(q-1) + w \leq 0$. In the first subcase, if the proposed equilibrium were to involve no trade with positive probability at some state m, then L would have a profitable deviation increasing the probability of trade. Hence, the only equilibrium is a FTE. In the second case, if there is no trade with positive probability at some state m, deviations will not always be profitable. In this case we are going to show that if there is mixing at some state, there is some other state with no trade with probability 1.

Case 1.1: $\Pi^*(q-1) + w > 0$. To show uniqueness we need to rule out equilibria in which there is no trade with positive probability in some state. First note that since $\Pi^*(1) > 0$, in every MPE we must have trade w.p. one at $m = 1$, for otherwise there is a sufficiently small $\epsilon > 0$ such that the leader could gain by raising her offer by $\epsilon > 0$, and having it be accepted for sure.

Now suppose that there is a MPE in which, for some state $m^* \in \{2, \ldots, q-1\}$, we have (i) $\gamma_m\alpha_m = 1$ for all $m < m^*$, and (ii) $0 < \gamma_{m^*}\alpha_{m^*} < 1$ (we study the case $m^* = q$ separately).

Because of mixing at m^*, we must have $s(m^*) = 0$, and thus

$$v(m^*) = v(m^*-1) + w_{out}(m^*-1) - w(m^*). \tag{45}$$

At the same time, the value for the leader is $v(m^*) = \gamma_{m^*}\alpha_{m^*}[\delta v(m^*-1) - p(m^*)] + (1 - \gamma_{m^*}\alpha_{m^*})\delta v(m^*)$. Solving for $v(m^*)$ and substituting $p(m^*) = \delta\left(w(m^*) - w^{out}(m^*-1)\right)$, we have

$$v(m^*) = \frac{\gamma_{m^*}\alpha_{m^*}\delta}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\left(v(m^*-1) + w^{out}(m^*-1) - w(m^*)\right) \tag{46}$$

From (45) and (46), it follows that $v(m^*) = 0$.

For L to make an offer in state m^*+1, we need $\delta v(m^*) - p(m^*+1) \geq \delta v(m^*+1)$. Given $v(m^*) = 0$, this becomes $w_{out}(m^*) \geq w(m^*+1)$, and by assumption on the equilibrium (trading with certainty in all $m < m^*$ and mixing at m^*), this condition is equivalent to

$$\frac{\delta\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}w_{out}(m^*-1) \geq H_{m^*+1}H_{m^*}\delta^2 w(m^*-1)$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta^{m^*-1}w \geq H_{m^*+1}H_{m^*}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)\delta^{m^*-1}w$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} \leq H_{m^*+1}H_{m^*}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)$$

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} \leq H_{m^*+1}\frac{r(m^*)\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left(\prod_{k=1}^{m^*-1}r(k)\right)$$

If $H_{m^*+1}=0$ then $\gamma_{m^*}\alpha_{m^*}=0$ and if $H_{m^*+1}\neq 0$ then we must have that

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} \leq \frac{r(m^*)\gamma_{m^*}\alpha_{m^*}}{1-r(m^*)\delta(1-\gamma_{m^*}\alpha_{m^*})}$$

but since $r(m^*) < 1$ this is only possible if $\gamma_{m^*}\alpha_{m^*}=0$, a contradiction with our initial assumption that $0 < \gamma_{m^*}\alpha_{m^*} < 1$. It follows that in any MPE that is not a FTE there is a $m^* > 1$ such that $\gamma_{m^*}\alpha_{m^*}=0$, and for all $m < m^*$ the equilibrium involves trade with certainty. Because trade stops at $m^* > 1$ we have

$$w^{out}(m^*) = w(m^*) = v(m^*) = 0$$

Consider then this proposed equilibrium. Because $\gamma_{m^*}\alpha_{m^*}=0$, either the leader does not make an offer at m^*, so that $\gamma_{m^*}=0$, or the follower does not accept the leader's offer, $\alpha_{m^*}=0$. Let's focus first in case in which the leader does not make the equilibrium offer: $\gamma_{m^*}=0$. Note that an offer $\widehat{p}=-\delta w^{out}(m^*-1)+\epsilon$ for $\epsilon>0$ is always accepted. By deviating and making this offer, the leader gets a payoff $\delta v(m^*-1)-\widehat{p}$. This is a profitable deviation if

$$\delta v(m^*-1)-\widehat{p} > \delta v(m^*) = 0$$

$$\delta\left(v(m^*-1)+w^{out}(m^*-1)\right) > \epsilon$$

$$\delta^{m^*}\times\left(\Pi^*(m^*-1)+w\right) > \epsilon$$

where the last line follows since there is trade in every $m < m^*$. Recall that $\Pi^*(m)$ is decreasing in m when $w<0$, so if $\Pi^*(q-1)+w\geq 0$ we must have that $\Pi^*(m^*-1)+w>0$ and therefore there is some $\epsilon>0$ that makes the offer $\widehat{p}$ preferable for L than not making an offer at $m^*<q$. Similar arguments show if there is some $m^*>1$ such that $\alpha_{m^*}=0$, when the leader offers $p(m^*)=\delta\left(w(m^*)-w^{out}(m^*-1)\right)$, there is a profitable deviation by increasing the offer slightly.

To finish this part of the proof let's focus in the case where $m^*=q$. If $0<\gamma_q\alpha_q<1$ (there is randomization) we must have $s(q)=v(q)=0$ and using (12) we get

$$v(q-1)+w^{out}(q-1) = w(q) = H_q\delta w(q-1)$$

$$\Pi^*(q-1)+w = H_q\delta\left(\prod_{k=1}^{q-1}r(k)\right)w$$

but this is not possible since $\Pi^*(q-1)+w>0$. Then, we need to rule out equilibria with $\gamma_q\alpha_q=0$ but with trade with probability 1 at all $m<q$. Since there is no trade at q, $v(q)=w(q)=0$, but in this case L has a profitable deviation. Let L make the following offer $\widehat{p}=-\delta w^{out}(q-1)+\epsilon$ for some $\epsilon>0$ which is accepted with probability 1. Thus, L obtains

$$\delta v(q-1)-\widehat{p}=\delta v(q-1)+\delta w^{out}(q-1)-\epsilon$$

and the leader is willing to make this offer with probability 1 if

$$\begin{aligned}\delta v(q-1)+\delta w^{out}(q-1)-\epsilon &> \delta v(q)\\ \delta v(q-1)+\delta w^{out}(q-1) &> \epsilon\\ \delta\left(\Pi^*(q-1)+w\right) &> \epsilon\end{aligned}$$

and again if $\Pi^*(q-1)+w>0$, the deviation is profitable

Case 1.2: $\Pi^*(q-1)+w\leq 0\leq \Pi^*(q)$. Using the same arguments as in Case 1.1 we can rule out any mixed strategy equilibria except for the possibility that $\gamma_{m^*}\alpha_{m^*}=0$ for some $m^*>1$. Here $s(m^*)=v(m^*)=w(m^*)=w^{out}(m^*)=0$, and then $w(m)=w^{out}(m)=v(m)=0$ for all $m>m^*$.

Case 2: $\Pi^*(q)<0$. In this case, as we've shown before, there is no FTE. We separate the analysis in two parts: (1) $0<\Pi^*(1)$ and (2) $\Pi^*(1)\leq 0$. In the first case, we have that there is trade for low states with probability 1 and potential mixing for high states. The second case is more straightforward since the only class of equilibria that is feasible involves mixing in every state. In both cases we are going to show that, if there is mixing, there is some state $m\leq q$ in which there is no trade with probability 1.

Case 2.1: $\Pi^*(q)<0<\Pi^*(1)$. We are going to show that the only other class of equilibria involves no trade at some $m^*\geq 1$. If there is no trade at $m=1$ we are done, so assume that there is trade at $m=1$.

Suppose first that $\Pi^*(q-1)\leq 0$. Since $\Pi^*(m)$ is decreasing in m, there is some $m^*<q$ such that $\Pi^*(m)<0$ for all $m\geq m^*$ but $\Pi^*(m)>0$ for all $m<m^*$. Since $s(m)=\delta^{m-1}(1-\delta)\Pi^*(m)\geq 0$ for all $m<m^*$, in any equilibrium with trade at $m=1$ we must have $\gamma_m=\alpha_m=1$ for all $m<m^*$. We will show that in equilibrium there is no trade at m^*. Assume towards a contradiction that there is trade with positive probability at m^*. Now, since $\Pi^*(m^*)<0$ by hypothesis, it has to be that $\gamma_{m^*}\alpha_{m^*}<1$. Thus $s(m^*)=0$, which with (12) implies that $v(m^*)=0$ and $w_{out}(m^*-1)=w(m^*)$. The same logic gives that for all $m>m^*$:

$$w_{out}(m-1)=w(m) \tag{47}$$

Note that using the recursive representation of the value functions for m^*+1 we have

$$\left(\frac{\gamma_{m^*}\alpha_{m^*}\delta}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\right)w_{out}(m^*-1)=\delta^2H_{m^*+1}H_{m^*}w(m^*-1)$$

and since for all $m<m^*$ we have trade with probability 1 in every meeting we can use (13) and (14) to get

$$\left(\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})}\right)=\delta H_{m^*+1}H_{m^*}\left[\prod_{k=1}^{m^*-1}r(k)\right]$$

Using

$$H_m \equiv \frac{r\,(m)\,\gamma_m\alpha_m}{1-r\,(m)\,\delta(1-\gamma_m\alpha_m)}$$

we have

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} = \frac{\gamma_{m^*+1}\alpha_{m^*+1}}{1-r\,(m^*+1)\,\delta(1-\gamma_{m^*+1}\alpha_{m^*+1})}\frac{\gamma_{m^*}\alpha_{m^*}}{1-r\,(m^*)\,\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left[\prod_{k=1}^{m^*} r(k)\right] \tag{48}$$

Note that

$$\frac{\gamma_{m^*+1}\alpha_{m^*+1}}{1-r\,(m^*+1)\,\delta(1-\gamma_{m^*+1}\alpha_{m^*+1})} \leq 1$$

so we must have

$$\frac{\gamma_{m^*}\alpha_{m^*}}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} \leq \frac{\gamma_{m^*}\alpha_{m^*}}{1-r\,(m^*)\,\delta(1-\gamma_{m^*}\alpha_{m^*})}\delta\left[\prod_{k=1}^{m^*} r(k)\right]$$

and if $\gamma_{m^*}\alpha_{m^*} > 0$, then we have

$$\frac{1-r\,(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} + r\,(m^*) \leq \delta\left[\prod_{k=1}^{m^*} r(k)\right]$$

and since $r(k) < 1$ we must have

$$\begin{aligned}\frac{1-r\,(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} + r\,(m^*) &< r\,(m^*)\\ \frac{1-r\,(m^*)}{1-\delta(1-\gamma_{m^*}\alpha_{m^*})} &< 0\end{aligned}$$

which is false so $\gamma_{m^*}\alpha_{m^*} = 0$.

Next suppose instead that $\Pi^*(q-1) > 0$. In this case $\Pi^*\,(m) > 0$ for all $m < q$, and thus $m^* = q$. Recall that for all $m < q$ we have that there is trade in every meeting. Assume towards a contradiction that there is trade with positive probability at q. Then $s(q) = 0$ and by (12) we have that $v(q) = 0$. Therefore, we must have

$$w(q) = \delta^{q-1}\left(v + \left[\sum_{l=1}^{q-1}\left(1-\delta\prod_{k=1}^{l} r(k)\right)\right]w + w\right) \tag{49}$$

Recalling that

$$w(q) = H_q\delta w\,(q-1)$$

and using (13) and the expression for H_q we have

$$w(q) = \frac{r(q)\gamma_q\alpha_q}{1-r(q)\delta(1-\gamma_q\alpha_q)}\delta\left[\prod_{k=1}^{q-1} r(k)\right]\delta^{q-1}w$$

Substituting in (49) we have

$$v + \left[\sum_{l=1}^{q}\left(1-\delta\prod_{k=1}^{l} r(k)\right)\right]w = -\frac{1-\delta r(q)}{1-r(q)\delta(1-\gamma_q\alpha_q)}(1-\gamma_q\alpha_q)\delta\left[\prod_{k=1}^{q} r(k)\right]w$$

Note that the left hand side is equivalent to $\Pi^*(q)$ and is by assumption negative but since $w < 0$, the right hand side is positive, which is a contradiction. Therefore there is no trade at q.

Case 2.2: $\Pi^*(1) \leq 0$. Note that $\Pi^*(1) \leq 0$ implies that $v + w < 0$, so there is always an equilibrium with no trade at any $m \leq q$. Since $w < 0$, implies that $\Pi^*(m)$ is decreasing we have that $\Pi^*(m) < 0$ for all $m > 1$ and the only other class of equilibria involves randomization at any $m > 1$; i.e $\gamma_m \alpha_m < 1$. Again following Case 2.1 we have that the only other class of equilibria involves no trade at some $m^* \geq 1$ and therefore $v(q) = w(q) = 0$. This concludes the proof. □

Proposition A.9. *Assume that $z \leq 0 < w$, and let*

$$\Pi_z^*(m) \equiv v + m\,(z - w) + \left[\sum_{l=1}^{m} \left(1 - \delta \left(\prod_{k=1}^{l} r(k)\right)\right)\right] w$$

(1) If $\Pi_z^*(q) \geq 0$ the unique MPE of the game is a FTE.
(2) If $\Pi_z^*(q-1) \geq 0 > \Pi_z^*(q)$, MPE are either "no trading equilibria" (NTE) or entail no trading with positive probability at q, and thus imply $v(q) = 0$. Thus, $\forall \kappa > 0$, if the leader has to pay an entry cost $\kappa > 0$, she doesn't enter.
(3) If $0 \geq \Pi_z^*(q-1) \geq \Pi_z^*(q)$, all MPE are NTE.

Proof. The proof is similar to that of Proposition 5.7 (click here for details). □

Competing for Loyalty: The Dynamics of Rallying Support

Matias Iaryczower and Santiago Oliveros

Online Appendix

Appendix B. Extensions

B.1. **Unobservable Trades.** In our model the state (m or $\vec{m}$ in monopoly and competition) is common knowledge. This has a first order effect on equilibrium outcomes. In any meeting between a leader and a follower, the follower's bargaining power is derived from his outside option. And what gives value to the outside option are free riding opportunities. Common knowledge about the state matters because the state affects the value of free riding opportunities, and thus agents' bargaining power.

The effect of the state on agents' bargaining power is potentially absent when past trades are not observable, and it *is* absent in a pooling equilibrium where the leaders' offer convey no information about the state. It follows that introducing non-observability of trades can change equilibrium payoffs (and outcomes) significantly. In fact, as we show below, non-observability of trades increases agents' bargaining power. But this is not the end of the story. The point is that non-observability of trades affects agents' bargaining power both in the monopoly and competitive games. Thus, the key for the comparison of followers' welfare under monopoly and competition is whether non-observability of trades affects agents' bargaining power differentially in competition and monopoly in a way that would upend the results with full observability.

To study this question, we analyze the monopoly and competitive games under the assumption that trades are not observed by non-participants, following Noe and Wang (2004).[23] Following our previous discussion, we focus on pooling FTE, in which leaders make the same offer to all followers, independently of the state. We also assume, as in Noe and Wang (2004), that agents don't learn from calendar time (agents don't update their belief about the state based on the period in which they are approached).

The plan is the following. We begin with some general results in the monopoly game. We show that followers' equilibrium payoffs in the game with non-observable trades is higher than in the benchmark model with observability. We then move to the competitive game and show that here too, non-observability affects equilibrium payoffs. We wrap up the analysis of both games by characterizing payoffs and proving existence of the pooling FTE in a three agent example. We finish by comparing followers' equilibrium payoffs in both games.

Our results offer two main conclusions. First, equilibrium payoffs of the non-observability games are indeed different than in the benchmark game. Second, still in this case, monopoly is preferred

[23]The setup of Noe and Wang (2004) differs from ours in two fundamental aspects. First – in the main part of the paper (with n agents) – Noe and Wang consider what is effectively a unanimity rule: a buyer transacts with n sellers, and gets a payoff of one if she buys all n goods, zero otherwise (the goods are perfect complements). Since unanimity gives veto power to each agent, this effectively eliminates free riding (recall that our result holds for all non-unanimous rules). Second, sellers only care about the money they obtain for selling the good. Hence they are in a pure private values case with no externalities.

to competition. In a nutshell, non-observability of the state does not alter the fact that free riding opportunities are larger under monopoly than under competition.

(i) Monopolistic Leadership with Non-Observability. Let $\rho(\cdot)$ denote followers' beliefs (i.e., $\rho(m)$ indicates the probability that a follower attaches to the leader being m uncommitted followers short of winning.) In a pooling FTE, $\rho(m) = 1/n$ for all m.[24]

Let $E\hat{w}^{out}$ denote a follower's expected continuation value of accepting an offer by the leader, and $E\hat{w}$ a follower's expected continuation value of rejecting this offer. The optimal relevant offer $\hat{p}$ by the leader in any state $m = 1, 2, \ldots, (n+1)/2$ satisfies

$$\delta E\hat{w}^{out} + \hat{p} = \delta E\hat{w}, \tag{50}$$

Let $\hat{w}^{out}(m)$ denote the continuation value of a committed follower in state m, as computed by an outside observer who knows m (i.e., computed under the assumption that the follower does not know the state). Similarly, let $\hat{w}(m)$ denote the continuation value of an uncommitted follower in state m, also as computed by an outside observer who knows m. Finally, let $\hat{v}(m)$ be the continuation value of the leader in state m. Note that since the leader observes past trades, $\hat{v}(m)$ is computed assuming that the leader knows the state.

The expected continuation value of a follower who accepts an offer is given by

$$E\hat{w}^{out} \equiv \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{\rho(m)}{\sum_{j=1}^{\frac{n+1}{2}} \rho(j)} \right) \hat{w}^{out}(m-1) \tag{51}$$

This expression incorporates the fact that having traded, the follower knows that he is one of the $\frac{n+1}{2}$ followers to trade with the leader (i.e., that $m = 1, 2, \ldots, (m+1)/2$). Note that $\hat{w}^{out}(m)$ is equivalent to $w^{out}(m)$ in the full information case, so

$$E\hat{w}^{out} = \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2}{n+1} \right) \delta^{m-1} w = \left(\frac{2}{n+1} \right) \left(\frac{1-\delta^{\frac{n+1}{2}}}{1-\delta} \right) w, \tag{52}$$

where we have used the fact that there is trade after every meeting.

Now consider the expected continuation value of a follower who rejects an offer, $E\hat{w}$. By (50), in any state m the leader offers a transfer that gives the follower a continuation value equal to the discounted continuation value of rejecting the offer, $\delta E\hat{w}$, so

$$\hat{w}(m) = \left(\frac{2}{n+2m-1} \right) \delta E\hat{w} + \left(\frac{n+2m-3}{n+2m-1} \right) \delta \hat{w}(m-1) \tag{53}$$

Solving this recursion,[25] we get

[24]As we will see below, this is not the case in the competitive game.

[25]Let $\beta(m) = \frac{2}{n+2m-1}$ denote the probability that a follower is chosen to face the leader, and write (53) as $\hat{w}(m) = \beta(m)\delta E\hat{w} + (1-\beta(m))\,\delta\hat{w}(m-1)$. It is easy to see that $\frac{1-\beta(m)}{1-\delta\beta(m)} = r(m)$ as defined in that text and that $\frac{1-\beta(m)}{\beta(m)} = \frac{1}{\beta(m-1)}$. Therefore, defining $x(m) \equiv \hat{w}(m)/\beta(m)$ we have that the difference equation that describes the value function $\hat{w}(m)$ is given by $x(m) = \delta E\hat{w} + \delta x(m-1)$. This is a linear, autonomous, first order difference equation, with general solution $x(m) = \frac{\delta}{1-\delta} E\hat{w} + C\delta^m$ and boundary

$$\hat{w}(m) = \left(\frac{2}{n+2m-1}\right)\left[\left(\frac{\delta}{1-\delta}\right)(1-\delta^m)\,E\hat{w} + \left(\frac{n-1}{2}\right)\delta^m w\right] \tag{54}$$

The expected continuation value for a follower of rejecting the offer is given by

$$E\hat{w} = \frac{1}{n}\sum_{m=1}^{\frac{n+1}{2}} \hat{w}(m) + \frac{n-1}{2n} w \tag{55}$$

where we used $\rho(m) = 1/n$. The second part is the probability of not meeting the leader and successfully free riding on all other followers. Substituting (54) in (55), we obtain

$$E\hat{w} = \frac{1 + \sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2\delta^m}{n+2m-1}\right)}{n - \frac{\delta}{1-\delta}\sum_{m=1}^{\frac{n+1}{2}} \left(\frac{2(1-\delta^m)}{n+2m-1}\right)} \left(\frac{n-1}{2}\right) w \tag{56}$$

which can be used to compute $\hat{w}(m)$ substituting in (54).

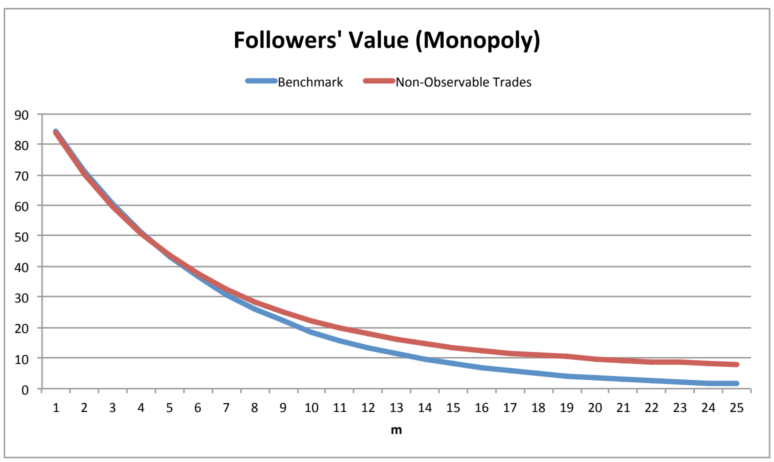


FIGURE B.1. Followers' equilibrium payoff with non-observable trades, $\hat{w}(m)$, and in the benchmark model, $w(m)$ ($\delta = 0.85$, $n = 49$, $w = 100$ and $v = 10$).

Figure B.1 plots $\hat{w}(m)$ together with the followers' payoff in the benchmark model, for $\delta = 0.85$, $n = 49$, $w = 100$ and $v = 10$. Consistent with your intuition, the plot shows that non-observability of trades raises followers' value.

condition given by $x(1) = \delta E\hat{w} + \frac{n-1}{2}\delta w$. Substituting,

$$x(m) = \left(\frac{\delta}{1-\delta}\right)(1-\delta^m)E\hat{w} + \left(\frac{n-1}{2}\right)\delta^m w,$$

and using the definition of $x(m)$, we obtain (54).

(ii) Existence and Characterization of Pooling FTE with 3 followers [Monopoly]. With $n = 3$, (56), (52) and (54) become

$$E\hat{w} = \left(\frac{6+\delta(3+2\delta)}{18-\delta(5+2\delta)}\right) w, \quad E\hat{w}^{out} = \left(\frac{1+\delta}{2}\right) w$$

and

$$\hat{w}(2) = \left(\frac{2+9\delta}{18-\delta(5+2\delta)}\right) \delta w$$

Up to this point, we have proceeded under the assumption that a Pooling FTE exists. We verify this here. Note that the leader is willing to make an offer in state m as long as

$$\delta\hat{v}(m-1) - \hat{p} \geq \delta\hat{v}(m). \tag{57}$$

An equilibrium with trade in every state requires that (57) holds in each state $m = 1, 2$. In order to calculate the expression for (57) we have that the value function for the leader is

$$\hat{v}(m) = \delta\hat{v}(m-1) - \hat{p},$$

where $\hat{p} = \delta[E\hat{w} - E\hat{w}^{out}]$ can be computed from (52) and (56). Again this is a linear, first order, autonomous, difference equation with complete solution given by:

$$\hat{v}(m) = \delta^m v - \frac{1-\delta^m}{1-\delta}\hat{p}$$

and therefore (57) is equivalent to

$$\hat{p} \leq \frac{\delta^m(1-\delta)}{(1-\delta^m)} v \tag{58}$$

Substituting from the expressions for $E\hat{w}$ and $E\hat{w}^{out}$ above, we have

$$\hat{p} = \left(\frac{6+\delta(3+2\delta)}{18-\delta(5+2\delta)} - \frac{1}{2}(1+\delta)\right) w = \frac{1}{2}\left(\frac{11\delta^2+2\delta^3-7\delta-6}{18-\delta(5+2\delta)}\right) w$$

Note that $f(\delta) = 11\delta^2 + 2\delta^3 - 7\delta - 6$ is convex, decreasing for $\delta = 0$ and $f(0) = f(1) = 0$. Therefore, $\hat{p} < 0$. Since $v > 0$, condition (57) holds for $m = 1, 2$ and there is a pooling FTE in the three player case under monopoly.

(iii) Competitive Leadership with Non-Observability. Let $E\hat{W}_\ell^{out}$ denote a follower's expected continuation value of accepting an offer by leader $\ell = A, B$, and $E\hat{W}$ a follower's expected continuation value of rejecting this offer. An optimal relevant offer by $\ell = A, B$ in state $\vec{m}$ verifies

$$\delta E\hat{W}_\ell^{out} + \hat{p}_\ell = \delta E\hat{W}. \tag{59}$$

The expectations

$$E\hat{W}_\ell^{out} = \sum_{m_A=1}^{\frac{n+1}{2}} \sum_{m_B=1}^{\frac{n+1}{2}} \rho(\vec{m})\hat{W}^{out}(\vec{m}^\ell), \quad E\hat{W} = \sum_{m_A=1}^{\frac{n+1}{2}} \sum_{m_B=1}^{\frac{n+1}{2}} \rho(\vec{m})\hat{W}(\vec{m}) \tag{60}$$

are computed with the followers' beliefs ρ. Note that even in a FTE, in the competitive game beliefs over states are not uniform. Since $\rho(\vec{m}) = \Pr(\vec{m} \mid \text{approached } k^{th}) \Pr(\text{approached } k^{th})$ and $\Pr(\text{approached } k^{th}) = 1/n$, then[26]

$$\rho(\vec{m}) = \frac{k!}{n} \frac{(\pi_A)^{\left(\frac{n+1}{2} - m_A\right)} (\pi_B)^{\left(k + m_A - \frac{n+1}{2}\right)}}{\left(\frac{n+1}{2} - m_A\right)! \left(k + m_A - \frac{n+1}{2}\right)!}$$

Note that in a FTE $\hat{W}^{out}(\vec{m})$ is equivalent to $W^{out}(\vec{m})$ as in the benchmark model with observable meetings, which pins down $E\hat{W}_\ell^{out}$. Now consider $E\hat{W}$. First, note that $\hat{W}(1,1) = \delta E\hat{W}$. For the boundary states in which $m_A = 1$ and $m_B \geq 2$ (a symmetric expression holds for $(m_A, 1)$, with $m_A \geq 2$), we have

$$\hat{W}(1, m_B) = \frac{\delta E\hat{W}}{m_B} + \frac{m_B - 1}{m_B} \Big(\pi_A \delta w_A + \pi_B \delta \hat{W}(1, m_B - 1) \Big) \quad \text{for } m_B \geq 2$$

Solving recursively,

$$\hat{W}(1, m_B) = \frac{\delta E\hat{W}(\vec{m})}{m_B} \sum_{j=0}^{m_B - 1} (\delta \pi_B)^j + \frac{\delta \pi_A w_A}{m_B} \sum_{j=0}^{m_B - 2} (m_B - j - 1)(\delta \pi_B)^j, \tag{61}$$

so that all boundary values again depend on $E\hat{W}(\vec{m})$. Note moreover that for all $\vec{m} \geq (2,2)$ we have

$$\hat{W}(\vec{m}) = \beta'(\vec{m})\, \delta E\hat{W}(\vec{m}) + \big(1 - \beta'(\vec{m})\big) \Big(\pi_A \delta \hat{W}(\vec{m}^A) + \pi_B \delta \hat{W}(\vec{m}^B) \Big) \tag{62}$$

Thus, *all* values again depend on $E\hat{W}(\vec{m})$. This observation highlights the effect of the non-observability of trades: when the offers are not contingent on the state the leader cannot extract as much surplus from critical followers, who in equilibrium are unaware of their position. This effect is then transmitted recursively to values at the beginning of the game.

(iv) Existence and Characterization of Pooling FTE in a 3-Follower Example [Competition]. To simplify computations, we assume that $w_A = w_B = w$, $\pi_A = \pi_B = 1/2$, and $\underline{v}_A = \underline{v}_B = 0$. With $n = 3$, and given our simplifying assumptions, (61) is

$$\hat{W}(1,2) = \hat{W}(2,1) = \frac{\delta E\hat{W}(\vec{m})}{2} \left(1 + \frac{\delta}{2}\right) + \frac{\delta w}{4}$$

[26]Note that in any state $\vec{m}$ in which follower i is approached in order k we have $m_A + m_B = n + 1 - k$, and therefore $m_B = n + 1 - k - m_A$ and $\frac{n+1}{2} - m_B = k + m_A - \frac{n+1}{2}$.

Using (62) we get

$$\hat{W}(2,2) = \frac{1}{3}\delta E\hat{W}(\vec{m}) + \frac{2}{3}\delta\hat{W}(1,2) = \frac{1}{3}\delta\left(E\hat{W}(\vec{m})\left(1+\delta+\frac{\delta^2}{2}\right)+\frac{\delta w}{2}\right),$$

and substituting in (60)

$$E\hat{W}(\vec{m}) = \frac{\delta}{3}\left[E\hat{W}(\vec{m})\times\left(\frac{4}{3}+\frac{7\delta}{12}+\frac{\delta^2}{6}\right)+w\left(\frac{1}{4}+\frac{\delta}{6}\right)\right]$$

Therefore,

$$\hat{W}(2,2) = \frac{1}{6}\left(\frac{42-6\delta+\delta^3}{36-\delta\left(16+7\delta+2\delta^2\right)}\right)\delta^2 w$$

and

$$E\hat{W} = \left(\frac{3+2\delta}{36-\delta\left(16+7\delta+2\delta^2\right)}\right)\delta w$$

We now establish existence of this equilibrium. Leader ℓ is willing to make an offer in state m iff $\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell} \geq \delta\hat{V}^{\ell}(\vec{m})$. Note that under a FTE, leader ℓ's value function is described by

$$\hat{V}^{\ell}(\vec{m}) = \pi_{\ell}\left(\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell}\right) + \pi_j\delta\hat{V}^{\ell}(\vec{m}^{j})$$

and thus ℓ is willing to make an offer in state m iff

$$\delta\hat{V}^{\ell}(\vec{m}^{\ell}) - \hat{p}_{\ell} \geq \frac{\delta\pi_j}{1-\delta\pi_{\ell}}\delta\hat{V}^{\ell}(\vec{m}^{j}) \tag{63}$$

where $\hat{p}_{\ell} = \delta[E\hat{W} - E\hat{W}^{out}_{\ell}]$.

For existence of a pooling FTE we need (63) to hold for all $\vec{m}$, which boils down to

$$\delta\hat{V}^{A}(\vec{m}^{A}) - \hat{p}_{A} \geq \frac{\delta}{2-\delta}\delta\hat{V}^{A}(\vec{m}^{B}) \tag{64}$$

Using that

$$E\hat{W}^{out}(\vec{m}^{A}) = \frac{3+\delta}{6}\delta w$$

we have

$$\hat{p}_A = \delta\left(\frac{3+2\delta}{36-\delta\left(16+7\delta+2\delta^2\right)} - \frac{3+\delta}{6}\right)\delta w < 0$$

Next, note that

$$\hat{V}^{A}(1,1) = \frac{(\delta\overline{v}_A - \hat{p}_A)}{2}, \quad \hat{V}^{A}(1,2) = \frac{(\delta\overline{v}_A - \hat{p}_A) + \delta\hat{V}^{A}(1,1)}{2},$$

$$\hat{V}^{A}(2,1) = \frac{\left(\delta\hat{V}^{A}(1,1) - \hat{p}_A\right)}{2}, \textit{and} \quad \hat{V}^{A}(2,2) = \frac{\left(\delta\hat{V}^{A}(1,2) - \hat{p}_A\right) + \delta\hat{V}^{A}(2,1)}{2}$$

Substituting, and noting that $\hat{p}_A \leq 0$, gives that (64) holds for all states $\vec{m} \neq (2,2)$. For $\vec{m} = (2,2)$, condition (64) is equivalent to

$$\frac{\delta^2}{2}\overline{v}_A + \frac{\delta(1-\delta)}{2-\delta}\left(\frac{\delta^2}{2}\overline{v}_A - \left(1+\frac{\delta}{2}\right)\hat{p}_A\right) \geq \hat{p}_A$$

and since $\hat{p}_A \leq 0$, condition (64) also holds for $\vec{m} = (2,2)$.

(v) Comparison of Followers' Equilibrium Payoffs. We are now ready to compare followers' equilibrium payoffs in both games. As in part 4 above, in the competitive game we assume that $w_A = w_B = w$, $\pi_A = \pi_B = 1/2$, and $\underline{v}_A = \underline{v}_B = 0$.

There are two alternative measures that we can consider: (i) the value of the game for the first follower to trade with the leader/s: $\hat{W}(2,2)$ and $\hat{w}(2)$, and (ii) the expected value of the game $E\hat{W}$ and $E\hat{w}$. Using the first measure we have that single leadership is preferred to competition if

$$\frac{42-6\delta+\delta^3}{36-\delta\left(16+7\delta+2\delta^2\right)}\delta < \frac{12+54\delta}{18-\delta(5+2\delta)}$$

which reduces to

$$-6\delta\left(230+176\delta+67\delta^2+15\delta^3\right) < 432+\delta^2(5+2\delta)\left(42-6\delta+\delta^3\right)$$

which is always true.

On the other hand, the comparison in expectation favors monopoly iff

$$\frac{3+2\delta}{36-\delta\left(16+7\delta+2\delta^2\right)}\delta w < \frac{6+\delta(3+2\delta)}{18-\delta(5+2\delta)} \times w$$

which reduces to

$$-(3+2\delta)\left(18+7\delta+2\delta^2\right)(1-\delta)\delta < 6\left[11+(1-\delta)\left(25+9\delta+2\delta^2\right)\right]$$

which again is true.

B.2. **Direct Competition.** In our model we assumed a sequential contracting setup, in which leaders and followers make deals in bilateral meetings. An important consequence of this assumption is that competition is "indirect" in the sense that competition affects trades through its effect on followers' outside option.[27]

Here we introduce the possibility of *direct competition*; i.e., that both leaders make simultaneous offers to the followers. To do this we consider a version of our game in which the follower who is selected to negotiate in state $\vec{m}$ meets with leader $j = A, B$ with probability $\pi_j = \pi > 0$, and meets with both leaders with probability $\pi_{AB} = 1 - 2\pi > 0$. This spans the level of direct competition from a situation in which direct competition is unlikely to a case in which it is pervasive. As we will show, our results are quite robust to the presence of direct competition, even for large π_{AB} when there are no frictions (for δ close to one).

Introducing direct competition requires that we **expand the state space** to include the set of principals that can make an offer to the follower at each meeting. Thus we let the *negotiation state* be $\eta \in \{A, B, AB\}$, where η denotes the set of principals that can make an offer to the follower on a meeting at a *step state* $\vec{m}$. The state is then $s = (\vec{m}, \eta)$, and the value functions $W(\vec{m}, \eta)$ and $V(\vec{m}, \eta)$.

The analysis of the game with direct competition introduces additional technical challenges. To attack this problem, we resort to a combination of analytical results and numerical analysis. In particular, we first characterize equilibrium payoffs recursively, expressing equilibrium payoffs in step states $\vec{m}$ as a function of quantities in *forward* step states $(\vec{m}^A, \vec{m}^B)$, imposing only the restriction that strategies are weakly undominated given continuation values.[28] (Click here to see the details of the derivation of the analytical results.) We then use these results to compare equilibrium payoffs under monopoly and competition numerically, for a variety of parameter values.

The gist of the effect of direct competition can be grasped by comparing optimal relevant offers in competitive and non-competitive negotiation states. In a noncompetitive negotiation state, $\eta = A, B$, an optimal relevant offer by leader j satisfies

$$p_j(\vec{m}, j) = -\delta[W(\vec{m}^j) - W(\vec{m})],$$

where $\delta[W(\vec{m}^j) - W(\vec{m})]$ is the follower's discounted gain of going from the current state $\vec{m}$ one step in the direction of leader j. With direct competition, instead, the winning leader j makes an offer

$$p_j(\vec{m}, AB) = \delta\left[(V_\ell(\vec{m}^\ell) - V_\ell(\vec{m}^j)) + (W(\vec{m}^\ell) - W(\vec{m}^j))\right]. \tag{65}$$

[27]This is also how competition among proposers enters in the vast majority of collective bargaining models, including all models in the tradition of Baron and Ferejohn (1989) and Chatterjee et al. (1993).
[28]The negotiation state with competition is similar to a first price auction with two bidders and complete information. As usual in these type of games, there is a continuum of equilibria in weakly dominated strategies in which both bidders bid the same amount, but in all equilibria the winner is the bidder with the highest willingness to pay. As is customary in the literature, we rule out these equilibria by focusing on strategies that are weakly undominated given continuation values, so the common bid is equivalent to the lowest willingness to pay.

This is the joint discounted surplus of the follower and the losing leader ℓ from going from the current state $\vec{m}$ one step in the direction of leader ℓ net of their surplus of going from the current state $\vec{m}$ one step in the direction of leader j.

Equation (65) makes clear the benefit that direct competition can bring to followers: the winning leader has to compensate the follower for the payoff him *and* the losing leader could be making if he were to go in the other direction. The transfer has to be this high to "price out" the competing leader ℓ from the contest. This introduces an additional effect which goes against the free-riding effect that benefits monopoly over competition.

Equation (65) also shows, however, that the potential benefit for a follower in state $\vec{m}$ depends on equilibrium payoffs in states $\vec{m}^{\ell}$ and $\vec{m}^{j}$ (having moved one step in ℓ and j's direction, respectively). This is important, because it implies that followers' rent extraction at later states will diminish the potential gain for followers in early states. Thus, whether the rents brought by direct competition can overturn the ranking of monopoly and competition in our benchmark model depends on whether the new rents are large enough and are sufficiently not competed away to overpower the free-riding effect which is still present in this model.

As it turns out, this can fail to happen even for a high probability that the negotiation state is competitive. This can be seen in Tables B.1 - B.5, which provide an extensive characterization of how direct competition affects our results in different environments. In each table, we compute the difference between the equilibrium payoff of an uncommitted follower in the competitive game and monopolistic games for a group size to $n = 21$, for given parameter values $w_A, w_B, \overline{v}_A \overline{v}_B, \delta$ (we fix $\underline{v}_A = \underline{v}_B = 0$). In each column, we fix a value of the probability that principals can make competing offers, π_{AB}. Each row denotes a value of the step state $m = 1, \ldots, 11$. For any given step state m, we compute the difference between the value of an uncommitted follower in the competitive game across the diagonal $W(m, m)$, and the value of an uncommitted follower in the monopolistic game, $w(m)$.

Table B.1. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 300$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	242.1	213.7	189.7	133.3	119.9	112.0	109.1	107.9	107.0
2	128.9	121.9	114.7	81.2	75.1	70.1	67.9	66.8	66.0
3	101.4	96.4	90.9	60.9	52.3	45.1	41.8	40.3	39.1
4	87.2	83.0	78.2	50.6	39.5	30.0	25.8	23.7	22.2
5	77.0	73.5	69.3	44.4	32.5	21.8	16.9	14.6	12.8
6	68.4	65.4	61.8	39.9	28.1	17.2	12.1	9.7	7.7
7	61.4	58.1	55.0	36.1	24.9	14.3	9.3	6.8	4.9
8	54.3	51.5	48.9	32.5	22.2	12.2	7.3	5.0	3.1
9	47.6	45.4	43.2	29.1	19.8	10.5	6.0	3.7	1.9
10	41.6	39.9	38.1	26.0	17.6	9.1	4.9	2.8	1.1
11	**36.2**	**34.9**	**33.4**	**23.1**	**15.7**	**8.0**	**4.1**	**2.1**	**0.6**

In table B.1 we fix $w_A = 300$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 30$, and $\delta = 0.85$. Here A represents a markedly better alternative than B for followers, while at the same time leader A also has a higher willingness to pay for winning than its competitor. The first column reports the results for the case in which direct competition only occurs with probability $\pi_{AB} = 0.01$. The monopoly differential at the beginning of the game (at $m = 11$) is positive (36.2), indicating that a monopoly of the better alternative A dominates competition. The advantage of monopoly increases as we move to states closer to completion, reflecting the higher probability of a win by the worst alternative. Columns 2 and above report the results of similar exercises for larger values of the probability of direct competition. The results show that while the monopoly differential decreases as direct competition becomes more prevalent, for these parameters monopoly dominates competition throughout, even with a probability of direct competition of $\pi_{AB} = 0.99$.

Table B.2. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 30$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	78.4	61.7	47.6	14.4	6.5	1.8	0.1	-0.6	-1.1
2	43.4	37.1	31.3	14.6	9.3	5.9	4.5	3.9	3.5
3	32.6	29.3	26.0	14.7	10.5	7.4	6.2	5.7	5.3
4	27.3	25.3	23.5	14.5	10.5	7.3	5.9	5.2	4.7
5	24.2	22.8	21.3	13.7	9.9	6.5	4.9	4.1	3.5
6	21.6	20.4	19.2	12.8	9.1	5.6	3.9	3.0	2.4
7	19.2	18.3	17.3	11.7	8.2	4.7	3.0	2.2	1.6
8	17.0	16.3	15.5	10.6	7.4	4.1	2.4	1.6	1.0
9	15.0	14.4	13.8	9.6	6.6	3.5	2.0	1.2	0.6
10	13.2	12.7	12.2	8.6	5.9	3.0	1.6	0.9	0.4
11	**11.5**	**11.1**	**10.7**	**7.6**	**5.2**	**2.7**	**1.4**	**0.7**	**0.2**

In table B.2 we reduce the value of a win by the attractive alternative A for the followers, setting $w_A = 100$ (all other parameters stay unchanged). When the probability of direct competition is low enough ($\pi_{AB} \leq 0.9$), monopoly dominates competition for all $m \leq 11$. Differently than in the first table, when the probability of direct competition is sufficiently large ($\pi_{AB} \geq 0.95$) the value of an uncommitted follower is greater in competition than under monopoly for the critical follower $m = 1$. However, even for these high levels of direct competition, the value differential reverts to favoring monopoly at the beginning of the game.

The reason for this is illustrated by the analytical results. The way in which direct competition helps followers is that in competitive negotiation states, the leader needs to pay the follower to dissuade him from favoring the other leader. This rent (eq.5) is increasing in the value differential for the losing leader of going one step towards her direction instead of the winner's direction, and the follower's value differential of going towards the losing leader's direction instead of the winner's. Since the last follower is able to extract the rent from the winner of direct competition at $m = 1$ (A in this case), this reduces the follower's value differential of going towards the loser's direction instead of the winner's at $m = 2$. For the parameters considered here, it is enough to overturn it completely by $m = 2$.

This is crucial to understand why the sole existence of competition is not sufficient to overturn our results. When the races are tight – which is what happens across the diagonal of the competitive game – followers' gains in later step states (say m') reduce leaders' willingness to pay in early step states ($m'' > m'$). This dynamic effect dampens the benefits of direct competition in the initial stages of the game.

TABLE B.3. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.85$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	74.6	46.2	22.3	-34.2	-47.6	-55.5	-58.4	-59.6	-60.5
2	42.0	31.1	21.1	-7.5	-16.2	-22.1	-24.5	-25.5	-26.3
3	33.5	27.0	20.9	3.0	-3.3	-6.3	-7.0	-7.2	-7.3
4	28.4	24.2	20.3	7.5	3.1	1.0	0.5	0.3	0.3
5	24.7	22.0	19.2	9.5	5.9	3.7	2.8	2.4	2.1
6	21.9	19.9	17.9	10.2	6.9	4.4	3.2	2.5	2.0
7	19.4	17.9	16.4	10.1	7.0	4.3	2.9	2.1	1.5
8	17.1	16.0	14.8	9.6	6.7	3.9	2.4	1.6	1.0
9	15.1	14.3	13.3	8.9	6.2	3.4	2.0	1.2	0.6
10	13.2	12.6	11.9	8.1	5.7	3.0	1.6	0.9	0.4
11	**11.6**	**11.1**	**10.5**	**7.3**	**5.1**	**2.6**	**1.4**	**0.7**	**0.2**

In table B.3 we increase the value of the a win for the principal favoring the unattractive alternative for followers (B) to $\overline{v}_B = 100$, maintaining all other parameters as in Table 2. This is the leader with the lower willingness to pay and who is also least preferred by the followers, so it is the leader that will lose on the competition stage across the diagonal. But because her willingness to pay is now higher, A needs to improve her offer to followers, who derive a larger benefit from direct competition. As expected, this increases the range of π_{AB} for which competition is preferred to monopoly for uncommitted followers at later stages of the game ($m = 1, 2, 3$). However, still monopoly beats competition for all $\pi_{AB} \leq 0.99$ at the initial state. In table B.4 we increase the discount factor from $\delta = 0.85$ to $\delta = 0.95$, maintaining all other parameters unchanged. This increases the range of states for which competition is preferred to monopoly for sufficiently high probability of direct competition. Still in each case the initial value differential favors monopoly.

TABLE B.4. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 70$, $\overline{v}_A = 200$, $\overline{v}_B = 100$, $\delta = 0.95$, $n = 21$.

	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	67.4	11.7	-16.9	-55.7	-61.8	-65.0	-66.1	-66.5	-66.9
2	41.5	15.4	0.6	-25.7	-31.9	-35.9	-37.6	-56.8	-56.8
3	35.5	17.2	6.4	-14.2	-18.2	-19.4	-19.3	-45.4	-43.3
4	32.1	18.6	9.9	-7.1	-9.7	-9.3	-8.2	-35.4	-29.8
5	30.0	19.7	12.4	-2.1	-4.0	-3.4	-2.4	-28.6	-19.6
6	28.9	20.6	14.3	1.5	-0.1	0.0	0.3	-6.6	-11.1
7	28.2	21.3	15.8	4.2	2.5	1.9	1.4	-2.5	-4.5
8	27.6	21.8	17.0	6.3	4.2	2.8	1.8	0.3	0.9
9	27.1	22.1	17.8	7.9	5.4	3.3	1.9	0.7	5.1
10	26.7	22.3	18.5	9.1	6.3	3.5	1.8	0.7	8.4
11	**26.2**	**22.4**	**19.0**	**9.9**	**6.8**	**3.5**	**1.7**	**0.4**	**11.1**

In table B.5 we further reduce the payoff differential between alternatives for the followers, increasing w_B to 90, *and* also increase $\overline{v}_B$ to 200, eliminating the differential in leaders' willingness to pay (We maintain all else constant as in table B.4.) Both of these changes increase the symmetry among alternatives. As the table shows, in this case competition beats monopoly even at the beginning of the game when the probability of direct competition is sufficiently high ($\pi_{AB} \geq 0.90$). In addition, competition is preferred to monopoly for followers at later stages in the game, even for a more moderate probability of direct competition (for $m \leq 7$ at $\pi_{AB} = 0.40$ and $m \leq 4$ at $\pi_{AB} = 0.10$).[29]

Table B.5. Differential between Followers' Payoffs in Monopoly of Best and Competition: $w_A = 100$, $w_B = 90$, $\overline{v}_A = 200$, $\overline{v}_B = 200$, $\delta = 0.95$, $n = 21$.

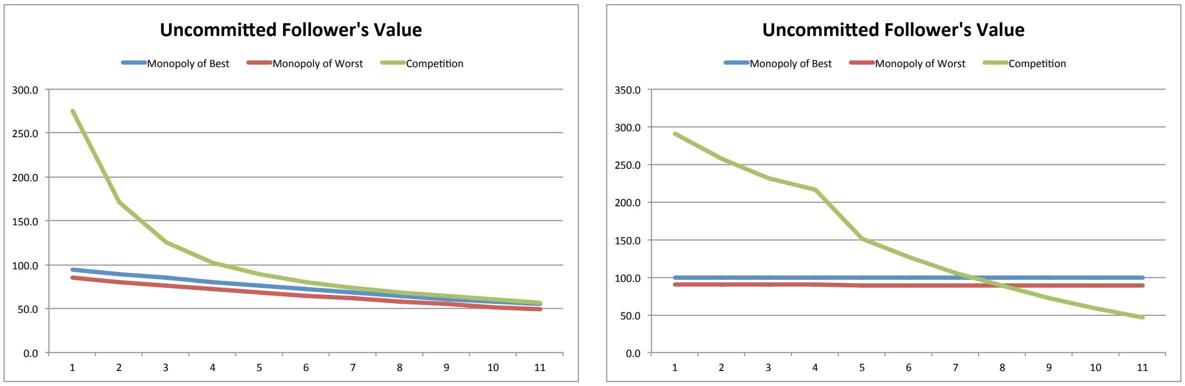
	π_{AB}								
m	**0.01**	**0.05**	**0.10**	**0.40**	**0.60**	**0.80**	**0.90**	**0.95**	**0.99**
1	48.2	-46.8	-95.5	-161.8	-172.1	-177.6	-179.5	-180.2	-180.8
2	28.5	-14.8	-37.3	-70.0	-75.9	-79.3	-80.5	-81.0	-81.4
3	24.7	-4.8	-19.9	-40.0	-41.7	-41.5	-41.0	-40.8	-40.7
4	23.5	1.2	-10.2	-24.0	-24.2	-23.0	-22.4	-22.2	-22.0
5	23.1	5.3	-3.7	-14.1	-13.9	-13.2	-13.1	-13.1	-13.1
6	23.0	8.4	1.0	-7.4	-7.5	-7.7	-8.1	-8.3	-8.6
7	23.0	10.8	4.7	-2.6	-3.3	-4.4	-5.2	-5.7	-6.1
8	23.0	12.8	7.5	0.8	-0.5	-2.3	-3.4	-4.0	-4.5
9	23.0	14.3	9.7	3.4	1.5	-0.9	-2.3	-2.9	-3.4
10	23.0	15.5	11.5	5.3	2.9	0.0	-1.4	-2.1	-2.6
11	**23.0**	**16.4**	**12.9**	**6.8**	**3.9**	**0.7**	**-0.9**	**-1.6**	**-2.1**

Finally, it is interesting to compare followers' welfare in monopoly and competition for δ approaching one (as frictions vanish). In Figure B.2 we do this for the parameters we considered in Table B.5 and a probability of direct competition of $\pi_{AB} = 0.95$, where we showed that competition beats monopoly even at the beginning of the game. The figure plots uncommitted followers equilibrium payoff in monopoly and competition as a function of the step state $m = 1, \ldots, 11$ for $\delta = 0.95$, as in table B.5 (left panel) and $\delta = 0.999$ (right panel). As the figure shows, with the high discount factor the result is reversed, and monopoly again beats competition at the beginning of the game.

Conclusions. Introducing the possibility of direct competition undoubtedly improves the standing of competition vis a vis monopoly. Because in simultaneous bidding both leaders are willing to increase their offers as long as there is a surplus, followers can extract additional rents from the winning leader, who has to raise the transfer enough to exhaust the surplus of the competitor. In a dynamic game, however, the benefit of direct competition is not shared equally among followers. In fact, because these additional rents are heavily captured by followers at the end stages in the game, leaders' willingness to pay to win in direct competition diminishes in early stages, where followers see a reduced advantage from direct competition.

Whether the possibility of direct competition can overturn the ranking of monopoly and competition in our benchmark model depends on whether new rents are large enough and are sufficiently

[29]Still, if given the choice to enter a monopoly or competitive game, followers would still choose monopoly as long as the probability of direct competition is not larger than 0.80.



$\delta = 0.95$ $\delta = 0.999$

FIGURE B.2. Followers' Value in Monopoly and Competition for $\delta = 0.95$ (left panel) and $\delta = 0.999$ (right panel). Parameters as in last column of Table B.5: $w_A = 100$, $w_B = 90$, $\overline{v}_A = 200$, $\overline{v}_B = 200$, $n = 21$, with probability of direct competition $\pi_{AB} = 0.95$.

not competed away to overpower the free-riding effect that we identified in the paper, and which is still present here. In particular, the race between the ability to capture rents in direct competition and the free-riding effect can benefit competition for some parameters. However, as long as both direct competition and bilateral bargaining are possible, the effect of free-riding on bargaining persists, and implies that for many parameter configurations monopoly beats competition even when direct competition is prevalent. We conclude that the mechanism we identify in the paper on the bargaining consequences of free-riding opportunities is still important in the presence of direct competition.

B.3. **Commitment to Reject.** In the paper we assumed that if a follower rejects an offer from a/the leader, the follower returns to the pool of uncommitted followers, and can possibly accept an offer at a later time (if one is extended to him). Here we study an extension of the model in which followers can choose to reject offers permanently, leaving the pool of uncommitted followers (i.e., followers can now accept, reject, or *leave*).

In this context the state space is multidimensional even in the single leader case. The reason is that when a supporter leaves he reduces the pool of uncommitted followers without increasing the support for the leader. Because this reduces the free-riding opportunities of the remaining uncommitted followers, we need to keep track of both the additional number of followers which the leader needs to win, m, and the number of remaining uncommitted followers, u. We also need to consider the possibility that the leader/s knowingly makes a low offer to reduce the number of uncommitted followers.

For simplicity, we consider a three-agent example. We show that when the leader/s value for winning is sufficiently large, the equilibrium payoff of an uncommitted follower in a MPE of the monopoly game is larger than his equilibrium payoff in the competitive game.

Consider first monopoly. From the same arguments that we used in the benchmark model, we know that for large v, the only candidate for a MPE is a full trading equilibrium (FTE). We therefore directly focus on equilibria of this class. We show that when v is large there exists a FTE, and the equilibrium payoff of an uncommitted follower is given by:

$$w(2,3) = (1+2\delta)\,(\delta^2/3)w$$

Proof. Consider first $(m,u) = (1,1)$, a state in which a single remaining uncommitted follower is pivotal for the decision (after one follower accepted and one follower left). Note that leaving gives the follower a payoff of zero, while rejecting an offer gives him a payoff of $\delta w(1,1)$. Then an optimal relevant offer satisfies

$$\delta w + p(1,1) = \max\{\delta w(1,1), 0\} \Rightarrow p\,(1,1) = \max\{\delta w(1,1), 0\} - \delta w.$$

Then $w(1,1) = \max\{\delta w(1,1), 0\} \Rightarrow w(1,1) = 0$. It follows that $p\,(1,1) = -\delta w$, giving the leader a payoff of $v(1,1) = \delta v - p(1,1) = \delta(v+w)$. Since $v(1,1) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(1,1) < v(1,1)$ – is not a profitable deviation. And since the follower is pivotal, deviating to a non-relevant offer $\tilde{p}\,(1,1) < p(1,1)$ gives her a payoff of zero, and is not profitable either.

Consider next the state in which exactly one follower has committed his support to the leader and nobody left; $(m,u) = (1,2)$. Note that the optimal relevant offer by the leader must verify

$$p\,(1,2) + \delta w = \delta \max\{w_{out}\,(1,1)\,, w\,(1,2)\}$$

Suppose first that $w\,(1,2) \geq w_{out}\,(1,1) = \delta w$. Then $p\,(1,2) = \delta w\,(1,2) - \delta w$, and

$$w(1,2) = \frac{1}{2}\delta w\,(1,2) + \frac{1}{2}\delta w \Rightarrow w(1,2) = \frac{\delta}{2-\delta}w < \delta w = w_{out}\,(1,1)\,,$$

a contradiction. Thus, the follower prefers to leave than to reject the offer, and we must have $w\,(1,2) \leq w_{out}\,(1,1) = \delta w$. Thus $p\,(1,2) = -\delta\,(1-\delta)\,w$, giving the leader a payoff $v(1,2) = \delta v - p(1,2) = \delta(v + (1-\delta)w) > 0$. Since $v(1,2) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(1,2) < v(1,2)$ – is not a profitable deviation. In this context, however, the leader could also deviate to making a non-relevant offer $\tilde{p}\,(1,2) < -\delta\,(1-\delta)\,w$ to transition to the state $(1,1)$. In this case she obtains a payoff $\delta v(1,1) = \delta^2(v+w)$. This is not a profitable deviation if $(1-\delta)v + (1-2\delta)w \geq 0$, which is always satisfied for large v. Then

$$w\,(1,2) = \frac{1}{2}\delta w_{out}\,(1,1) + \frac{1}{2}\delta w = \frac{1}{2}\delta(1+\delta)w$$

Consider next the state $(m,u) = (2,2)$, reached after the first follower chose to leave. Note that in this situation, both followers are critical (and hence there are no free riding opportunities). Thus leaving gives a follower a payoff of zero. An optimal relevant offer then satisfies

$$p\,(2,2) + \delta w_{out}\,(1,1) = 0 \Rightarrow p\,(2,2) = -\delta^2 w,$$

giving the leader a payoff of $v(2,2) = \delta v(1,1) - p(2,2) = \delta^2(v+2w) > 0$. As before, deviating to not making an offer only delays this outcome and is not a profitable deviation. And since the follower is pivotal, deviating to a non-relevant offer $\tilde{p}\,(2,2) < p(2,2)$ gives her a payoff of zero, and is not profitable. Therefore $w(2,2) = 0$, and $w_{out}(2,2) = \delta^2 w$.

Finally, consider the initial state $(2,3)$. Note that the optimal relevant offer by the leader must verify

$$p\,(2,3) + \delta w_{out}\,(1,2) = \delta \max\left\{w\,(2,3)\,, w_{out}\,(2,2)\right\}$$

Assume first that $w\,(2,3) \geq w_{out}\,(2,2)$. Then $p\,(2,3) = \delta[w\,(2,3) - w_{out}\,(1,2)]$, and

$$w(2,3) = \frac{1}{3}\delta w\,(2,3) + \frac{2}{3}\delta w(1,2) \Rightarrow w(2,3) = \frac{\delta^2(1+\delta)}{(3-\delta)}w < \delta^2 w = w_{out}\,(2,2)\,,$$

a contradiction. Thus, the follower prefers to leave than to simply reject the offer, and we must have $w\,(2,3) \leq w_{out}\,(2,2) = \delta^2 w$. Thus

$$p\,(2,3) = \delta w_{out}\,(2,2) - \delta w_{out}(1,2) = -(1-\delta)\delta^2 w,$$

giving the leader a payoff

$$v(2,3) = \delta v(1,2) - p(2,3) = \delta^2(v + 2(1-\delta)w) > 0.$$

Since $v(2,3) > 0$, deviating to not making an offer – which gives the leader a payoff of $\delta v(2,3) < v(2,3)$ – is not a profitable deviation. In this context, however, the leader could also deviate to making a non-relevant offer $\tilde{p}\,(2,3) < -(1-\delta)\delta^2 w$ to transition to the state $(2,2)$. In this case she obtains a payoff $\delta v(2,2) = \delta^3(v+2w)$. This is not a profitable deviation if $v(1-\delta)+2(1-2\delta)w \geq 0$, which is always satisfied for large v. Then

$$w(2,3) = \frac{1}{3}\delta^3 w + \frac{2}{3}\delta w(1,2) \Rightarrow w(2,3) = (1+2\delta)\,(\delta^2/3)w$$

□

We now consider followers' equilibrium payoffs in the competitive game. In this case the state is $(\vec{m}, u)$. As before, we assume that leaders have a high valuation for winning. We also assume that both leaders prefer to retain the status quo rather than losing to the competition; i.e.,

$\underline{v}_A < 0$ and $\underline{v}_B < 0$.[30] Under this conditions, we show that there is a full trading equilibrium, and that the followers' payoffs are given by

$$W(2,2,3) = \left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B\right].$$

Proof. Consider first the state $(\vec{m}, u) = (1,1,1)$, in which there is only one uncommitted follower, and both leaders need the support of only one additional follower to win. Suppose the follower meets with leader $\ell = A, B$, who makes him an offer. Note that leaving gives the follower a payoff of zero, while rejecting the offer gives him a payoff of $\delta W(1,1,1)$. Then an optimal relevant offer by ℓ satisfies

$$\delta w + p_\ell(1,1,1) = \max\{\delta W(1,1,1), 0\} \Rightarrow p_\ell(1,1,1) = \max\{\delta W(1,1,1), 0\} - \delta w,$$

Since this is the same for both leaders, we have that $W(1,1,1) = \max\{\delta W(1,1,1), 0\} \Rightarrow W(1,1,1) = 0$. It follows that $p_\ell(1,1,1) = -\delta w$, giving the leader a payoff of $V_\ell^\ell(1,1,1) = \delta\overline{v}_\ell - p_\ell(1,1,1) = \delta(\overline{v}_\ell + w) > 0$. Note that $\ell's$ payoff in state $(\vec{m}, u) = (1,1,1)$ after leader $j \neq \ell$ meets with the follower is $V_\ell^j(1,1,1) = \underline{v}_\ell < V_\ell^\ell(1,1,1)$. Thus, deviating to not making an offer gives the leader a payoff of

$$\delta V_\ell(1,1,1) = \delta[\pi_\ell V_\ell^\ell(1,1,1) + \pi_j V_\ell^j(1,1,1)] < V_\ell^\ell(1,1,1)$$

and is not profitable. Now consider a non-relevant offer, $\tilde{p}_\ell(1,1,1) < p_\ell(1,1,1)$. Since the follower is indifferent between rejecting or leaving, we have to consider two possibilities. If he leaves, the leader gets a payoff of zero. If he rejects, the leader gets a payoff of $\delta V_\ell(1,1,1)$. In both cases, this gives the leader a lower payoff than what she obtains in equilibrium, and thus a non-relevant offer is not a profitable deviation. To summarize, we have

$$\begin{aligned}
W(1,1,1) &= 0 \\
V_A(1,1,1) &= \delta\pi_A(\overline{v}_A + w_A) + \delta\pi_B\underline{v}_A \\
V_B(1,1,1) &= \delta\pi_A\underline{v}_B + \delta\pi_B(\overline{v}_B + w_B) \\
W_{out}(1,1,1) &= \delta(\pi_A w_A + \pi_B w_B)
\end{aligned}$$

Consider next the state $(\vec{m}, u) = (1,2,1)$. In this case there is one uncommitted follower, but B is two steps away from winning. It follows that by getting the follower's support, B can only assure that the status quo will prevail, but can not win. In this case leaving gives the follower a payoff of zero, so the best outside option is to reject, giving a payoff of $\delta W(1,2,1)$. Thus the optimal relevant offers satisfy

$$\begin{aligned}
p_B(1,2,1) &= \delta W(1,2,1) \\
p_A(1,2,1) + \delta w_A &= \delta W(1,2,1)
\end{aligned}$$

[30]The assumption that leaders have a high valuation for winning implies that in equilibrium meetings result in trades in all but two states. The exception is the state $(\vec{m}, u) = (1,2,1)$ – and symmetrically, the state $(\vec{m}, u) = (2,1,1)$ – in which leader B cannot win (she needs two additional supporters, but there is only one uncommitted follower) but can force a tie. Here equilibrium behavior depends on whether B prefers to retain the status quo rather than losing to A, or vice versa. For concreteness, we assume that both leaders prefer to retain the status quo rather than losing to the competition, i.e., $\underline{v}_A < 0$ and $\underline{v}_B < 0$. We then show that our main result is unchanged if leaders prefer the status quo to losing to the competition.

Leader $\ell = A, B$ prefers to make this offer instead of passing iff

$$\begin{aligned} 0 - p_B(1,2,1) &\geq \delta V_B(1,2,1) \\ \delta \overline{v}_A - p_A(1,2,1) &\geq \delta V_A(1,2,1) \end{aligned}$$

or equivalently

$$\begin{aligned} V_B(1,2,1) + W(1,2,1) &\leq 0 \\ V_A(1,2,1) + W(1,2,1) &\leq \overline{v}_A + w_A \end{aligned}$$

Note that in equilibrium

$$\begin{aligned} W(1,2,1) &= \pi_A \delta W(1,2,1) + \pi_B \delta W(1,2,1) = 0 \\ V_A(1,2,1) &= \pi_A (\delta \overline{v}_A - p_A(1,2,1)) = \pi_A \delta (\overline{v}_A + w_A) \\ V_B(1,2,1) &= \pi_A \delta \underline{v}_B \end{aligned}$$

Thus substituting,

$$\begin{aligned} \pi_A \delta \underline{v}_B &\leq 0 \\ \overline{v}_A(1 - \pi_A \delta) + w_A(1 - \pi_A \delta) &\geq 0 \end{aligned}$$

Note that the first inequality is satisfied iff $\underline{v}_B \leq 0$, while the second is satisfied for large $\overline{v}_A$. Thus, leaders prefer to make relevant offers than to pass. Note moreover that if leader ℓ deviates to a non-relevant offer, she gets $\delta V_\ell(1,2,0) = 0$, which is not a profitable deviation. Analogously, in state $(\vec{m}, u) = (2,1,1)$, we have

$$\begin{aligned} W(2,1,1) &= 0 \\ V_B(2,1,1) &= \pi_B \delta (\overline{v}_B + w_B) \\ V_A(2,1,1) &= \pi_B \delta \underline{v}_A \end{aligned}$$

Next, consider the state $(\vec{m}, u) = (1,2,2)$, where two followers remain uncommitted, A is one step from winning, and B is two steps from winning. Optimal relevant offers verify

$$\begin{aligned} p_A(1,2,2) + \delta w_A &= \delta \max\{W_{out}(1,2,1), W(1,2,2)\} \\ p_B(1,2,2) + \delta W_{out}(1,1,1) &= \delta \max\{W_{out}(1,2,1), W(1,2,2)\} \end{aligned}$$

Assume first that the equilibrium is such that $W(1,2,2) \geq W_{out}(1,2,1)$ which implies that

$$W(1,2,2) = (\delta/2) W(1,2,2) + \frac{1}{2}\pi_A \delta w_A + \frac{1}{2}\pi_B \delta W(1,1,1) \Rightarrow W(1,2,2) = \frac{\pi_A \delta}{2 - \delta} w_A.$$

Since we assume that $W(1,2,2) \geq W_{out}(1,2,1)$ it must be that

$$\frac{\pi_A \delta}{2 - \delta} w_A \geq \pi_A \delta w_A + \pi_B \delta W_{out}(1,1,0) = \pi_A \delta w_A \Rightarrow \delta \geq 1,$$

which is a contradiction. Therefore $W(1,2,2) < W_{out}(1,2,1)$, so that leaving is the relevant outside option. The offers are then given by

$$\begin{aligned} p_A(1,2,2) &= -\delta \left(\frac{1 - \delta}{1 - \delta \pi_B} \right) w_A \\ p_B(1,2,2) &= -\delta^2 \pi_B \left(w_B - \frac{\delta \pi_A}{1 - \delta \pi_B} w_A \right) \end{aligned}$$

Equilibrium payoffs are therefore

$$\begin{aligned}
W(1,2,2) &= \frac{1}{2}\delta W_{out}(1,2,1) + \frac{1}{2}\left(\pi_A \delta w_A + \pi_B \delta W(1,1,1)\right) \\
W_{out}(1,2,2) &= \pi_A \delta w_A + \pi_B \delta W_{out}(1,1,1) \\
V_A(1,2,2) &= \pi_A \left(\delta \overline{v}_A - p_A(1,2,2)\right) + \pi_B \delta V_A(1,1,1) \\
V_B(1,2,2) &= \pi_A \delta \underline{v}_B + \pi_B \left(\delta V_B(1,1,1) - p_B(1,2,2)\right)
\end{aligned}$$

or substituting,

$$\begin{aligned}
W(1,2,2) &= \frac{1}{2}\pi_A \delta w_A (1+\delta) \\
W_{out}(1,2,2) &= \pi_A \delta w_A + \pi_B \delta^2 [\pi_A w_A + \pi_B w_B] \\
V_A(1,2,2) &= \pi_A \delta \left(\overline{v}_A + \left(\frac{1-\delta}{1-\delta\pi_B}\right) w_A\right) + \pi_B \delta^2 [\pi_A(\overline{v}_A + w_A) + \pi_B \underline{v}_A] \\
V_B(1,2,2) &= \pi_A \delta \underline{v}_B + \pi_B \delta^2 \left([\pi_A \underline{v}_B + \pi_B(\overline{v}_B + w_B)] + \pi_B \left(w_B - \frac{\delta\pi_A}{1-\delta\pi_B} w_A\right)\right)
\end{aligned}$$

Each leader prefers to make a relevant offer than to pass iff

$$\begin{aligned}
\delta \overline{v}^A - p_A(1,2,2) &\geq \delta V_A(1,2,2) \\
\delta V_B(1,1,1) - p_B(1,2,2) &\geq \delta V_B(1,2,2)
\end{aligned}$$

Substituting, these are

$$\begin{aligned}
(1 - \delta\pi_A(1+\delta\pi_B))\overline{v}_A &\geq A \\
(1-\delta\pi_B)\overline{v}_B &\geq B
\end{aligned}$$

for some constants A and B that do not depend on $\overline{v}_A, \overline{v}_B$. Now $1 > \delta\pi_A(1+\delta\pi_B)$ iff $\delta\pi_A[1+\delta(1-\pi_A)] < 1$. LHS is increasing in δ, so it is enough to show that $\pi_A[2-\pi_A] \leq 1$. But the LHS of this expression is maximized at $\pi_A = 1$, attaining a value of 1. It follows that $\delta\pi_A(1+\delta\pi_B) < 1$, and hence that both inequalities are satisfied for large $\overline{v}_A, \overline{v}_B$.

Moreover, each leader prefers to make a relevant offer than to make a non-relevant offer iff

$$\begin{aligned}
\delta \overline{v}^A - p_A(1,2,2) &\geq \delta V_A(1,2,1) \\
\delta V_B(1,1,1) - p_B(1,2,2) &\geq \delta V_B(1,2,1)
\end{aligned}$$

or substituting,

$$\begin{aligned}
\overline{v}^A (1-\pi_A\delta) + \left(\frac{1-\delta}{1-\delta\pi_B}\right) w_A &\geq \pi_A \delta w_A \\
(\overline{v}_B + w_B) + \left(w_B - \frac{\delta\pi_A}{1-\delta\pi_B} w_A\right) &\geq 0
\end{aligned}$$

which again hold for high $\overline{v}_A, \overline{v}_B$, and we are done.

Analogously,

$$\begin{aligned}
W(2,1,2) &= \frac{1}{2}\pi_B\delta w_B(1+\delta) \\
W_{out}(2,1,2) &= \pi_B\delta w_B + \pi_A\delta^2[\pi_A w_A + \pi_B w_B] \\
V_B(2,1,2) &= \pi_B\delta\left(\overline{v}_B + \left(\frac{1-\delta}{1-\delta\pi_A}\right)w_B\right) + \pi_A\delta^2[\pi_B(\overline{v}_B + w_B) + \pi_A\underline{v}_B] \\
V_A(2,1,2) &= \pi_B\delta\underline{v}_A + \pi_A\delta^2\left([\pi_B\underline{v}_A + \pi_A(\overline{v}_A + w_A)] + \pi_A\left(w_A - \frac{\delta\pi_B}{1-\delta\pi_A}w_B\right)\right)
\end{aligned}$$

Consider next the state $(2,2,2)$, where there are two uncommitted followers and both leaders are still two steps away from winning. Since leaving gives a follower a payoff of $\delta W_{out}(2,2,1) = 0$, the relevant outside option is given by $\delta W(2,2,2)$ which implies that optimal relevant offers will satisfy

$$\begin{aligned}
p_A(2,2,2) + \delta W_{out}(1,2,1) &= \delta W(2,2,2) \\
p_B(2,2,2) + \delta W_{out}(2,1,1) &= \delta W(2,2,2)
\end{aligned}$$

It follows that

$$W(2,2,2) = \frac{1}{2}\delta W(2,2,2) + \frac{1}{2}\delta(\pi_A W(1,2,1) + \pi_B W(2,1,1))$$

and using $W(1,2,1) = W(2,1,1) = 0$, we have $W(2,2,2) = 0$, so that

$$\begin{aligned}
p_A(2,2,2) &= -\delta W_{out}(1,2,1) \\
p_B(2,2,2) &= -\delta W_{out}(2,1,1)
\end{aligned}$$

and equilibrium payoffs are given by

$$\begin{aligned}
W(2,2,2) &= 0 \\
W_{out}(2,2,2) &= \delta(\pi_A W_{out}(1,2,1) + \pi_B W_{out}(2,1,1)) \\
V_A(2,2,2) &= \pi_A(\delta V_A(1,2,1) - p_A(2,2,2)) + \pi_B\delta V_A(2,1,1) \\
V_B(2,2,2) &= \pi_A\delta V_B(1,2,1) + \pi_B(\delta V_B(2,1,1) - p_B(2,2,2))
\end{aligned}$$

which after substituting, become

$$\begin{aligned}
W(2,2,2) &= 0 \\
W_{out}(2,2,2) &= (\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B \\
V_A(2,2,2) &= (\delta\pi_A)^2(\overline{v}_A + w_A) + (\delta\pi_A)^2 w_A + (\delta\pi_B)^2\underline{v}_A \\
V_B(2,2,2) &= (\delta\pi_B)^2(\overline{v}_B + w_B) + (\delta\pi_B)^2 w_B + (\delta\pi_A)^2\underline{v}_B
\end{aligned}$$

Leaders prefer to make these offers to passing iff

$$\begin{aligned}
\delta V^A(1,2,1) - p_A(2,2,2) &\geq \delta V_A(2,2,2) \\
\delta V^B(2,1,1) - p_B(2,2,2) &\geq \delta V_B(2,2,2)
\end{aligned}$$

Substituting, these are

$$\begin{aligned}
\delta^2(1-\delta\pi_A)\pi_A(\overline{v}_A + w_A) - \pi_B\delta^2\pi_B\delta\underline{v}_A + \delta(1-\delta\pi_A)W_{out}(1,2,1) &\geq 0 \\
\delta^2(1-\delta\pi_B)\pi_B(\overline{v}_B + w_B) - \pi_A\delta^2\pi_A\delta\underline{v}_B + \delta(1-\delta\pi_B)W_{out}(2,1,1) &\geq 0
\end{aligned}$$

which always hold for high $\overline{v}_A, \overline{v}_B$. Similarly, it is easy to check that a deviation to making a non-relevant offer is not profitable either.

Finally, consider the initial state $(\vec{m}, u) = (2,2,3)$. First note that the offers must verify

$$\begin{aligned} p_A(2,2,3) + \delta W_{out}(1,2,2) &= \delta \max\{W(2,2,3), W_{out}(2,2,2)\} \\ p_B(2,2,3) + \delta W_{out}(2,1,2) &= \delta \max\{W(2,2,3), W_{out}(2,2,2)\} \end{aligned}$$

Assume first that $W(2,2,3) \geq W_{out}(2,2,2)$. Then

$$W(2,2,3) = \frac{1}{3}\delta W(2,2,3) + \frac{2}{3}(\pi_A \delta W(1,2,2) + \pi_B \delta W(2,1,2)),$$

and substituting,

$$W(2,2,3) = \frac{1}{(3-\delta)}\left((\delta\pi_A)^2 w_A(1+\delta) + (\delta\pi_B)^2 w_B(1+\delta)\right)$$

Since we have assumed that $W(2,2,3) \geq W_{out}(2,2,2)$, we need to verify that

$$\frac{1}{(3-\delta)}\left((\delta\pi_A)^2 w_A(1+\delta) + (\delta\pi_B)^2 w_B(1+\delta)\right) \geq (\delta\pi_A)^2 w_A + (\delta\pi_B)^2 w_B$$

which is false. Therefore, we must have that $W(2,2,3) \leq W_{out}(2,2,2)$ and the optimal relevant offers are

$$\begin{aligned} p_A(2,2,3) &= \delta[W_{out}(2,2,2) - W_{out}(1,2,2)] \\ p_B(2,2,3) &= \delta[W_{out}(2,2,2) - W_{out}(2,1,2)] \end{aligned}$$

The leaders prefer to make these offers to passing iff

$$\begin{aligned} \delta V_A(1,2,2) - p_A(2,2,3) &\geq \delta V_A(2,2,3) \\ \delta V_B(2,1,2) - p_B(2,2,3) &\geq \delta V_B(2,2,3) \end{aligned}$$

or substituting, iff

$$\begin{aligned} \overline{v}_A \pi_A[(1-\delta\pi_A)(1+\delta\pi_B) - (\delta)^2 \pi_B \pi_A] &\geq A \\ \overline{v}_B \pi_B[(1-\delta\pi_B)(1+\delta\pi_A) - (\delta)^2 \pi_A \pi_B] &\geq B \end{aligned}$$

where A and B are constants that do not depend on $\overline{v}_A, \overline{v}_B$. Now, $(1-\delta\pi_A)(1+\delta\pi_B) - (\delta)^2\pi_B\pi_A \geq 0$ iff $1+\delta \geq 2\delta\pi_A(1+\delta(1-\pi_A))$. The RHS of this inequality is maximized for a value of $\pi_A = (1+\delta)/2\delta$. Substituting, the previous inequality becomes $\delta \leq 1$. It follows that leaders prefer to make a relevant offer to passing for high $\overline{v}_A, \overline{v}_B$.

Similarly, the leaders prefer making relevant offers to making not relevant offers iff

$$\begin{aligned} \delta V_A(1,2,2) - p_A(2,2,3) &\geq \delta V_A(2,2,2) \\ \delta V_B(2,1,2) - p_B(2,2,3) &\geq \delta V_B(2,2,2) \end{aligned}$$

Substituting, we have

$$\begin{aligned} \overline{v}_A \delta \pi_A [1 + \delta(1-2\pi_A)] &\geq A \\ \overline{v}_B \delta \pi_B [1 + \delta(1-2\pi_B)] &\geq B \end{aligned}$$

where A and B are constants that do not depend on $\overline{v}_A, \overline{v}_B$. Since $1+\delta(1-2\pi_\ell) \geq 0$ for all $\delta, \pi_\ell \in (0,1)$, these inequalities always hold for high $\overline{v}_A, \overline{v}_B$, and we are done. The value for the uncommitted follower is then

$$W\left(2,2,3\right)=\left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A+(\delta\pi_B)^2 w_B\right]$$

□

We can now compare followers values in monopoly and competition. In monopoly

$$w(2,3)=(1+2\delta)\left(\delta^2/3\right)w$$

and in competition

$$W\left(2,2,3\right)=\left(\frac{1+2\delta}{3}\right)\left[(\delta\pi_A)^2 w_A+(\delta\pi_B)^2 w_B\right]$$

Let $\max\{w_A,w_B\}=w$. A monopoly of the best alternative is preferred to competition for all $w_A,w_B>0$ iff

$$(\pi_A)^2+(\pi_B)^2\leq 1$$

which is always true.

In the comparison above we maintained the assumption that leaders have a high valuation for winning. We also assumed that both leaders prefer to retain the status quo rather than losing to the competition; i.e., $\underline{v}_A<0$ and $\underline{v}_B<0$. These assumptions imply that while in state $(\vec{m},u)=(1,2,1)$ B cannot win (B needs two additional supporters, but there is only one uncommitted follower), it still makes a relevant offers in order to force a tie. Similarly, A makes a relevant offer in state $(\vec{m},u)=(2,1,1)$. The main result, however, doesn't depend on this assumption. Proceeding similarly as above we can show that if instead leaders prefer the status quo to losing to the competition, followers value is given by

$$W\left(2,2,3\right)=\frac{1}{3}\delta^2\left\{\left(\frac{1+\delta(1+\pi_A)}{1-\delta\pi_B}\right)(\pi_A)^2 w_A+\left(\frac{1+\delta(1+\pi_B)}{1-\delta\pi_A}\right)(\pi_B)^2 w_B\right\}$$

It follows that a monopoly of the best alternative is preferred to competition for all $w_A,w_B>0$ iff

$$(1+2\delta)\geq\left(\frac{1+\delta(1+\pi_A)}{1-\delta\pi_B}\right)(\pi_A)^2+\left(\frac{1+\delta(1+\pi_B)}{1-\delta\pi_A}\right)(\pi_B)^2$$

or equivalently,

$$\left(\frac{1+\delta(1+\pi_A)}{(1+2\delta)}\right)\frac{1}{1-\delta\pi_B}(\pi_A)^2+\left(\frac{1+\delta(1+\pi_B)}{(1+2\delta)}\right)\frac{1}{1-\delta\pi_A}(\pi_B)^2\leq 1$$

Note that since $\frac{1+\delta(1+\pi_\ell)}{(1+2\delta)}<1$, it is enough to show that

$$\frac{1}{1-\delta\pi_B}(\pi_A)^2+\frac{1}{1-\delta\pi_A}(\pi_B)^2\leq 1$$

But $1-\delta\pi_B>\pi_A$ and $1-\delta\pi_A>\pi_B$, so

$$\frac{1}{1-\delta\pi_B}(\pi_A)^2+\frac{1}{1-\delta\pi_A}(\pi_B)^2<\frac{1}{\pi_A}(\pi_A)^2+\frac{1}{\pi_B}(\pi_B)^2=\pi_A+\pi_B=1,$$

which establishes the result.

B.4. **Contingent Payments.** In the paper we assumed that leaders offer instantaneous cash transfers in exchange for a commitment of support. Transfers that occurred in the past are sunk, and hence do not affect the incentives in subsequent periods. Alternatively one can assume that the leader and the follower agree on a contingent transfer in exchange for support; a "partnership" offer instead of a buyout. This in fact seems the most appropriate assumption in some applications, as in the case of endorsements by party elders in presidential primaries. In this case candidates negotiate with party elders their support, but they do so in exchange of future promises.

Substituting cash for promises can change the conditions for existence of a fully competitive equilibrium, but does not alter the conclusions regarding the ranking of competition and monopoly. Let $\tilde{W}_{out}(\vec{m}'|p_j(\vec{m}))$ denote the value in state $\vec{m}' = (m'_A, m'_B)$ of a committed follower locked with a promise $p_j(\vec{m})$ acquired towards leader j in state $\vec{m} = (m_A, m_B)$. Note that

$$\begin{aligned}\tilde{W}_{out}(\vec{m}'|p_A(\vec{m})) &= \sum_t \Pr\left(j \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right) \delta^t [w_j + p_j(\vec{m})] \\ &\quad + \sum_t \Pr\left(\ell \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right) \delta^t w_\ell \\ &= W_{out}(\vec{m}') + \underbrace{\sum_t \Pr\left(j \text{ wins in } t \text{ periods} \,|\, \vec{m}'\right) \delta^t p_j(\vec{m})}_{\tilde{p}_j(\vec{m}', \vec{m})},\end{aligned} \tag{66}$$

where $W_{out}(\vec{m}')$ denotes the value of a committed follower in state $\vec{m}'$ in the cash game, and $\tilde{p}_j(\vec{m}', \vec{m})$ gives the expected value of the contingent transfer $p_j(\vec{m})$ in state $\vec{m}'$. Note then that the value function $\tilde{W}_{out}(\vec{m}'|p_\ell(\vec{m}))$ is separable in transfers and the value derived from implementing the alternative. Thus when ℓ meets an uncommitted follower in state $\vec{m}$, she offers a contingent payment $p_\ell(\vec{m})$ such that

$$\tilde{p}_\ell(\vec{m}^\ell, \vec{m}) + \delta W_{out}(\vec{m}^\ell) = \delta \tilde{W}(\vec{m}) \tag{67}$$

This implies that the continuation payoff of a follower after he meets one of the leaders is $\delta\tilde{W}(\vec{m})$ no matter what, and therefore the recursive representation of $\tilde{W}(\vec{m})$ is given by (16) as in the "cash" game, so that $\tilde{W}(q, q) = W(q, q)$; i.e., the value of the uncommitted follower at the beginning of the promises game is equal to the value in the cash game. This moreover implies by (67) that the expected value of the payment in the promise game is the same as in the cash game.

Now, to evaluate existence of a fully competitive equilibrium (for our large n results), we need to consider the value of the leader. And in this regard there is in fact a crucial difference with the benchmark cash model. Note that since promises are executed if and only when the leader wins, present exchanges now affect the incentives for future exchanges and must be incorporated on the value function. In particular, the relevant state in the promises game is composed of the number of additional followers that each leader needs in order to win, as before, but now also the stock of promises that a leader brings to the table when meeting another follower.

This difference complicates the algebra, but does not alter our main results. To see this note that after a leader wins, she obtains a payoff composed of a direct benefit $\overline{v}_\ell$ and a transfer from/to all committed followers. These two components are, indeed, additively separable. Moreover, this property still holds recursively, which implies that the value function of the leader in any state – now with the stock of promises as part of the state – is also additively separable in the utility for winning and the promises collected if and when she wins. It follows immediately that Proposition 5.1 extends to this case and a fully competitive equilibrium exists for sufficiently high $\overline{v}$ or $\overline{v} - \underline{v}$. A similar argument holds for the monopoly case, and the welfare comparison in the paper holds.

B.5. **Simultaneous Contracting in the Monopolistic Model.** Here we compare simultaneous and sequential contracting in the public goods model with a single alternative. To consider the static setting, we follow Segal and Whinston (2000). Segal and Whinston consider two contracting environments: one in which the leader can make discriminatory offers (allowing $p_i \neq p_j$ for followers i, j), and one in which the leader cannot discriminate among followers, so that she has to offer a single offer p. We consider each in turn.

As a benchmark, recall that in the sequential monopoly game, the leader's equilibrium payoff is given by

$$v\left(m\right) = \delta^m v + \left[\sum_{l=1}^{m} \left(1 - \delta \prod_{k=1}^{l} r(k)\right)\right] \delta^m w \tag{68}$$

As we explained in the paper, in the limit as $\delta \to 1$ followers' bargaining power is maximized, and the leader has no ability to extract rents, so that $v\left(m\right) \to v$. For $\delta < 1$, however, followers' bargaining power decreases, and the leader can in fact extract rents from the followers. The effect of reducing the discount factor on the leader's value at the beginning of the game then exhibits a tradeoff between a higher ability to extract rents from followers and a larger loss from discounting. When w is small, the effect of discounting dominates, and the leader's value increases monotonically with δ approaching v as $\delta \to 1$. When w is larger, the rent extraction effect dominates for high δ, and the leader's equilibrium payoff is maximized for a $\tilde{\delta} \in (0, 1)$. For low enough δ, of course, the discounting effect dominates, and the leader's value falls monotonically with δ, with $v\left(m\right) \to 0$ as $\delta \to 0$. In this case the simultaneous game obviously gives the leader a larger payoff than the sequential offer game.

Consider next the static game with *nondiscriminatory offers*. First, note that there exists an equilibrium in which all followers accept offers $p \geq 0$ and reject offers $p < 0$, and the leader offers $p = 0$. For any $p \geq 0$ all followers accept, so a follower i cannot gain by deviating to reject (would only lose p). For $p < 0$, all followers reject in equilibrium. If i were to deviate and accept, she would lose p. Given followers' strategy, the leader cannot gain by offering $p' \neq 0$: any $p' > 0$ would only mean larger transfers to the followers, and $p' < 0$ would lead her to lose. Note that neither leader nor followers are using weakly dominated strategies. In this equilibrium, the leader obtains a payoff of v, and does not extract rents from the followers.

There also exists an equilibrium in which the leader obtains a payoff $v + \frac{n+1}{2}w$. Suppose that for any $p \in [-w, 0)$ followers $1, 2, \ldots, (n+1)/2$ accept, and $(n+3)/2, \ldots, n$ reject, for $p \geq 0$ all followers accept, and for all $p < -w$ all followers reject. The leader offers $p = -w$. Consider first $p \in [-w, 0)$. Followers $i \geq (n+3)/2$ don't have an incentive to deviate, for the leader is already winning, and would only do worst by accepting the offer. Followers $i \leq (n+1)/2$ are pivotal, and thus as long as $p \geq -w$, have no profitable deviations. As before, for $p \geq 0$ or $p < -w$, followers have no profitable deviations. The leader doesn't have a profitable deviation. In equilibrium, she obtains a payoff $v + \frac{n+1}{2}w$. If she were to offer $p < -w$, she would get $0 < v + \frac{n+1}{2}w$. If she were to offer $p \in (-w, 0)$ she would only be reducing rent extraction. If she were to offer $p \geq 0$ she would get $v < v + \frac{n+1}{2}w$.

Next consider the static game with discriminatory offers. As in the previous case, with discriminatory offers there is a SPE in which the leader can extract all the surplus from $(n+1)/2$ followers. In particular, suppose that the leader offers $p = -w$ to a set I of $(n+1)/2$ followers, and $\tilde{p} < -w$ to a set O of $(n-1)/2$ followers, and that followers accept any offer $p_i \geq -w$ and reject any offer $p_i < -w$. Followers in O do not have a profitable deviation, since accepting gives them a payoff $w - \tilde{p} < 0$. Followers $i \in I$ get an equilibrium payoff of zero. But since each $i \in I$ is pivotal in equilibrium, rejecting doesn't improve his payoff. The leader cannot gain by proposing a different $\vec{p}$: whenever more than $(n+1)/2$ followers accept offers $p_i < 0$ for sure, any one follower would prefer to deviate and reject the offer. And increasing offers in I can only hurt the leader since it would decrease rents and not improve the probability of winning. The leader wins with probability one, and makes a payoff $v + w(n+1)/2$.

Is there an equilibrium without WDS in which the leader only makes v? Suppose all followers accept any offer $p_i \geq 0$ and reject any offer $p_i < 0$. Suppose the leader offers 0 to a set I of $(n+1)/2$ followers and $p \in (-w, 0)$ to a set O of $(n-1)/2$ followers. Consider a follower $i \in O$. In equilibrium, he gets w. By deviating and accepting the offer, he gets $w + p_i < w$. The followers in I don't have incentives to deviate either. In equilibrium the leader wins with probability one, and then gets a payoff v. Given the strategy of the followers, she cannot profitably deviate. This strategy profile thus is a SPE. Moreover, note that no player uses a weakly dominated strategy.

Both with discriminatory and non-discriminatory offers, there is an equilibrium in which the leader makes a payoff v and an equilibrium in which the leader makes a payoff $v + w(n+1)/2$. Note that the parenthesis in (68) is $\left(1 - \delta \prod_{k=1}^{l} r(k)\right) < 1$. Substituting,

$$v\left(q\right) < \delta^{q}\left[v + \left(\frac{n+1}{2}\right)w\right] < v + \left(\frac{n+1}{2}\right)w$$

Since $v < v(q)$ for w, δ large, it follows that in this case there is an equilibrium in the static game in which the leader obtains a payoff that is larger than her payoff in the unique equilibrium with sequential offers, but also equilibria in which the opposite holds.

Conclusion. For low δ and/or sufficiently small w, the leader's payoff with simultaneous offers is higher than in the sequential game. For w, δ large, instead, both with discriminatory and non-discriminatory offers there is an equilibrium in which the leader would prefer simultaneous to sequential offers, but also equilibria in which the opposite holds.