

Web Appendix: Contracts as a barrier to entry in markets with non-pivotal buyers

Özlem Bedre-Defolie ^{*†} Gary Biglaiser [‡]

January 16, 2017

Abstract

In this Appendix we extend our model to an alternative setup where the intrinsic preference parameter is interpreted as a consumer's mismatch value from the entrant's product relative to the incumbent's. We also extend the main findings to a more general distribution of preference parameter, s , that satisfies the Increasing Hazard Rate Property and has a log-concave cumulative distribution function.

*We would like to thank Jim Anton, Felix Bierbrauer, Meghan Busse, Dominik Grafenhofer, Michal Grajek, Paul Heidhues, Martin Hellwig, Bruno Julien, Fei Li, Simon Loertscher, Claudio Mezzetti, Markus Reisinger, Andrew Rhodes, Tommaso Valletti, Keith Waehrer, Florian Zettelmeyer, participants at many conferences, three anonymous referees, and the editor for helpful comments. The authors declare that they have no relevant or material financial interests that relate to the research described in this paper.

[†]European School of Management and Technology (ESMT), Berlin, ozlem.bedre@esmt.org.

[‡]University of North Carolina, gbiglais@email.unc.edu

1 Mismatch value model

We consider a two-period model of entry. In the first period there is only one firm, the incumbent (I), and in the second period the incumbent faces one entrant (E). Mass 1 of consumers are willing to buy one unit in each period. The value of consuming the incumbent's good in each period is v , while the value of consuming the entrant's good is $v - s$. Consumers incur mismatch value s of the entrant's product relative to the incumbent's, where s is uniformly distributed over $[0, \theta]$. We interpret s as a consumer's perception of how inferior the entrant's product is compared to the incumbent's. Following Chen [1997], consumers learn their preference parameter s at the beginning of period 2. Firms never observe s and know only its distribution.

We assume that the entrant is more efficient than the incumbent in production. Let c_I and c_E denote the marginal cost of the incumbent and the entrant, respectively. The efficiency advantage of the entrant is denoted by $\Delta c \equiv c_I - c_E > 0$.

The timing of the contracting is the following:

Period 1 The incumbent offers a long-term (LT) contract specifying three prices: p_{I1} is the price for buying one unit in period 1, p_{I2} is the price for buying an additional unit from the incumbent in period 2, and d is the breakup fee to be paid by the buyer who signed the incumbent's LT contract and does not buy from it in period 2. The incumbent offers a Most-Favored-Nation clause (MFN) making it free for consumers to switch from its long-term contract to the spot contract in period 2.¹ Consumers decide whether to accept or reject the LT contract.

Period 2 Consumers learn their match value s . Simultaneously, the incumbent offers a spot price p_{I2}^S and the entrant offers a price p_E . Consumers who signed the incumbent's LT contract and decide to buy a unit from it are able to purchase at the incumbent's lowest price (the minimum of p_{I2} and p_{I2}^S) due to an MFN clause. Consumers who signed the incumbent's LT contract and switch to the entrant, pay p_E to the entrant and pay d to the incumbent. Consumers who signed the incumbent's LT contract and do not buy in period 2 just pay d to the incumbent. Consumers who did not sign the incumbent's LT contract choose between p_{I2}^S , p_E and no purchase.

We assume that consumers' valuation from the product is sufficiently high so that consumers will always buy a product in period 2 in equilibrium.

Assumption 1 $v > \max\{\frac{2\theta + c_E + 2c_I}{3}, 4\theta + c_E\}$.

¹We will demonstrate that the incumbent prefers to offer an MFN clause in equilibrium.

The first term ensures that the market is covered when the incumbent accommodates the entrant and the second term ensures that the market is covered, out-of-equilibrium, when the incumbent forecloses the entrant, all consumers who did not sign the LT contract will make a purchase in period 2. We look for a Subgame Perfect Nash Equilibrium where the firms never price below their marginal costs.

Efficient benchmark: In the efficient outcome, all consumers buy a unit from the incumbent in period 1, consumers with high match value ($s \geq \Delta c$) purchase an additional unit from the incumbent, and those with low match values ($s < \Delta c$) switch and buy a unit from the entrant in period 2.

2 Equilibrium Analysis

In equilibrium the incumbent's spot price should not be below the second unit price of the LT contract, $p_{I2}^S \geq p_{I2}$, since otherwise the incumbent would have an incentive to undercut p_{I2} , which would make p_{I2} irrelevant since consumers would buy at p_{I2}^S .

First we discuss consumers' period 2 purchasing decisions. If a consumer did not sign the incumbent's LT contract, she buys a unit from the incumbent if her mismatch value of the entrant's product is greater than the difference between the incumbent's spot price and the entrant's price: $s \geq p_{I2} - p_E$, and the incumbent's price is lower than the valuation of buying a unit from the incumbent: $p_{I2} \leq v$. On the other hand, if her mismatch value is small enough, $s < p_{I2} - p_E$, she buys from the entrant as long as the entrant's price is lower than the valuation of buying a unit from the entrant: $p_E \leq v - s$. Otherwise, she buys nothing.

If a consumer signed the incumbent's LT contract, she buys a unit from the incumbent if her mismatch value of the entrant's product is greater than the difference between the incumbent's lowest period 2 price and the entrant's price plus the breakup fee: $s \geq p_{I2} - p_E - d$. On the other hand, if her mismatch value is small enough, $s < p_{I2} - p_E - d$, she buys from the entrant as long as the entrant's price plus the breakup fee is lower than the valuation of buying a unit from the entrant: $p_E + d \leq v - s$. Otherwise, she buys nothing.

Consumers' beliefs about the other consumers' behaviour affect the expected price of the entrant, p_E^* . This might generate multiplicity of equilibria depending on how many consumers signed the LT contract. We first prove the existence of an equilibrium where all consumers sign the incumbent's LT contract and then show that this is the unique equilibrium.

If everyone signed the incumbent's LT contract, in period 2 equilibrium the entrant will behave like a Stackelberg follower, since the incumbent's LT contract's second unit price

determines its second period price.

A consumer's outside option to signing the incumbent's LT contract is the expected surplus from consumption only in period 2

$$EU_{nosignI} = v - p_{I2} Prob(s \geq p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E^*} (s + p_E^*) \frac{1}{\theta} ds. \quad (1)$$

where p_E^* is a function of p_{I2} and d . The consumer's outside option is therefore endogenous.

The incumbent can use breakup fees as a tool to affect consumers' beliefs about what the entrant's price will be. In period 1, the incumbent has two options:

Option 1: It can choose a breakup fee low enough to accommodate entry: $p_{I2} - d \geq c_E$. In the equilibrium of this sub-game some (or all) consumers buy a unit from the entrant in period 2. A consumer's expected utility from signing the incumbent's LT contract is

$$EU_{signI} = v - p_{I1} + v - p_{I2} Prob(s \geq p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^* + d) \frac{1}{\theta} ds, \quad (2)$$

which is the net surplus of consuming a unit from the incumbent in period 1 plus the expected surplus of consuming a unit in period 2: Consumers buy a unit from the incumbent with the probability that the incumbent's differentiation value is high enough, $s \geq p_{I2} - p_E - d$. Otherwise, they buy a unit from the entrant at cost $p_E^* + d$.

Consumers prefer to sign the incumbent's LT contract if and only if the expected surplus from signing it is greater than the outside option: $EU_{signI} \geq EU_{nosignI}$. The incumbent has to compensate consumers for the expected amount of breakup fee payments by lowering p_{I1} to induce them to sign the LT contract. This is because in equilibrium consumers buy one unit in each period. Breakup fees are pure transfers between consumers and the incumbent; they cannot be used as a tool to shift rent from the entrant. Hence, for the second period equilibrium only $p_{I2} - d$ matters, but not each of these prices separately. The incumbent is the Stackelberg leader when setting $p_{I2} - d$ and E is the follower: $p_E^*(p_{I2} - d)$.

In equilibrium, the incumbent sets prices such that consumers get exactly their expected outside option: $EU_{signI} = EU_{nosignI}$. We demonstrate in the Appendix that if $\Delta c > 2\theta$, the incumbent cannot profitably compete against the entrant, so sets $p_{I2}^* - d^* = c_I$, which accommodates entry since $c_I > c_E$. The entrant sets $p_E^* = c_I - \theta$, efficiently selling to all consumers in period 2 and capturing its cost advantage after compensating consumers for the highest mismatch value. The incumbent captures its static monopoly profit by collecting $p_{I1}^* + d^* = v$ upfront.² However, if $\Delta c \leq 2\theta$, the incumbent can profitably sell to some

²The incumbent is indifferent between any level of d as long as $d < \Delta c$, that is, breakup fees are washed out in the incumbent's profit since the incumbent has to compensate consumers ex-ante for expected amount

consumers in period 2 and its equilibrium profit from accommodating entry is

$$\Pi_I^* = v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta} + \frac{\theta}{2}. \quad (3)$$

Option 2: The incumbent can foreclose the entrant by setting a high enough breakup fee: $p_{I2} - d < c_E$. In the equilibrium of this sub-game, the entrant cannot profitably attract any consumer from the incumbent's LT contract and competes for the consumers who did not sign it. These are zero measure of consumers, since we look for an equilibrium where all consumers signed the LT contract of the incumbent. In the spot market the incumbent does not undercut its LT contract's second unit price, since lowering price below p_{I2} would lead to a loss from measure one of consumers and some gains from measure zero of consumers. The entrant would behave as Stackelberg follower reacting to the LT contract's second unit price. As a result, consumers' outside option of not signing the LT contract, which is given by (1), is above zero and decreasing in the entrant's marginal cost.

A consumer's expected utility from signing the incumbent's LT contract is

$$EU_{signI}^F = 2v - p_{I1} - p_{I2}. \quad (4)$$

In the Appendix we show that if $\Delta c \leq 2\theta$ the incumbent's profit from foreclosure is

$$\Pi_I^{*F} = v - c_I + 2\theta - \Delta c, \quad (5)$$

which is decreasing in the entrant's cost advantage. More importantly, the incumbent's equilibrium profit from foreclosure is decreasing faster than profit from accommodating entry, (3). Comparing these profits we prove our main result:

Proposition 1 *In the unique equilibrium all consumers sign the incumbent's long-term contract.*

- *If $\Delta c > 2\theta$, all consumers efficiently buy a unit from the entrant in period 2. The equilibrium profits and consumer utility are*

$$\begin{aligned} \Pi_I^* &= v - c_I, \Pi_E^* = \Delta c - \theta, \\ U^* &= v - c_I + \theta/2. \end{aligned}$$

- *If $\Delta c \leq 2\theta$, there is inefficient foreclosure of the entrant. The equilibrium profits and*

of breakup fees to convince them to sign the LT contract.

consumer utility are

$$\begin{aligned}\Pi_I^* &= v - c_I + 2\theta - \Delta c, \Pi_E^* = 0, \\ U^* &= v - c_E - 2\theta.\end{aligned}$$

Note that in equilibrium either the incumbent or the entrant sells to all consumers in period 2. We will discuss below why breakup fees are crucial to have this bang-bang allocation of consumers in period 2.

Recall that the efficiency requires all consumers to switch to the entrant whenever $\Delta c > \theta$. Proposition 3 illustrates that when $\Delta c \leq 2\theta$ the entrant is inefficiently foreclosed. Hence, the distortion from foreclosure rises in Δc until Δc exceeds 2θ . Alternatively, when the highest mismatch value to the entrant's product, θ , increases, the range of inefficient foreclosure expands.

We show in the Appendix that the equilibrium is unique regardless of what consumers believe about the others' behaviour. Suppose each consumer believes that $0 \leq \alpha < 1$ fraction of consumers sign the incumbent's LT contract. We show that the expected utility from signing it is strictly greater than the expected utility from not signing it. Hence, even if consumers would benefit from coordinating and not-signing the incumbent's contract, they unilaterally prefer to sign it, regardless of what they believe about the others' behaviour. This gives us the unique equilibrium where all consumers sign the incumbent's LT contract.

3 Critical factors for the inefficient foreclosure

Like in the benchmark model of the paper, several assumptions are critical for the foreclosure result: a breakup fee and a Most-Favored-Nation (MFN) clause in the incumbent's long-term contract, non-pivotal buyers, entrant market power. We discuss here only the role of breakup fees, since the other factors remain to be critical due to the same arguments as the benchmark.

3.1 Breakup fees

When the incumbent is not allowed to use a breakup fee, there are no inter-temporal effects, since any consumer who signed the incumbent's LT contract could buy at the lowest price of the incumbent or switch to the entrant in period 2. This means that the incumbent and the entrant compete for all consumers (regardless of the number of consumers who signed the incumbent's LT contract). The solution to the simultaneous differentiated duopoly

competition determines the second period equilibrium prices. The consumer gets the same expected surplus from period 2 consumption whether she signs the incumbent's LT contract or not, and this does not depend on what she believes about the other consumers' behavior. This implies that each consumer wants to sign the incumbent's contract if and only if $p_{I1} \leq v$. At the optimal solution the incumbent sets $p_{I1}^* = v$ as if it was a static monopolist. From this we show the equilibrium when breakup fees are prohibited as:

Proposition 2 *If breakup fees are prohibited, there are no inter-temporal effects. In the unique equilibrium all consumers sign the incumbent's long-term contract.*

- *If $\Delta c > 2\theta$, all consumers efficiently buy from the entrant in period 2.*
- *If $\Delta c \leq 2\theta$, the entrant sells to some positive measure (< 1) of consumers.*
 - *If $\frac{\theta}{2} \leq \Delta c < 2\theta$, too few consumers buy from the entrant.*
 - *If $\Delta c < \frac{\theta}{2}$, too many consumers buy from the entrant.*

Efficiency requires that consumers switch to the entrant if and only if $s < \Delta c$. Proposition 2 implies that if $\Delta c > 2\theta$, all consumers efficiently buy from the entrant. However, if $\Delta c \leq 2\theta$, we show in the Appendix that in equilibrium consumers of type $s < \frac{\theta + \Delta c}{3}$ buy from the entrant. At $\Delta c = \frac{\theta}{2}$, the marginal type is exactly the difference in cost and so we get efficient allocation in equilibrium. As Δc decreases from 2θ down to $\theta/2$, the price difference is less than the cost difference and hence too few people buy from the entrant. For values of Δc smaller than $\theta/2$, the equilibrium price difference is larger than the cost difference and so too many people buy from the entrant.

To further understand the intuition behind the result first note that the difference between the incumbent's and the entrant's price in equilibrium, $p_{I2}^* - p_E^* = \frac{\theta + \Delta c}{3}$, determines the marginal consumer type in period 2, which is increasing in Δc . When the entrant becomes more efficient (Δc increases), the entrant's price decreases more than the incumbent's second unit price, so the marginal type increases. Alternatively, when the incumbent becomes more inefficient (Δc increases), the incumbent's second period price increases more than the entrant's price, so the marginal type increases.

Policy implications of banning breakup fees: Now we analyze under which conditions prohibition of breakup fees improves efficiency.

Corollary 1 *When a regulator can ban breakup fees,*

- *the ban is inconsequential for the equilibrium allocation if $\Delta c > 2\theta$;*

- the ban improves welfare if $\frac{\theta}{5} \leq \Delta c \leq 2\theta$;
- the ban reduces welfare if $\Delta c < \frac{\theta}{5}$.

In the case where the entrant is very efficient, $\Delta c > 2\theta$, Propositions 3 and 2 show that in equilibrium all consumers efficiently buy from the entrant and so breakup fees do not matter. When $\Delta c \leq 2\theta$, the second-period welfare with breakup fees is $W^* = v - c_I$. The second-period welfare without breakup fees is

$$W_{d=0}^* = v - Pr\left(s < \frac{\theta + \Delta c}{3}\right)c_E - Pr\left(s \geq \frac{\theta + \Delta c}{3}\right)c_I - \int_0^{\frac{\theta + \Delta c}{3}} \frac{s}{\theta} ds. \quad (6)$$

It is then straightforward to show that when $\frac{\theta}{5} \leq \Delta c \leq 2\theta$ the welfare without breakup fees is larger than the welfare when breakup fees are allowed. A prohibition of breakup fees changes the firms' pricing and so the allocation of consumers in equilibrium. As a result, whether banning breakup fees improves welfare depends on the efficiency advantage of the entrant relative to the incumbent. When the entrant's efficiency advantage is very low, banning breakup fees is detrimental to the allocative efficiency. This is because without breakup fees the entrant will have too many consumers Proposition 2, and it is more efficient for the incumbent to serve all consumers. However, when the entrant's efficiency advantage is very high but low enough for the incumbent to make sales in period 2, then banning breakup fees is an efficient regulatory intervention. If it is possible for a regulator to control the level of breakup fees, e.g., via a binding cap on breakup fees, then the regulator could in principle implement the efficient allocation. This would clearly require the regulator to have a great deal of knowledge on all relevant market features.

4 Extension: More general distribution of the preference parameter

Assume that consumers' mismatch value from the entrant's product relative to the incumbent's, s , is distributed over $[0, \theta]$ with pdf $f(\cdot)$ and cdf $F(\cdot)$. Assume also that $F(\cdot)$ is log-concave and satisfies the Increasing Hazard Rate Property (IHRP), which is equivalent to $\frac{1-F}{f}$ being decreasing, that is,

$$\frac{-f^2 - (1-F)f'}{f^2} < 0 \Leftrightarrow f^2 + (1-F)f' > 0. \quad (7)$$

The log-concavity of $F(\cdot)$ means that $\log(F(s))$ is concave:

$$[\log(F(s))]'' = \left(\frac{f}{F}\right)' = \frac{f' \cdot F - f^2}{F^2} < 0 \Leftrightarrow f' \cdot F - f^2 < 0 \quad (8)$$

Note that if f' is positive $F(\cdot)$ satisfies the *IHRP*, but might fail to be log-concave. If f' is negative, $F(\cdot)$ is log-concave, but might fail to satisfy the *IHRP*. We keep other assumptions and features of the model unchanged.³

Proposition 3 *If $F(s)$ is log-concave and satisfies the Increasing Hazard Rate Property (IHRP), there exists an equilibrium where all consumers sign the incumbent's long-term contract.*

- If $\Delta c > \theta + \frac{1}{f(\theta)}$, all consumers efficiently buy a unit from the entrant in period 2. The equilibrium profits and consumer utility are

$$\begin{aligned} \Pi_I^* &= v - c_I, \Pi_E^* = \Delta c - \theta, \\ U^* &= v - c_I + \theta - E[s]. \end{aligned}$$

- If $\Delta c \leq \theta + \frac{1}{f(\theta)}$, there is inefficient foreclosure of the entrant.

Appendices

Proof of Proposition 1: In period 1 the incumbent has two options: accommodate the entrant by setting $p_{I2} - d \geq c_E$ or foreclose the entrant by $p_{I2} - d < c_E$. We now derive the incumbent's expected profit from each option to determine the incumbent's equilibrium strategy.

Period 2: If none of the consumers signed the incumbent's LT contract in period 1, the incumbent and the entrant are differentiated competitors for all consumers in period 2 and the solution to the differentiated duopoly competition determine the equilibrium prices. The entrant's demand will be $D_E = \text{Prob}(s < p_{I2}^S - p_E)$ and the incumbent's demand will be $D_{I2} = \text{Prob}(s \geq p_{I2}^S - p_E)$. The incumbent sets p_{I2}^S by maximizing its second period profit

$$\Pi_{I2} = (p_{I2}^S - c_I) \frac{\theta - p_{I2}^S + p_E}{\theta}.$$

³We note that the equilibrium characterized in Proposition 1 is unique symmetric equilibrium.

The best-reply of the incumbent to the entrant's price is $p_{I2}^{S*}(p_E) = \frac{\theta + p_E + c_I}{2}$ if at that price it has some positive demand: $p_{I2}^{S*}(p_E) - p_E < \theta$. Otherwise, the incumbent cannot compete against the entrant and sets $p_{I2}^{S*} = c_I$.

The entrant sets p_E by maximizing its profit

$$\Pi_E = (p_E - c_E) \frac{p_{I2}^S - p_E}{\theta}.$$

The best-reply of the entrant to the incumbent's spot price is $p_E^*(p_{I2}^S) = \frac{p_{I2}^S + c_E}{2}$ if at this price the entrant does not attract all consumers: $p_{I2}^S - p_E^*(p_{I2}^S) < \theta$. Otherwise, the entrant sets $p_E^*(p_{I2}^S) = p_{I2}^S - \theta$ and sells to all consumers.

If $\Delta c < 2\theta$, the interior solution to the duopoly competition is

$$\begin{aligned} p_{I2}^{S*} &= \frac{2\theta + c_E + 2c_I}{3}, p_E^* = \frac{\theta + 2c_E + c_I}{3} \\ D_{I2}^{S*} &= \frac{2\theta - \Delta c}{3\theta}, D_E^* = \frac{\theta + \Delta c}{3\theta}. \end{aligned} \quad (9)$$

We verify that $v > p_{I2}^{S*} = \frac{2\theta + c_E + 2c_I}{3}$ by Assumption 1 and so the incumbent gets some consumers.

If $\Delta c \geq 2\theta$, the equilibrium prices and demands are

$$p_{I2}^{S*} = c_I, p_E^* = c_I - \theta, D_{I2}^{S*} = 0, D_E^* = 1. \quad (10)$$

In equilibrium we must have $p_{I2} \leq p_{I2}^{S*}$, since otherwise the incumbent would have a profitable deviation from p_{I2} to implement p_{I2}^{S*} in period 2 and so p_{I2} would not be paid by any consumer. Given $p_{I2} \leq p_{I2}^{S*}$, the entrant's best-reply will be $p_E^* = \frac{p_{I2} + c_E}{2}$.

Suppose that all consumers signed the incumbent's first period contract where the incumbent sets $p_{I2} - d \geq c_E$ allowing the entrant to have some sales in period 2. Given the period 1 contract, the entrant's demand in period 2 is $D_E = Prob(s < \min\{p_{I2}^S, p_{I2}\} - p_E - d)$ and the incumbent's period 2 demand is $D_{I2}^S = Prob(s \geq \min\{p_{I2}^S, p_{I2}\} - p_E - d)$. By solving the problem of the incumbent and the entrant, we show that if $\Delta c < 2\theta$, the interior solution to the duopoly competition is the same as (23) noting that the net price of buying from the incumbent is equal to its spot price when none of the consumers signed the LT in period 1: $p_{I2}^{S*} - d = \frac{2\theta + c_E + 2c_I}{3}$. Similarly, if $\Delta c \geq 2\theta$, the equilibrium prices and demands will be the same as (22) where the net price of the incumbent is equal to its spot price when none of the consumers signed the LT in period 1: $p_{I2}^{S*} - d = c_I$.

In equilibrium we must have $p_{I2} \leq p_{I2}^{S*}$, since otherwise the incumbent would have a profitable deviation from p_{I2} to implement p_{I2}^{S*} in period 2 and so p_{I2} would not be paid

by any consumer. Given $p_{I2} \leq p_{I2}^{S*}$ and $p_{I2} - d \geq c_E$, the entrant's best-reply will be $p_E^* = \frac{p_{I2} - d + c_E}{2}$.

Period 1: We will first show that there exists an equilibrium where each consumer signs the incumbent's long-term contract and then show that this is the unique equilibrium for any belief that each consumer has on the others' behaviour.

Suppose that each consumer expects every other consumer to sign the incumbent's first period contract. A consumer's expected utility if she does not sign the incumbent's first contract is

$$\begin{aligned} EU_{nosignI} &= v - p_{I2} \text{Prob}(s \geq p_{I2} - p_E^*) - p_E^* \text{Prob}(s < p_{I2} - p_E^*) \\ &= v - p_{I2} + \frac{(p_{I2} - p_E^*)^2}{2\theta}, \end{aligned} \quad (11)$$

where the entrant's best-reply is $p_E^* = \frac{p_{I2} - d + c_E}{2}$. A consumer's expected utility if she signs the incumbent's first contract is equal to

$$EU_{signI} = 2v - p_{I1} - p_{I2} \text{Prob}(s \geq p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^* + d) \frac{1}{\theta} ds.$$

The incumbent maximizes its profit subject to the consumers' participation constraint, $EU_{signI} \geq EU_{nosignI}$, the second period price equilibrium, $p_{I2} \leq p_{I2}^{S*}$, and the constraint that ensures some positive sales by the entrant, $p_{I2} - d < c_E$:

$$\max_{p_{I1}, p_{I2}, d} \Pi_I = [p_{I1} - c_I + (p_{I2} - c_I) \text{Prob}(s \geq p_{I2} - p_E^* - d) + d \text{Prob}(s < p_{I2} - p_E^* - d)]$$

subject to:

- (i) $EU_{signI} \geq EU_{nosignI}$,
- (ii) $p_{I2} \leq p_{I2}^{S*}$
- (iii) $p_{I2} - d \geq c_E$

At the optimal solution the incumbent sets the highest p_{I1} satisfying the participation constraint, (i):

$$p_{I1}^* = v + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta} - p_{I2} \text{Prob}(s \geq p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^* + d) \frac{1}{\theta} ds.$$

Replacing the latter into the incumbent's profit we rewrite its problem:

$$\max_{p_{I2}, d} \Pi_I = [v - c_I + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta} - c_I \text{Prob}(s \geq p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^*) \frac{1}{\theta} ds]$$

subject to (ii) and (iii). Given that $p_E^* = \frac{p_{I2} - d + c_E}{2}$ the first-order conditions with respect to p_{I2} and d are, respectively

$$\begin{aligned} 2p_{I2} &= d + c_I + c_E + 2\theta, \\ 2d &= p_{I2} - c_I. \end{aligned}$$

If $\Delta c < 2\theta$ the solution to the previous first-order conditions gives us the equilibrium prices:

$$\begin{aligned} p_{I2}^* &= \frac{c_I + 2c_E + 4\theta}{3}, d^* = \frac{2\theta - \Delta c}{3} \\ p_E^* &= \frac{\theta + 2c_E + c_I}{3} \end{aligned}$$

since the unconstrained optimal prices satisfy the second period price constraint: $p_{I2}^* = p_{I2}^{S*}$,⁴ and the constraint ensuring some positive sales to the entrant: $p_{I2}^* - d^* = \frac{c_E + 2c_I + 2\theta}{3} > c_E$ given that $c_I > c_E$ and $\theta > 0$. On the other hand, if $\Delta c \geq 2\theta$, the incumbent cannot compete against the entrant, so sets $p_{I2}^* - d^* = p_{I2}^{S*} - d^* = c_I$ and $c_I - d^* > c_E$ (accommodating the entrant), and the entrant reacts by setting $p_E^* = c_I - \theta$. In this case, the entrant sells all consumers in period 2 (this is efficient) and the incumbent captures its static monopoly profit, $\Pi_I^* = v - c_I$, by collecting $p_{I1}^* = v - d^*$ upfront.⁵

The incumbent's equilibrium profit if it accommodates the entrant is

$$\Pi_I^* = v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta} + \frac{\theta}{2}, \quad (12)$$

and the consumers' equilibrium utility is

$$U^* = v - \frac{\theta + 2c_E + c_I}{3} - \frac{\theta}{2}. \quad (13)$$

We now derive the incumbent's highest profit if it forecloses the entrant to see whether/when we could have foreclosure in equilibrium. If the incumbent sets $p_{I2} - d < c_E$, the entrant cannot profitably attract any consumer from the incumbent's first contract and therefore competes for the consumers who did not sign the incumbent's contract. The entrant's best-reply price is therefore $p_E^* = \frac{p_{I2}^{S*} + c_E}{2}$. The incumbent's optimal spot price is $p_{I2}^{S*} = p_{I2}$ since lowering price below p_{I2} would lead to a margin loss from measure 1 of consumers and a market share gain from measure 0 of consumers (given that consumers can switch between

⁴Recall that if $\Delta c < 2\theta$, $p_{I2}^{S*} = \frac{c_I + 2c_E + 2\theta}{3} + d$, and if $\Delta c \geq 2\theta$, $p_{I2}^{S*} - d = c_I$.

⁵In this case, the incumbent is indifferent between any level of d as long as $d < \Delta c$, that is, breakup fees are washed out in the incumbent's profit since the incumbent has to compensate consumers ex-ante for expected amount of breakup fees to convince them to sign the LT contract.

the incumbent's plans at no cost, MFN). Hence, in equilibrium of the second period we have $p_{I2}^{S*} = p_{I2}$ and $p_E^* = \frac{p_{I2} + c_E}{2}$.

A consumer's expected utility from signing the incumbent's first contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2},$$

since she expects to buy from the incumbent in period 2 when she signs the incumbent's first period contract. A consumer's expected utility if she does not sign the incumbent's first contract is again given by (24).

The incumbent maximizes its profit subject to the consumers' participation constraint and the foreclosure constraint:

$$\max_{p_{I1}, p_{I2}, d} \Pi_I^F = [p_{I1} - c_I + p_{I2} - c_I]$$

subject to:

- (i) $EU_{signI} \geq EU_{nosignI},$
- (ii) $p_{I2} - d < c_E.$

At the optimal solution the incumbent sets the highest p_{I1} satisfying the participation constraint:

$$p_{I1}^* = v - \frac{(p_{I2} - p_E^*)^2}{2\theta}.$$

Replacing the latter into the incumbent's profit we rewrite its problem as:

$$\max_{p_{I2}, d} \Pi_I^F = \left[v - 2c_I + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta} \right]$$

st. (ii).

Given that $p_E^* = \frac{p_{I2} + c_E}{2}$ the first-order condition with respect to p_{I2} determines the equilibrium price:

$$p_{I2}^* = 4\theta + c_E.$$

and the incumbent sets a sufficiently high breakup fee to foreclose the entrant: $d^* > 4\theta$. The equilibrium price of the entrant and upfront fee of the incumbent are then respectively

$$p_E^* = 2\theta + c_E, p_{I1}^* = v - 2\theta.$$

The incumbent's profit from foreclosure is

$$\Pi_I^{F*} = v - c_I + 2\theta - \Delta c. \quad (14)$$

Comparing the latter profit with the incumbent's highest profit without foreclosure (12) we conclude that the incumbent prefers to foreclose the entrant if and only if $\Delta c < 2\theta$.

To sum up, if consumers believe that all the other consumers sign the incumbent's contract, the following unique equilibrium prevails:

- If $\Delta c < 2\theta$, all consumers sign the incumbent's long-term contract and continue buying from the incumbent in period 2, that is, the incumbent forecloses the entrant. The equilibrium prices, demands and payoffs are then

$$\begin{aligned} p_{I1}^* &= v - 2\theta, d^* > 4\theta, p_{I2}^* = p_{I2}^{S*} = 4\theta + c_E, p_E^* = 2\theta + c_E \\ D_{I1}^* &= D_{I2}^* = 1, D_E^* = 0 \\ \Pi_I^* &= v - c_I + 2\theta - \Delta c, \Pi_E^* = 0. \end{aligned} \quad (15)$$

- If $\Delta c \geq 2\theta$, all consumers sign the incumbent's long-term contract and switch to the entrant in period 2. The equilibrium prices, demands and payoffs are then

$$\begin{aligned} p_{I1}^* &= v - d^*, d^* < \Delta c, p_{I2}^* = p_{I2}^{S*} = c_I + d^*, p_E^* = c_I - \theta \\ D_{I1}^* &= 1, D_{I2}^* = 0, D_E^* = 1 \\ \Pi_I^* &= v - c_I, \Pi_E^* = \Delta c - \theta. \end{aligned} \quad (16)$$

Uniqueness:

We will now show that the previously characterized equilibrium is the unique equilibrium for any belief of consumers about the others' behavior. To do so suppose that each consumer believes that $\alpha < 1$ measure of consumers sign the incumbent's first period contract and $1 - \alpha > 0$ of consumers do not sign the long-term contract.

Consider first the case of $\Delta c < 2\theta$. Given the incumbent sets $p_{I2} - d < c_E$, consumers believe that in the second period locked-in consumers will continue buying from the incumbent and there will be differentiated competition between the incumbent and the entrant for the consumers who did not sign the incumbent's long-term contract. So they believe that in the second period the demands will be

$$\begin{aligned} D_{I2} &= \alpha + (1 - \alpha)Pr(s > p_{I2}^S - p_E) \\ D_E &= (1 - \alpha)Pr(s < p_{I2}^S - p_E). \end{aligned}$$

The incumbent maximizes its second period profit, $\Pi_{I2} = (p_{I2}^S - c_I)D_{I2}$, and so its best-reply spot price will be

$$p_{I2}^{S*} = \frac{\alpha\theta}{2(1-\alpha)} + \frac{c_I + p_E + \theta}{2}.$$

Similarly, the entrant maximizes its profit, $\Pi_E = (p_E - c_E)D_E$, and so its best-reply will be

$$p_E^* = \frac{p_{I2}^S + c_E}{2}.$$

The simultaneous solutions to these best-replies imply that the second period prices will be

$$\begin{aligned} p_{I2}^{S*} &= \frac{2\alpha\theta}{3(1-\alpha)} + \frac{2c_I + c_E + 2\theta}{3}, \\ p_E^* &= \frac{\alpha\theta}{3(1-\alpha)} + \frac{c_I + 2c_E + \theta}{3}. \end{aligned} \quad (17)$$

if the incumbent's spot demand is positive at these prices, that is, if $\alpha < \frac{2\theta - \Delta c}{3\theta - \Delta c}$. However, if $\alpha \geq \frac{2\theta - \Delta c}{3\theta - \Delta c}$, the incumbent prefers not to compete (for non-locked-in consumers) against the entrant and not to undercut its long-term contract's second period price: $p_{I2}^{S*} = p_{I2}$. In this case the foreclosure equilibrium prevails since consumers' expected outside option will be the same as the one when they all believe that the others sign the incumbent's contract and the rest of the analysis will be the same as above.

Now, we analyze whether the foreclosure equilibrium is unique if $\alpha < \frac{2\theta - \Delta c}{3\theta - \Delta c}$. In this case, a consumer's expected surplus from not signing the incumbent's long-term contract will be

$$EU_{nosignI} = v - p_{I2}^{S*} + \frac{(p_{I2}^{S*} - p_E^*)^2}{2\theta},$$

where p_{I2}^{S*} and p_E^* are given by (17). Suppose that the incumbent offers the equilibrium long-term contract, $p_{I1}^* = v - 2\theta$, $d^* > 4\theta$, $p_{I2}^* = 4\theta + c_E$. If a consumer signs this contract, she expects to pay p_{I2}^{S*} in period 2 since $4\theta + c_E > p_{I2}^{S*}$ and she can switch to the spot contract of the incumbent at no cost. But then the consumer's expected surplus from signing the incumbent's long-term contract will be

$$EU_{signI} = 2v - p_{I1} - p_{I2}^{S*}.$$

Consumers sign the incumbent's long-term contract if and only if $EU_{signI} > EU_{nosignI}$ or

$$\begin{aligned} v - p_{I1} &> \frac{(p_{I2}^{S*} - p_E^*)^2}{2\theta} \\ \iff p_{I1} &< v - \frac{(\theta + \Delta c)^2}{18\theta} - \frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2}. \end{aligned}$$

which is the case at the equilibrium price, $p_{I1}^* = v - 2\theta$, if and only if

$$\frac{(\theta + \Delta c)^2}{18\theta} + \frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2} < 2\theta.$$

Observe that $\frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2}$ is increasing in α and when α goes to the upper bound, this fraction goes to $\frac{(2\theta - \Delta c)^2}{18\theta}$.

$$\lim_{\alpha \rightarrow \frac{2\theta - \Delta c}{3\theta - \Delta c}} \frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2} = \frac{(2\theta - \Delta c)^2}{18\theta}.$$

The latter implies that, for $\alpha < \frac{2\theta - \Delta c}{3\theta - \Delta c}$,

$$\frac{(\theta + \Delta c)^2}{18\theta} + \frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2} < \frac{(\theta + \Delta c)^2}{18\theta} + \frac{(2\theta - \Delta c)^2}{18\theta} = \frac{5\theta^2 - 2\theta\Delta c + 2\Delta c^2}{18\theta}.$$

Besides, observe that the fraction $\frac{5\theta^2 - 2\theta\Delta c + 2\Delta c^2}{18\theta}$ is maximized at $\Delta c = \frac{\theta}{2}$ and the maximum value of this fraction is $\frac{\theta}{4}$. But then we prove that

$$\frac{(\theta + \Delta c)^2}{18\theta} + \frac{(\alpha\theta)^2}{18\theta(1-\alpha)^2} < \frac{\theta}{4} < 2\theta.$$

and so $EU_{signI} > EU_{nosignI}$ if the incumbent offers the equilibrium long-term contract. Hence, the foreclosure equilibrium we described in (15) is the unique equilibrium if $\Delta c < 2\theta$.

Finally, if $\Delta c \geq 2\theta$ and the incumbent offers $p_{I1}^* = v - d^*$, $d^* < \Delta c$, $p_{I2}^* = c_I + d^*$, consumers anticipate that in the second period all consumers buy from the entrant at price $p_E^* = c_I - \theta$ and expect to get disutility $\frac{\theta}{2}$ from buying the entrant's product in period 2 (this is true regardless of what they believe about the others' behaviour). Hence, consumers' expected surplus from not signing the long-term contract is $EU_{nosignI} = v - c_I + \frac{\theta}{2}$ and the expected surplus from signing the offered contract is

$$EU_{signI} = 2v - p_{I1} - p_E^* - \frac{\theta}{2} = v - c_I + d^* + \frac{\theta}{2},$$

Consumers are strictly better-off if they sign the incumbent's long-term contract, $EU_{signI} > EU_{nosignI}$, if and only if $d^* > 0$, which the incumbent can offer in the equilibrium contract, as

the incumbent is indifferent between any level of d^* such that $d^* < \Delta c$. Hence, this concludes that the efficient equilibrium we described in (29) is the unique equilibrium if $\Delta c \geq 2\theta$ and $d^* > 0$.

Proof of Proposition 2: When the incumbent's long-term contract cannot have a breakup fee, the incumbent and the entrant are differentiated competitors for all consumers in period 2 and the solution to the differentiated duopoly competition determine the equilibrium prices. This is true regardless of who signed the incumbent's first period contract given that consumers could switch from the incumbent's long-term contract to the entrant at no cost. The entrant's demand will be $D_E = Prob(s < p_{I2}^S - p_E)$ and the incumbent's demand will be $D_{I2} = Prob(s > p_{I2}^S - p_E)$.

The entrant's best-reply to the incumbent's spot price is $p_E^* = \frac{p_{I2}^S + c_E}{2}$ if $p_{I2}^S \leq \min\{2\theta + c_E, v\}$.⁶ If $p_{I2}^S > v$, the entrant will be local monopoly with price $p_E^* = \frac{v + c_E}{2}$ and demand $D_E^* = \frac{v - c_E}{2\theta}$ (this will not happen in equilibrium). If $2\theta + c_E < p_{I2}^S \leq v$, the entrant sells all consumers at $p_E^* = p_{I2}^S - \theta$. Similarly the incumbent's best-reply to the entrant's price is $p_{I2}^{S*} = \frac{\theta + p_E + c_I}{2}$ if $p_{I2}^S \leq \min\{\theta + c_I, v\}$. If $p_E > v$, the incumbent will be the local monopoly with price $p_{I2}^{S*} = \frac{v + c_E}{2}$ and demand $D_{I2}^{S*} = \frac{v - c_E}{2\theta}$. If $\theta + c_I < p_E \leq v$, the incumbent sells all consumers by setting $p_{I2}^{S*} = p_E$. If $\Delta c < 2\theta$, the interior solution to the duopoly competition is

$$\begin{aligned} p_{I2}^{S*} &= \frac{2\theta + c_E + 2c_I}{3}, p_E^* = \frac{\theta + 2c_E + c_I}{3} \\ D_{I2}^{S*} &= \frac{2\theta - \Delta c}{3\theta}, D_E^* = \frac{\theta + \Delta c}{3\theta} \end{aligned} \quad (18)$$

We verify that $v > p_{I2}^{S*} = \frac{2\theta + c_E + 2c_I}{3}$ by Assumption 1 and so there is active competition between the incumbent and entrant. If $\Delta c \geq 2\theta$, the equilibrium prices are $p_{I2}^{S*} = c_I$ and $p_E^* = c_I - \theta$, and the entrant serves all consumers: $D_{I2}^* = 0$, $D_E^* = 1$.

In equilibrium we must have $p_{I2} \leq p_{I2}^{S*}$, since otherwise the incumbent would have a profitable deviation from p_{I2} to implement p_{I2}^{S*} in period 2 and so p_{I2} would not be paid by any consumer. Given $p_{I2} \leq p_{I2}^{S*}$, the entrant's best-reply will be $p_E^* = \frac{p_{I2} + c_E}{2}$.

We now look for an equilibrium where all consumers signed the incumbent's first contract. Suppose that each consumer expects every other consumer to sign the incumbent's first period contract. The solution of period 1 differs from the previous section analysis since a consumer's outside option to the incumbent's contract is now her expected surplus from buying a unit in period 2 where she buys from the more efficient entrant and gets $v - s - p_E^*$ if her s is low relative the price difference between the firms and buys from the incumbent

⁶It cannot be the case that $c_E > p_{I2}^S$ in equilibrium since the incumbent is less efficient than the entrant.

and gets $v - p_{I2}$ if her s is high relative to the price difference (given that we have $p_{I2} = p_{I2}^{S*}$ in equilibrium):

$$\begin{aligned} EU_{nosignI} &= v - p_{I2} Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E^*} (s + p_E^*) \frac{1}{\theta} ds, \\ &= v - p_{I2} + \frac{(p_{I2} - p_E^*)^2}{2\theta}. \end{aligned} \quad (19)$$

Observe that period 1 has no effect on period 2 competition, since a consumer's expected utility in case she signs the incumbent's first contract is equal to

$$\begin{aligned} EU_{signI} &= v - p_{I1} + v - p_{I2} Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E} (s + p_E^*) \frac{1}{\theta} ds \\ &= v - p_{I1} + EU_{nosignI}. \end{aligned}$$

Hence, a consumer wants to sign the incumbent's contract if and only if $p_{I1} \leq v$. At the optimal solution the incumbent sets $p_{I1}^* = v$ (as if it was static monopoly) and the second period prices are given by the equilibrium of the differentiated duopoly competition, (18). In equilibrium consumers get their expected outside option:

$$\begin{aligned} EU_{nosignI}^* &= v - p_{I2}^* + \frac{(p_{I2}^* - p_E^*)^2}{2\theta} \\ &= v - \frac{2\theta + c_E + 2c_I}{3} + \frac{(\theta + \Delta c)^2}{2\theta}. \end{aligned}$$

Observe that this is the unique equilibrium, since each consumer expects to get exactly the same second period payoff as her outside option in case she signs the incumbent's contract, and so she prefers to sign the incumbent's first contract as long as $p_{I1} \leq v$. This is true regardless of her beliefs about the others' behaviour.

Proof of Proposition 3 In period 1 the incumbent has two options: accommodate the entrant by setting $p_{I2} - d \geq c_E$ or foreclose the entrant by $p_{I2} - d < c_E$. Like in the main model where $F(\cdot)$ was the uniform distribution, we derive the incumbent's expected profit from each option to determine the incumbent's equilibrium strategy.

Period 2: If none of the consumers signed the incumbent's LT contract in period 1, the incumbent and the entrant are differentiated competitors for all consumers in period 2 and the solution to this competition determines the equilibrium prices. The entrant's demand is $D_E = Pr(s < p_{I2}^S - p_E) = F(p_{I2}^S - p_E)$ and its best-reply price is the one maximizing its profit, $\Pi_E = (p_E - c_E)F(p_{I2}^S - p_E)$. The solution to the first-order condition gives the

entrant's best-reply to the incumbent's price, $p_E^*(p_{I2}^S)$,

$$\frac{d\Pi_E}{dp_E} = F(p_{I2}^S - p_E) - (p_E - c_E)f(p_{I2}^S - p_E) = 0, \quad (20)$$

if at this solution the entrant does not attract the entire market: $p_{I2} - p_E^*(p_{I2}^S) < \theta$. Otherwise, the entrant sets the maximum price at which it can sell to all consumers: $p_E^*(p_{I2}^S) = p_{I2}^S - \theta$. The incumbent's second period demand is $D_{I2} = 1 - F(p_{I2}^S - p_E)$ and its optimal price is the one maximizing its profit, $\Pi_{I2} = (p_{I2}^S - c_I)[1 - F(p_{I2}^S - p_E)]$. The solution to the first-order condition gives the incumbent's best-reply to the entrant's price, $p_{I2}^{S*}(p_E)$,

$$\frac{d\Pi_{I2}}{dp_{I2}^S} = 1 - F(p_{I2}^S - p_E) - (p_{I2}^S - c_I)f(p_{I2}^S - p_E) = 0, \quad (21)$$

if at this solution the incumbent has some positive demand: $p_{I2}^{S*}(p_E) - p_E < \theta$. Otherwise, the incumbent cannot compete against the entrant and sets $p_{I2}^{S*} = c_I$.

Define function $h(x) = \Delta c + \frac{1 - 2F(x)}{f(x)}$. We first show that $h(x)$ is decreasing since $\frac{1 - 2F(x)}{f(x)}$ is decreasing due to the IHRP and the log-concavity of $F(\cdot)$:

$$\begin{aligned} \left(\frac{1 - 2F(x)}{f(x)}\right)' &= \frac{-2f^2 - (1 - 2F)f'}{f^2} = \frac{-2f^2 + Ff' - (1 - F)f'}{f^2} \\ &= \frac{Ff' - f^2 - (f^2 + (1 - F)f')}{f^2} < 0. \end{aligned}$$

The latter inequality holds since we have $f^2 + (1 - F)f' > 0$ by the IHRP [inequality (7)] and $f'F - f^2 < 0$ by the log-concavity of $F(\cdot)$ [inequality (8)]. Moreover, $h(x)$ is a continuous function and intersects the vertical axis at a positive point: $h(0) = \Delta c + \frac{1}{f(0)} > 0$. Hence, the fix point of $h(x)$ exists and is unique. Let x^* denote the fix point of $h(x)$: $x^* = h(x^*)$. Using equilibrium equations (21)-(20) we show that x^* corresponds to the difference between the incumbent's spot price and entrant's price in equilibrium: $x^* = p_{I2}^{S*} - p_E^*$ if $x^* < \theta$, that is, if the incumbent has some positive sales in period 2. If $x^* \geq \theta$ the incumbent's period 2 demand is zero and the entrant sells all consumers in period 2. Given that x^* is the fixed point of $h(x)$ we now determine when $x^* = \theta$:

$$\begin{aligned} x^* = \theta &\Leftrightarrow \theta = h(\theta) = \Delta c + \frac{1 - 2F(\theta)}{f(\theta)} \Leftrightarrow \theta = \Delta c - \frac{1}{f(\theta)} \\ &\Leftrightarrow \Delta c = \theta + \frac{1}{f(\theta)} \end{aligned}$$

If $\Delta c \geq \theta + \frac{1}{f(\theta)}$, then $x^* \geq \theta$ which means that the incumbent cannot profitably compete against the entrant, so sets its spot price at its marginal cost and the entrant serves the entire market:

$$p_{I2}^{S*} = c_I, p_E^* = c_I - \theta. \quad (22)$$

If $\Delta c < \theta + \frac{1}{f(\theta)}$, p_{I2}^{S*} and p_E^* are the solution to (21) and (20), such that

$$p_{I2}^{S*} = c_I + \frac{1 - F(x^*)}{f(x^*)}, p_E^* = c_E + \frac{F(x^*)}{f(x^*)} \text{ where } x^* = \Delta c + \frac{1 - 2F(x^*)}{f(x^*)}, \quad (23)$$

The incumbent's second period demand is then positive: $D_{I2}^S = 1 - F(x^*) > 0$, and the entrant's demand is $D_E = F(x^*) < 1$.

Suppose now that all consumers signed the incumbent's first period contract. If the incumbent sets $p_{I2}^S \geq p_{I2}$ and $p_{I2} - d \geq c_E$, the entrant can sell profitably in period 2 and its demand is $D_E = F(p_{I2} - p_E - d)$. The entrant's best-reply price is then $p_E^*(p_{I2} - d)$, which is the entrant's best reply, (20), to $p_{I2} - d$, given that $p_{I2} - d - p_E^*(p_{I2} - d) < \theta$. However, if $p_{I2} - d - p_E^*(p_{I2} - d) \geq \theta$, the entrant sets $p_E^* = p_{I2} - d - \theta$ and sells to all consumers.

If the incumbent sets $p_{I2}^S < p_{I2}$ and $p_{I2}^S - d \geq c_E$, the entrant can sell profitably in period 2 and its demand is $D_E = Prob(s < p_{I2}^S - p_E - d) = F(p_{I2}^S - p_E - d)$ and the incumbent's period 2 demand is $D_{I2}^S = 1 - F(p_{I2}^S - p_E - d)$. By solving the problem of the incumbent and the entrant, we show that if $\Delta c < \theta + \frac{1}{f(\theta)}$, the interior solution to the duopoly competition is the same as (23) except that the net price of buying from the incumbent is equal to its spot price when none of the consumers signed the LT in period 1: $p_{I2}^{S*} - d = c_I + \frac{1 - F(x^*)}{f(x^*)}$. Similarly, if $\Delta c \geq \theta + \frac{1}{f(\theta)}$, the equilibrium prices and demands will be the same as (22) except that the net price of the incumbent is equal to its spot price when none of the consumers signed the LT in period 1: $p_{I2}^{S*} - d = c_I$.

Period 1: We show that there exists an equilibrium where each consumer signs the incumbent's long-term contract. In equilibrium we must have $p_{I2} \leq p_{I2}^{S*}$, since otherwise the incumbent would have a profitable deviation from p_{I2} to implement p_{I2}^{S*} in period 2 and so p_{I2} would not be paid by any consumer.

Suppose that each consumer expects every other consumer to sign the incumbent's first period contract. A consumer's expected utility if she does not sign the incumbent's first

contract is

$$\begin{aligned}
EU_{nosignI} &= v - p_{I2} \text{Prob}(s \geq p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E^*} (s + p_E^*) f(s) ds \\
&= v - p_{I2} [1 - F(p_{I2} - p_E^*)] - \int_0^{p_{I2} - p_E^*} (s + p_E^*) f(s) ds,
\end{aligned} \tag{24}$$

where the consumer expects the entrant's price to be $p_E^*(p_{I2} - d)$ (as she expects all the other consumers to sign the LT contract). Recall that $p_E^*(p_{I2} - d)$ is the solution to (20) where the incumbent's second period net price is $p_{I2} - d$ given that $p_{I2} - d - p_E^*(p_{I2} - d) < \theta$. However, if $p_{I2} - d - p_E^*(p_{I2} - d) \geq \theta$, the entrant's expected price is $p_E^* = p_{I2} - d - \theta$, where the entrant sells to all consumers.

Option 1: We derive the incumbent's maximum payoff if it accommodates the entrant by setting a sufficiently low breakup fee: $p_{I2} - d < c_E$. A consumer's expected utility if she signs the incumbent's first contract is equal to

$$EU_{signI} = 2v - p_{I1} - p_{I2} \text{Prob}(s \geq p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^* + d) f(s) ds.$$

The incumbent maximizes its profit subject to the consumers' participation constraint, $EU_{signI} \geq EU_{nosignI}$, the second period price equilibrium, $p_{I2} \leq p_{I2}^{S*}$, and the constraint that ensures some positive sales by the entrant, $p_{I2} - d < c_E$:

$$\max_{p_{I1}, p_{I2}, d} \Pi_I = [p_{I1} - c_I + (p_{I2} - c_I) \text{Prob}(s \geq p_{I2} - p_E^* - d) + d \text{Prob}(s < p_{I2} - p_E^* - d)]$$

subject to:

- (i) $EU_{signI} \geq EU_{nosignI}$,
- (ii) $p_{I2} \leq p_{I2}^{S*}$
- (iii) $p_{I2} - d \geq c_E$

At the optimal solution the incumbent sets the highest p_{I1} satisfying the participation constraint, (i):

$$\begin{aligned}
p_{I1}^* &= v + (p_E^* - p_{I2}) [F(p_{I2} - p_E^*) - F(p_{I2} - p_E^* - d)] - dF(p_{I2} - p_E^* - d) \\
&\quad - \int_0^{p_{I2} - p_E^* - d} s f(s) ds + \int_0^{p_{I2} - p_E^*} s f(s) ds.
\end{aligned}$$

Replacing the latter into the incumbent's profit we rewrite it as:

$$\begin{aligned} \Pi_I(p_{I2}, d, p_E^*) &= [v - c_I + p_{I2} - c_I - (p_E^* - c_I)F(p_{I2} - p_E^* - d) - (p_{I2} - p_E^*)F(p_{I2} - p_E^*) \\ &\quad - \int_0^{p_{I2} - p_E^* - d} s f(s) ds + \int_0^{p_{I2} - p_E^*} s f(s) ds \end{aligned} \quad (25)$$

The incumbent maximizes the latter profit subject to (ii) and (iii).

Recall that p_E^* is the solution to (20) where the incumbent's second period net price is $p_{I2} - d$:

$$F(p_{I2} - d - p_E) - (p_E - c_E)f(p_{I2} - d - p_E) = 0, \quad (26)$$

The first-order condition with respect to p_{I2} is

$$\begin{aligned} \frac{d\Pi_I}{dp_{I2}} &= \frac{\partial\Pi_I}{\partial p_{I2}} + \frac{\partial\Pi_I}{\partial p_E^*} \frac{dp_E^*}{dp_{I2}} = 0, \\ &= -\frac{\partial\Pi_I}{\partial d} - \frac{\partial\Pi_I}{\partial p_E^*} \frac{dp_E^*}{dd} + 1 - F(p_{I2} - p_E^*) = 0. \end{aligned}$$

and the first-order condition with respect to d is

$$\frac{d\Pi_I}{dd} = \frac{\partial\Pi_I}{\partial d} + \frac{\partial\Pi_I}{\partial p_E^*} \frac{dp_E^*}{dd} = 0, \quad (27)$$

Replacing the latter condition into the former, we find that in the unconstrained solution the incumbent would set

$$p_{I2}^* - p_E^* = \theta.$$

But then using the definition of p_E^* , (26), we get

$$\begin{aligned} p_{I2}^* &= \theta + c_E + \frac{F(\theta - d)}{f(\theta - d)} \\ p_E^* &= c_E + \frac{F(\theta - d)}{f(\theta - d)}. \end{aligned}$$

Given $\Delta c < \theta + \frac{1}{f(\theta)}$, the unconstrained solution described above gives us the equilibrium prices if constraints (ii) and (iii) satisfied at those prices.

Constraint (ii) requires $p_{I2}^* \leq p_{I2}^{S*}$, that is, (given that $p_{I2}^{S*} = d + c_I + \frac{1-F(x^*)}{f(x^*)}$), as we derive

above.)

$$\begin{aligned} \theta + c_E + \frac{F(\theta - d)}{f(\theta - d)} &\leq d + c_I + \frac{1 - F(x^*)}{f(x^*)} \iff \\ \theta - d + \frac{F(\theta - d)}{f(\theta - d)} &\leq x^* + \frac{F(x^*)}{f(x^*)} \iff \theta - d \leq x^* \end{aligned}$$

Constraint (iii) requires $p_{I2}^* - p_E^* - d > c_E$, that is, $\theta - d > c_E$. Hence, if at the unconstrained solution (27) evaluated at (p_{I2}^*, p_E^*) if $\theta - x^* \leq d < \theta - c_E$, the unconstrained solution will be the equilibrium of this case.

On the other hand, if $\Delta c \geq \theta + \frac{1}{f(\theta)}$, the incumbent cannot compete against the entrant, so sets $p_{I2}^* - d^* = p_{I2}^{S*} - d^* = c_I$ and $c_I - d^* > c_E$ (accommodating the entrant), and the entrant reacts by setting $p_E^* = c_I - \theta$. In this case, the entrant sells all consumers in period 2 (this is efficient) and the incumbent captures its static monopoly profit, $\Pi_I^* = v - c_I$, by collecting $p_{I1}^* = v - d^*$ upfront.⁷

Option 2: We now derive the incumbent's profit if it forecloses the entrant to see whether/when we could have foreclosure in equilibrium. If the incumbent sets $p_{I2} - d < c_E$, the entrant cannot profitably attract any consumer from the incumbent's first contract and therefore competes for the consumers who did not sign the incumbent's contract. The entrant's best-reply price is therefore $p_E^*(p_{I2}^S)$, which is the solution to (20). The incumbent's optimal spot price is $p_{I2}^{S*} = p_{I2}$ since lowering price below p_{I2} would lead to a margin loss from measure 1 of consumers and a market share gain from measure 0 of consumers (given that consumers can switch between the incumbent's plans at no cost, MFN). Hence, in equilibrium of the second period we have $p_{I2}^{S*} = p_{I2}$ and $p_E^*(p_{I2})$. A consumer's expected utility from signing the incumbent's first contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2},$$

since she expects to buy from the incumbent in period 2 when she signs the incumbent's long-term contract. A consumer's expected utility if she does not sign the incumbent's first contract is again given by (24).

The incumbent maximizes its profit subject to the consumers' participation constraint

⁷In this case, the incumbent is indifferent between any level of d as long as $d < \Delta c$.

and the foreclosure constraint:

$$\max_{p_{I1}, p_{I2}, d} \Pi_I^F = [p_{I1} - c_I + p_{I2} - c_I]$$

subject to:

$$(i) \quad EU_{signI} \geq EU_{nosignI},$$

$$(ii) \quad p_{I2} - d < c_E.$$

At the optimal solution the incumbent sets the highest p_{I1} satisfying the participation constraint:

$$p_{I1}^* = v - (p_{I2} - p_E^*)F(p_{I2} - p_E^*) + \int_0^{p_{I2} - p_E^*} sf(s)ds$$

Replacing the latter into the incumbent's profit we rewrite it as:

$$\Pi_I^F(p_{I2}, p_E^*) = v - c_I + p_{I2} - c_I - (p_{I2} - p_E^*)F(p_{I2} - p_E) + \int_0^{p_{I2} - p_E} sf(s)ds. \quad (28)$$

Comparing the foreclosure profit (28) with the profit from accommodating the entrant when $\Delta c < \theta + \frac{1}{f(\theta)}$, (25), we find that the foreclosure profit is higher than the profit of accommodating the entrant for given (p_{I2}, p_E) :

$$\Pi_I^F(p_{I2}, p_E) - \Pi_I^{NF}(p_{I2}, d, p_E) = (p_E - c_I)F(p_{I2} - p_E - d) + \int_0^{p_{I2} - p_E - d} sf(s)ds > 0.$$

Moreover, the foreclosure profit increases in p_E and the entrant's price in the foreclosure outcome (option 2) is greater than its price at the no foreclosure outcome (option 1) for any p_{I2} and $d \geq 0$: $p_E^*(p_{I2}) > p_E^*(p_{I2} - d)$. Hence, we conclude that for any p_{I2} when $\Delta c < \theta + \frac{1}{f(\theta)}$

$$\Pi_I^F(p_{I2}, p_E^*(p_{I2})) > \Pi_I^{NF}(p_{I2}, d, p_E^*(p_{I2} - d)).$$

At the optimal foreclosure scenario the incumbent maximizes $\Pi_I^F(p_{I2}, p_E^*(p_{I2}))$ subject to $p_{I2} - d < c_E$, which can be satisfied by a high enough d without affecting the objective function (as the foreclosure profit does not depend on d). However, at the optimal solution of accommodating the entrant the incumbent maximizes $\Pi_I^{NF}(p_{I2}, d, p_E^*(p_{I2} - d))$ subject to two constraints (ii) and (iii), which will result in a lower profit than unconstrained solution. As a result, we conclude that the optimal foreclosure profit is strictly higher than the optimal profit from accommodating the entrant when $\Delta c < \theta + \frac{1}{f(\theta)}$.

To sum up, if consumers believe that all the other consumers sign the incumbent's contract, the following prevails in an equilibrium:

- If $\Delta c < \theta + \frac{1}{f(\theta)}$, all consumers sign the incumbent's long-term contract and continue

buying from the incumbent in period 2, that is, the incumbent forecloses the entrant.

- If $\Delta c \geq \theta + \frac{1}{f(\theta)}$, all consumers sign the incumbent's long-term contract and switch to the entrant in period 2. The equilibrium prices, demands and payoffs are then

$$\begin{aligned} p_{I1}^* &= v - d^*, d^* < \Delta c, p_{I2}^* = p_{I2}^{S*} = c_I + d^*, p_E^* = c_I - \theta \\ D_{I1}^* &= 1, D_{I2}^* = 0, D_E^* = 1 \\ \Pi_I^* &= v - c_I, \Pi_E^* = \Delta c - \theta. \end{aligned}$$

References

Yongmin Chen. Paying customers to switch. *Journal of Economics & Management Strategy*, 6(4):877–897, 1997.