# Online appendix for: Firm-Related Risk and Precautionary Saving Response

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In this appendix we first prove the inconsistency of OLS estimates of Euler equation (2) and then the consistency of the IV estimate using as instruments the variances of the firm permanent and transitory shocks. Lastly we provide summary statistics of the estimation sample as well as first stages of the IV regressions.

## 1 Bias of OLS and consistency of IV estimates of the two Euler equations

Euler equation (1) (omitting the other controls and suppressing the $i$ and $t$ indexes) is

$$\Delta c = \alpha V_c + \zeta$$

where $\alpha$ is the parameter of interest, $V_c$ is the conditional variance of consumption growth, and $\zeta$ is the consumption innovation.

Consumption risk is related to income risk plus other types of risk that don't depend on labor market:

$$V_c = \theta V_y + V_\eta$$

where $\theta < 1$ means there is some (self-)insurance.

There are two "measurement" error problems. The first is that we observe consumption volatility and wage volatility, not consumption risk and wage risk. In particular:

$$\widetilde{V}_c = V_c + \xi$$

where $\widetilde{V}_c = (\Delta c)^2$, $V_c = E\left((\Delta c)^2\right)$ and $\xi$ is the innovation in the consumption variance. Moreover, the (residual) wage variance is:

$$\widetilde{V}_y = H + \rho_f F$$

But part of the $H$ variability is not risk, but choice, so wage risk is really:

$$V_y = \rho_h H + \rho_f F$$

What people typically use is $\widetilde{V}_y = V_y + \varepsilon$, where $\varepsilon = (1 - \rho_h) H$.

An OLS regression of $\Delta c$ on $\widetilde{V}_c$ (Euler equation (2)) has a classical measurement error problem:

$$\begin{aligned}
\frac{cov\left(\Delta c, \widetilde{V}_c\right)}{var\left(\widetilde{V}_c\right)} &= \frac{cov\left(\alpha \widetilde{V}_c + \zeta - \alpha\xi, \widetilde{V}_c\right)}{var\left(\widetilde{V}_c\right)} \\
&= \alpha - \alpha \frac{cov\left(\xi, \widetilde{V}_c\right)}{var\left(\widetilde{V}_c\right)} \\
&= \alpha \frac{var\left(V_c\right)}{var\left(\widetilde{V}_c\right)} \\
&< \alpha
\end{aligned}$$

An OLS regression of $\Delta c$ on $\widetilde{V}_y$ (Euler equation (2)) yields :

$$\begin{aligned}
\frac{cov\left(\Delta c, \widetilde{V}_y\right)}{var\left(\widetilde{V}_y\right)} &= \frac{cov\left(\alpha V_c + \zeta, H + \rho_f F\right)}{var\left(H + \rho_f F\right)} \\
&= \frac{cov\left(\alpha\left(\theta V_y + V_\eta\right) + \zeta, H + \rho_f F\right)}{var\left(H + \rho_f F\right)} \\
&= \frac{cov\left(\alpha\left(\theta\left(\rho_v H + \rho_f F\right) + V_\eta\right) + \zeta, H + \rho_f F\right)}{var\left(H + \rho_f F\right)} \\
&= \frac{cov\left(\alpha\theta\rho_v H + \alpha\theta\rho_f F + \alpha V_\eta + \zeta, H + \rho_f F\right)}{var\left(H + \rho_f F\right)} \\
&= \alpha\theta \frac{\rho_h V_H + \rho_f^2 V_F}{V_H + \rho_f^2 V_F} \\
&< \alpha\theta
\end{aligned}$$

which does not identify $\alpha$ both because of the measurement error and because of the presence of self-insurance.

The IV regression of Euler equation (2) that uses the firm variance as an instrument fixes the measurement error problem but not the sufficient statistics problem:

$$
\begin{aligned}
\frac{cov\left(\Delta c, F\right)}{cov\left(\widetilde{V}_y, F\right)} &= \frac{cov\left(\alpha V_c + \zeta, F\right)}{cov\left(H + \rho_f F, F\right)} \\
&= \frac{cov\left(\alpha\left(\theta V_y + V_\eta\right) + \zeta, F\right)}{cov\left(H + \rho_f F, F\right)} \\
&= \frac{cov\left(\alpha\left(\theta\left(\rho_h H + \rho_f F\right) + V_\eta\right) + \zeta, F\right)}{cov\left(H + \rho_f F, F\right)} \\
&= \frac{cov\left(\alpha\theta\rho_h H + \alpha\theta\rho_f F + \alpha V_\eta + \zeta, F\right)}{cov\left(H + \rho_f F, F\right)} \\
&= \frac{\alpha\theta\rho_f V_F}{\rho_f V_F} \\
&= \alpha\theta
\end{aligned}
$$

The IV on Euler equation (2) that uses the firm-related variances as instruments for realized consumption volatility corrects the bias and identifies $\alpha$:

$$
\begin{aligned}
\frac{cov\left(\Delta c, F\right)}{cov\left(\widetilde{V}_c, F\right)} &= \frac{cov\left(\alpha V_c + \zeta, F\right)}{cov\left(\widetilde{V}_c, F\right)} \\
&= \frac{cov\left(\alpha\left(\theta V_y + V_\eta\right) + \zeta, F\right)}{cov\left(\theta V_y + V_\eta + \xi, F\right)} \\
&= \frac{cov\left(\alpha\left(\theta\left(\rho_h H + \rho_f F\right) + V_\eta\right) + \zeta, F\right)}{cov\left(\theta\left(\rho_h H + \rho_f F\right) + V_\eta + \xi, F\right)} \\
&= \frac{cov\left(\alpha\theta\rho_h H + \alpha\theta\rho_f F + \alpha V_\eta + \zeta, F\right)}{cov\left(\theta\rho_h H + \theta\rho_f F + V_\eta + \xi, F\right)} \\
&= \frac{\alpha\theta\rho_f V_F}{\theta\rho_f V_F} \\
&= \alpha
\end{aligned}
$$

Hence, jointly the IV estimates of Euler equation (1) and (2) allow to identify $\alpha$ and $\theta$.

# 2 Additional Tables

Table A1: Summary statistics

| | Mean | Std. Dev. |
|---|---|---|
| $c_{ict}$ | 32,554.6 | 22,100.22 |
| $\Delta \ln c_{it}$ | 0.056 | 0.485 |
| $\widetilde{V}_{cit}$ | 0.238 | 0.408 |
| $\widetilde{V}_{yit}$ | 0.043 | 0.087 |
| $F_{jt}^{T}$ | 0.021 | 0.065 |
| $F_{jt}^{P}$ | 0.029 | 0.07 |
| Years of education | 12.905 | 2.368 |
| Fraction married | 0.457 | 0.498 |
| Male | 0.815 | 0.388 |
| Age | 46.199 | 8.743 |
| Year | 2005.9 | 2.563 |
| Family size | 2.306 | 1.345 |
| Children | 0.634 | 0.973 |

*Notes*: The table reports summary statistics for the estimation sample of 327,518 individuals. Values in 2011 USD.

Table A2: First stages of IV regressions

| | $\widetilde{V}_y$ | $\widetilde{V}_c$ |
|---|---|---|
| $F_{it}^{T}$ | 0.0510*** | 0.0290** |
| | (0.0024) | (0.0111) |
| $F_{it}^{P}$ | 0.0270*** | 0.0324*** |
| | (0.0021) | (0.0103) |
| Year FE | Yes | Yes |
| Age polynomial | Yes | Yes |
| $\Delta$ children | Yes | Yes |
| Years of education | Yes | Yes |
| Observations | 327,518 | 327,518 |

*Notes*: The table reports the first stages of the IV estimates in Table 1 of the marginal effect of wage and consumption risk on the growth of consumption, using two instruments - the variance of transitory and permanent shocks to firm's value added. Clustered standard errors are in brackets. Coefficient significance: *** at 1 % or less; ** at 5 %; * at 10 %.