Optimal Life-Cycle Investing with Flexible Labor Supply: A Welfare Analysis of Life-Cycle Funds

Francisco J. Gomes, Laurence J. Kotlikoff and Luis M. Viceira[1]

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[1]Gomes: London Business School and CEPR. Email: *fgomes@london.edu*. Kotlikoff: Boston University and NBER. Email:*kotlikof@bu.edu*. Viceira: Harvard Business School, Boston MA 02163, CEPR and NBER. Email: *lviceira@hbs.edu*. We thank ...Viceira also thanks the Division of Research at HBS for generous financial support.

We investigate optimal consumption, asset accumulation and portfolio decisions in a life-cycle model with flexible labor supply. Using this model, we also investigate the welfare costs of constraining portfolio allocations over the life cycle to mimic popular default investment choices in defined-contribution pension plans.

Most prior work on life-cycle investing has treated labor earnings as exogenous (Viceira, 2001, Cocco, Gomes, and Maenhout, 2005, Gomes, Kotlikoff, and Viceira, 2006, Gomes and Michaelides 2005). As such, it has focused on the bond-like feature of labor earnings—the fact that these resources are not closely correlated with the returns to equities—while ignoring the insurance feature of variable labor supply —the ability of investors who do poorly on the market to hedge their losses by working and earning more. Our work considers this second aspect of labor earnings and studies not only how labor supply affects portfolio choice, but also how portfolio choice affects labor supply. Our framework is a realistically calibrated life-cycle model with wage rate uncertainly, variable labor supply, and portfolio choice over safe bonds and risky equities.

Our analysis reinforces prior findings that equities are the preferred asset for young households, with the optimal share of equities generally declining prior to retirement. However, variable labor materially alters pre-retirement portfolio choice by significantly raising optimal equity holdings. Post retirement, however, the optimal equity share increases as households spend down their financial assets, leaving bond-like pension benefits to increasingly dominate household resources.

Our derived pre-retirement optimal portfolio allocation is similar to the holdings of "life-cycle" or "target retirement" funds, which are replacing money market and stable funds as the default portfolio in many defined-contribution plans (Bodie and Treussard 2007, Viceira 2008). As we show,it is highly costly for moderately risk averse investors to invest their savings only in stable value funds. In contrast, the welfare losses from investing in balanced funds (the stock-bond mix is fixed) and life-cycle funds are much smaller and, indeed, negligible in the case of life-cycle funds that follow the average optimal asset allocation path the investor would choose if unconstrained. Interestingly, constraining portfolio choice affects asset accumulation, but has a relatively small effect on labor supply.

Ours is not the first study to incorporate flexible labor supply over the life cycle. Low (2005) and French (2005) explore optimal consumption in a realistically calibrated life-cycle model, but ignore portfolio choice. Bodie, Merton, and Samuelson (1992) consider portfolio choice, but assume wages are perfectly correlated with stock returns. Chan and Viceira (2000) also consider portfolio choice but in a less realistic setting.

I. Model

Agents work their first K periods and live a maximum of T periods. Lifespan is uncertain, with p_j denoting the probability of surviving to date j given survival to date $j-1$. Preferences are given by

$$U = E_1 \sum_{t=1}^{T} \delta^{t-1} \left(\prod_{j=0}^{t-2} p_j \right) \frac{(C_t L_t^{\alpha})^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\delta < 1$ is the discount factor, L_t is time-t leisure, C_t is time-t consumption, $\gamma > 0$ is the coefficient of relative risk aversion with respect to consumption, and α is a leisure preference parameter. Leisure is measured as a fraction of total available time and satisfies $L_t \in [\underline{L}, 1]$, where $\underline{L}$ is minimum leisure time (set to 1/3 below). Note that for γ greater than 1— our case of interest—, marginal utility of consumption decreases with leisure, thus making leisure and consumption substitutes. With these modified Cobb-Douglas preferences, labor supply is invariant to secular changes in the real wage in accord with U.S. experience.

There are two ways to invest – in riskless bonds with constant gross real return $\overline{R}_f$, and in risky stock, with gross real return R_t. Log stock returns are normally distributed, with mean $\mu + \overline{r}_f$ and variance σ_R^2, where $\overline{r}_f = \ln \overline{R}_f$.

Investors hold B_t and S_t dollars of each asset respectively, and face borrowing and short-sales constraints, so that $B_t \geq 0$ and $S_t \geq 0$. Letting π_t denote the proportion of assets invested in stocks at time t, these constraints imply that $\pi_t \in [0, 1]$ and that wealth is non-negative.

Finally we use R_t^p to denote the after-tax net return on the portfolio held from period t to period $t + 1$, i.e.,

$$R_t^p \equiv 1 + (1 - \tau_C)(\pi_t R_t + (1 - \pi_t)\overline{R}_f - 1) \ , \tag{2}$$

where τ_C is the uniform tax rate applied to all asset income. We ignore tax-exempt retirement accounts, since our focus is on asset allocation, not tax-efficient asset location (see Dammon, Spatt, and Zhang, 2004).

II. Wealth accumulation

A. Working Life

The investor starts period t with wealth W_t. He then observes his wage rate w_t and makes work ($N_t = 1 - L_t$), consumption (C_t), and investment (π_t) decisions. We treat housing and other durables consumption expenditures (h_t) as exogenous, "off-the-top" spending and subtract it from the measure of disposable income.[2] Agents face proportional income taxes. This preserves the scalability/homogeneity of the model and limits the number of state variables. In particular, we assume that labor income is taxed at a rate τ_L, that retirement income is taxed at a rate τ_{SS}, and, as noted, that asset income is taxed at a rate τ_C.

[2] Assuming investors save to make a downpayment on a house early in life doesn't materially affect our findngs.

Under these assumptions, the investor's financial wealth at the end of working period t is given by

$$W_{t+1} = R^p_{t+1} \left(W_t + (1 - h_t) (1 - \tau_L) w_t N_t - C_t \right), \tag{3}$$

where w_t is the time-t wage.

The log of wages follows the process

$$\ln w_t = f(t) + v_t + \varepsilon_t, \tag{4}$$

where $f(t)$ is a deterministic function of age, v_t is a permanent component given by

$$v_t = v_{t-1} + u_t, \tag{5}$$

u_t is distributed as $N(0, \sigma^2_u)$, and ε_t is a transitory shock uncorrelated with u_t, which is distributed as $N(0, \sigma^2_\varepsilon)$. The innovation to the permanent component of the wage rate (u_t) can be correlated with the return to equity R_t, with coefficient ρ.

B. Retirement

During retirement ($t > K$), wealth accumulation follows

$$W_{t+1} = R^p_{t+1} \left(W_t + (1 - h_t) (1 - \tau_{SS}) Y - C_t \right), \tag{6}$$

where Y denotes social security income, which is taxed at a rate τ_{SS}. We assume that the log of social security income is a fraction λ of the average lifetime labor earnings that the agent would have obtained had he worked full time during his working life:

$$Ln(Y) = \lambda \frac{\sum_{t=1}^{K} (f(t) + v_t)}{K} \overline{N}, \tag{7}$$

where $\overline{N}$ denotes full time labor supply.

Retirement age and the level of social security benefits are exogenous. In practice, social security income depends on the individual's average earnings in his 35 highest earnings years. French (2003) notes that this provides incentives to retire at age 65 and to increase labor supply over the working life. Thus our simplified assumption should be viewed to a first-order approximation to the incentives built into the Social Security system.[3]

III. Optimization Problem

The agent maximizes (1) with respect to C_t, L_t, and π_t, subject to (2)-(7), $C_t \geq 0$, $L_t \in [\underline{L}, 1]$, and $\pi_t \in [0, 1]$. There are four state variables: age (t), wealth (W_t), and the permanent and transitory components of the wage rate ($\exp(v_t)$, and $\exp(\varepsilon_t)$). However, our assumptions of homothetic preferences and linear tax rates make the model scale free with respect to the permanent component of wages $\exp(v_t)$; i..e, if this state variable doubles, all choice variables double. This allows us to eliminate one state variable by normalizing wealth and the choice variables by $\exp(v_t)$. The model is solved via backward induction using grid search, cubic value function interpolations, and Gaussian quadrature.

IV. Calibration

A. Wage Process and Labor Supply

Agents are initially age 21, retire at 65, and die for sure at age 100. Prior to this age we use the mortality tables of the National Center for Health Statistics to parameterize the conditional survival probabilities, p_j for $j = 1, ..., T$. We set the discount factor δ to 0.97 and the coefficient of relative risk aversion γ to 5. Following Low (2005), we choose

[3] Letting social security income depend on past labor supply decisions—specifically, average past labor supply, introduces a computationally costly extra state variable, but makes little difference to the results.

α so that the average labor supply over the life cycle matches the average male hours of work per year reported in the Consumer Expenditure Survey – 2080 hours per annum. Assuming a time endowment of 100 hours per week and that $\alpha = 0.9$, average lifetime labor supply equals 0.374. We take the housing expenditure profile $(\{h_t\}_{t=1}^{T})$ from Gomes and Michaelides (2005).

The mean equity premium (in levels) is set at 4.0% per annum, the risk-free rate is set at 1.0% p.a., and the annualized standard deviation of innovations to the risky asset is set at 20.5%. This equity premium is lower than the historical equity premium based on a comparison of average stock and T-bill returns, but accords with the forward-looking estimates reported in Fama and French (2002). Higher premiums generate unrealistically high equity portfolio shares.

The tax rate is 30% on labor income (τ_L) and 15% on retirement income (τ_R) to 15%. Asset income is taxed at a 20% rate (τ_C). These rates roughly match effective income tax rates faced by a typical household.

In order to calibrate the wage income process (4)-(5) we combine the wage profile reported in Fehr, Jokisch and Kotlikoff (2005), which we use for the deterministic age-dependent component of wages[4], with the estimates of σ_u and σ_ε of 10.95% and 13.89% reported in Cocco, Gomes and Maenhout (2005).[5] The implied wage growth rates over the life cycle generated by this function exhibit an inverted-U shape are comparable to average total income growth rates in the PSID data. We also assume a zero correlation between stock returns and innovations in the permanent component of wages (ρ). Finally, we set the replacement ratio λ equal to 68.8% of labor supply at age 65.

[4] Specifically we use their earnings function $E(a, 2)$, given in equation (9) of their paper, with parameter λ equal to 0. In this function, the argument a denotes age, and 2 denotes the middle income class.

[5] Following Carroll (1997), we divide the estimated standard deviation of transitory income shocks by 2 to account for measurement error.

B. Baseline Results

Figures 1, 2, and 3 show baseline results. Figure 1 plots average paths of optimal consumption, income and financial assets over the life cycle, all relative to permanent income; Figure 2 plots the average path of the optimal allocation to stocks as a percentage of financial wealth; and Figure 3 plots average optimal labor supply before retirement, which occurs at age 65, as a fraction of available hours.

Overall, Figures 1 and 2 show consumption, income, asset accumulation, and asset allocation patterns that are qualitatively similar to those assuming fixed labor supply (Cocco, Gomes, and Maenhout 2005, Gomes, Kotlikoff, and Viceira 2006). In particular, consumption, income and wealth accumulation exhibit an inverted-U shaped pattern over the life-cycle, while the share of stocks in the portfolio exhibits a U-shaped pattern.

Figure 3 helps explain the life-cycle pattern of labor income. This figure shows that, consistent with the patterns observed in the data (French 2005, Low 2005), the investor chooses a declining pattern of labor supply over the live cycle after an initial period of slightly increasing labor supply. This pattern, together with the pattern in the wage rate, which in our model as in the data exhibits an inverted-U shape, results in income increasing steadily until the investor is in his late thirties, and decreasing smoothly until he reaches retirement age. At that point income drops by roughly 35 percent drop as social security starts replacing labor earnings.

Figure 1 shows that, consistent with the empirical evidence, consumption slighlty declines as the investor starts increasing leisure late in his working life, and falls more sharply at retirement, when leisure increases dramatically. Asset accumulation exhibits an inverted U-shape, but assets peak much later than labor income. Asset grow rapidly until the investor is in his mid-fifties, at which point he starts dissaving. The rapid accumulation of assets through middle age reflects concern about wage uncertainty and the presence of liquidity constraints. But portfolio choice also matters here. Figure 2 shows that the investor is optimally fully invested in stocks until his early thirties. At that point the optimal portfolio share of stocks declines steadily until it reaches a minimum of about 45% at retirement age, and increases monotonically afterwards. Thus while the share of stocks declines steadily during the working life of the investor, it is still very high on average, thus contributing to a rapid growth in asset values along the mean optimal path.

The risk characteristics of the investor's human wealth—the present discounted value of the investor's future earnings and pension income—and the life-cycle path of assets and human capital explain the patterns in portfolio shares over the life cycle. Uncertainty about future wages makes human capital risky. However, wage uncertainty is uncorrelated with stock market uncertainty, and the investor can offset adverse shocks to wages or to financial wealth by increasing his labor supply. This makes human capital equivalent to an implicit investment in a relatively safe asset. Thus the investor optimally tilts his portfolio towards stocks, particularly early in the life-cycle when human capital is largest relative to financial wealth. As the investor accumulates assets and his human capital is depleted, he optimally decreases the allocation to stocks. This trend reverses in retirement, when the investor starts depleting his assets rapidly and the value of safe pension income becomes increasingly important relative to financial assets.

The optimal portfolio allocation to stocks over the life cycle generated by our realistically calibrated model is qualitatively similar to the asset allocation path built into self-rebalacing life-cycle mutual funds (Viceira 2008). Thus our realistic calibration life-cycle portfolio decisions and labor supply decisions provides support for this approach to saving for retirement. However, our calibrated model does not provide support for the type of asset allocation strategy to which these funds converge at retirement. This is a strategy with constant portfolio allocations. Instead, our model suggests that investors receiving pension income should increase their allocation to stocks as they age as the spend down their assets but experience no diminution of social security income. Please not, however, that our model does not account for potentially large financial liabilities generated by healthcare costs in retirement, which are likely to reduce the investor's willingness to invest in stocks in retirement.

Finally, the optimal asset allocation path shown in Figure 2 is an average path. In practice our investor finds it optimal to deviate from this path as the relevant state variables change. By contrast, an individual who saves for retirement using a life-cycle fund is precommitting to a fixed course of asset allocation over his life cycle. We examine in the next section the welfare cost to such a fixed course, but one that follows the average optimal asset allocation path.

C. Comparative Statics

Our baseline model assumes that the investor makes optimal decisions about consumption (or savings), portfolio and labor supply decisions subject to liquidity constraints and maximum labor supply constraints. We now examine the impact on investor's welfare and on optimal decision-making of imposing additional constraints. We examine two main sets of constraints, fixed labor supply constraints and portfolio constraints, and report the results in Table 1. Each panel reports average optimal consumption, wealth accumulation, labor supply, labor income and portfolio allocation to stocks for a specific case (left side of the panel) as well as changes in these variables relative to the baseline case (right side of the panel). To save space, we report average values of these variables across age ranges. Panel A in Table 1 reports results for our baseline case.

Panel B in Table 1 reports optimal consumption, asset accumulation and allocation to stocks when labor supply is fixed. A comparison of Panel A with Panel B shows that the optimal allocation to stocks is more conservative when labor supply is held fixed. This results from the fact that financial wealth relative to future labor income is higher in that case. To understand this pattern, note that Panel B shows that labor income is lower early in life than in the case with flexible labor supply, and higher closer to retirement. This is expected given the roughly declining pattern in optimal labor supply over the life cycle. Interestingly, the individual also chooses a lower level of consumption early in life, which together with higher labor earnings lead to significantly larger wealth accumulation during his working life. This wealth accumulation results in more conservative portfolio allocations over the life cycle, and it sustains higher consumption in retirement.

These results suggest that the ability to increase labor supply acts as an important buffer against future income uncertainty. When we eliminate this extra choice variable, the individual is forced to accumulate extra savings to increase his buffer stock and behaves more conservatively in his portfolio decisions. The welfare loss from not being able to adjust labor supply optimally is very large. Relative to our baseline model, the investor would be willing to give up 82% of his first-year expected labor income to be able to optimally adjust his labor supply. Note that we use first-year labor income as a benchmark for our welfare computations instead of consumption as it is standard in this literature because we also have leisure entering the utility function. In a model without leisure the welfare loss in this case would probably correspond to about 4% of annual consumption, but in our model we can't make those calculations.

Panels C through Panel F examine the impact on consumption, wealth accumulation and labor supply of constrained portfolio allocations. These allocations mimic investments in a bond (or "stable value") fund (Panel C), two balanced funds (Panel D and Panel E), and a life-cycle fund (Panel F), and thus let us explore the welfare costs of popular default choices for defined contribution plans.

Panel C reports results for the case that constrains the investor to invest only in bonds. This is the case considered in prior research on life cycle consumption with flexible labor supply (French 2005, Low 2005). Thus it provides a useful point of comparison for our baseline case. This case is also relevant for its practical relevance, since until recently the preferred default investment choice in defined contribution plans was a money market fund or a stable value fund. Relative to the case where the individual has stocks available for investment, this case leads to significantly lower asset accumulation and consumption over the life-cycle, particularly at retirement, and to substantial welfare losses, in the order of 46% of first-year labor income.[6]

Panel D and Panel E examine the case where investors can hold stocks, but only in fixed proportions of their financial wealth—50% and 60% respectively. Balanced funds typically follow this type of fixed-proportion asset allocation strategy with constant rebalancing (Viceira 2008). Relative to our baseline case, this constrained case leads to smaller loses in consumption and wealth accumulation than the case with no stock investment at all. Overall welfare losses are also substantially smaller, at 4.8% and 7.3% of first-year labor income respectively. Interestingly, the 60/40 stock-bond fixed allocation produces larger welfare losses than the 50/50 stock-bond allocation.

Finally, Panel F examines the case where the investor follows a strategy of constantly rebalancing his portfolio using weights that change with age. These weights equal the optimal average allocation in the unconstrained case (see Panel A), which for ages below the retire-

[6]Note that in our model the individual invests in an inflation-indexed bond fund, while in reality the default investment choice in defined contribution plans has been a nominal money market fund or a nominal stable fund which are subject to real interest rate risk and short- and medium-term inflation risk. Thus our calibration likely underestimates the welfare losses from constraining portfolio choice.

ment age mimics the strategy typically followed by life-cycle—or target retirement—funds (Viceira, 2008). This strategy is the one that produces minimal deviations in consumption and wealth accumulation with respect to the baseline case, and results in the smallest welfare loss, at 2.4% of first-year labor income. We have also computed, but not reported here to save space, the welfare losses for each of these cases when labor supply is fixed. These losses are generally large, but comparable to those with flexible labor supply.

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TABLE 1

	Optimal values					Change relative to baseline case (%)				
Age	**C**	**W**	**L**	**Y**	**π**	**C**	**W**	**L**	**Y**	**π**
					A. Baseline case					
21-30	0.5189	0.4997	0.5614	0.6215	0.9951	-	-	-	-	-
31-40	0.6376	1.9291	0.5718	0.7389	0.8284	-	-	-	-	-
41-50	0.7068	3.4128	0.6175	0.7347	0.5776	-	-	-	-	-
51-65	0.7076	3.8109	0.7330	0.5518	0.4719	-	-	-	-	-
66-80	0.4389	1.5616	1.0000	0.2674	0.6780	-	-	-	-	-
81-100	0.2908	0.1158	1.0000	0.2657	0.9130	-	-	-	-	-
					B. Fixed labor supply (Welfare loss = 82.00% of first-year labor income)					
21-30	0.4520	0.5417	-	0.5654	0.9930	-12.9%	8.4%	-	-9.0%	-0.2%
31-40	0.5571	2.1672	-	0.6818	0.7719	-12.6%	12.3%	-	-7.7%	-6.8%
41-50	0.6667	4.0607	-	0.7201	0.5212	-5.7%	19.0%	-	-2.0%	-9.8%
51-65	0.7171	5.5328	-	0.6191	0.4001	1.3%	45.2%	-	12.2%	-15.2%
66-80	0.5727	3.1693	-	0.2674	0.5303	30.5%	103.0%	-	0.0%	-21.8%
81-100	0.3278	0.2956	-	0.2657	0.8434	12.7%	155.3%	-	0.0%	-7.6%
					C. Flexible labor supply and 100% bond allocation (Welfare loss = 45.94% of first-year labor income)					
21-30	0.5153	0.4893	0.5577	-	-	-0.7%	-2.1%	-0.6%	-	-
31-40	0.6242	1.8653	0.5686	-	-	-2.1%	-3.3%	-0.5%	-	-
41-50	0.6931	3.2490	0.6068	-	-	-1.9%	-4.8%	-1.7%	-	-
51-65	0.6923	3.5796	0.7189	-	-	-2.2%	-6.1%	-1.9%	-	-
66-80	0.4192	1.3504	1.0000	-	-	-4.5%	-13.5%	0.0%	-	-
81-100	0.2819	0.0724	1.0000	-	-	-3.1%	-37.5%	0.0%	-	-
					D. Flexible labor supply and fixed 50/50 stock/bond allocation (Welfare loss = 4.84% of first-year labor income)					
21-30	0.5182	0.4908	0.5599	-	0.5000	-0.1%	-1.8%	-0.3%	-	-
31-40	0.6344	1.8709	0.5698	-	0.5000	-0.5%	-3.0%	-0.3%	-	-
41-50	0.7034	3.3097	0.6155	-	0.5000	-0.5%	-3.0%	-0.3%	-	-
51-65	0.7049	3.7545	0.7300	-	0.5000	-0.4%	-1.5%	-0.4%	-	-
66-80	0.4364	1.5471	1.0000	-	0.5000	-0.6%	-0.9%	0.0%	-	-
81-100	0.2893	0.1096	1.0000	-	0.5000	-0.5%	-5.3%	0.0%	-	-
					E. Flexible labor supply and fixed 60/40 stock/bond allocation (Welfare loss = 7.25% of first-year labor income)					
21-30	0.5188	0.4907	0.5601	-	0.6000	0.0%	-1.8%	-0.2%	-	-
31-40	0.6356	1.8731	0.5708	-	0.6000	-0.3%	-2.9%	-0.2%	-	-
41-50	0.7042	3.3299	0.6178	-	0.6000	-0.4%	-2.4%	0.0%	-	-
51-65	0.7075	3.8121	0.7323	-	0.6000	0.0%	0.0%	-0.1%	-	-
66-80	0.4407	1.6074	1.0000	-	0.6000	0.4%	2.9%	0.0%	-	-
81-100	0.2919	0.1253	1.0000	-	0.6000	0.4%	8.2%	0.0%	-	-
					F. Flexible labor supply and fixed optimal asset allocation (Welfare loss = 2.42% of first-year labor income)					
21-30	0.5189	0.4997	0.5614	-	0.9951	0.0%	-1.1%	-0.2%	-	-
31-40	0.6376	1.9291	0.5718	-	0.8284	-0.1%	-0.9%	0.1%	-	-
41-50	0.7068	3.4128	0.6175	-	0.5776	-0.1%	-0.6%	0.1%	-	-
51-65	0.7076	3.8109	0.7330	-	0.4719	0.0%	-0.3%	-0.2%	-	-
66-80	0.4389	1.5616	1.0000	-	0.6780	-0.1%	0.3%	0.0%	-	-
81-100	0.2908	0.1158	1.0000	-	0.9130	0.2%	5.5%	0.0%	-	-

Figure 1. Optimal Consumption, Wealth and Income

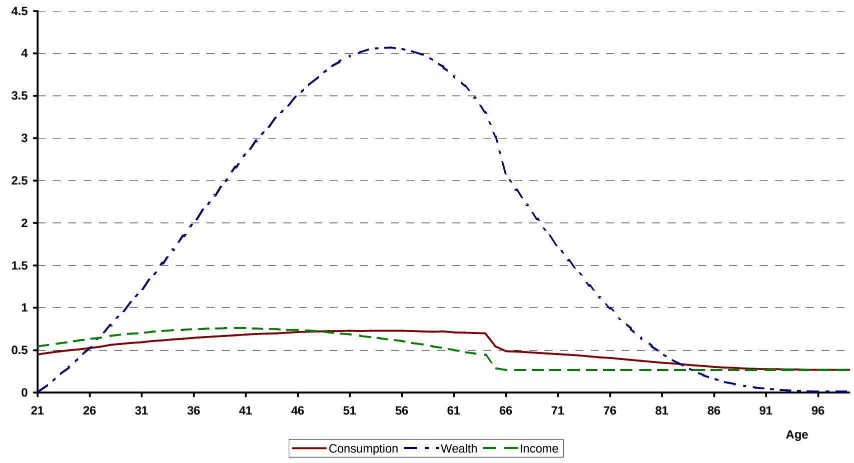

Figure 2. Optimal Portfolio Share Invested in Stocks

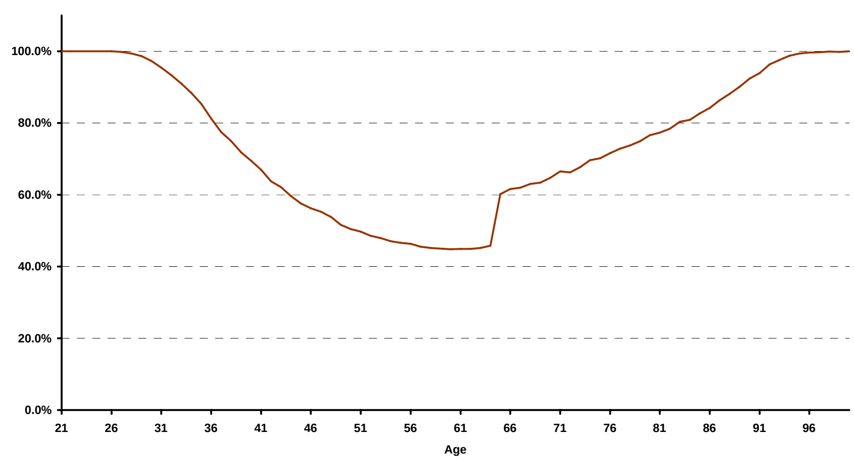

Figure 3. Optimal Leisure